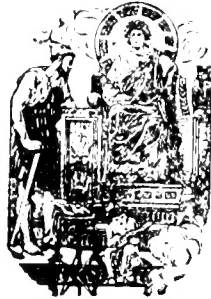




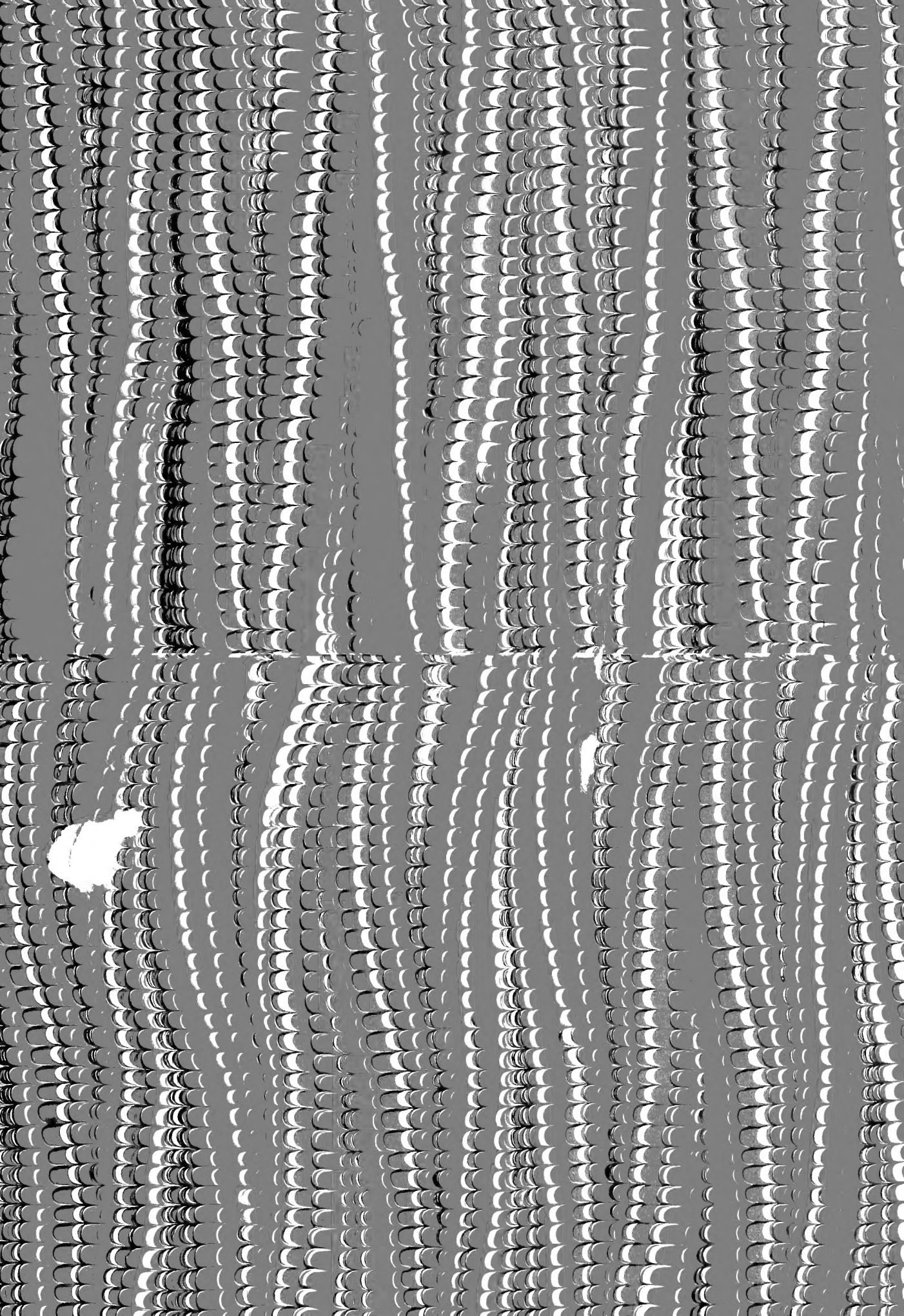
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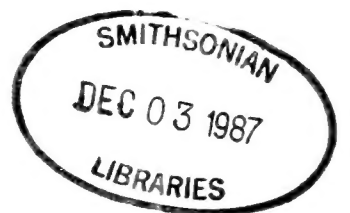
VOL. XIX.



EVERY MAN IS A VALUABLE MEMBER OF SOCIETY, WHO, BY HIS OBSERVATIONS, RESEARCHES, AND EXPERIMENTS, PROCURES KNOWLEDGE FOR MEN.—SMITHSON.

CITY OF WASHINGTON:
PUBLISHED BY THE SMITHSONIAN INSTITUTION.

MDCCCLXXIV.



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ADVERTISEMENT.

THIS volume forms the nineteenth of a series, composed of original memoirs on different branches of knowledge, published at the expense, and under the direction, of the Smithsonian Institution. The publication of this series forms part of a general plan adopted for carrying into effect the benevolent intentions of JAMES SMITHSON, Esq., of England. This gentleman left his property in trust to the United States of America, to found, at Washington, an institution which should bear his own name, and have for its objects the “*increase and diffusion* of knowledge among men.” This trust was accepted by the Government of the United States, and an Act of Congress was passed August 10, 1846, constituting the President and the other principal executive officers of the general government, the Chief Justice of the Supreme Court, the Mayor of Washington, and such other persons as they might elect honorary members, an establishment under the name of the “SMITHSONIAN INSTITUTION FOR THE INCREASE AND DIFFUSION OF KNOWLEDGE AMONG MEN.” The members and honorary members of this establishment are to hold stated and special meetings for the supervision of the affairs of the Institution, and for the advice and instruction of a Board of Regents, to whom the financial and other affairs are intrusted.

The Board of Regents consists of three members *ex officio* of the establishment, namely, the Vice-President of the United States, the Chief Justice of the Supreme Court, and the Mayor of Washington, together with twelve other members, three of whom are appointed by the Senate from its own body, three by the House of Representatives from its members, and six persons appointed by a joint resolution of both houses. To this Board is given the power of electing a Secretary and other officers, for conducting the active operations of the Institution.

To carry into effect the purposes of the testator, the plan of organization should evidently embrace two objects: one, the increase of knowledge by the addition of new truths to the existing stock; the other, the diffusion of knowledge, thus increased, among men. No restriction is made in favor of any kind of knowledge; and, hence, each branch is entitled to, and should receive, a share of attention.

The Act of Congress, establishing the Institution, directs, as a part of the plan of organization, the formation of a Library, a Museum, and a Gallery of Art, together with provisions for physical research and popular lectures, while it leaves to the Regents the power of adopting such other parts of an organization as they may deem best suited to promote the objects of the bequest.

After much deliberation, the Regents resolved to divide the annual income into two parts—one part to be devoted to the increase and diffusion of knowledge by means of original research and publications—the other part of the income to be applied in accordance with the requirements of the Act of Congress, to the gradual formation of a Library, a Museum, and a Gallery of Art.

The following are the details of the parts of the general plan of organization provisionally adopted at the meeting of the Regents, Dec. 8, 1847.

DETAILS OF THE FIRST PART OF THE PLAN.

I. TO INCREASE KNOWLEDGE.—*It is proposed to stimulate research, by offering rewards for original memoirs on all subjects of investigation.*

1. The memoirs thus obtained, to be published in a series of volumes, in a quarto form, and entitled “Smithsonian Contributions to Knowledge.”

2. No memoir, on subjects of physical science, to be accepted for publication, which does not furnish a positive addition to human knowledge, resting on original research; and all unverified speculations to be rejected.

3. Each memoir presented to the Institution, to be submitted for examination to a commission of persons of reputation for learning in the branch to which the memoir pertains; and to be accepted for publication only in case the report of this commission is favorable.

4. The commission to be chosen by the officers of the Institution, and the name of the author, as far as practicable, concealed, unless a favorable decision be made.

5. The volumes of the memoirs to be exchanged for the Transactions of literary and scientific societies, and copies to be given to all the colleges, and principal libraries, in this country. One part of the remaining copies may be offered for sale; and the other carefully preserved, to form complete sets of the work, to supply the demand from new institutions.

6. An abstract, or popular account, of the contents of these memoirs to be given to the public, through the annual report of the Regents to Congress.

II. TO INCREASE KNOWLEDGE.—*It is also proposed to appropriate a portion of the income, annually, to special objects of research, under the direction of suitable persons.*

1. The objects, and the amount appropriated, to be recommended by counsellors of the Institution.

2. Appropriations in different years to different objects; so that, in course of time, each branch of knowledge may receive a share.

3. The results obtained from these appropriations to be published, with the memoirs before mentioned, in the volumes of the Smithsonian Contributions to Knowledge.

4. Examples of objects for which appropriations may be made:—

(1.) System of extended meteorological observations for solving the problem of American storms.

(2.) Explorations in descriptive natural history, and geological, mathematical, and topographical surveys, to collect material for the formation of a Physical Atlas of the United States.

(3.) Solution of experimental problems, such as a new determination of the weight of the earth, of the velocity of electricity, and of light; chemical analyses of soils and plants; collection and publication of articles of science, accumulated in the offices of Government.

(4.) Institution of statistical inquiries with reference to physical, moral, and political subjects.

(5.) Historical researches, and accurate surveys of places celebrated in American history.

(6.) Ethnological researches, particularly with reference to the different races of men in North America; also explorations, and accurate surveys, of the mounds and other remains of the ancient people of our country.

I. TO DIFFUSE KNOWLEDGE.—*It is proposed to publish a series of reports, giving an account of the new discoveries in science, and of the changes made from year to year in all branches of knowledge not strictly professional.*

1. Some of these reports may be published annually, others at longer intervals, as the income of the Institution or the changes in the branches of knowledge may indicate.

2. The reports are to be prepared by collaborators, eminent in the different branches of knowledge.

3. Each collaborator to be furnished with the journals and publications, domestic and foreign, necessary to the compilation of his report; to be paid a certain sum for his labors, and to be named on the title-page of the report.

4. The reports to be published in separate parts, so that persons interested in a particular branch, can procure the parts relating to it, without purchasing the whole.

5. These reports may be presented to Congress, for partial distribution, the remaining copies to be given to literary and scientific institutions, and sold to individuals for a moderate price.

The following are some of the subjects which may be embraced in the reports:—

I. PHYSICAL CLASS.

1. Physics, including astronomy, natural philosophy, chemistry, and meteorology.
2. Natural history, including botany, zoology, geology, &c
3. Agriculture.
4. Application of science to arts.

II. MORAL AND POLITICAL CLASS.

5. Ethnology, including particular history, comparative philology, antiquities, &c.
6. Statistics and political economy.
7. Mental and moral philosophy.
8. A survey of the political events of the world; penal reform, &c.

III. LITERATURE AND THE FINE ARTS.

9. Modern literature.
10. The fine arts, and their application to the useful arts.
11. Bibliography.
12. Obituary notices of distinguished individuals.

II. TO DIFFUSE KNOWLEDGE.—*It is proposed to publish occasionally separate treatises on subjects of general interest.*

1. These treatises may occasionally consist of valuable memoirs translated from foreign languages, or of articles prepared under the direction of the Institution, or procured by offering premiums for the best exposition of a given subject.

2. The treatises to be submitted to a commission of competent judges, previous to their publication.

DETAILS OF THE SECOND PART OF THE PLAN OF ORGANIZATION.

This part contemplates the formation of a Library, a Museum, and a Gallery of Art.

1. To carry out the plan before described, a library will be required, consisting, 1st, of a complete collection of the transactions and proceedings of all the learned societies of the world; 2d, of the more important current periodical publications, and other works necessary in preparing the periodical reports.

2. The Institution should make special collections, particularly of objects to verify its own publications. Also a collection of instruments of research in all branches of experimental science.

3. With reference to the collection of books, other than those mentioned above, catalogues of all the different libraries in the United States should be procured, in order that the valuable books first purchased may be such as are not to be found elsewhere in the United States.

4. Also catalogues of memoirs, and of books in foreign libraries, and other materials, should be collected, for rendering the Institution a centre of bibliographical knowledge, whence the student may be directed to any work which he may require.

5. It is believed that the collections in natural history will increase by donation, as rapidly as the income of the Institution can make provision for their reception; and, therefore, it will seldom be necessary to purchase any article of this kind.

6. Attempts should be made to procure for the gallery of art, casts of the most celebrated articles of ancient and modern sculpture.

7. The arts may be encouraged by providing a room, free of expense, for the exhibition of the objects of the Art-Union, and other similar societies.

8. A small appropriation should annually be made for models of antiquity, such as those of the remains of ancient temples, &c.

9. The Secretary and his assistants, during the session of Congress, will be required to illustrate new discoveries in science, and to exhibit new objects of art; distinguished individuals should also be invited to give lectures on subjects of general interest.

In accordance with the rules adopted in the programme of organization, each memoir in this volume has been favorably reported on by a Commission appointed

for its examination. It is however impossible, in most cases, to verify the statements of an author; and, therefore, neither the Commission nor the Institution can be responsible for more than the general character of a memoir.

The following rules have been adopted for the distribution of the quarto volumes of the Smithsonian Contributions:—

1. They are to be presented to all learned societies which publish Transactions, and give copies of these, in exchange, to the Institution.
2. Also, to all foreign libraries of the first class, provided they give in exchange their catalogues or other publications, or an equivalent from their duplicate volumes.
3. To all the colleges in actual operation in this country, provided they furnish, in return, meteorological observations, catalogues of their libraries and of their students, and all other publications issued by them relative to their organization and history.
4. To all States and Territories, provided there be given, in return, copies of all documents published under their authority.
5. To all incorporated public libraries in this country, not included in any of the foregoing classes, now containing more than 10,000 volumes; and to smaller libraries, where a whole State or large district would be otherwise unsupplied.

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PROBLEMS OF ROTARY MOTION

PRESENTED BY

THE GYROSCOPE,
THE PRECESSION OF THE EQUINOXES,
AND
THE PENDULUM.

BY

BREVET MAJ.-GEN. J. G. BARNARD,
COLONEL OF ENGINEERS U. S. A., A. M., LL. D., MEMBER OF NATIONAL ACADEMY OF SCIENCES.

[ACCEPTED FOR PUBLICATION, OCTOBER, 1871.]

ADVERTISEMENT.

THE three following papers were read at intervals before the National Academy of Sciences, and subsequently presented to the Smithsonian Institution for publication.

JOSEPH HENRY,
Secretary Smithsonian Institution.

THE PRECESSION OF THE EQUINOXES AND NUTATION AS RESULTING FROM THE THEORY OF THE GYROSCOPE.

In a paper published in the American Journal of Science, in 1857, and in Barnard's American Journal of Education¹ [No. 9] of the same year, I remarked:—

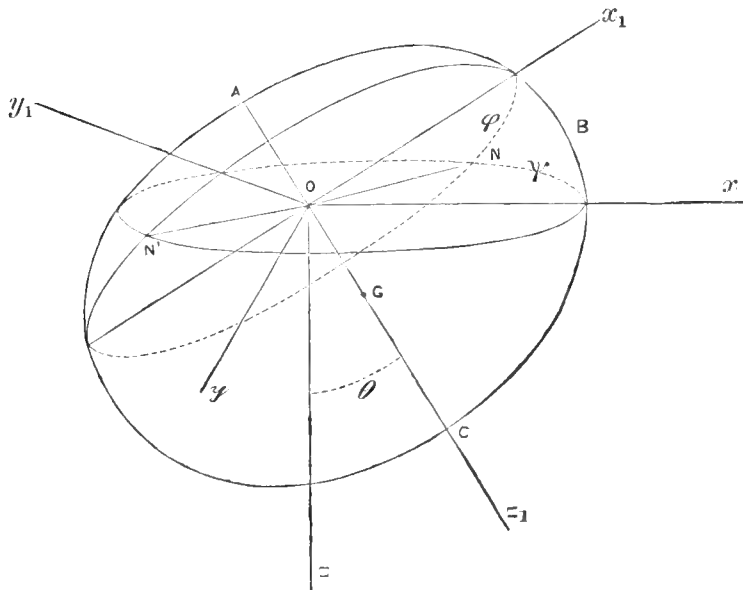
“The analogy between the minute motions of the gyroscope and that grand phenomenon exhibited in the heavens, the ‘precession of the equinoxes,’ is often remarked. In an ultimate analysis, the phenomena, doubtless, are identical,” &c.

It is the object of the present paper to deduce the analytical expressions of this phenomenon directly from the theory of the gyroscope.

A brief summary of the processes used and results arrived at in the paper referred to is necessary as a preliminary.

Let A, B, C, D (Fig. 1) be a solid body of any shape, retained by the fixed point O (within or without its mass). $Ox, Oy,$ and Oz are the three co-ordinate axes,

Fig. 1.



fixed in space, to which the motion of the body is referred. Ox_1, Oy_1, Oz_1 are the three *principal axes* belonging to the point O , and which, of course, partake of the

¹ “The Phenomena of the Gyroscope Analytically Examined.”

body's motion. The position of the body at any instant of time is determined by those of the moving axes.

For the purpose of determining the positions of the axes $Ox_1, Oy_1,$ and $Oz_1,$ with reference to the (fixed in space) axes $Ox, Oy, Oz,$ three auxiliary angles are used.

If we suppose the moving plane of $x_1 y_1,$ at the instant considered, to intersect the fixed plane of xy in the line NN' and call the angle $xON=\psi,$ and the angle between the planes xy and $x_1 y_1$ (or the angle $zOz_1)=\theta,$ and the angle $NOx_1=\phi$ (in the figure these angles are supposed *acute* at the instant taken), these three angles will determine the positions of the axes Ox_1, Oy_1, Oz_1 (and hence of the body) at any instant, and will themselves be functions of the time. and the rotary velocities about the axes of $x_1, y_1,$ and $z_1,$ may be expressed in terms of them and of their differential coefficients.

When a body is a solid of revolution, revolving with an angular velocity $n,$ about its axis of figure, and acted upon by the accelerating force of gravity (the fixed point O being in the axis of figure), the general equations of rotary motion (by processes fully developed in the paper referred to)¹ take the form

$$1. \quad \begin{cases} \sin^2 \theta \frac{d\psi}{dt} = \frac{Cn}{A} (\cos \theta - \cos \omega) \\ \sin^2 \theta \frac{d\psi^2}{dt^2} + \frac{d\theta^2}{dt^2} = \frac{2Mg\gamma}{A} (\cos \theta - \cos \omega) \\ d\phi = ndt + \cos \theta d\psi \end{cases}$$

In which

M is the mass of the body.

A its moment of inertia about an equatorial axis through $O.$

C " " " " its axis of figure.

g the force of gravity.

γ the distance OG from centre of gravity to the point of support.

ω the initial value of $\theta,$ or its value at the instant when the body has no other motion than the rotation n about its axis of figure.

Eliminating $\frac{d\psi}{dt}$ between the first two equations (1), and putting

$$2. \quad \frac{A}{M\gamma} = \lambda, \text{ and } \frac{C^2 n^2}{2A^2 g} = \frac{2\beta^2}{\lambda}, \text{ we get}$$

$$3. \quad \sin^2 \theta \frac{d\theta^2}{dt^2} = \frac{2g}{\lambda} [\sin^2 \theta - 2\beta^2 (\cos \theta - \cos \omega)] (\cos \theta - \cos \omega),$$

and the first equation (1) becomes

$$4. \quad \sin^2 \theta \frac{d\psi}{dt} = 2\beta \sqrt{\frac{g}{\lambda}} (\cos \theta - \cos \omega)$$

The quantity $\beta = \frac{Cn}{2A\sqrt{g}} = \frac{Cn}{2\sqrt{AM\gamma g}},$ may be very great in consequence of the rotary velocity, $n,$ being great, or (n being small) in consequence of the ratio

¹ The analysis therein used was mostly taken from Poisson as far as equations (of that paper) (9), (10), (11), corresponding to (7), (8), (9) of this; but the subsequent developments were original.

$\frac{C}{2\sqrt{AM\gamma g}}$, being very great. In the phenomena of the gyroscope, the first condition obtains; in the case of the earth, attracted by the sun or moon (u being small), it is easy to show that the alternative condition is fulfilled.¹

Putting $\frac{d\theta}{dt}$ equal to zero in equation (3) we get $\theta = \omega$ for the *maximum* of θ , and for the *minimum*, the equation,

$$\cos \theta = -\beta^2 + \sqrt{1 + 2\beta^2 \cos \omega + \beta^4}$$

in which, if β is very great, the value of $\cos \theta$ differs but slightly from that of $\cos \omega$. Hence by introducing a new variable u , equal to $\omega - \theta$, and deducing the values of $d\theta$ and (by development) of $\sin^2 \theta$ and $\cos \theta$ (neglecting the higher powers of u) and substituting in (3) and (4), they become (omitting, as relatively small, $\cos \omega$ in the factor $\cos \omega + 4\beta^2$).

$$5. \quad \sqrt{\frac{g}{\lambda}} dt = \frac{du}{\sqrt{2u \sin \omega - 4\beta^2 u^2}}$$

$$6. \quad \frac{d\psi}{dt} = 2\beta \left(\frac{g}{\lambda}\right)^{\frac{1}{2}} \frac{u}{\sin \omega}$$

Equation (5) gives by integration and putting $\beta \left(\frac{g}{\lambda}\right)^{\frac{1}{2}} = k$.

$$7. \quad u = \frac{1}{2\beta^2} \sin \omega \sin^2 kt$$

which substituted in (6) gives

$$8. \quad \frac{d\psi}{dt} = \frac{1}{\beta^2} k \sin^2 kt$$

$$9. \quad \psi = \frac{1}{2\beta^2} kt - \frac{1}{4\beta^2} \sin 2kt$$

If we make $\omega = 90^\circ$, $\sin \omega = 1$, in equation (6), deduce the value of dt , and substitute in (5) we get,

$$10. \quad d\psi = \frac{u du}{\sqrt{\frac{1}{2\beta^2} u - u^2}}$$

the differential equation of the cycloid, generated by a circle of which the diameter is $\frac{1}{2\beta^2}$, and having a chord $\frac{\pi}{2\beta^2}$.

¹ For the earth the moments of inertia, A and C , with reference to principal axes through the centre, differ very little. The value of β may therefore be approximately written $\frac{n}{2\sqrt{AM\gamma g}}$, and the denominator is to be replaced (17) by $\frac{L}{\sin \theta}$. Substitute the value of L (19) and put, for the sun, $\frac{S}{r^3} = n_1^2$ (25), and the value of β becomes $\frac{n}{2n_1 \sqrt{3(C-A) \cos \theta}}$, which is very large.

The value as depending on the moon's attraction is (28a), $\frac{n}{2n_2 \sqrt{3(C-A) \cos \theta}}$, of the same order of magnitude as before.

If the value of ω is not 90° , the diameter of this circle will be $\frac{1}{2\beta^2} \sin \omega$; but the quantity $\frac{\pi}{2\beta^2}$ then measures an angle of an arc of a small circle having a radius $= \sin \omega$; and the chord of the curve is reduced in the same proportion as its sagitta, and the curve is still a cycloid.

The axis of figure *gyrates* therefore about the vertical through O as if it was attached to the circumference of a small circle of the minute diameter specified, the centre of which circles moves with a uniform horizontal velocity, which velocity is the *mean* rate of gyration; or the motion may be compared to that of a cone having its vertex at O and diameter of base $= \frac{1}{2\beta^2} \sin \omega$ (the axis of figure constituting an element of this cone) rolling upon a fixed conical surface, all the elements of which make, with the vertical, the angle ω ; but this imaginary cone *is not fixed in the body* (save in the exceptional case of the moments of inertia A and C being equal). For the rotary velocity of the body is n , while that of the cone is $2\beta \sqrt{\frac{g}{\lambda}} = \frac{Cn}{A}$.

Since the rotary velocities of the body and cone are different, the *instantaneous axis* cannot move along the chord of the cycloid, *nor with uniform velocity*.

The common methods of investigating the Precession of the Equinoxes, founded upon the incipient rate of motion of the instantaneous axis, involve this error, which does not become apparent, simply because the moments of inertia A and C are, for the earth, so nearly equal.

The instantaneous axis will describe a prolate cycloid having the same chord as the common one $\left(\frac{\pi}{2\beta^2}\right)$, and a sagitta $= \frac{1}{2\beta^2} \sin \omega \left(\frac{A}{C} - 1\right)$.

The mean rate of gyration is given by the coefficient of t in equation (9); it is $\frac{1}{2\beta} \left(\frac{g}{\lambda}\right)^{\frac{1}{2}}$, or substituting values (2) for β and λ ;

$$11. \quad \frac{Mg\gamma}{Cn}$$

Thus far I have supposed that at the origin of time, or at the moment when the accelerating force commenced to act, the body had no other motion than a rotation, n , about its axis of figure. It remains to prove, that if, at this instant, there are *small* (compared to n) velocities about either or both the other principal axes, the rate of gyration will be the same.

The solution will be perfectly general if we suppose at this instant a velocity, m , about the axis of x only, and assume at same moment $\theta = \omega$, $\phi = 90^\circ$, we should get, instead of equations (3) and (4), the two following:—

$$12. \quad \sin^2 \theta \frac{d\theta^2}{dt^2} = \left[\frac{2Mg\gamma}{A} \sin^2 \theta - \frac{2Cmn}{A} \sin \omega - \frac{C^2 n^2}{A^2} (\cos \theta - \cos \omega) \right. \\ \left. - m^2 (\cos \theta + \cos \omega) \right] (\cos \theta - \cos \omega)$$

$$\sin^2 \theta \frac{d\psi}{dt} = \frac{Cn}{A} (\cos \theta - \cos \omega) + m \sin \omega$$

Substituting in these $\omega - u$ for θ , rejecting all small quantities of the second order (among which is m^2), and introducing β and λ (see equations 2)

$$13. \quad \frac{\lambda}{g} \frac{du^2}{dt^2} = 2u \left(\sin \omega - 2\beta \sqrt{\frac{\lambda}{g}} m \right) - 4\beta^2 u^2$$

$$14. \quad \frac{d\psi}{dt} = 2\beta \sqrt{\frac{g}{\lambda}} \frac{u}{\sin \omega} + \frac{m}{\sin \omega}$$

The integral of (13) (using k with its already given value) in (7) is

$$15. \quad u = \frac{1}{2\beta^2} \left(\sin \omega - \frac{2\beta^2}{k} m \right) \sin^2 kt$$

Substitute in (14) and integrate

$$16. \quad \psi = \frac{1}{2\beta^2} kt - \left(\frac{1}{4\beta^2} - \frac{m}{2k \sin \omega} \right) \sin 2kt$$

The coefficient t in (16) is identical with that of equation (9), $= \frac{Mg\gamma}{Cn}$, showing that although the character of the gyratory motion is altered, and the axis of figure, instead of moving on a common cycloid (which forms cusps) and coming periodically to rest, moves along a *prolate* cycloid or even *without undulation*, yet the rate of gyration is unchanged.

If $\omega = 90^\circ$ and $m = \frac{Cn}{Mg\gamma}$, u and $\frac{du}{dt}$ become zero for all values of t , and the body gyrates horizontally without nutation.¹

In all that precedes, the revolving body has been supposed retained by a fixed point in its axis of figure, but *not* at its centre of gravity, while the accelerating force, being gravity itself, acts through that centre.

If, instead, the fixed point by which the body is retained is the centre of gravity, and the accelerating or disturbing forces any other whatever (provided their direction is invariable and their resultant acts through a fixed point of the axis), the

¹ This is the case referred to in the preceding paragraph in which the moment of the accelerating force (or couple) is equal to that of (what I have styled in the work before referred to) the "deflecting force," which has for its value the expression $\frac{Cnm}{M\gamma}$.

That this case should arise, a determinate relation between m , n , and g , expressed by the equation $\frac{mn}{g} = \frac{M\gamma}{C}$, is necessary.

When this relation exists, the movement may be represented by the rolling of a conical surface (the locus, in the body, of the instantaneous axis), described about the axis of figure with the angle (approximately) equal to $\frac{Mg\gamma}{Cn^2}$, upon another, all of the elements of which make, with the vertical, the angle $\omega - \frac{Mg\gamma}{Cn^2}$, (when ω is not 90° , the centrifugal as well as the deflecting force affects the relation between m , n , and g).

But no such relation is *essential* to the gyration expressed by (11); and, in the case of the precession of the equinoxes, the supposition of rolling cones is not realized. There are, probably, no two instants of time at which the precessional movements of the axis are identically the same.

equations will be precisely the same, the moments of inertia A and C , referring to principal axes through the centre of gravity, and Mg expressing the intensity of the resultant of the forces, and γ the distance from the centre of gravity of the point through which the resultant acts.

In expression (11) $Mg\gamma$ is the moment of the force with respect to the point O , divided by the sine of the angle (θ) which its direction makes with the axis of figure. Denote that moment by L . Then the expression for the velocity of gyration (11) becomes,

$$17. \quad \frac{L}{Cn \sin \theta}$$

If the body in question, like the earth, is acted upon by forces, the resultant of which does not pass through its centre of gravity, its movements about that centre are precisely the same as if that centre were fixed; in other words, it will gyrate about the line connecting its centre and the origin of the force with a velocity denoted by expression (11). In the case of the earth, however, the direction of the disturbing force and its moment are constantly changing, and I have to assume something not proved in what foregoes, viz., that the *elementary gyration at each moment of time* will be likewise expressed by (11); an assumption *not* (probably) strictly true, since, when the forces are constant in direction and intensity, equation (14) shows (the value of u , equation (15) being substituted) that the gyratory velocity, though its mean is always expressed by (11), varies at each instant unless the value of m has a certain relation to that of k .

Since the integral of these varying elementary displacements shows, under all circumstances of constantly directed force (though these elementary motions of the axis exhibit all possible directions with regard to that of the force), a mean rate of gyration expressed by (11), we may assume that the fact will hold good though the direction and moment of the force change.¹

In the case of the earth there is probably no instant of time at which it is revolving exactly about its axis of figure; the quantity m has, for it, in all cases, a finite (though exceedingly small) value; neither observation nor (scarcely) analysis can detect the minute diurnal (nearly) nutations which belong to the diurnal cycloidal movement; and hence the presumption that the gyration is *at all instants* perpendicular, or nearly so, to the direction of the force, and hence that even its elementary values vary little from expression (11).²

¹ Such an assumption is made in all the investigations not, like Laplace's, purely analytical, without always giving the true grounds on which it should be based.

² In reality, if the moment L remains the same for different values of θ , the elementary displacement produced by the gyration is independent of θ , for, though the expression $\frac{L}{Cn \sin \theta}$ varies inversely as $\sin \theta$, yet the radius of the small circle on which the displacement takes place increases in like proportion. Again, that a revolving body should gyrate around a given axis it is not *necessary* that the accelerating force should be always parallel in direction to that axis, but that it should remain in the moving plane through the axis of figure and the given axis. The general equations of rotation would be the same.

Let a , be the equatorial diameter of the earth.

b , be the polar diameter.

ρ , its variable density.

C , its moment of inertia about the polar axis.

A , its moment of inertia about an equatorial one.

$e = \frac{a-b}{a}$, earth's ellipticity.

\mathcal{S} , the absolute attractive force of the sun, or its attraction upon a unit of mass at a unit's distance.

r , the mean distance of centres of sun and earth. x, y, z being rectangular co-ordinates of any element of the earth's mass, dm ; the origin being the earth's centre, the axis of z the polar one, of x an equatorial one in a plane passing through the sun's centre, of y an equatorial one perpendicular to this plane.

The moment of the sun's attractive force upon the earth is shown in various works on precession (*vide* Mr. Airy's "Figure of the Earth," Encyc. Metropolitana) to be (θ being the angle of earth's axis with line drawn to sun).

$$18. \quad \frac{3\mathcal{S}}{r^3} \iiint \rho (x^2 - z^2) dx dy dz \sin \theta \cos \theta$$

the integral being taken through the spheroid.

The quantity under the signs of integration may be written

$\rho (x^2 + y^2) dm - \rho (y^2 + z^2) dm$, the integral of the first term of which is C , and of the second A .

Hence the moment of the sun's force (18)

$$19. \quad \frac{3\mathcal{S}}{r^3} (C - A) \sin \theta \cos \theta = L$$

Hence the gyration produced upon the earth by the sun's force about the line of its direction is (17)

$$20. \quad \frac{3\mathcal{S}}{r^3 n} \frac{C - A}{C} \cos \theta; \text{ and in time } dt,$$

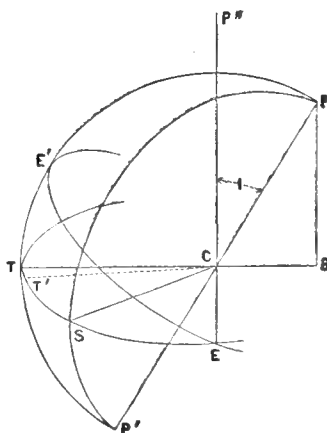
$$21. \quad \frac{3\mathcal{S}}{r^3 n} \frac{C - A}{C} \cos \theta dt$$

Let EST be a great circle in the plane of the ecliptic, EE' an equatorial one, $PE'TP'$ a great circle through the tropics, PSP' one through the sun in any position, C the centre of the earth, and PCP' its axis. SCP' is the angle θ . If the sun moves in the ecliptic from E (the equinox) towards T with an angular velocity n , $n_1 t$ will be the value of the arc ES . In the spherical triangle $P'ST$, right-angled at T , we have $\cos P'S$ (or $\cos \theta$) = $\cos P'T \cos TS$ = $\sin TE' \sin SE$.

TE' is the inclination of the equator to the ecliptic; call this I . Then

$$\cos \theta = \sin I \sin n_1 t$$

Fig. 2.



The elementary gyration about the line SC will be therefore, (21)

$$22. \quad \frac{3S}{nr^3} \frac{C-A}{C} \sin I \sin n_1 t \, dt.$$

If this rotation about SC is decomposed into components about the lines TC and EC , they will be

$$23. \quad \frac{3S}{nr^3} \frac{C-A}{C} \sin I \sin^2 n_1 t \, dt.$$

$$24. \quad \frac{3S}{nr^3} \frac{C-A}{C} \sin I \sin n_1 t \cos n_1 t \, dt.$$

The component (23) represents a rotation of the pole about TC , the radius of its motion being PB , or $\cos I$. To obtain the actual value as an arc of a great circle, of this minute displacement, it must be multiplied by $\cos I$; and to refer this to the pole of the ecliptic as angular motion, it must be divided by $\sin I$. Performing these operations, integrating, and remembering that by Kepler's laws

$\frac{S}{r^3} = \frac{4\pi^2}{T^2} = n_1^2$ (T =number of units of time in one year), we get, for precession,

$$25. \quad \frac{3}{2} \frac{n_1^2}{n} \frac{C-A}{C} \cos I t - \frac{3}{4} \frac{n_1}{n} \frac{C-A}{C} \cos I \sin 2n_1 t.$$

And for nutation

$$26. \quad \frac{3}{4} \frac{n_1}{n} \frac{C-A}{C} \sin I \cos 2n_1 t.$$

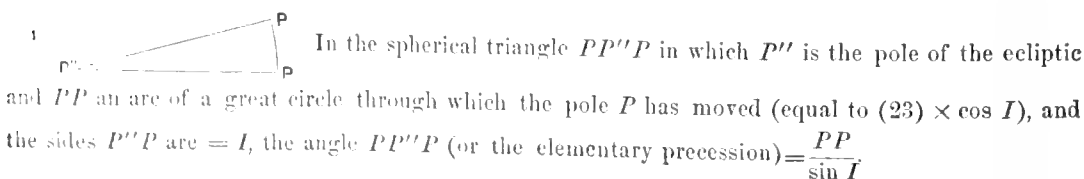
The first term of (25) is the mean solar precession; making $t = \frac{2\pi}{n_1}$, it gives for the annual solar precession

$$27. \quad 3 \frac{n_1}{n} \frac{C-A}{C} \cos I.$$

Expression (26) is the solar nutation, and the second term of (25) gives the equation of the equinoxes in longitude, or the fluctuating term of the precession corresponding to the nutation.

These expressions correspond to those obtained by the ordinary solutions. They differ from most of them, however, in having C in the denominator instead of A , an error of those solutions I have alluded to before, which, however real, analytically, exerts no important influence on the result.

By the above method the precession is the integral of the components of gyration about a solstitial diameter of the ecliptic, which line itself, by the process of precession, has an angular motion equal to that precession, the real effect



being a revolution about an axis (the pole of the ecliptic) perpendicular to the plane of that angular motion. In other words, if we integrate directly equation (23) and make $t = \frac{2\pi}{n_1}$, and $\frac{S}{r^3} = n_1^2$, we shall get

$$28. \quad 3 \frac{n_1}{n} \pi \frac{C-A}{C} \sin I;$$

and this will be the total angular motion of the pole P about the solstitial line TC in one revolution of the sun; but by this very motion of the pole the equinoxes have moved an angle measured by *this displacement referred to the pole of the ecliptic*—that is, by the angle expressed by (27)—and the solstitial line TC has of course, undergone the same movement, and the next annual gyration will be about the consecutive line $T' C$, and so on; producing a continuous motion of the pole P about the pole of the ecliptic P'' .

To obtain the precession due to the moon, it is necessary to substitute in (19) for $\frac{S}{r^3}$, $\frac{M^1}{(r)^3}$, in which M^1 is the attractive force of the moon and (r) its mean distance.

But T^1 (time of moon's revolution) is, by Kepler's laws, $\frac{2\pi(r)^{\frac{3}{2}}}{(M^1 + E)^{\frac{1}{2}}} = \frac{2\pi(r)^{\frac{3}{2}}}{[(1+\gamma)M^1]^{\frac{1}{2}}} = \frac{2\pi}{n_2}$ (calling the mean angular velocity of the moon n_2 and the ratio of earth's mass to that of moon's mass, γ) (28a); hence $\frac{M^1}{(r)^3} = \frac{n_2^2}{1+\gamma}$.

If i is the inclination of the moon's orbit to the equator during any one revolution (regarded as constant for that time), we should obtain for the precession and nutation, *referred to the pole of the moon's orbit*, expressions analogous to (25) and (26).

Although the moon's disturbing effect, as above expressed, is almost exactly double that of the sun, yet the larger divisor n_2 , introduced by integration, renders the value of (26) and of the fluctuating term of (25) very small for the moon—say about $\frac{1}{6}$ th the corresponding values for the sun. Hence these terms are usually disregarded in the lunar expressions.

The elementary precession due to the moon *about the pole of its own orbit* would be by (25)

$$29. \quad \frac{3}{2} \frac{n_2^2}{n(1+\gamma)} \cdot \frac{C-A}{C} \cos i dt.$$

From this, by the usual methods, can be deduced the real precession and nutation. But it will be more in harmony with the object of this paper, and indeed more elegant, to reduce the *gyration* produced by the moon directly to precession and nutation.

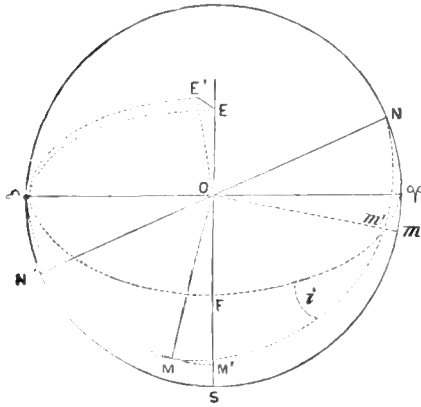
If we substitute for n , $\frac{n_2}{1+\gamma}$, and $\sin i$ for $\sin I$, in (28) we shall get, for the total gyration *about the line of greatest declination*, produced by one revolution of the moon in its orbit, the expression:

$$30. \quad 3 \frac{n_2}{n(1+\gamma)} \pi \frac{C-A}{C} \sin i.$$

In which the line of greatest declination is regarded as stationary during the single revolution, and taking a consecutive position for the next; but it will be in harmony with the fact, and allowable, to regard the line as in continuous motion and the above amount of gyration to be uniformly spread over the time $\frac{2\pi}{n_2}$, of the revolution, producing thus an elementary gyration, in the time dt , of

$$31. \quad \frac{3}{2} \frac{n_2^2}{n(1+\gamma)} \frac{C-A}{C} \sin i dt.$$

Let $\varphi S \triangleq$ be a great circle in the plane of the ecliptic; $\varphi \triangleq$ the line of equinoxes, NON' the line of moon's nodes, $\varphi F \triangleq$ the equator, and $Nm'MN'$ the moon's orbit crossing the equator at m' . The line of the moon's maximum declination, OM , will be 90° from the line Om' .



The pole E of the earth is supposed to undergo a displacement by gyration about OM represented by EE' ; the precession produced will be the angle EOE' ; the nutation, the angle $E \triangleq E'$.

In the spherical triangle $Nm'\varphi$ the angle at N is $= I$, the inclination of moon's orbit to ecliptic; the angle at φ is the supplement of I (inclination of the equator) and the angle at m' is i (or the variable inclination of the moon's orbit to the

equator), and the side φN is $= n_3 t$ (calling the angular velocity of the moon's node n_3); therefore,

$$32. \quad \cos i = \cos I \cos I + \sin I \sin I \cos n_3 t$$

and

$$33. \quad \text{tang } m'\varphi = \frac{\sin n_3 t}{\sin I \cot I - \cos I \cos n_3 t}.$$

In the spherical triangle $mm'\varphi$

$$34. \quad \text{tang } m\varphi = \cos I \text{ tang } m'\varphi$$

OS is the line of maximum declination of the sun, or the solstitial diameter of the ecliptic about which the annual gyration produced by the sun is made. As the inclination I of the moon's orbit is small, the arc MM' , drawn through $M \triangleq$, is approximately equal to $m\varphi$, and the angle $MO \triangleq$ differs immaterially from the complement of $m\varphi$; hence by (33) and (34)

$$35. \quad \text{tang } MM' = \frac{\cos I \sin n_3 t}{\sin I \cot I - \cos I \cos n_3 t} = \frac{\cos I \sin I \sin n_3 t}{\sin I \cos I - \cos I \sin I \cos n_3 t}.$$

If the gyration about OM (31) is decomposed into components about OM and $O \triangleq$, we shall have for the first (calling the coefficient of $\sin i dt$, K)

$$36. \quad K \sin i \cos MM' dt,$$

and for the second

$$37. \quad K \sin i \sin MM' dt.$$

To refer the *displacement* expressed by (36) to the pole of the ecliptic O , as angular motion, we must (see note page 8) multiply by $\cos M'F$ or (from which it differs but slightly) $\cos i$ and divide by $\sin I$; we thus obtain for the elementary *precession*

$$38. \quad \frac{K}{\sin I} \cos MM' \sin i \cos i dt,$$

and for nutation

$$39. \quad K \sin MM' \sin i dt.$$

The maximum value of the arc MM' is about $11^\circ 56'$; the line MO describing during an entire revolution of the moon's nodes an elliptical cone about OS of which the minor semi-diameter (SM) is $5^\circ 8\frac{1}{2}'$ (about), and the semi-major $11^\circ 56'$. By conceiving the elementary motion of the pole of the earth (or its gyration) as at each instant about the line MO , as it makes its conical revolution, the undulating nature of that motion, or the "nutation," is easily conceived.

Approximate values of sine and cosine of MM' may be determined from (35); which, substituted with those of i (32) in (38) and (39), will enable us to integrate and obtain very accurate expressions for the lunar precession and nutation.⁽¹⁾ But these expressions, nearly free from errors of approximation, may be more elegantly determined as follows: When the angle of the moon's orbit with the equator is minimum, the angle $i=I-I'$; when maximum $i=I+I'$ (epochs corresponding to $n_3t=0$ and $n_3t=\pi$); the angle MM' is zero, and the corresponding *rates* of precession are by (38)

$$(a) \quad K \frac{\sin 2(I-I')}{2 \sin I} \text{ for } n_3t=0.$$

$$(b) \quad K \frac{\sin 2(I+I')}{2 \sin I} \text{ for } n_3t=\pi.$$

When $n_3t=\frac{1}{2}\pi$, we have

$$\cos MM' = \frac{\sin I}{\sqrt{1-\cos^2 I \cos^2 I'}}, \quad \sin MM' = \frac{\cos I \sin I'}{\sqrt{1-\cos^2 I \cos^2 I'}}$$

$\cos M'F = \cos I \cos^2 I'$, $\cos i = \cos I \cos I'$, $\sin i = \sqrt{1-\cos^2 I \cos^2 I'}$ (the three first being the residuals of exact analytical expressions after omission of quantities of inappreciable magnitude).

Hence, by (38), the rate of precession for $n_3t=\frac{1}{2}\pi$ is

$$(c) \quad K \cos I \cos^2 I' *$$

Assume the formula for precession to be

$$K(Pt + P' \sin n_3t + P'' \sin 2n_3t);$$

* When the moon's orbit intersects the ecliptic in a solstitial line, the elementary precession $K \cos i$ (29) about its own pole is reduced, with but slight error, to the same about the ecliptic pole, by simple multiplication by $\cos I'$: a result coinciding with the above.

⁽¹⁾ See Additional Notes, p. 51.

the rates of precession will be

$$\begin{aligned} (a') & K(P + n_3 P' + 2n_3 P'') \quad \text{for } n_3 t = 0 \\ (b') & K(P - n_3 P' + 2n_3 P'') \quad \text{“ } n_3 t = \pi \\ (c') & K(P - 2n_3 P'') \quad \text{“ } n_3 t = \frac{1}{2}\pi \end{aligned}$$

Equating (a) to (a'), &c., we deduce

$$P = \frac{1}{2} \cos I (\cos^2 I' + \cos 2I') = \cos I \left(1 - \frac{3}{2} \sin^2 I'\right)$$

$$P' = -\frac{1}{2n_3} \frac{\sin 2I' \cos 2I}{\sin I} = -\frac{1}{n_3} \cos I \sin 2I' \cot 2I$$

$$P'' = -\frac{1}{4n_3} \cos I (\cos I' - \cos 2I') = -\frac{1}{4} n_3 \cos I \sin^2 I'.$$

Hence the formula for precession may be written

$$44. \quad \frac{3}{2n} \frac{n_2^2}{(1+\gamma)} \frac{C-A}{C} \cos I \left[\left(1 - \frac{3}{2} \sin^2 I'\right) t - \frac{1}{n_3} \sin 2I' \cot 2I \sin n_3 t - \frac{1}{4n_3} \sin^2 I' \sin 2n_3 t \right].$$

Similarly we would get for the nutation

$$45. \quad \frac{3}{2n} \frac{n_2^2}{(1+\gamma)} \frac{C-A}{C} \cos I \frac{\sin I'}{n_3} \cos n_3 t.$$

The ratio of actual lunar precession to what it would be were the moon's orbit in the ecliptic, is therefore expressed by

$$1 - \frac{3}{2} \sin^2 I' = 0.99 \text{ (very nearly).}^1$$

The third term of (44) indicates a slight periodical variation from the true elliptic motion referred to in the next paragraph. There should be a corresponding term in (45) which may be obtained by the same process, but they are both too minute to enter into computations.

¹ It is worthy of remark that the formulæ of Laplace [3100] and [3101] (Bowditch) contain no such coefficient qualifying the mean lunar precession, though one is found in all the more popular solutions; neither do they contain the term (quite minute) in $2n_3 t$ of (44), but, on the other hand, contain terms in $2n_2 t$ (corresponding to the terms in $2n_1 t$ of 25 and 26), which, referred to in the fourth par. (page 9), are generally omitted as inappreciable.

If we multiply the coefficient of $\sin n_3 t$ in (44) by $\sin I$, we shall have the value as an arc of a great circle of this fluctuating displacement called *the equation of the equinoxes in longitude*. The coefficient thus modified will represent the minor semi-diameter, and that of the nutation proper (45), the major semi-diameter of the ellipse of nutation; they have the ratio $\cos 2I : \cos I$ nearly. This ellipse has its major axis (equal to about $18''$ of arc) directed towards the pole of the ecliptic. The period of its description $\frac{2\pi}{n_3}$ is that of the revolution of the moon's nodes. At

the same time a minute ellipse of semi-annual nutation due to the sun is superimposed upon this. It has its longer axis (about one second of arc) likewise directed to the pole of the ecliptic. The smaller axis is to the major as $\cos I : 1$.

It is easy to show that the precession caused by the sun and moon is equal (with slight difference due to the ratio we have just been considering) to what it would be if those bodies were uniformly distributed in solid rings over circles (in the plane of the ecliptic) about the centre of the earth, having radii equal to their mean distances. In this case, unless there was a particular relation between the couple producing the initial rotation of the earth and that arising from the attraction of the two rings, there would be an extremely minute nutation, of which the period

would be (see equation 9) $\frac{2\pi}{2k} = \frac{2\pi}{n} \frac{A}{C}$; which is almost identical with the sidereal

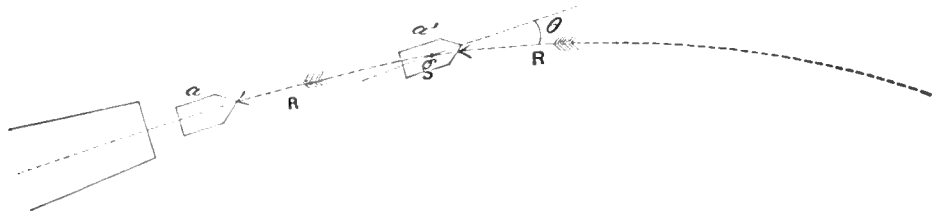
day $\left(\frac{2\pi}{n}\right)$, the ratio being $\frac{A}{C} : 1$. If we suppose the primitive rotation of the earth

to be that alone, about its axis of figure n , then the nutation will exhibit the common cycloidal motion of equation (10); but its total amount would be but about $\frac{1}{55}$ second of arc.

An explanation of the deviation of rifled projectiles will be found in what is said (pages 8 and 9) in reference to the conversion of gyration about a shifting axis into precession about an axis perpendicular to the plane of motion of the first. Elongated rifled projectiles, while they maintain almost unaltered their "angle of elevation," are found to deviate with great uniformity from the vertical plane of projection, in a direction corresponding to the twist of the gun, while spherical projectiles (fired from rifled guns) having precisely the same rotary motion, do not so deviate; showing that the cause of the phenomenon is something else than the direct action of friction or pressure of the air.

In the remarks made in the paragraphs just referred to, it is explained how the consecutive small annual gyrations about the line from the centre of the earth to the sun (in the tropics) become, in their integral, the movement which we call precession, about an axis perpendicular to the plane of motion of that line, inasmuch as each small primary gyration causes a corresponding shifting of the line about which it takes place. Something very similar occurs to produce the deviation of elongated projectiles. In issuing from the gun, the resultant of the atmospheric resistance (denoted by the arrow R) coincides with the axis of the projectile, and it has no other effect than to retard the motion of translation; but the action of gravity causes the trajectory to curve downwards, and the direction of the atmospheric resistance becomes oblique to the axis of the projectile (at a'), and (in

almost all forms of projectiles) passes *above*, not through, the centre of gravity g . We have then the essential conditions of gyration, viz., a solid of revolution revolving rapidly about its axis, and a dynamic "couple" (*i. e.*, the inertia of the



projectile's motion of translation acting through its centre of gravity, and the resistance of the air acting through a point of the axis more or less distant from g) tending to turn the projectile upwards about a horizontal (or "equatorial") axis through g , and there is in fact, at each instant, an elementary gyration about a line through g , parallel to R (the atmospheric resistance). If this line retained an invariable direction, the integral effect of these elementary gyrations would be to revolve *down* the axis of the projectile, and we should ultimately find it assuming horizontal and even sub-horizontal directions. But such cannot be the case; the direction of the axis is no sooner deviated, laterally, from its original direction, than a (nearly) corresponding change takes place in the direction of the resistance R (since from the elongated form of the projectile, the direction of its motion follows pretty nearly that of its axis) and in that of the line (parallel to R) about which gyration takes place. The integral of such a series of elementary gyrations, accompanied by a *corresponding* horizontal angular motion of the line about which they take place, is angular motion about a line perpendicular to the plane in which that line shifts direction, that is, about a vertical. Hence the vertical direction (or "elevation") of the axis of the projectile remains constant, or nearly so, while its horizontal direction undergoes a progressive angular precession (if I may so term it), and the deviation of rifled projectiles is thus seen to have analogy with the precession of the equinoxes.¹

In what precedes I do not profess to throw new light on a subject so thoroughly studied as the Precession of the Equinoxes; my object has been rather to make evident the analogy that exists between "the minute motions of the gyroscope and that grand phenomenon exhibited in the heavens," and to show how a common analysis applies to both.

¹ It is quite probable that there are other causes of deviation, the friction of the air being (in case of long ranges) one. Experimental facts are needed for a full discussion of this subject.

MOTIONS OF FREELY SUSPENDED AND GYROSCOPIC PENDULUMS, AND
ON THE PENDULUM AND GYROSCOPE AS EXHIBITING
THE ROTATION OF THE EARTH.

LET the point of suspension of the pendulum be taken as the origin of rectangular co-ordinates, the axis of z vertical (downwards), and those of x and y in the plane of the horizon, the former directed to the east, the latter to the north.

The forces which act on the pendulum (considered as concentrated in its centre of oscillation) are

1st. Gravity, the resultant of the earth's attraction, and of the centrifugal force of its rotation.

2d. The tension of the pendulum cord.

3d. The force of inertia, the components of which are represented by the differential coefficient $\frac{d^2x}{dt^2}$, $\frac{d^2y}{dt^2}$, $\frac{d^2z}{dt^2}$; and

4th. The disturbing forces arising from the earth's rotation.

5th. The resistance of the air.

If we represent the length of the string by l , its tension by N , and the force of gravity by g , and neglect the forces named in the 4th and 5th categories, we shall have the three equations:

$$(1) \quad \begin{aligned} \frac{d^2x}{dt^2} &= -\frac{Nx}{l} \\ \frac{d^2y}{dt^2} &= -\frac{Ny}{l} \\ \frac{d^2z}{dt^2} &= -\frac{Nz}{l} + g. \end{aligned}$$

The forces due to the earth's rotation which disturb the *relative* motions of a projectile, or any material particle moving near the earth's surface, have been expressed by Poisson (*Journal de l'Ecole Polytechnique* Cahier, 26) as follows:—¹

¹ These expressions are perfectly general and applicable to *all* problems which involve relative motions near the earth's surface, whether of solids or fluids. They (or their equivalent) appear in the work of Laplace (*Mec. Cel.*, Vol. IV.), as well as of Poisson (from whom I have quoted them), in the investigations of the motions of projectiles. It is somewhat extraordinary that both of these great analysts should have failed to perceive their remarkable application in the motions of the gyroscope and pendulum, to the exhibition of the earth's rotation, although the latter has seemed to desire such an exhibition in the sentence: "Quoique la rotation de la terre soit maintenant établie

$$\begin{aligned}
 X &= 2n \left(\frac{dy}{dt} \sin \lambda + \frac{dz}{dt} \cos \lambda \right) \\
 Y &= -2n \frac{dx}{dt} \sin \lambda \\
 Z &= -2n \frac{dx}{dt} \cos \lambda.
 \end{aligned}
 \tag{2}$$

In which λ is the latitude and n the angular velocity of rotation of the earth.

These analytical expressions make their appearance in the transformations of the equations of motion, near the earth's surface, expressed in co-ordinates referring to *fixed* axes, into others referring to the moving axes which are used in this analysis. But these forces can be obtained and their origin better understood by the following considerations.

The centrifugal force of a material point at rest on the earth's surface at the given latitude will be $\frac{n^2 r^2 \cos^2 \lambda}{r \cos \lambda}$ (r being the earth's radius). If it has a small relative velocity $\frac{dx}{dt}$ to the east, the centrifugal force will become $\frac{\left(n r \cos \lambda + \frac{dx}{dt} \right)^2}{r \cos \lambda}$. Subtracting the former expression from the latter (omitting $\frac{dx^2}{dt^2}$) we get for the centrifugal force arising from the relative velocity $\frac{dx}{dt}$ the expression $2n \frac{dx}{dt}$. The component of this in the direction of the axis of y will be $-2n \sin \lambda \frac{dx}{dt}$, which corresponds to the value of Y (equations 2) of Poisson, and is to be added to the second member of the second of equations (1).

The component of the force just calculated in the direction of the axis of Z is

$$-2n \cos \lambda \frac{dx}{dt}.$$

This force corresponding to Poisson's value of Z is to be added to the second member of the third of equations (1).

A body moving on a meridian of the earth's surface from south to north will have the moment of its quantity of motion, with reference to the earth's axis, diminished; in virtue of which it will press with a certain force towards the east

avec toute la certitude que les sciences physiques comportent, cependant une *preuve directe* de ce phénomène doit intéresser les géomètres et les astronomes." The former, in making a partial application to the pendulum, of his investigations, absorbed, apparently, in the single object of proving that the accuracy of the instrument as a measure of time was not affected, has inadvertently assumed that the disturbing force normal to the plane of oscillation is "trop petite pour écarter sensiblement le pendule de son plan et avoir aucune influence appréciable sur son mouvement." (*Journal de l'Ecole Poly. Cahier, 26, p. 24.*) It is true, indeed, that the force he mentions, even if permitted free action, will have but an inappreciable influence upon the time, and none whatever when, as in the chronometer, the plane is constrained to fixedness; but the effect is *cumulative* in changing the azimuth of the freely suspended pendulum.

These same disturbing forces, introduced along with the attractions of the sun, moon, and earth, into the general equations of equilibrium of fluids, produce in a very simple manner the differential equations for the tidal motions. (*Vide American Journal of Science, 1860.*)

and the moment of the force thereby developed is equal to $\frac{d}{dt}$ (the moment of its quantity of motion). This moment for the pendulum in any latitude λ , is $nr^2 \cos^2 \lambda$, of which the differential coefficient, taken with reference to λ as a function of t , is $-2nr^2 \sin \lambda \cos \lambda \frac{d\lambda}{dt}$, which is the moment of the force required. Dividing by the radius of rotation, $r \cos \lambda$, we have $+2nr \sin \lambda \frac{d\lambda}{dt}$ for the expression of the force which (acting positively in the direction of the axis of x) is to be taken with the plus sign.

In the system of rectangular co-ordinates which I am using $\frac{dy}{dt}$ corresponds to the velocity expressed by $r \frac{d\lambda}{dt}$; and substituting it therefor, we have for a disturbing force in the direction of the axis of x the expression

$$2n \sin \lambda \frac{dy}{dt}$$

In almost precisely the same way it may be shown that a body falling towards the centre of the earth with a velocity $\frac{dz}{dt}$ will have the moment of its quantity of motion diminished by $2nr \cos^2 \lambda \frac{dz}{dt}$, giving rise to the force

$$2n \cos \lambda \frac{dz}{dt}$$

The sum of these two expressions constitutes the disturbing force X of Poisson, and is to be added to the second member of the first of equations (1), and these equations become¹

$$(3) \quad \begin{aligned} \frac{d^2x}{dt^2} + \frac{Nx}{l} &= 2n \sin \lambda \frac{dy}{dt} + 2n \cos \lambda \frac{dz}{dt} \\ \frac{d^2y}{dt^2} + \frac{Ny}{l} &= -2n \sin \lambda \frac{dx}{dt} \\ \frac{d^2z}{dt^2} + \frac{Nz}{l} &= g - 2n \cos \lambda \frac{dz}{dt} \end{aligned}$$

¹ There are really other disturbing forces (comparatively slight indeed) than the X Y and Z of Poisson (equation 2), as appears from the following considerations:—

Draw a line through the origin of co-ordinates parallel to the axis of the earth, and project the moving body on the plane of yz . The distance of the projection from the line will be $y \sin \lambda + z \cos \lambda$, the distance of the body from the plane of yz being x : hence there will be a centrifugal force relatively to this line, due to the earth's rotation, tending to increase the ordinates x y z by its components

$$\begin{aligned} n^2 x \\ n^2 \sin \lambda (y \sin \lambda + z \cos \lambda) \\ n^2 \cos \lambda (y \sin \lambda + z \cos \lambda) \end{aligned}$$

With these expressions added, respectively, to the second members of equations (3), they correspond to those found in Carmichael (Calcul. of Operations), who quotes from Galbraith and Houghton (Proc. R. Irish Acad., 1851). They express forces of the second order in minuteness, compared with those expressed by equations (2), and, insensible in their effects, are neglected in all discussions. They are noticed here only to recognize their existence and to show their origin.

Since $x^2 + y^2 + z^2 = l^2$, we have $x dx + y dy + z dz = 0$. Hence multiplying equations (3), respectively, by dx , dy , and dz , and adding, we have

$$\frac{dx d^2x + dy d^2y + dz d^2z}{dt^2} = g dz$$

$$(3)_2 \quad \frac{dx^2 + dy^2 + dz^2}{dt^2} = 2gz + c$$

This expression is independent of n , and the velocity at any point of the path depends, in the same way as does that of a pendulum vibrating over a motionless earth, upon the height of fall. The plane of vibration of the chronometer pendulum is maintained in a fixed relative position, thereby differing from a "freely suspended" pendulum. It will be seen hereafter, in treating of the gyroscope pendulum, that the forces which maintain this relative fixedness are equivalent to a force varying directly as the angular velocity, applied at the centre of gravity, normally to the path. Such a force will have no influence upon the velocity. Hence the time of vibration of the chronometer pendulum is not affected by the earth's rotation, nor by the azimuth angle of the plane of vibration.¹

Multiplying the first of equations (3) by y , and the second by x , and adding, we get:—

$$y \frac{d^2x}{dt^2} - x \frac{d^2y}{dt^2} = 2n \sin \lambda \left(y \frac{dy}{dt} + x \frac{dx}{dt} \right) + 2n \cos \lambda y \frac{dz}{dt}$$

Integrating:—

$$(4) \quad y \frac{dx}{dt} - x \frac{dy}{dt} = n \sin \lambda (x^2 + y^2) + C + 2n \cos \lambda \int y dz$$

The above (4) expresses that the moment of the quantity of motion about the axis of z is equal to a constant C (depending upon any arbitrarily given initial value) increased by what is due to the constant angular motion $n \sin \lambda$, and by the area $2 \int y dz$ (in the case of ordinary plane vibration this is the projection on

¹ This conclusion is not invalidated by the introduction of the disturbing forces of the order n^2 referred to in note to p. 17, for, since the arc of vibration of the chronometer pendulum is exceedingly small, z may be considered as equal to l , the pendulum's length, and y as very minute. Those forces will thence be

$$\begin{aligned} & n^2 x \\ & \frac{1}{2} n^2 l \sin 2\lambda \\ & n^2 l \cos^2 \lambda \end{aligned}$$

The third of these is an *increment to gravity*, and the first tends to prolong vibrations in the prime vertical. The second is null in its effects, since, being always positive, it retards the vibration in one direction as much as it accelerates it in the other. But they are all *inappreciably* minute, the last being, for the seconds pendulum, an increment to the force of gravity at the equator of about $\frac{1}{1,800,000,000}$, decreasing the time of vibration by about $\frac{1}{3,600,000,000}$. The first has the contrary tendency to increase the time of vibrations if made in a prime vertical (or any other plane than a meridian), but its effect is equally inappreciable even when (as in the prime vertical) it is a maximum.

the plane of yz of the circular segment included between the arc and chord of vibration), multiplied by $n \cos \lambda$. This multiplier, which is the component of the earth's rotation about a diameter of the earth normal to that passing through the locality, indicates that the term $2n \cos \lambda \int y dz$ expresses a disturbance produced by this complementary component. As this term for vibratory motion is small and periodic, passing through nearly equal positive and negative values in the course of a double vibration, it follows that $n \sin \lambda$ expresses the mean increment of angulation, or, in other words, that, to the plane or spherical vibrations exhibited by the pendulum over a motionless earth, there is, superadded, in consequence of this rotation, a uniform azimuthal motion measured by the earth's rotating velocity multiplied by the sine of the latitude. This is the material fact or peculiar feature of the freely suspended pendulum, and we see that it is exhibited by equation (4) generally for all ordinary vibrations, whether plane or spherical. We shall see hereafter, however, that the disturbing term of equation (4) $2n \cos \lambda \int y dz$ expresses a tendency to a like motion about the complementary axis, and that, on the supposition of an infinite velocity, this tendency may be realized, and the plane of motion, by the joint effect of the two components, turn around a parallel to the earth's axis, with an angular velocity equal and contrary in direction to n .

In the case of *very small* deviations from the vertical, the equations (3) may be solved as follows: The variations of z then become of the second order of minuteness compared with those of x and y , and omitting them we have between x and y and their differentials the relations

$$(4)_2 \quad \begin{aligned} \frac{d^2x}{dt^2} + \frac{Nx}{l} - 2n \sin \lambda \frac{dy}{dt} &= 0 \\ \frac{d^2y}{dt^2} + \frac{Ny}{l} + 2n \sin \lambda \frac{dx}{dt} &= 0 \end{aligned}$$

the integrals of which are (Gregory Examp. p. 390),

$$(5) \quad \begin{aligned} x &= +\Sigma (D \cos \beta t + E \sin \beta t) \\ y &= -\Sigma (D \sin \beta t - E \cos \beta t) \end{aligned}$$

in which to β is given both the values obtained from the quadratic equation

$$a_0 - a_2 \beta^2 = a_1 \beta^*$$

in which $a_0 = \frac{g}{l}$, $a_1 = -2n \sin \lambda$, $a_2 = 1$; hence solving the quadratic

$$\beta = -\frac{1}{2} \frac{a_1}{a_2} \pm \sqrt{a_0 \pm \frac{1}{4} \left(\frac{a_1}{a_2}\right)^2}$$

Substituting the values of a_0 , &c., and omitting the second term under the radical as inappreciably small, we have

* This equation, $\frac{g}{l} - \beta^2 = 2n \sin \lambda \beta$, can be got by substituting the integrals $x = C \cos (\beta t - \epsilon)$, $y = C \sin (\beta t - \epsilon)$ in the given equations.

$$\beta = n \sin \lambda \pm \sqrt{\frac{g}{l}}$$

Representing these two values by β_1 and β_2 , equations (5) may be put in the form (by writing for D_1 and E_1 , $C_1 \cos \varepsilon_1$ and $C_1 \sin \varepsilon_1$, &c., and reducing)

$$\begin{aligned} x &= C_1 \cos(\beta_1 t - \varepsilon_1) + C_2 \cos(\beta_2 t - \varepsilon_2) \\ y &= -C_1 \sin(\beta_1 t - \varepsilon_1) - C_2 \sin(\beta_2 t - \varepsilon_2) \end{aligned}$$

Assuming for $t=0$, $x=0$ and $\frac{dy}{dt}=0$, it will give $\varepsilon_1 = \varepsilon_2 = \frac{1}{2} \pi$, and the above become

$$\begin{aligned} x &= C_1 \sin \beta_1 t + C_2 \sin \beta_2 t \\ y &= C_1 \cos \beta_1 t + C_2 \cos \beta_2 t \end{aligned}$$

Instead of the arbitrary constants C_1 and C_2 we may write $\frac{1}{2}(A+B)$ and $\frac{1}{2}(A-B)$, at the same time substituting the values of β_1 and β_2 and developing; by which the preceding equations become (putting $n \sin \lambda = n'$)

$$\begin{aligned} (6) \quad x &= A \cos \sqrt{\frac{g}{l}} t \sin n' t + B \sin \sqrt{\frac{g}{l}} t \cos n' t \\ y &= A \cos \sqrt{\frac{g}{l}} t \cos n' t - B \sin \sqrt{\frac{g}{l}} t \sin n' t \end{aligned}$$

If we transfer the co-ordinates now referring to (relatively) *fixed* axes, to others *moving* with the relative angular velocity n' , that is, if we transfer to axes making at any instant the angle $n \sin \lambda t$ with the fixed ones, the new co-ordinates will have the values

$$\begin{aligned} x' &= x \cos n' t - y \sin n' t \\ y' &= x \sin n' t + y \cos n' t \end{aligned}$$

or, substituting values of x and y ,

$$\begin{aligned} x' &= B \sin \sqrt{\frac{g}{l}} t \\ y' &= A \cos \sqrt{\frac{g}{l}} t \end{aligned}$$

From which we may obtain

$$A^2 x'^2 + B^2 y'^2 = A^2 B^2$$

which is the equation of an ellipse, having A and B for semi-transverse and semi-conjugate axes. If $B=0$ the ellipse becomes a right line, hence the earth's rotation causes an azimuthal motion of this line, or of the axes of the ellipse if the motion is elliptical, equal to the component of that rotation about the local axis and in the reverse direction.

The motions of the "gyroscope pendulum," which is but the ordinary gyroscope with an exceedingly long arm (or distance γ , of my analysis, from the point of support in the axis to the centre of gravity), are indicated by equations precisely similar to the above, deduced from an identical analysis; always assuming, as in the solutions just given, that the arcs of vibration are small, so that *vertical* motions

may be disregarded. To prove this, I refer to my expression (*h*) and the context, in my analysis of the "Gyroscope."¹

$$(a) \quad -\frac{Cn}{\gamma M} v_s$$

for the deflecting force, as I call it (a force due to the rotation of the disk with angular velocity *n*, and acting, at the centre of gravity, normally to the plane of angular motion of the disk-axis, or of the arm of the gyroscope pendulum), in which *M* is the mass, *C* its moment of inertia about the disk-axis, γ the distance from its centre of gravity to point of suspension, and v_s the angular velocity.

Disregarding the vertical motions represented by $\frac{dz}{dt}$ on account of the smallness of the arcs, $\frac{1}{l} \frac{dx}{dt}$ and $\frac{1}{l} \frac{dy}{dt}$ (I substitute *l* for the γ mentioned above) would represent very nearly the components of angular velocity of the centre of gravity. Substituting these for v_s we shall get the components of the "deflecting force," and the equations of motion will be,

$$\begin{aligned} \frac{d^2x}{dt^2} + \frac{Nx}{l} &= -\frac{Cn}{l^2M} \frac{dy}{dt} \\ \frac{d^2y}{dt^2} + \frac{Ny}{l} &= +\frac{Cn}{l^2M} \frac{dx}{dt} \\ \frac{d^2z}{dt^2} + \frac{Nz}{l} &= +g \end{aligned}$$

These equations are identical in all but the value of the coefficients with (3), when transformed to (3)₂, under the same license. Of course the motions of the gyroscope pendulum would have the same solutions, the mean azimuthal motion of the nodes of its orbit being expressed by half the coefficient of $\frac{dy}{dt}$, $-\frac{Cn}{2l^2M}$, or, since the moment of inertia *A*, of the gyroscope, with reference to a principal axis through the point of support, is (*l* being supposed to be very large compared to the dimensions of the disk) very nearly l^2M , the mean azimuthal motion is more simply expressed by $\frac{Cn}{2A}$.

This may be more generally proved as follows: The first of the general differential equations (equations 4 of my analysis) of gyroscopic motion is

$$\sin^2 \theta \frac{d\psi}{dt} = \frac{Cn}{A} (\cos \theta - c)$$

in which θ , counted from the inferior vertical, is the variable inclination, and ψ the azimuth angle of the disk-axis or pendulum arm, and *c* a constant depending on initial values of θ and $\frac{d\psi}{dt}$.

¹ See American Journal of Science, 1857, and Barnard's American Journal of Education, 1857.

Develop $\cos \theta = -(1 - \sin^2 \theta)^{\frac{1}{2}}$ and we get

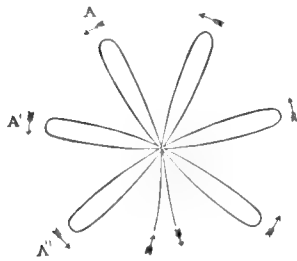
$$\frac{d\psi}{dt} = -\frac{Cn}{2A} \left(1 + 2\frac{1-c}{\sin^2 \theta} - \frac{1}{4} \sin^2 \theta - \frac{1}{8} \sin^4 \theta - \&c. \right)$$

For ordinary ranges of pendulum vibration the terms involving positive powers of $\sin \theta$ (which express an excess of nodal motion for large excursions) may be omitted, and we have

$$\frac{d\psi}{dt} = -\frac{Cn}{2A} + \frac{Cn}{A} \frac{1-c}{\sin^2 \theta}$$

Thus we see that for small vibrations, whether spherical or plane, the azimuthal motion is made up of a uniform progression of the nodes $\frac{Cn}{2A}$, and a fluctuating term which represents the angular velocity *in the orbit*. Indeed we have, in the second term, the motions of the spherical pendulum.

If we suppose the pendulum to have been propelled from a state of rest in the vertical, $\frac{d\psi}{dt}$ must have a finite value when θ is indefinitely small, and c must hence be unity. Hence we see that at the very outset the initial value $\frac{Cn}{2A}$ must be attributed to $\frac{d\psi}{dt}$, and that the pendulum reacquires it at every return excursion, that is, whenever θ diminishes indefinitely. Hence the pendulum continues to pass



through the vertical at every return. The horizontal projection of the curve would be a series of loops radiating from a common centre. For each complete vibration the integral of $\frac{Cn}{2A} dt$ would represent the entire angular motion of the nodal axis (much exaggerated in the diagram) from A to A' , &c., and the integral of the remaining terms should be 2π . These loops are in fact but the path a pencil attached to a common pendulum would trace upon a paper beneath, turning with uniform angular velocity about the projection of the point of suspension.

Though the numerator of the fraction $\frac{Cn(1-c)}{A \sin^2 \theta}$ becomes zero for the case just considered, it is evident that, at the moment of passing through the vertical, the limiting value must be considered infinity, and that the integral through the infinitely short time of passage must be π ; for the azimuthal position undergoes, at that instant, an increment (or decrement) of a semi-circumference. There is an identical case in the spherical pendulum. Regarding plane as the final limit of narrowing spherical vibrations, it is evident that the azimuthal velocity of passage by the vertical becomes very great and has its limit infinity when they pass *through* the vertical.

This expression $\frac{Cn}{2A}$ (equal to $\beta \sqrt{\frac{g}{\lambda}}$ nearly) is a very different thing from the "mean precession" of the gyroscope, $\frac{1}{2\beta} \sqrt{\frac{g}{\lambda}}$, given in my analysis. The latter is

the mean azimuthal motion of the body itself, the former that of the nodes of its orbit. The latter is strictly true only for very great values of β ; the former, rightly interpreted, is always true, though it has no special applicability except for (as in the gyroscope pendulum) small values of β . The harmony of the two expressions is easily shown.

Practically the gyroscope is made up of not only a rotating disk, but a non-rotating frame. In estimating the "deflecting force," therefore, in the expression (α) C should apply to the disk alone, and M and γ to the entire mass of disk frame and stem.

The solutions that have been given of equations (4)₂ for the freely suspended pendulum are restricted to very small motions; the following is general.

Transfer equations (4) to polar co-ordinates by substituting for x, y, z the values¹

$$\begin{aligned} x &= l \sin \phi \sin \theta \\ y &= l \cos \phi \sin \theta \\ z &= l \cos \theta \end{aligned}$$

in which ϕ denotes the azimuth of the pendulum measured from the north, and θ its deviation from the vertical, and we get

$$(7) \quad \frac{d\phi}{dt} = n \sin \lambda + \frac{C}{l^2 \sin^2 \theta} - \frac{2n \cos \lambda}{\sin^2 \theta} \int \cos \phi \sin^2 \theta \, d\theta$$

in which C is a constant depending on arbitrary initial values of $\frac{d\phi}{dt}$, the final term corresponding to the last term of (4).

At the equator we have $\lambda=0$, and the azimuthal velocity expressed by the third term of (7) becomes $-\frac{2n}{\sin^2 \theta} \int \cos \phi \sin^2 \theta \, d\theta$, which being periodic, produces but very minute change in the plane of vibration. If the pendulum is propelled, from a state of rest in the vertical, in the direction measured by the angle ϕ from the meridian, this angle will be but very slightly affected by the minute values of the above expression during the *outward* excursion, and the increment which $\frac{d\phi}{dt}$ receives will be *almost* exactly neutralized (*quite* so if $\cos \phi$ were absolutely invariable) during the return, and the angular velocity due to the term will again become zero; which cannot happen unless the pendulum *again* pass through the vertical on its return, in which case ϕ will be as little varied during the return; (*otherwise* ϕ will, during the return, pass through all possible values from 0 to $\frac{1}{2}\pi$, and integration is impracticable). Hence we may assume ϕ as constant, and, as in any other latitude, the term in question is, multiplied by $\cos \lambda$, the same as at the equator, we may generally integrate that term *for plane vibrations*, considering ϕ constant, and putting $C=0$. Equation 7 thus becomes,

$$(8) \quad \frac{d\phi}{dt} = n \sin \lambda - n \cos \lambda \frac{\theta - \sin \theta \cos \theta}{\sin^2 \theta}$$

¹ The following analysis, as far as equation (9), is modified from Galbraith and Houghton. Proc. R. I. Acad.

If the amplitudes of the vibration are very minute, so that θ may be substituted for $\sin \theta$ in (7), the integral becomes

$$(9) \quad \frac{d\phi}{dt} = n \sin \lambda - \frac{2}{3} n \cos \lambda \cos \phi \theta$$

expressing the precession in azimuth, $n \sin \lambda$, precisely as it results from the former analysis (equation 6). The slight periodic disturbance expressed by the second term of (9) escaped that analysis, however, owing to the omission of the terms involving dz .

In the above integrals the angle ϕ must be taken at 180° greater or less on one side of the vertical than on the other, and the parts of $\frac{d\phi}{dt}$ expressed by it will have contrary signs. Hence the curve described in each complete excursion, disregarding the superadded uniform azimuthal motion expressed by the first term of the second member of (9), will have the form of an excessively attenuated lemniscate, or figure of 8.¹⁾

For greater amplitudes equations (8) will apply until θ becomes nearly equal to 180° ; if θ equals or exceeds 180° , it *cannot be assumed* that the pendulum will pass through the zenith (the condition for ϕ to remain nearly constant), and the integral becomes inapplicable and erroneous.

The foregoing integrals involve the condition that the pendulum shall pass through the vertical, and imply that vibration is induced by propulsion from a state of rest in the vertical. But, in the usual form of the experiment for exhibiting the rotation of the earth, the pendulum starts from a state of *relative* rest at the extremity of the initial vibratory arc.

If we disregard the symbolic integral of (7), as may be done, since the minute periodic disturbance it measures has no influence upon the permanent azimuthal motion, that equation will become

$$(a) \quad \frac{d\phi}{dt} = n \sin \lambda + \frac{C}{l^2 \sin^2 \theta}$$

The second term of the second member is identically the equation of the "spherical pendulum." The latter, we know, exhibits an azimuthal motion of the apsides of its orbit, very minute when C is small, but incomparably greater than the horary azimuthal motion when, C being large, the conjugate dimensions of the orbit approaches equality to the transverse, and of which the limit corresponding to perfect equality of these dimensions is for one vibration,

$$(a)_2 \quad \pi \left[\frac{2}{\sqrt{1+3 \cos^2 \theta_2}} - 1 \right]$$

as may be deduced from the expression for U_π , par. 731, Peirce, *Analyt. Mech.*, or from expression [79] and [83] of *Méc. Cél.* (Bowditch), by making $a=b$ and determining the corresponding values of c and dt .

In the case under consideration the value of C will be determined by making $\frac{d\phi}{dt} = 0$ for the commencement of motion, $\theta = \theta_2$, hence

$$C = -n l^2 \sin \lambda \sin^2 \theta_2$$

¹⁾ See Additional Notes, p. 51.

The pendulum will not move in a plane passing through the vertical, but on a conical surface differing slightly from such a plane, and there will ensue a slight apsidal motion reverse to, and diminishing, the apparent horary motion.

(b) The equation $\frac{d\phi}{dt} = \frac{C}{l^2 \sin^2 \theta} = \frac{C}{l^2 - z^2}$ is usually solved by the aid of elliptic integrals (*vide* Prof. Peirce's *Analyt. Mech.*, p. 418); but for present objects the ordinary processes of integration are preferable.

From equations (3)₂ and (4) may easily be deduced, neglecting terms containing n , and bearing in mind that

$$x^2 + y^2 = l^2 - z^2, \quad x \, dx + y \, dy = -z \, dz,$$

$$(c) \quad dt = \frac{-l \, dz}{\mp \sqrt{(l^2 - z^2)(c + 2gz) - C^2}}$$

in which the upper or lower sign of the radical is to be taken according as dz is positive or negative, that is, as the pendulum is descending or ascending. The quantity under the radical may be put in the form (*vide* *Mec. Celeste*, Bowditch, Vol. I. p. 52).

$$2g(a-z)(z-b) \left(z + \frac{f}{2g} \right)$$

in which

$$\frac{f}{2g} = \frac{l^2 + a \, b}{a + b}$$

$$(d) \quad C^2 = \frac{2g(l^2 - a^2)(l^2 - b^2)}{a + b}$$

$$c = 2g \frac{(l^2 - a^2 - ab - b^2)}{a + b}$$

a and b being the greatest and least values of z .

If now we transfer the origin of co-ordinates to the lowest point of the spherical surface by substituting for z , a , and b , $l-u$, $l-a$, $l-\beta$, and replace C and dt in (b) by the values above found, we shall have (putting $l + \frac{f}{2g} = \rho$)

$$d\phi = \frac{1}{2} \sqrt{\frac{(l^2 - a^2)(l^2 - b^2)}{a + b}} \frac{l \, du}{u(2l-u)[(\rho-u)(u-a)(\beta-u)]^{\frac{1}{2}}}$$

the varying sign of the radical being understood. If we develop the two factors $(2l-u)^{-1}$ and $(\rho-u)^{-\frac{1}{2}}$, and multiply the results, we shall have

$$d\phi = \sqrt{\frac{(l^2 - a^2)(l^2 - b^2)}{a + b}} \int \frac{l \, du}{u[-\alpha\beta + (\alpha + \beta)u - u^2]^{\frac{1}{2}}} \left[\frac{1}{2l\rho^3} - \frac{1}{4l\rho^3} \left(\frac{1}{l} + \frac{1}{\rho} \right) u \right. \\ \left. + \frac{1}{8l\rho^3} \left(\frac{1}{l^2} + \frac{1}{l\rho} + \frac{3}{2\rho^2} \right) u^2 + \frac{1}{16l\rho^3} \left(\frac{1}{l^3} + \frac{1}{l^2\rho} + \frac{3}{2l\rho^2} + \frac{15}{6\rho^3} \right) u^3 + \&c. \right]$$

Strike out the common factor l , and remove the factor $2\rho^{\frac{1}{2}}$ into the denominator of the first radical factor (which factor then becomes $\sqrt{(l-a)(l-b)} = \sqrt{\alpha\beta}$), and the above integral becomes (*vide* *Hirsch*, *Integral Tables*, pp. 160-164), writing U for $-\alpha\beta + (\alpha + \beta)u - u^2$,

$$\begin{aligned}
 \phi = & \frac{1}{2} \cos^{-1} \left(\sqrt{\frac{\alpha\beta}{(\alpha+\beta)^2 - 4\alpha\beta}} \cdot \frac{\sqrt{U}}{u} \right) + \frac{1}{4} \sqrt{\alpha\beta} \left(\frac{1}{l} + \frac{1}{\rho} \right) \cos^{-1} \frac{2\sqrt{U}}{\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}} \\
 & + \frac{1}{8} \sqrt{\alpha\beta} \left(\frac{1}{l^2} + \frac{1}{l\rho} + \frac{3}{2\rho^2} \right) \left[-\sqrt{U} + \frac{1}{2}(\alpha+\beta) \cos^{-1} \frac{2\sqrt{U}}{\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}} \right] \\
 (c) \quad & + \frac{1}{16} \sqrt{\alpha\beta} \left(\frac{1}{l^3} + \frac{1}{l^2\rho} + \frac{3}{2l\rho^2} + \frac{15}{16\rho^3} \right) \left[-\left(-\frac{1}{2}u + \frac{3}{4}(\alpha+\beta) \right) \sqrt{U} + \right. \\
 & \left. \left(\frac{3}{8}(\alpha+\beta)^2 - \frac{1}{2}\alpha\beta \right) \cos^{-1} \frac{2\sqrt{U}}{\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}} \right] + \&c., + \text{cons't.}
 \end{aligned}$$

In order that the arcs in the above expression should continually increase with the time, the positive or negative sign must be applied to the radical \sqrt{U} according as u is increasing or diminishing: taken from $u=\alpha$ to $u=\beta$ (or the converse), the arcs all become $=\pi$, and the non-circular functions vanish.

Hence the azimuthal angle passed over by the pendulum in its motion from a lowest to a highest point of its orbit (or the converse) is expressed by

$$\begin{aligned}
 \phi_{\frac{1}{2}\pi} = & \frac{1}{2}\pi \left[1 + \frac{1}{2} \sqrt{\alpha\beta} \left(\frac{1}{l} + \frac{1}{\rho} \right) + \frac{1}{4} \sqrt{\alpha\beta} \left(\frac{1}{l^2} + \frac{1}{l\rho} + \frac{3}{2\rho^2} \right) \frac{\alpha+\beta}{2} \right. \\
 (f) \quad & + \frac{1}{8} \sqrt{\alpha\beta} \left(\frac{1}{l^3} + \frac{1}{l^2\rho} + \frac{3}{2l\rho^2} + \frac{15}{6\rho^3} \right) \left(\frac{3(\alpha+\beta)^2}{8} - \frac{1}{2}\alpha\beta \right) \\
 & \left. + \frac{1}{16} \sqrt{\alpha\beta} \left(\frac{1}{l^4} + \frac{1}{l^3\rho} + \frac{3}{2l^2\rho^2} + \frac{15}{6l\rho^3} + \frac{105}{24\rho^4} \right) \left(\frac{5(\alpha+\beta)^3}{16} - \frac{3\alpha\beta(\alpha+\beta)}{4} \right) + \&c. \right]
 \end{aligned}$$

The sum of the terms after unity included in the brackets is the ratio by which the azimuth angle exceeds a quadrant; or, if the integral is taken through an entire revolution (relatively to the *apsides*), it, multiplied by 2π , is angle of advance of the apsidal per revolution.

For motion nearly oscillatory, of whatever amplitude (*i. e.*, α being small and β arbitrarily large), or for spherical motions of considerable amplitude (α and β taken within limits not exceeding say one-third of l , corresponding to a swing of over 90°), ρ , always greater than $2l$, differs but slightly from that magnitude. Giving ρ that value, and taking the angle ϕ for a complete vibration, or a semi-apsidal revolution, we have the formula,

$$(g) \quad \phi_\pi = \pi \left[1 + \frac{3}{2} \frac{\sqrt{\alpha\beta}}{2l} + \frac{1.3.5}{1.2.4} \frac{\sqrt{\alpha\beta} \cdot \frac{1}{2}(\alpha+\beta)}{(2l)^2} + \&c. \right]$$

If α and β are both small and nearly equal, and θ the angle of which they are the versed sine, then $\frac{\sqrt{\alpha\beta}}{l} = \frac{1}{2} \sin^2 \theta$, and the apsidal motion corresponding to the second term of the above (the following terms neglected) becomes $\frac{3}{8} \pi \sin^2 \theta$; agreeing with the expression $(a)_2$ on p. 24, when developed for the same case.

If the pendulum moves *nearly* horizontally in a great circle, that is, if $\alpha+\beta=2l$ and $\alpha\beta=l^2$ (nearly), then ρ , l' , and ρ' are each infinitely great, and (f) becomes

$$\phi_{\frac{1}{2}\pi} = \frac{1}{2}\pi \left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{16} + \frac{1}{32} + \&c. \right] = \pi$$

which denotes that the line of apsides moves through a quadrant while the pendulum is passing, through 180° azimuth, from a highest to a lowest point; in other words, that the highest and lowest points are diametrically opposite, and the apsides are *apparently* stationary. This theorem is true (as shown by Prof. Peirce), whatever be the inclination of the great circle, though it cannot be *generally* made evident by the above formulæ.

If in (c) we make transformations and substitutions already described, develop the factor $(\rho-u)^{-1}$, integrate between the limits $u=\alpha$, $u=\beta$, and double the result, we shall have for the time of one vibration, or one semi-orbital revolution,

$$(h) \quad T=2\pi \frac{l}{\sqrt{2g\rho}} \left[1 + \left(\frac{1}{2}\right)^2 \frac{\alpha+\beta}{\rho} + \left(\frac{1.3}{2.4}\right)^2 \left(\frac{\alpha+\beta}{\rho^2}\right)^2 - \frac{1.3}{2.4} \frac{\alpha\beta}{2\rho^2} + \left(\frac{1.3.5}{2.4.6}\right)^2 \frac{(\alpha+\beta)^3}{\rho^3} - \frac{1.3.5}{2.4.6} \frac{3\alpha\beta(\alpha+\beta)}{4\rho^3} + \&c. \right]$$

and if $\alpha=0$, that is, if the motion is purely oscillatory, then $\rho=2l$, and this becomes

$$(i) \quad T=\pi \sqrt{\frac{l}{g}} \left[1 + \left(\frac{1}{2}\right)^2 \frac{\beta}{2l} + \left(\frac{1.3}{2.4}\right)^2 \left(\frac{\beta}{2l}\right)^2 + \&c. \right]$$

and more generally for any value of α and β not exceeding the versed sine of thirty or forty degrees:

$$(j) \quad T=\pi \sqrt{\frac{l}{g}} \left[1 + \left(\frac{1}{2}\right)^2 \frac{\alpha+\beta}{2l} - \frac{1.3}{2.4} \frac{\alpha\beta}{(2l)^2} + \left(\frac{1.3}{2.4}\right)^2 \left(\frac{\alpha+\beta}{2l}\right)^2 + \&c. \right]$$

When α and β are both small, as in most pendulum experiments, the terms after the first in the brackets may be omitted, and we have the ordinary expression for the time $\pi \sqrt{\frac{l}{g}}$.

In the expression (j), omitting all the terms in brackets after the second, it is evident that that term will measure the apsidal motion for the time $\pi \sqrt{\frac{l}{g}}$; and hence that the total integral of equation (a) taken through that time will be,

$$\pi \left[n \sin \lambda \sqrt{\frac{l}{g} \pm \frac{3\sqrt{\alpha\beta}}{4l}} \right]$$

and if we denote by ϕ' the angle of azimuth of the apsidal line measured from its initial direction, we shall have, at any time t , substituting for α and β , $l(1-\cos \theta_1)$ and $l(1-\cos \theta_2)$,

$$(k) \quad \phi' = \left[n \sin \lambda \pm \frac{3}{4} \sqrt{\frac{g}{l}} \sqrt{(1-\cos \theta_1)(1-\cos \theta_2)} \right] t$$

or, since θ_1 is always small,

$$(l) \quad \phi' = \left[n \sin \lambda \pm \frac{3}{4} \sqrt{\frac{g(1-\cos \theta_2)}{2l}} \sin \theta_1 \right] t$$

The angle θ_2 being given, θ_1 is determined for the *ordinary* pendulum experiment by the consideration that the constant C , of which the value is found p. 24, is the moment of the quantity of motion. As c is very minute, the actual velocity at the

lowest point will be, sensibly, $\sqrt{2gl(1-\cos\theta_2)}$, and hence the moment at that point will be $l \sin\theta_1 \sqrt{2gl(1-\cos\theta_2)} = C = -n^2 \sin\lambda \sin^2\theta_2$, from which deducing the value of $\sin\theta_1$ and substituting in (*k'*), we have

$$(l) \quad \phi' = n \sin\lambda \left(1 - \frac{3}{8} \sin^2\theta_2\right) t$$

The second term in brackets is the retardation from the true horary motion; it being implied, of course, that the amplitude of oscillation is preserved unimpaired. Though small, this retardation would be sensible, especially if θ_2 had a considerable magnitude, say eight or ten degrees, though inappreciable for very small oscillations.

Practically, the resistance of the air constantly diminishes the value of θ_2 , and failure to procure a perfect state of rest to the pendulum before it is set free, or currents of air, may give quite different values to C and c , and determine the character of the orbital motion to be progressive instead of retrograde; and it is generally observed that the conjugate dimension of the orbit increases (probably owing to the resistance of the air) as θ_2 diminishes. In this way the apsidal motion due to the orbit may acquire a value quite considerable compared to the proper horary motion, which will apparently be sensibly retarded or accelerated. By observing, at any period of the experiment, the value of θ_1 and θ_2 and the direction of the orbital motion, the coefficient of t in the formula (*k'*) will give the theoretical rate of azimuthal motion at that instant.

If the orbital motion is retrograde and

$$(m) \quad \sin\theta_1 = \frac{4}{3} n \sin\lambda \sqrt{\frac{2l}{g(1-\cos\theta_2)}}$$

the line of the apsides would be stationary. For Columbia College, where $n \sin\lambda = 00004747$, with a pendulum of 26 feet in length, and a value of θ_2 of 6° , this would give $\theta_1 = 3'3$, the actual semi-axes of the projection of the orbit being about two feet nine inches and one-third of an inch.

It would generally be sufficiently accurate to substitute for $1 - \cos\theta_2$, $\frac{1}{2} \sin^2\theta_2$, by which formula (*k'*) would become more simply,

$$(n) \quad \phi' = \left[n \sin\lambda \pm \frac{3}{8} \sqrt{\frac{g}{l}} \sin\theta_1 \sin\theta_2 \right] t$$

in which it is seen that the deviation from the proper horary motion is proportional to the area of the projection of the orbit.

The above, or (*k'*), expresses the azimuthal motion as it would be were there no other forces acting than those included in the investigation, in which case θ_1 and θ_2 would be invariable. In point of fact there are practically numerous disturbing forces, of which, however, the resistance of the air is the most considerable, and through which θ_1 and θ_2 are incessantly changing, and it is not improbable that the *change of shape* of the orbit may, in itself, cause some variation of direction of the line of apsides:¹ a matter which cannot be decided until the problem of the spherical pendulum is solved with the resistance taken into account.

¹ An investigation of the simpler case of the plane elliptical motion of a body attracted by a central force proportional to the distance, in a medium which resists either directly or as the square

To get a clearer idea of what is expressed by the periodic term of (7), $-2n \cos \lambda \int \cos \phi \sin^2 \theta d\theta$ (which corresponds to the integral $\int y dz$ of (4)), we must revert to the latter equation. Conceive the pendulum propelled from a state of rest in the vertical, with a very great angular velocity, denoted by v , in the plane of the meridian. Were the earth motionless, it would continue to whirl in this plane, passing through the zenith at every revolution. Introduce the element of the earth's rotation, and the two terms of equation (4) containing n take effect, by the first of which the plane of revolution moves in azimuth with the angular velocity $n \sin \lambda$. The second expressing that there will be an increase of the moment of the quantity of motion about the vertical after a time T proportionate to the area generated in that time $\int y dz$. Under these conditions this area is cumulative, and at the end of one revolution expresses the area of the circle of radius l . Let us suppose that the plane of motion turns about a line parallel to the complementary terrestrial axis with an angular velocity $n \cos \lambda$. At the end of the time T (supposed very small) the plane will make with the meridian the angle $n \cos \lambda T$, and as the quantity of motion in its own plane is vl , its moment referred to a vertical axis will, from zero, have become $vl^2 \sin (n \cos \lambda T)$, or, substituting the small arc for its sine,

$$v l^2 n \cos \lambda T$$

But T , for one revolution, is expressed by $\frac{2\pi}{v}$ hence the above becomes

$$2n \cos \lambda \pi l^2$$

The area of the circle which is generated in the same time is πl^2 and is expressed by the integral $\int y dz$, and it is easy to show for each successive revolution that the area $\int y dz$ multiplied by $2n \cos \lambda$ corresponds to an increment of the moment of quantity of motion about the vertical which it would receive from a turning of the plane about the complementary axis through the angle $n \cos \lambda t$.

Hence, for the particular case under consideration, the second term of second member of equation 4 expresses an angular motion about the complementary axis of which $n \cos \lambda$ is the velocity. The resultant of this, and the azimuthal component, is rotation about an axis parallel to that of the earth, and opposite in direction to the earth's rotation.

The above theorem can be analytically demonstrated. The quantity N , expressive of the tension of the cord, is made up of the centrifugal force due to the pendulum's relative angular motion and of the variable component of the force of gravity (neglecting, as we have done, quantities of the order n^2). If this centrifugal force is so great that the component of gravity may be neglected, $\frac{N}{l}$ will

of the velocity, indicates no apsidal motion accompanying the decrease of parameters of the orbit. Neither, however, does it indicate the enlargement of the minor axis (initially very small) so universally observed in the pendulum experiments.

reduce to the constant v^2 (v being the impressed angular velocity of pendulum movement), and the second and third of equations (3) will yield the equation

$$\frac{d^2}{dt^2}(y \cos \lambda - z \sin \lambda) = -v^2 (y \cos \lambda - z \sin \lambda) - g \sin \lambda$$

the integral of which is

$$y \cos \lambda - z \sin \lambda = -\frac{g \sin \lambda}{v^2} + A \cos vt + B \sin vt$$

The above is independent of n ; $y \cos \lambda - z \sin \lambda$ is the value of the new co-ordinate of y' when the axis of y is changed to parallelism to that of the earth. Hence, as thus transformed, this co-ordinate is unaffected by the earth's rotation, the plane of pendulum motion must turn (if it turns at all) about such an axis.

The assumption for $t=0$, of $y=0$, $z=l$, $\frac{dy}{dt}=lv$, $\frac{dz}{dt}=0$, gives

$$\begin{cases} A = -\left(l - \frac{g}{v^2}\right) \sin \lambda \\ B = l \cos \lambda \end{cases}$$

$$(10) \quad \text{hence} \quad y \cos \lambda - z \sin \lambda = -\frac{g}{v^2} \sin \lambda (1 - \cos vt) + l \sin (vt - \lambda)$$

The second term of the second member of the above gives precisely the value which the first member would have, were the plane of pendulum motion stationary or were it turning with *any* angular velocity about an axis parallel to the earth's. The first term is, owing to the assumed high value of v , very minute, and is *periodic*, the period being equal to $\frac{2\pi}{v}$, the time of the pendulum's revolution in its circular orbit. Owing to its minuteness and periodicity, this term may be neglected, and equation (10) becomes

$$y \cos \lambda - z \sin \lambda = l \sin (vt - \lambda)$$

to satisfy which, and at the same time the three differential equations (3) (omitting g , as we have found reason to do in (10), and also omitting terms containing n^2 , in the developments), requires the following values for the co-ordinates:—

$$\begin{aligned} x &= l \sin nt \cos (vt - \lambda) \\ y &= l [\cos \lambda \sin (vt - \lambda) + \sin \lambda \cos nt \cos (vt - \lambda)] \\ z &= l [-\sin \lambda \sin (vt - \lambda) + \cos \lambda \cos nt \cos (vt - \lambda)] \end{aligned}$$

Changing the axes of y and z by turning them through the angle λ , we should have for new co-ordinates,

$$\begin{aligned} x' &= l \sin nt \cos (vt - \lambda) \\ y' &= l \sin (vt - \lambda) \\ z' &= l \cos nt \cos (vt - \lambda) \end{aligned}$$

If we now change the plane of yz by moving it through the variable angle nt about the axis of y' , we get,

$$\begin{aligned} x'' &= 0 \\ y'' &= l \sin (vt - \lambda) \\ z'' &= l \cos (vt - \lambda) \\ y''^2 + z''^2 &= l^2 \end{aligned}$$

These values show that the plane of pendulum revolution turns about an axis parallel to the earth's with the relative angular velocity n ; or, in other words, that the plane preserves its parallelism to itself in space.

If we had a *succession of pendulums* rigidly connected with each other, the disturbing effect of gravity would be eliminated. Such a succession would be simply a *gyroscope*, and the gyroscope, mounted in gimbals and set running in a meridian plane, would exhibit the apparent rotation of its disk around an axis parallel to the earth's axis equal and contrary to that of the earth; which is simply saying that as the earth revolves the plane of the disk maintains its parallelism to itself, and if we suppose its axis directed at a star in the plane of the equator, it would follow that star so long as the rotation of the disk is sustained.¹

The pendulum experiment, in its ordinary form, exhibits not the whole rotation of the earth, but only one component of it; the component which belongs to an axis passing through the locality. It is perhaps quite as interesting and important, as being the only experimental demonstration we can have of a principle difficult of comprehension, but as fundamental to mechanics, since its enunciation by Euler, as the corresponding one of the decomposition of linear velocities, viz., that of the decomposition into distinct components, of rotary velocities. The plane of the pendulum appears to turn relatively to the surface of the earth simply because the earth turns just so much underneath it, the earth really *revolving* about the local axis with a certain calculable component of velocity. The earth turns at the same time with another component of velocity about another axis (the complementary one), and the joint effect, or the resultant of the two components, is the rotation about the polar axis. The second component, very great as we approach the equator, where the first vanishes entirely, is not exhibited by the pendulum, and is only detected by analysis as a slight disturbance. Convert the pendulum into the gyroscope, however, and this second component appears equally with the first.

¹ Owing to the friction of the gimbals, there would be, *practically*, besides the motion above described, a motion of the axis in the plane of the meridian, north or south, according to the direction of the disk rotation; this angular motion might be greater or less than the equatorial motion, but would be, with a well-constructed apparatus, independent of it, at least for the brief time during which a gyroscope experiment would last.

INTERNAL STRUCTURE OF THE EARTH CONSIDERED AS AFFECTING
THE PHENOMENA OF PRECESSION AND NUTATION.

THE equations of precession and nutation are, as is well known, entirely independent of any particular law of density, and are functions only of the absolute values of the moments of inertia about the equatorial and polar axes A and C ,¹ and are independent indeed of the figure of the earth, except so far as it affects the values of these moments.

Moreover, if the earth, instead of being solid throughout, is (as supposed by most geologists) a solid shell inclosing a fluid nucleus, it is only necessary (leaving out of consideration the pressure that may be exerted on the interior surface by the fluid) that the shell should have these moments of inertia. Mr. Poinsot² has obtained as the results of calculation for a homogeneous spheroid, values of precession and nutation identical with those of observation, by taking the ellipticity at $\frac{1}{308.65}$ and η (the ratio of mass of moon to that of the earth), at $\frac{1}{88}$.

We have, assuming a uniform density, indicating by a and b the equatorial and axial radii, and by e the ellipticity:—

$$\begin{aligned}
 C &= \frac{8}{15} \pi a^4 b = (\text{approx.}) \frac{8}{15} \pi b^5 (1 + 4e) \\
 A &= \frac{4}{15} \pi (a^4 b + a^2 b^3) = \frac{8}{15} \pi b^5 (1 + 3e) \\
 (46) \quad \frac{C-A}{C} &= \frac{e}{1+4e} = e - 4e^2
 \end{aligned}$$

If e is taken at $\frac{1}{308.65}$ then $\frac{C-A}{C} = \frac{1}{312.7}$. But all meridian measurements of

¹ I assume, of course, the equality of all moments of inertia, A , about the equatorial axes, and overlook all questions as to the non-symmetry of the earth with respect to its axis of figure or to the equator; for, in fact, neither the rotation of the earth nor any observable celestial phenomena reveal it.

² *Connaissance des temps*, 1853.

the earth indicate an ellipticity greater rather than less than $\frac{1}{300}$;* and the latest determination makes it $\frac{1}{295}$ † by giving which value to e we obtain $\frac{C-A}{C} = \frac{1}{299}$.

Therefore the observed precession is to that which would result in a homogeneous spheroid, from the formulas, with the latest determined value of e introduced, as 299 : 311.7, provided the relative mass of the moon be but $\frac{1}{88}$ ‡.

The value of $\frac{C-A}{C}$ would be the same for a homogeneous *shell* of which the interior surface had the *same ellipticity*, e , as the exterior; or it would be the same for a shell of which all the elementary strata had the same ellipticity, in which the density, constant through each stratum, should vary according to any law, from stratum to stratum. The ratio of 299 : 312.7, so nearly unity ($= 0.96 = \frac{24}{25}$ nearly), while the ratio of mean to surface density of the earth is so high, indicates nearly uniform ellipticity of stratification, and hence fluidity of *origin*; while, on the other hand, the considerable inequalities in the equatorial axes indicated in the note below are incompatible with the hypothesis of *actual* fluidity beneath a *thin* crust, and are, to the measure of their probability, a disproof of it.

The effect upon the axial movements of such a shell which would result from the pressures of an internal fluid has been made the subject of an elegant mathematical investigation by W. Hopkins, F.R.S., in the Philosophical Transactions of 1839-40-42. On the supposition of a uniform density of shell and fluid, and the same ellipticity for inner and outer surfaces of the shell, the precession will be the same

* Airy, "Figure of the Earth," Encyc. Metrop.; Guillemin: Mädler, Am. Journ. of Science, Vol. 30, 1860, makes the polar compression of greatest meridian $\frac{1}{292.109}$
of smallest meridian $\frac{1}{302.004}$

(Article translated by C. A. Schott, U.S. Coast Survey, from Prof. Heis' "Astronomie, Météorologie et Géographie," Nos. 51, 52. 1859.)

† Appendix "Figure of the Earth" to the "Comparisons of Standards of Length," published 1866 by the British Ordnance Survey, gives for a "spheroid of revolution," $\frac{a-c}{c} = \frac{1}{295}$; for a spheroid of three axes, $\frac{a-c}{c} = \frac{1}{285.97}$, $\frac{b-c}{c} = \frac{1}{313.38}$, $\frac{a-b}{c} = \frac{1}{3269.5}$. The probabilities of the latter supposition to the former being 154 : 138.

‡ There is yet great uncertainty as to the relative mass of the moon, and as long as that point is unsettled, so is also the ratio of observed to calculated precession. Laplace, from observations of the tides at Brest, fixed it at $\frac{1}{75}$, which number is adopted by Pontécoulant. Former determinations from the observed nutations make it $\frac{1}{80.75}$; but $\frac{1}{87}$ was the determination from the coefficient of nutation of Lindenau. Guillemin gives $\frac{1}{88}$, and these two last numbers coincide nearly with that used by Poinsot. A discussion by Mr. Wm. Ferrell, member of National Academy of Sciences, of tidal observations made for a series of years at the port of Boston, as well as those at Brest, gives results confirmatory of the larger ratio of Laplace. Serret (Annales de l'Observatoire Imp. 1859) assumes $\frac{1}{83}$ and deduces $\frac{C-A}{C} = \frac{1}{306} = 0.00327$. These ratios are adopted by Thomson and Tait, §§ 803, 828. Archdeacon Pratt ("Figure of the Earth," 4th ed. 1871) adheres to Laplace's determination.

and the nutation essentially the same as for a homogeneous spheroid; but for the actual case of a heterogeneous fluid contained in a heterogeneous shell he finds that the ellipticity of the inner surface of the shell must be less than that of the exterior in the proportion of the *observed* precession to the precession of a homogeneous spheroid of same external ellipticity, a proportion which he assumes to be $\frac{7}{8}$, in accordance to the then received numbers for ellipticity, and for the mass of the moon.

The fulfilment of the condition, in the actual constitution of the earth, is improbable; for the isothermal surfaces are in all probability (and indeed by his own mathematical conclusions) of progressively *greater* ellipticity from the surface inwards. He finds, however, the requisite decrease of ellipticity in the fluid surfaces of *equal density*, assuming the well-known hypothetical law of density of Laplace $\rho = A \frac{\sin qb}{b}$; but we know not the influence which pressure has upon solidification, and it seems probable that the interior surface of the shell would conform nearly to the surfaces of equal temperature. The demand which he makes for 800 or 1000 miles thickness of shell is therefore a minimum (for the data used), while the more *probable* result is a much greater thickness or even entire solidity. But however elegant may be Mr. Hopkins' analysis, the basis of the structure is but slender, and those results have not been generally accepted as fully decisive of the question.

Sir Wm. Thomson, F.R.S., brings forward (Philosophical Transactions, 1863; also Thomson's and Tait's Treatise on Natural Philosophy, 1867) arguments against the popular theory of a thin crust, which are more forcible. A thin crust would itself undergo tidal distortion, and the height of the *apparent* tides of the ocean be thereby much reduced, while the *actual* precession would be diminished in the same ratio; that is, the differential forces of the sun and moon would expend themselves in producing these solid tides instead of producing precession.

From a theoretical investigation (given in a separate paper in the same volume)¹ of the deformation experienced by a homogeneous elastic spheroid under the influence of any external attracting force, he arrives at the result that, if the earth had no greater rigidity than steel or iron, it would yield about $\frac{2}{3}$ as much to tide-producing influences as if it had no rigidity—more than $\frac{3}{4}$ as much if its rigidity did not exceed that of glass. Moreover, the apparent ocean tides (or difference of high and low water level) would be (if H is the measure for a perfectly rigid earth) $0.59 H$, if the earth had the rigidity of iron or steel only; $\frac{1}{4\frac{1}{2}} H$ if it had that of glass.

As to *precession*, the centrifugal force of the crowns of the tidal elongation would balance $\frac{7}{9}$ of the dynamic couple resulting from the sun's or moon's attraction if the earth had only the rigidity of glass, and $\frac{2}{3}$ if it had only that of steel.

“That the effective tidal rigidity, and what we call the precessional effective

¹ See also “Treatise on Natural Philos.,” § 852 *et seq.*

rigidity of the earth, may be several times as much as that of iron (which would make the phenomena, both of the tides and precession, sensibly the same as if the earth were perfectly rigid), it is enough that the actual rigidity should be several times as great as the actual rigidity of iron throughout 2000 or more miles thickness of crust."

A theorem fundamental to the establishment of the above propositions is, that a revolving spheroid destitute of rigidity, a homogeneous fluid one, for instance, would have no precession. Sir W. Thomson does not mathematically demonstrate this theorem, but by use of an hypothesis gives an elegant illustration of its truth, for which, though to me it is convincing, I prefer to substitute the following demonstration.

Such a spheroid, all the particles of which revolve about an axis with a common angular velocity n , and attract each other by the law of universal gravitation, would have the form of an ellipsoid of revolution, the ellipticity of its meridional section being $\frac{5}{4} \frac{n^2}{g}$.* (See "Figure of the Earth," Encyc. Metrop., par. 33, by Prof. Airy.) Attracted by the sun, its tides would be expressed by the terms of [2316] Méc. Cél., Book IV (Bowditch). Of these three terms, the first (a function of the declination *only* of the attracting body) and the third (the semi-diurnal oscillation) express tidal elevations symmetrically distributed on each side of the equator, which would, hence, exert no influence through the centrifugal forces of their masses, upon precession. The second therefore, or *the diurnal tide*, is alone to be considered.

Conceive a meridian plane passed through the sun at any declination, the "couple" exerted by its attraction would be exerted wholly to turn the spheroid about an equatorial axis normal to this plane. We have therefore to investigate what dynamic couple, with reference to this same axis, will be exerted by the centrifugal force of the diurnal tidal protuberance. As the calculation involves the state of things at but a single instant of time, the angle, $nt + \varpi - \psi$, may be written ϖ and counted from the meridian of the sun: ρ , the uniform density of the fluid, taken as unity. The height, y , of the diurnal tide will be expressed for all parts of the spheroid by

$$(17) \quad y = \frac{15S}{2r^3g} \sin \theta \cos \theta \sin \lambda \cos \lambda \cos \varpi \dagger$$

in which λ is the polar distance or complement of the latitude of the locality, and θ the declination of the sun. If, with Laplace, we put $\cos \lambda = \mu$, and $\sin \lambda = \sqrt{1 - \mu^2}$, the mass of the elementary column of height y will be $y d\mu d\varpi$, and its centrifugal

* g being the force of gravity at the equator of the hypothetical spheroid.

† The expression, in the original, for the diurnal oscillation, is

$$\frac{3L}{r^3g} \left(1 - \frac{3}{5\rho}\right) \sin V \cos V \sin \theta \cos \theta \cos (nt + \varpi - \psi)$$

The notation of my paper on the precession of the equinoxes is substituted, and the assumed value of ρ introduced.

force (the radius of the spheroid being taken at unity, and the variation, assumed slight, due to ellipticity, disregarded) $n^2 y d\mu d\varpi \sqrt{1-\mu^2}$. The component of this tending to tilt the spheroid about the axis in question is $n^2 y d\mu d\varpi \sqrt{1-\mu^2} \cos \varpi$, and its moment $n^2 y d\mu d\varpi \mu \sqrt{1-\mu^2} \cos \varpi$.

Substituting the value of y (47), the above becomes

$$n^2 \frac{15S}{2r^3 g} \sin \theta \cos \theta d\mu d\varpi \mu^2 (1-\mu^2) \cos^2 \varpi$$

Integrating, first with reference to μ from $\mu=-1$ to $\mu=+1$, then with reference to ϖ from 0 to 2π , we get, as the expression for the couple due to "the centrifugal force of the crowns of the tidal elongation," resisting the sun's action,

$$(48) \quad 2\pi \frac{n^2 S}{g r^3} \sin \theta \cos \theta$$

We have found (19) for the moment of the sun's force, producing precession, the expression

$$\frac{3S}{r^3} (C-A) \sin \theta \cos \theta$$

and (46), $(C-A) = \frac{8}{15} \pi e$ (b being taken at unity) and e , as already stated, is for a homogeneous fluid spheroid $= \frac{5}{4} \frac{n^2}{g}$. Making these substitutions, the above expression becomes identical with (48). The precessional force of the sun is, therefore, exactly neutralized by the centrifugal force of the tidal swelling.

The theorem could, doubtless, be demonstrated for a revolving fluid spheroid in equilibrium, of which the density of the strata varies. Without extending any further the mathematical analysis, it will be sufficient to remark that the calculation of the tidal elevations is, identically, that of equilibrium of form of the revolving body subjected to a foreign attraction, and in the calculation the motion of rotation is disregarded, and the centrifugal force, which expresses its entire effect upon the form, alone considered. Under this point of view, equilibrium of form is, necessarily, equilibrium (or stability) of *position*. For if any effective turning force exists, it must, in order not to interfere with equilibrium of form, either be so distributed as to give each individual particle of the spheroid its proper relative quantity of turning motion, or it must be a distorting force. The first alternative cannot be admitted; the second is excluded by the hypothesis of equilibrium. Hence, there can be no turning (or precessional) force.

The accuracy of the foregoing analysis is complete,¹ except that the consideration of *relative motion* of the particles is excluded. But Laplace shows (p. 604, Vol. II, Bowditch) that as the depth of the ocean increases, the expressions for the tidal

¹ There are slight errors of approximation: 1st, in the tidal expression (47) itself; 2d, in the above integration which disregards the variation of the radius; and, 3d, in the value of $C-A$. They neutralize each other in the final result.

oscillations given by the dynamic theory approximate rapidly to those of the "equilibrium theory," with which, when the depth is very great, or the spheroid wholly fluid, they are essentially identical. Moreover, he shows (p. 219, Vol. I) that the *vertical* motions of the particles, when the depth is small, may be disregarded. When the spheroid is wholly fluid, *all* the relative motions of the particles are of the same order as the vertical ones and exceedingly minute; and the forces of inertia thereby developed are insensible compared with those we have been considering.¹

By parity of reasoning the truth of Sir W. Thomson's propositions concerning a solid but yielding spheroid is made evident; for exactly in the same ratio to the tides of a fluid spheroid that the solid tidal elevations are produced (the actual ellipticity of the earth being nearly that of equilibrium with the centrifugal forces), will the precessional couple due to the tide-producing attraction be neutralized by their centrifugal action.² That a thin solid crust, such as geologists generally assume, would yield and exhibit tidal elongations, seems without calculation very probable; but if Sir W. Thomson is correct as to the rigidity required in even a wholly solid earth, the hypothesis of a thin crust must be abandoned, and it would seem indeed that rigidity several times as great as the actual rigidity of iron throughout 2000 or more miles thickness of crust would be incompatible with a very high internal temperature.

Without having recourse to Sir W. Thomson's profound analysis, the necessity, in order that there shall be no sensible solid tidal wave, of a very high rigidity

¹ The foregoing demonstration does not conflict with Laplace's theorem that ocean tides do not affect the precession; for his theorem applies only to a shallow ocean over a rigid nucleus, of which ocean the precessional couple, by altered attractions, pressures, and centrifugal forces due to generation of living forces in the fluid, is transferred to the nucleus. I have already alluded to the minuteness of the motions of the particles of a fluid spheroid. The remarks apply, *à fortiori*, to those of an elastic solid. *Vibratory* motions, properly speaking, cannot exist, for the elastic forces extremely minute are always held (sensibly) in equilibrium by the distorting forces. The solid surface would oscillate in the same sense that the ocean tides oscillate, *i. e.*, by a "forced" tide-wave.

² "It is interesting to remark," say Thomson and Tait (§ 848, "Treatise, &c."), "that the popular geological hypothesis of a thin shell of solid material, having a hollow space within it filled with liquid, involves two effects of deviation from perfect rigidity which would influence in opposite ways the amount of precession. The comparatively easy yielding of the shell must render the effective moving couple due to sun and moon much smaller than it would be if the whole interior were solid, and, on this account, must tend to diminish the amount of precession and nutation. But the effective moment of inertia of a thin solid shell, containing fluid in its interior, would be much less than that of the whole mass if solid throughout; and the tendency would be to much greater amounts of precession and nutation on this account."

The co-efficient of precession of the "thin solid shell" would be (p. 34) the same, nearly, as that of the spheroid of which the homogeneous strata have the same ellipticity. Its precession-resisting couple (48) due to tidal distortion would be just what is necessary to develop its proportional influence upon the precession *of that shell*, upon which the fluid contents can exert influence only through their pressure. This is identically Prof. Hopkins' problem. The *thin* shell of popular geological hypothesis would, however, be subject to tidal distortions scarcely inferior in magnitude to those of a wholly fluid spheroid; by which, as we have seen, the sun and moon's "moving-couple" is wholly neutralized throughout the whole spheroid.

for the earth, may be made evident from the following considerations: A rod of steel extending towards the sun from the centre to the surface of the earth, would be elongated by the differential force of the sun's attraction $0^{\text{ft}}.975$, or one foot, nearly. The height of the solar tide of a homogeneous fluid spheroid is $1^{\text{ft}}.355$; but the *mutual attraction* of the elevated particles produces $0^{\text{ft}}.793$ of this, and the remaining $0^{\text{ft}}.542$ is the proper measure of the *direct* action of the solar force. In the case of the rod the elastic forces of the steel *alone* are considered; in the spheroid gravitation is the sole binding force. The maximum extension of the rod per unit of length would be expressed by the decimal .000000055 corresponding to a tensile force of 1.87 lbs. (taking the coefficient of elasticity at 34 millions lbs.) per square inch.⁽¹⁾ The necessity of the extreme rigidity demanded by Sir W. Thomson is recognized when it is seen how excessively minute would be the elastic forces developed in the production of distortion, in a rigid earth spheroid, commensurable with fluid tide-waves.¹

In a paper "On the Secular Cooling of the Earth" (Trans. R. S. E., 1862, and Appendix to "Treatise, &c."), Sir W. Thomson applies a solution of Fourier to the determination of the interior temperature and its rate of increase downwards,

⁽¹⁾ See Additional Notes, p. 51.

¹ M. Delaunay, President of the French Academy, after quoting (Comptes rendus 1868) from the paper of Sir W. Thomson to which I have already referred, the results of Hopkins and some corroborating remarks from Sir W. Thomson's paper (referred to above), says: "Ainsi, on le voit, l'objection mise en avant par M. Hopkins, contre les idées généralement admises par les géologues sur la fluidité intérieure du globe terrestre, est regardée par plusieurs savants anglais comme parfaitement fondée. Je suis d'un avis diamétralement opposé: je crois que l'objection de M. Hopkins ne repose sur aucun fondement réel." M. Delaunay then refers to an experiment made under his direction with a glass vase 0^m 24 in diameter, as furnishing decisive proof that the "viscosity" of a liquid as perfectly fluid as water even, is sufficient to cause it to take up the rotary motions of its enveloping shell, provided that those motions are relatively slow, as are those which constitute the precession and nutation of the earth; and he goes on to say: "Hence it does not appear to me possible to admit that the effect of the perturbing forces to which precession and nutation are due extend only to a portion of the mass of the terrestrial globe; the entire mass ought to be carried along (entraînée) by the perturbing actions, whatever may be the magnitude attributed to the interior fluid portion, and consequently the consideration of the phenomena of precession and nutation can furnish no datum for estimating the greater or less thickness of the solid crust of the globe."

M. Delaunay seems to be unaware that Sir W. Thomson coincides with Prof. Hopkins only in *this* (as the sequel of the very paper quoted shows), that he demands a great thickness of crust, and, moreover, that the interior, to the depth of this crust, shall be not merely "solid," but possessing a rigidity "several times as great as that of iron." I have endeavored to show that Sir W. Thomson's argument is irrefragable; but, based upon wholly different considerations, it is certain that no degree of "viscosity" assigned to an internal liquid will refute it.

I have remarked, at the outset of this discussion, that Prof. Hopkins' results "have not been generally accepted as decisive;" but I cannot admit that, as a test of their tenability, the experiment of M. Delaunay possesses the crucial character which he attributes to it. Viscosity, considered as an accelerating force tending to impart to a fluid the rotary motions of an enveloping shell, is directly proportional to the surface of contact, and inversely to the *mass* of contained liquid; in other words, it varies inversely as the diameter of the enveloping shell. The effect of viscosity of the fluid contents of the earth compared to those contained in a similar spherical envelope of only ten inches diameter, would be expressed (nearly enough) by the fraction $\frac{1}{40,000,000}$.

assuming a uniform primitive melting temperature of 7000° Fah., and a lapse of 100 millions of years since the cooling process commenced.

“The rate of increase of temperature from the surface downwards would be sensibly $\frac{1}{51}$ of a degree per foot for the first 100,000 feet or so. Below that depth the rate of increase per foot would begin to diminish sensibly. At 400,000 feet it would have diminished to about $\frac{1}{141}$ of a degree per foot. At 800,000 feet it would have diminished to less than $\frac{1}{50}$ of its initial value, that is to say, to less than $\frac{1}{2550}$ of a degree per foot; and so on, rapidly diminishing. Such is, on the whole, the most probable representation of the earth’s present temperature, at depths of from 100 feet, where the annual variations cease to be sensible, to 100 miles, below which the whole mass, or all except a nucleus cool from the beginning, is (whether liquid or solid) probably at, or very nearly at, the proper melting temperature for the pressure at each depth.”

The high rigidity demanded is difficult to conceive of in connection with a temperature in the solidified mass “at or near” that of melting, extending downwards indefinitely towards the centre of the earth. Hence Poisson’s reasoning, which results in showing that the earth to have become thoroughly cooled and to have been subsequently reheated, *superficially*, harmonizes better with the demand for rigidity than that of Leibnitz, which supposes it to be now cooling from a (throughout) incandescent liquid state. In the latter case the law of actual temperature as deduced from Fourier’s formulæ (as expressed above by Sir W. Thomson) would extend to the centre; in the former case only to the unmelted portion or to the “nucleus cool from the beginning.”

Referring to the increase of temperature, with depth observed in mines, &c., Poisson remarks: “Fourier et ensuite Laplace ont attribué ce phénomène à la chaleur d’origine que la terre conserverait à l’époque actuelle et qui croîtrait en allant de la surface au centre, de tel sort qu’elle fût excessivement élevée vers le centre * * * en vertu de cette chaleur initiale la température serait aujourd’hui de plus de 2000 degrés à une distance de la surface égale seulement au centième du rayon; au centre elle surpasserait 200,000 degrés. * * * Mais quoique cette explication ait été généralement adoptée, j’ai exposé, dans mon ouvrage, les difficultés qu’elle présente, et qui m’ont paru la rendre inadmissible, * * * je crois avoir démontré que la chaleur développée par la solidification de la terre a du se dissiper pendant la durée de ce phénomène, et que *depuis longtemps il n’en subsiste plus aucune trace.*” (Théorie de la Chaleur.)

Poisson, as is well known, attributes the increase of temperature, with depth, observed in the earth’s crust, to the passage, at a remote period, of the solar system through hotter stellar regions, the temperature of which, he argues, should differ from place to place. Even a hypothetical case of the illustrious author, conforming to his theory, of an increased superficial temperature, 5000 centuries ago, of 200° C. diminishing by cooling (by transition to cooler stellar regions) to 5° , 500 centuries ago, and, subsequently, to the actual mean temperature, would scarcely meet the present demands for time, of palæontology; while the determinations of the conductivity of the earth’s crust made near Edinburgh show, according to Sir W. Thomson, a necessity for increments of temperature of 25° , 50° , and 100°

(Fah.), for past periods of only 1250, 5000, and 20,000 years, and authorize him to pronounce Poisson's hypothesis impossible, without destruction of life, or relegation of the date to so remote an era as to demand an intensely heated stellar region.

A reheating *after* solidification which should again fuse the surface to great depths would be, it seems to me, as "inadmissible" for the origin of observed subterranean temperatures, as the heat originally "developed by the solidification of the earth." Hence, the hypothesis of a reheating to fusion of the surface by impact of meteoric bodies would be likewise excluded by Poisson's theory of internal temperatures.

In the paper referred to (p. 39), Sir Wm. Thomson discusses the probable circumstances of solidification of the earth, assuming the known crust-materials (granite, &c.), in a molten state, as the constituent, and reasons that in consequence of the great condensation of granite in freezing, solidification must commence at the centre, and that "there could be no complete permanent incrustation all round the surface "till the globe is solid, with, possibly, the exception of irregular, comparatively "small spaces of liquid;" such separation of constituents in the process of crystallization taking place in all liquids composed of heterogeneous materials, and, indeed, is observable in the lava of modern volcanoes. He infers from the probable phenomena developed, into the discussion of which he goes at some length, "results sufficiently great and various to account for all that we see at present, and "all that we learn from geological investigation, of earthquakes, of upheavals, and "subsidences of solid, and of eruptions of melted rock."

Still we would, if possible, find reason to attribute a lower than "the proper melting temperature" to the solidified interior. Ice, indeed, preserves its rigidity unimpaired up to the point of fusion, and there may be a few other substances that have the like property; but it seems to be an exceptional one. The known constituents of the earth's crust certainly do not possess it, at least under ordinary pressures. If, as suggested by Prof. Joseph Le Conte (*Am. Journal of Science*, Nov. Dec. 1872), the "conductivity" be increased by pressure and *condensation*, such diminished temperatures may obtain.

NEW ADDENDUM.¹

A FEW words are in place here concerning the results of the late Prof. Hopkins' investigation (against which M. Delaunay's objections, (note, page 38,) are especially directed), briefly stated, pages 34, 35. They are as follows: First FOR HOMOGENEOUSNESS. "Supposing the earth to consist of a homogeneous spheroidal shell (the ellipticities of the outer and inner surfaces being the same) filled with a fluid mass of the same uniform density as the shell;" then, "the precession will be the same, whatever be the thickness of the shell, as if the whole earth were homogeneous and solid."

SECOND, FOR HETEROGENEOUSNESS, his result may be thus expressed:

$$(a) \quad P_1 - P' = P_1 \left\{ 1 - \frac{\varepsilon}{\varepsilon_1} \left(1 + \frac{s}{1 + \frac{h}{q^2 - 1}} \right) \right\};$$

"where P_1 denotes the precession of a solid homogeneous spheroid of which the ellipticity $=\varepsilon_1$, that of the earth's exterior surface, and P' the precession of the earth, supposing it to consist of an interior heterogeneous fluid contained in a heterogeneous spheroidal shell, of which the interior and exterior ellipticities are respectively ε and ε_1 , the transition being immediate from the entire solidity of the shell to the perfect fluidity of the interior mass."

In the multiplier of $\frac{\varepsilon}{\varepsilon_1}$, second member of (a), q is the ratio of external to internal polar radius of the shell; s depends on the varying ellipticity and density of the strata of equal density of the shell; h depends on the density of the fluid interior. For a thin crust the coefficient in question is unity nearly; for a thick one it will be somewhat greater if ε be less than ε_1 .

It cannot fail to be observed that, under the conditions just before expressed for *homogeneous*—*i. e.*, equality of external and internal ellipticities—we get from the formula (s becoming zero) *the same result, i. e.*, $P' = P_1$, as for that case.

In accordance with rational hypothesis as to the internal condition of the earth, equalities of ellipticities for the surfaces of a *thin* crust (and corresponding equality of densities), or closely approximate equalities would be expected. The necessity for a *thick* crust arises, therefore, from the alleged discrepancy between the observed and calculated annual precessions (50 seconds and 57 seconds), which, according to Prof. Hopkins, makes $\frac{P_1 - P'}{P_1} = \frac{1}{8}$, nearly, assuming the moon's mass $\frac{1}{70}$, and the earth's ellipticity $\frac{1}{300}$. (The real discrepancy is probably very much less. See page 34, *et sequentia*.)

¹ The original ADDENDUM, hurriedly written while the work was in the printer's hands, has been, in what follows, somewhat modified and amplified.

If, in applying the expression (ϵ), the symbolic fraction, for a first approximation, be omitted, we have, according to above assumption of discrepancy, $\epsilon = \frac{7}{8}\epsilon_1$. This value of ϵ will be *in excess*; hence, the thickness of crust deduced from it will err the other way, and a determination on this basis will give a thickness which must, *in fact*, be exceeded.

The limit of solidity, proceeding inwards, may and probably does depend upon both temperature and pressure. Isothermal surfaces Prof. Hopkins finds to have *increasing* ellipticities. Surfaces of equal pressure, deduced from the hypothetical law of density, $A \frac{\sin qb}{b}$, have diminishing ellipticities, and if $qb_1 = 150^\circ$ the above law agrees sufficiently well with the actual ellipticity and ratio of surface to mean density of the earth. This law for $\epsilon = \frac{7}{8}\epsilon_1$ demands a thickness of crust of $\frac{1}{4}$ the radius, or 1000 miles. This is a *minimum*, since the actual surface of solidification (lying between this and the corresponding isothermal surface) would have greater (and hence too great) ellipticity.

Before commenting upon this application, and upon the real meaning of the formula, I return to the case of homogeneousness. Some of the results arrived at by the analysis of Prof. Hopkins may be illustrated by the following considerations: The fluid spheroid, treated of p. 36, is subjected, by the attraction of the sun, to the distortion expressed by (47). This distortion, as shown by the form of the expression, is equivalent to an exceedingly slight rotational displacement¹ of figure about an equatorial axis, such as would be caused by displacing through a still more minute angle the planes of diurnal rotation. It is one of the beautiful results of the analysis to show that the change in the direction of the centrifugal force due to this slight obliquity of the planes of rotation is equivalent to turning forces at all points of the fluid exactly proportional to their distances from the equatorial axis.

Let now a rigid shell, exactly conforming internally to the external surface of the fluid, be applied, and the whole turned back until the planes of rotation are restored to perpendicularity to their axis; the precessional effect of the attracting body now operates upon the whole mass; for there are no longer counteracting tidal protuberances. If we take that part of (47) which is due to the *direct* action of the sun, viz., $\frac{3S}{r^3g} \sin \theta \cos \theta \sin \lambda \cos \lambda \cos \varpi$ (for, the protuberances being repressed by the shell, the pressures on its interior which replace them will arise only from the direct action), and estimate it as a pressure and calculate the elementary couples for an internal ellipticity, e , we shall find the integral couple (and this corresponds with Prof. Hopkins' result) to be identical with (48) viz., exactly that due to the

¹ The required angle of the displacement is the height of the tidal wave (47), for $\varpi = 0$, divided by $\frac{dr}{d\lambda}$ for an ellipse of ellipticity, e , ($2e \sin \lambda \cos \lambda$). Prof. Hopkins shows that the corresponding divergence of the planes from perpendicularity develops a couple $= \frac{4}{15} \pi n^2 e$ multiplied by the sine of twice this arc (or twice the arc itself). Performing the operations we get, $\frac{24}{15} \pi e \frac{S}{r^3} \sin \theta \cos \theta$, in which we have the solar couple (19) and (48), which causes the displacement, since $e = \frac{15}{8\pi} (C - A)$

couple which the sun would exert on the fluid mass considered as a solid.¹ It would increase the precessional force of the shell in the ratio $\frac{q^5}{q^5 - 1}$ of the analysis. By virtue of this pressure the fluid tends to transform its own precession into an augmented precession of the shell.

It requires, however, but an extremely minute angular separation of the axes of the shell and fluid to generate counter-pressures equivalent to those which caused the separation.² The divergence cannot, therefore, be progressive, but is simply a minute oscillation of the two axes, or a rotation around each other. In the latter form it appears in the analysis which, otherwise, gives to the internal fluid mass a precession *identical with that of the enveloping shell*.

Prof. Hopkins confines his analysis for the case of homogeneousness to equal ellipticities for the bounding surfaces of the shell. Excepting the case of sphericity for the inner surface, the result would be the same—viz., an unchanged precession, however the ellipticities might differ.

I now return to the formula (a) and remark, that it is an inaccurate expression for a slight difference ($P_1 - P'$) due to the fact that the spheroid is *heterogeneous*—that it is *not capable of being made a test of internal fluidity, or a measure of thickness of crust*.

I have already shown that for homogeneousness the couple due to pressure on the inner surface of the shell is identical with the sun-couple upon the fluid mass solidified, a result approximately true (as will be shown hereafter) if the density of the fluid strata vary. Hence, if we take the sum of the sun-couple exerted on a shell of interior and exterior ellipticities, ε and ε_1 , and of the pressure-couple developed in the fluid,³ and divide by the moment of inertia of the entire mass and by ω , we shall have the rate of gyration of the entire mass considered as a solid.

Referring to Prof. Hopkins' analysis and symbolism, the quotient will be⁴

$$(r) \quad \frac{3}{2} \frac{\mu}{r^3 \omega} \sin 2\Delta \frac{\int_0^{a_1} \rho' \frac{d(a^5 \varepsilon')}{da'} da' + 2a^3 \varepsilon \int_0^a \rho' a' da'}{\sigma(a_1) = \int_0^{a_1} \rho' \frac{da'^5}{da'}}.$$

¹ The lever arm is also $2e \sin \lambda \cos \lambda$. Multiply the above by this arm, by g , by the elementary surface $d\mu d\alpha$, and, again, by $\cos \alpha$, and we get the elementary component tending to tilt the shell. The integral, with proper substitutions, is equivalent again to (19) or (48).

² There is another process which may take effect in neutralizing internal pressure. I have remarked (last par. p. 6), that, considered as a perfectly rigid body, the precessional motions of the earth cannot be *precisely* those assumed. In fact, our imperfect integrals of the conditional differential equations present the anomaly of a varied motion in which the generating force *does no work*; no yielding to the tilting couple having place. There are necessarily some, too minute to be detected, nutational movements. In case the precessional force were augmented by so large a ratio as $\frac{q^5}{q^5 - 1}$ would be for a *thin* shell, *these* nutational movements would surpass in magnitude those necessary to generate the required counteracting pressures.

³ I use provisionally Prof. Hopkins' computations for this, involving $\int_0^a \rho' a' da'$; its erroneous-ness will appear hereafter.

⁴ The symbols μ , Δ , ω , correspond to S , θ , n , of p. 7; ρ' is the density of stratum, solid or fluid, for which i' is the ellipticity, and a' the polar radius; a_1 is the external, and a the internal polar radius of the shell.

Denote the moment of inertia of the entire spheroid by $I_1 = \frac{8}{15} \pi \sigma (a)$

“ “ “ “ “ “ shell “ $I' = \frac{8}{15} \pi [\sigma(a_1) - \sigma(a)]$

“ “ “ “ “ “ nucleus “ $I'' = \frac{8}{15} \pi \sigma (a)$

Then $\sigma (a_1) = \frac{I_1}{I'} (\sigma (a_1) - \sigma (a))$

and the above expression, reduced to precession, will become

$$(x) \quad \frac{I'}{I_1} \left(1 + s + \frac{h}{q^5 - 1} \right) P \quad (1)$$

Prof. Hopkins gets for the precession of the same spheroid considered as fluid within the shell, (his symbolic abbreviations used in both cases)

$$(y) \quad \frac{(\gamma_2)}{(\gamma_1) + (\gamma_2)} \left(1 + s + \frac{h}{q^5 - 1} \right) P$$

In this last expression (γ_1) and (γ_2) denote coefficients of gyration which one and the same couple (*i. e.* the centrifugal force, by pressure on the shell and by reaction on the fluid mass—the assumption being made that the latter, having its proportionate force on each particle, gyrates as a solid) produce upon the shell and fluid mass respectively. They should be therefore inversely proportional to the respective moments of inertia of the shell and nucleus, rendering the expressions (x) and (y) identical.

But this apparent identity is brought about by assuming that Prof. Hopkins' expression for the pressure-couple on the shell arising from the sun's attraction on the fluid to be identical (or at least approximately so) with that which would be exerted on the same heterogeneous fluid solidified; by which assumption I introduce

in (x), $\frac{h}{q^5 - 1}$, (which is Prof. Hopkins' symbolic abbreviation of

$$\frac{2a^3 \int_0^a \rho' a' da'}{\sigma(a_1) - \sigma(a)} \Bigg): \text{ instead of } \frac{\int_0^a \rho' \frac{da'^5 \epsilon'}{da'}}{\epsilon [\sigma(a_1) \sigma(a)]}$$

which latter expression belongs to the case just specified, of the *solidified* fluid.

Now in the case of nature—*i. e.* the earth with the received hypothetical laws of density and ellipticity, the two expressions differ in a ratio (about 4 : 3) so greatly exceeding unity, as to forbid the assumption of approximate equality. The error of the first expression will be better appreciated by referring to the quan-

* The interpretation of (x) and (y) is obvious. P is the coefficient of precession for a *homogeneous* shell of uniform ellipticity ϵ ; instead thereof let the shell be *heterogeneous* with same internal surface, but of an *external* ellipticity ϵ_1 . P for such a shell will have a fractional increment denoted by the ratio s . By the pressure of the internal fluid the precessional coefficient of this shell will be still further increased by a ratio denoted (a result of the analysis) by $\frac{h}{q^5 - 1}$. But the shell is constrained to carry along and to take up a common precession with the nucleus, and the coefficient will be thereby diminished in a ratio $\frac{I'}{I_1}$ of (x) or (according to Prof. Hopkins) the corresponding expression of (y). Since $P = P_1 \frac{\epsilon}{\epsilon_1}$, expression (a) is readily deducible from (y).

ties (γ_1) and (γ_2) in (y). The brief account given, p. 43, will show how, as resulting from the analysis, a rotating homogeneous fluid enveloped and confined by a shell reacts, with a *practical* rigidity conferred by rotation, against the shell, (when the respective axes of rotation are slightly separated) and thereby receives an angular motion "precisely as if it were solid," (Phil. Trans., 1839, p. 394). It is clear that, through this interaction, the shell, likewise, must receive an angular motion, and that these several angular motions must be in inverse ratio to the respective movements of inertia of shell and nucleus; and so, for homogeneousness, the analysis makes them. When we come to heterogeneousness, the same *modus operandi* is (and rightly) attributed to the fluid; and again, most clearly, the relative angular motions of shell and fluid should be in above-mentioned inverse ratio; whereas their ratio is quite differently computed to be $\frac{h}{q^5 - 1}$ of which the value has just been given. The error (for there is clearly one) is in the computation of the pressure-couple developed in the fluid and exerted upon the shell by centrifugal force, when their axes of rotation are slightly separated.

The same error of computation (exhibiting itself by the identical symbol, $\frac{h}{q^5 - 1}$) enters into the expression for the pressure-couple developed by solar attraction, and introduces *again* the above symbol as the third term of the second factor of (y).

Without going into lengthy discussion, it is sufficient to remark that both the centrifugal force and the foreign attraction produce in the strata of equal density of a heterogeneous fluid, *special configurations*, and expend themselves in so doing; and, moreover, that the prior establishment of these forms of equilibrium is assumed, and necessarily assumed, in the analysis. It is, therefore, unwarrantable to integrate (as is done) through the fluid mass these forces, as free forces, to get its pressure upon the shell.¹

I have shown, I think, that in the expression (y) the first factor should be corrected to be, as it is in (x), $\frac{I'}{I_1}$.

The correction consists in substituting for $\frac{(\gamma_1)}{(\gamma_2)}$ in the *first* factor of (y), $\frac{\int_0^a \rho' \frac{da^5}{da} da'}{\sigma(a_1) - \sigma(a)}$ instead of $\frac{h}{q^5 - 1}$ [or its value, p. 45]. The *same* correction for $\frac{h}{q^5 - 1}$, introduced into the second factor, would require $\varepsilon \int_0^a \rho' \frac{da^5}{da} da'$ to be substituted for the second term of numerator of (r). But that (r) should belong to entire solidity we require $\int_0^a \rho \frac{da^5}{da} da'$.⁽²⁾ Now these quantities differ inappreciably, for taking the entire

¹ It is obvious that the taking account of the internal motions by which the configurations produced by foreign attraction adapt themselves to the diurnal rotation, does not meet the point made.

² Although the errors in the two cases have the same expression, it does not follow that the corrections should be *identical*. The configurations of strata of like density due to foreign attraction are superinduced on the *previously established* configurations due to the centrifugal force, which have the varying ellipticity ε' . The correction should involve this ellipticity, and it is probable that the above symbol is, if we disregard the slight *internal motions*, the true one. Indeed, I think I may venture to affirm, that, given a heterogeneous fluid wholly enveloped by a rigid bounding surface,

integrals from zero to a_1 and multiplying by $\frac{8}{15}\pi$, the last becomes (Thomson and Tait, § 825),

$$\frac{2}{3}Ma_1^2(\varepsilon_1 - \frac{1}{2}m) = \mathfrak{C} - \mathfrak{H}, \text{ for the earth as actually constituted.}$$

And the first (deduced from Hopkins, Phil. Trans., 1840, pp. 203 and 204), is (nearly)

$$\frac{1}{3}Ma_1^2\varepsilon_1 = C - A, \text{ for same spheroid with uniform internal ellipticities.}$$

(M = mass of the earth).

[The value of the first, using the constants of density of Archdeacon Pratt, "Figure of the Earth," 4th ed., p. 113, is somewhat *less* than this last expression.] Now $m = \frac{1}{2}\frac{1}{89}$ (ratio of centrifugal force to gravity) and hence $2(\varepsilon_1 - \frac{1}{2}m)$ is very little less than ε_1 . For a fluid *nucleus*, the inequality would be still less.

Hence it appears that both (x) and (y) (when corrected) express very nearly the precession of the solidified earth; and, moreover, that the effect upon precession due to the variation of internal ellipticity is very small, the precession of the earth considered as rigid being, essentially, that corresponding to uniform ellipticity; or what is the same thing, that of a *homogeneous spheroid of its external form*.

This also appears in the comparison of the value of $\frac{C-A}{A}$, as established from observation, and the resulting *calculated* ellipticity. The first is .00327 and the second $\frac{1}{2}\frac{1}{97}$ (Thomson and Tait, § 828). Now a homogeneous spheroid of the latter ellipticity would have for $\frac{C-A}{C}$ a value ($e - 4e^2$) of .00332; a difference of about $\frac{1}{6}\frac{1}{6}$. Variations in the constants which enter into the expressions for internal density give rise to variations in the *calculated* ellipticity—and, of course, in the resulting precession; but if the *external* ellipticity is defined by a rigid shell, the effect of *internal* variation is, in the case in hand, almost *nil*. Hence, had the hypothetical consolidation of the earth, of p. 43, been carried to the very centre, no material approximation to the desired correction of $\frac{1}{6}$ in the calculated precession would have been found.¹ In fact, the problem for heterogeneity

subjected to a foreign attraction, and a condition of static equilibrium assumed, the pressure-couple exerted by the fluid on the shell cannot differ from that which the attraction would exert on the solidified fluid.

In the case of homogeneity, I have arrived (p. 43: the results, though based on an ellipticity corresponding to fluid equilibrium, hold good for any *small* ellipticity) at the exact expression for the pressure-couple, from the function expressing the tidal protuberance due to the foreign attraction. The tidal configuration of the heterogeneous earth, wholly liquefied, would result from the transcendental analysis of Hopkins, pp. 203, 204, or of Thomson and Tait, § 822-824, and the maximum height would be one foot, very nearly; but the pressure function cannot be readily deduced. It would depend on gravity and [§ 825] "the value of $C - A$ may be determined solely from a knowledge of surface or external gravity, or from the figure of the sea level without any data regarding the internal distribution of density."

¹ It is curious, to say the least, that there should be ground for the remark that the expressions (a) and (y), which latter, with its author's valuation of $\frac{(\gamma_1)}{(\gamma_2)}$, may be written $\left\{ 1 + \frac{s}{1 + \frac{h}{q^2 - 1}} \right\} P$, give,

was practically solved under Prof. Hopkins' treatment of it when it was shown that, for homogeneousness, the precession for internal fluidity was the same as for solidity, and when it appeared that the analysis applied would exhibit the action of the heterogeneous fluid as if disposed in strata of like ellipticity with that of the inner shell surface. The erroneous computation of pressures has merely the effect of exaggerating in a like ratio the densities of all the fluid strata. Hence, and hence only, a resulting precession differing materially from that which would result from solidity.

In conclusion, I remark, 1st. The analysis of Prof. Hopkins, in its application to a homogeneous fluid and shell, seems to establish (and the result is confirmed by its harmony with tidal phenomena as developed in p. 43) that the rotation imparts to the fluid a *practical rigidity*¹ by which it reacts upon the shell as if it were

with decreasing internal ellipticities, for fluidity of nucleus *less precession than would belong to solidity*. This is obvious since his $\frac{h}{q^2-1}$ is greater than the ratio $\frac{I''}{I'}$ (nearly) which should take its place on the latter hypothesis.

¹ I do not concur with Sir William Thomson in the opinions quoted in note, p. 38, from Thomson and Tait, and expressed in his letter to Mr. G. Poulett Scrope ("Nature," February 1st, 1872), so far as regards fluidity, or imperfect rigidity, within an infinitely rigid envelope. I do not think the rate of precession would be affected.

That no increase arises from fluidity I have endeavored to show; and it is unquestionably a corollary of Prof. Hopkins' investigations. As regards imperfect rigidity, Sir William Thomson bases his argument upon the assumption that "the whole would not rotate as a rigid body round one 'instantaneous axis' at each instant, but the rotation would take place internally, round axes deviating from the axes of external figure, by angles to be measured in the plane through it and the line perpendicular to the ecliptic in the direction towards the latter line. These angular deviations would be greater and greater the more near we come to the earth's centre. * * * * * Hence the moment of momentum round the solstitial line would be sensibly less than if the whole mass rotated round the axis of figure."

If I do not misunderstand his language, Sir William Thomson assumes that the same *bending* distortion which would ensue from the application of a couple to the external portions of a non-rotating spheroid, would, equally and *identically*, take place in a rotating one: thus causing the angle made by the planes of the external rings of matter and the solstitial line to be increased; with a corresponding diminution of the component of *Cn* about this line.

In the case specified by him (an extreme one) while sensible and important nutational movements would ensue, the mean *precession* would be insensibly affected; but I do not think precisely *such* elastic yielding would take place.

As an extreme case of an infinitely rigid and infinitely *thin* shell containing matter completely destitute of rigidity, take the fluid spheroid of p. 36, and conceive it enveloped by such a shell. It is *still*, as shown, p. 43, susceptible (and susceptible *only*) of the extremely minute deflections of its planes of rotation by which precession is completely annihilated. Confer now upon the contents of the shell rigidity, uniform, or varying from surface to centre, continuously or discontinuously, in any arbitrary manner, and you have every possible case of imperfectly rigid matter contained within a perfectly rigid crust. I can attribute no other effect to the conferred rigidity than a restoration of the *lost* precession—in whole or in part; nor can I suppose the shell enveloping imperfectly rigid matter to change its obliquity *more* than that which contains the fluid; regard being had to conditions of *equilibrium* without reference to living forces generated.

I must remark that this hypothetical case, though as admissible for argument as any other form of "preternaturally rigid" crust, is exceptional. With a shell of *finite* moment of inertia, having some comparable relation to that of the fluid contents, the precession, instead of being annihilated, would be that due to the entire mass

a solid mass, while its pressure imparts to the shell the requisite couple to preserve the precession unchanged.

2d. The same practical rigidity is, with entire reason, attributed to the heterogeneous fluid by which (leaving out of view minute *relative* oscillations which do not affect the mean resultant in other natural phenomena and should not in this) the shell and fluid take a common precession.

3d. The two masses retaining their configurations, mutual relations, and rotary velocities, essentially unaltered by the hypothesis of internal fluidity, it would be a violation of fundamental mechanical principles were the resulting precession not identical with that due to the entire mass considered as solid.

4th. The common and identical precession of fluid and shell resulting from the analysis, is indispensable to any conception of *precession* for the earth as composed of thin shell and fluid; for otherwise internal equilibrium would be destroyed and the "figure of the earth" cease to have any assignable expression. The entire mass, fluid and solid must (without invoking the aid of "viscosity"), be "carried along in the precessional motion of the earth." The analysis I have examined demonstrates the possibility and exhibits the *rationale* of such a community of precession, but fails in the attempt to exhibit a test of the existence or absence of internal fluidity.

5th. The powerful pressures that would be exerted upon a *thin* and rigid shell would probably produce in it noticeable nutational movements;¹ while if the shell be *not* of a rigidity far surpassing that of the constituents of the cognizable crust, the "precessional motion of the earth" would, owing to the neutralizing effect of tidal protuberances, scarcely be observable.

¹ *Vide* p. 44, and note 2: without reference to conventional "Nutation" which is but a form of *precession*. In connection with these *relative* motions of shell and fluid, it is in place to allude to the "Vindication of Mr. Hopkins' method against the strictures of M. Delaunay," by the late Archdeacon Pratt ("Figure of the Earth," 4th ed., p. 132). He reasons, that, if at any moment, the crust and fluid be arranged as to density, "exactly as if they had been hitherto one solid mass and be moving alike, this state cannot possibly continue." For the shell will be acted upon not only by the foreign attraction but by the fluid pressure, and will "begin to move quicker," with a precession due to *both* the thence arising couples. That this should not occur requires, he estimates, the counteracting centrifugal force of a tidal protuberance, in a crust supposed 100 miles thick, of *seventy-four feet*.

The writer does not seem to be aware that the author whom he vindicates finds no such relative acceleration of the shell (*vide* p. 44, § 2, of this Addendum) as resulting from the pressure; and, strangely, for an authority on the "Figure of the Earth," fails to recognize that "an elevation of the outer surface of the crust"—that is a tidal distortion—of *a single foot*, would relieve the shell from all pressure. This is, perhaps, a natural result of the use of an expression (Prof. Hopkins') for the pressure which disregards the influence of "Figure."

ADDENDUM TO NOTE I, PAGE 38.

THE apparent antagonism between the theorem of the text and that of Laplace suggests a few additional words. The theorem of Laplace is that "in whatever manner the waters of the ocean act upon the earth, either by their attraction, their pressure, their friction, or by the various resistances which they suffer, they communicate to the axis of the earth a motion which is *very nearly equal* to that it would acquire from the action of the sun and moon upon the sea, if it form a solid mass with the earth." (Méc. Cél., Bowditch [3345].)

The theorem is demonstrated in two distinct, quite different, manners. The last demonstration is founded upon the principle of the "conservation of areas;" and as the result of *this* demonstration the proposition is stated in the above quoted words.

The first demonstration is purely analytical, and, after stating that "this fluid" (*i. e.* of the ocean) "acts upon the terrestrial spheroid by its pressure and by its attraction," Laplace proceeds to find the analytical expressions for the precession and nutation-producing couples due to this pressure and to this attraction as they are modified by the attraction of the sun and moon upon the fluid. He then proceeds to calculate these couples for the material substance of the ocean, considered as rigidly connected (or forming a solid mass) with the earth. He finds the couples, so calculated, respectively, identical in the two cases, and epitomizes the result as follows: "the phenomena of the precession of the equinoxes and the nutation of the earth's axis are *exactly the same* as if the sea form a solid mass with the spheroid which it covers." [3287.]

But this demonstration is limited by the assumption that "the sea wholly covers the terrestrial spheroid or nucleus, that is of a regular depth, and suffers no resistance from the nucleus;" and *both* demonstrations imply an ocean of (relatively) small depth.

Under the last *mentioned* treatment of the subject the proposition of Laplace and that which I demonstrate are but the extreme phases exhibited by the solution of a problem, according as the datum be that the depth of the sea is minute (in which case its entire precession-producing couple, not effectively exerted upon its own mass, is *almost* wholly *transferred* to the solid nucleus); or that the nucleus is very small, in which case the lost precession-producing couple of the fluid is but in small part transferred to the nucleus—or wholly disappears with the vanishing of the latter.

I think, however, that the last-mentioned (first in point of order) demonstration of Laplace is not as general as the language quoted [3287] would indicate. The omission of variations of the radius-vector, R , in all the integrations gives rise to errors which do not seem to me to be identical in the two processes by which the couples are calculated, when the variation of depth is very small.

An apt illustration of the above remarks is derived from the supposition—as admissible as any other—that, the depth, γ , is constant. In this case, *whatever be the ellipticity* of the solid nucleus, the value of y (the height of the diurnal or precession-affecting tide, see [2253] [3333] and also p. 36) is zero, and, of course, the couples [3272] and [3273] become zero—as will be found by performing the integrations in those equations. So, of course, do expressions [3284] and [3285] become zero, with γ made constant. But these last *should not* be zero except for the case of sphericity of the nucleus.¹ The expressions do not seem to me capable of sustaining the inference which I have quoted [3287], when the depth of the sea is uniform; a case which most naturally presents itself to the mind.

¹ A shell of slight internal ellipticity and small *uniform* thickness has a precessional coefficient of four-fifths the value of that of a shell bounded by surfaces of equal ellipticity. Hence *in general* the variations of R (or the ellipticity) produce effects insensible compared to those of the depth.

Mr. Airy (Tides and Waves, Art. 127) bases his demonstration of the theorem exclusively upon the principle of the conservation of areas, remarking at the outset, "if the earth and sea were so entirely disconnected that one of them could revolve for any length of time with any velocity, increasing or diminishing in any manner, while the other could revolve with any other velocity changing in any other manner, we could pronounce nothing as to the effect of the fluctuation" (tidal) "upon precession."

A spheroidal nucleus wholly covered by an ocean of uniform depth, suffering no resistance, does not seem to me to lack much for fulfilling the above conditions.

If *velocities* are generated in the waters of the ocean by solar (or lunar) attraction, the centrifugal forces due to them might be looked to (though not alluded to by Laplace) as agents for transferring, from the fluid to the nucleus, the precession-producing couples due to the fluid mass, especially in the above hypothetical case. It will be found, however, by reference to the expressions [2260], that they give rise to no couple, and are, moreover, very minute.

The motion which the displacements [2260] [2261] indicate is a slight oscillation of the axis of the fluid envelope, moving as a solid, about the axis of the nucleus, the angular distance between these axes being slightly less than 2 seconds: it is, I presume, that which a non-rotating shell would have were the attracting body, with constant distance and declination, to move, with angular velocity n , in right ascension. In the case in hand it is the fluid shell which revolves and, suffering no change of form, would be itself affected by its proper precessional couple to the exclusion of the oscillation above described.

ADDITIONAL NOTES.

NOTE TO PAGE 11.

^o The process indicated is a more legitimate carrying out of the methods peculiar to this paper than what follows in the text. The tangent of MM' (35) may be (approx.) taken for the sine, and the cosine taken constant at unity, as may also be the $\cos I'$.

From (32) we may calculate by developing and neglecting terms in which $\sin^2 I'$ enters

$$\sin i = (1 - \cos^2 i)^{\frac{1}{2}} = \sin I - \sin I' \cos I \cos n_3 t$$

$$\sin i \cos i = \sin I \cos I - \sin I' \cos 2I \cos n_3 t - \frac{1}{2} \sin^2 I' \sin 2I \cos^2 n_3 t$$

Introducing these values in (38) and (39), and integrating we get expressions identical with 44 and 45, except a (practically) immaterial difference in the coefficient of t in the first which becomes $1 - \frac{1}{2} \sin^2 I'$ instead of $1 - \frac{2}{3} \sin^2 I'$.

NOTE TO PAGE 24.

^o The foregoing interpretation of the symbolic integral in (7), adopted with hesitation from authors cited, is based on assumed constancy of the angle ϕ ; but this angle necessarily varies, slowly indeed, but progressively, by the azimuthal motion measured by $n \sin \lambda$. The conditions for the formation of a lemniscate are not, therefore, rigidly fulfilled. It will be found, however, taking into account a complete excursion, that the slight increment which will enure to the moment of the quantity of motion, $\sin^2 \theta \frac{d\phi}{dt}$, from this cause on one side of the vertical, will be neutralized on the other, in consequence of the opposing signs of $\cos \phi$, in opposite azimuths; or, at least, the *resultant* increment or decrement will be a quantity of the second order in minuteness, and hence, affecting only in the same degree the azimuthal motion $\frac{d\phi}{dt}$.

NOTE TO PAGE 39.

^o The differential attraction of the sun on any length $d\chi$ of the rod, at distance χ from the earth's centre is $\left(\frac{S}{(r-\chi)^2} - \frac{S}{r^2} \right) d\chi$, r being sun's distance. Integrate from $\chi = \chi$ to $\chi = R$ (the earth's radius).

$$(1) \quad S \left(\frac{1}{r-R} - \frac{R}{r^2} - \frac{1}{r-x} + \frac{x}{r^2} \right)$$

The above divided by the coefficient of elasticity E will give the elongation per unit of length at any point. Multiply by $d\alpha$ and integrate from $\alpha = 0$ to $\alpha = R$, and the total elongation is

$$\frac{S}{E} \left\{ \frac{R}{r-R} - \frac{R^2}{2r^2} + \log \left(1 - \frac{R}{r} \right) \right\}.$$

Since $\log \left(1 - \frac{R}{r} \right) = -\frac{R}{r} - \frac{1}{2} \frac{R^2}{r^2} - \frac{1}{3} \frac{R^3}{r^3} - \&c.$, the foregoing will reduce, approximately to

$$\frac{2}{3} \frac{S}{E} \frac{R^3}{r^3}.$$

If $M =$ earth's mass, $g =$ gravity at its surface, and $\frac{1}{n}$ the ratio of M to S , and m the ratio of length of rod of weight E (per square inch section) to R , we shall have: $S = ngR^2$, $E = mgR$, and the above expression for total elongation becomes:

$$(2) \quad \frac{2}{3} \frac{n}{m} R \frac{R^2}{r^3}$$

Take $\frac{R}{r} = \frac{4000}{92000000} = \frac{1}{23000}$; $R = 4000 \times 5280$ feet; $n = 316000$; $m = .472$ (the latter value based on $E = 34$ millions lbs. per square inch, and a steel rod of that section to weigh 3.4 lbs. per foot length) and the total elongation (2) becomes $0^{\text{e}}.975$. The *maximum* extension per unit of length is at the centre, and is found by putting $\alpha = 0$ in (1) and dividing by E . It is $\frac{S}{E} \frac{R^2}{r^3} = \frac{n}{m} \frac{R^2}{r^3} = .000000055$, indicating a strain of $1^{\text{b}}.87$ per square inch. The ratio of the *total* elongation (2) to the total length of the rod R is two-thirds of the above, indicating *about* that ratio for the ellipticities of superficial and central strata of a steel globe distorted by the sun's attraction; a result thus rudely calculated which differs little from that given in Thomson and Tait, § 837.

A CONTRIBUTION

TO THE

HISTORY OF THE FRESH-WATER ALGÆ

OF

NORTH AMERICA.

BY

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A D V E R T I S E M E N T .

THE following memoir was referred for examination to Dr. John Torrey and Dr. F. A. P. Barnard, of Columbia College, New York. They recommended its publication provided certain changes were made in the manuscript. These having been made by the author, the work is published as a part of the series of "Smithsonian Contributions to Knowledge."

JOSEPH HENRY,

Secretary, S. I.

WASHINGTON, October, 1872.

P R E F A C E.

OF all the various branches of Natural History, none has been more enthusiastically and more successfully prosecuted in the United States than Botany. The whole field has been most thoroughly occupied, save only as regards certain of the lower cryptogams, and amongst the latter, it is the *fresh-water Algæ* which alone can be said to have been almost totally neglected. In this fact lies my apology for offering to the scientific public the following memoir.

In doing this, so far from thinking that the work contains no error, I hasten to disarm criticism, and to ask with solicitude for a favorable reception, in view of the difficulties of the investigation, which I have conducted alone, and almost unaided.

The investigation was first undertaken in connection with my elementary studies of *Materia Medica* and *Therapeutics*, and has since been prosecuted at intervals amidst the distractions of medical teachings and practice, and in some cases without immediate access to authorities. The field covered is so wide that it is almost impossible to exhaust it, and, if it were not for rapidly increasing professional engagements, I would gladly devote more time to the subject; but, as it is, I must leave to others to carry on the work thus begun.

While saying this, it is but just to state that nothing here published has been done hastily, but that all is the result of arduous and conscientious investigation.

A very large part of my material has been of my own gathering, and was studied whilst fresh; but I am indebted to several persons for aid by collections.

First of all, I desire to offer my thanks to Dr. J. S. Billings, U. S. A., and to Professor Ravenel, of South Carolina; to the former for assistance in various ways, and for collections made near Washington City; to the latter for very large collections made in Texas, South Carolina, and Georgia. I am also indebted to Mr. C. F. Austin for a large collection gathered in Northern New Jersey, to Mr. William Canby for some beautiful specimens obtained in Florida, to Professor Sereno Watson for Rocky Mountain plants, and to Dr. Frank Lewis for a number of White Mountain desmids.

These various collections were partly dried and partly preserved in a watery solution of carbolic acid or of acetate of alumina, both of which I have found more or less satisfactory preservatives.

The present investigations embrace all families of the fresh-water algæ except the *Diatomaceæ*, which, as every one knows, are so numerous as to constitute in

themselves a special study. As I have paid no attention to these plants, they are of course not included in this memoir.

In the synonymy I have generally followed Prof. Rabenhorst. The original descriptions of the forms, especially those of the older authorities, are very frequently so meagre and obscure, that the species cannot be recognized by them with any certainty. Prof. Rabenhorst has gone over the ground most carefully, with access to the whole literature of the subject and probably to all extant type specimens, and his decisions are, no doubt, as accurate as the circumstances will allow. To attempt to differ from them, to go behind his work to the original sources and make fresh interpretations, would cause endless confusion. I have, therefore, nearly always contented myself with his *dictum*, and have referred to him as the authority for the names used.

The following references were omitted through a misunderstanding from the first portion of the text.

- Page 14. *Calosphaerium dubium*, GRUNNOW. RABENHORST, Flora Europ. Algarum, Sect. I. p. 55.
 " 15. *Merismopedia convoluta*, BRÉBISSON. RABENHORST, Flora Europ. Algarum, Sect. I. p. 58.
 " 18. *Oscillaria chlorina*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. I. p. 97.
 " 18. *O. Fröhlichii*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. I. p. 109.
 " 19. *O. nigra*, VAUCHER. RABENHORST, Flora Europ. Algarum, Sect. I. p. 107.
 " 19. *O. limosa*, AGARDII. RABENHORST, Flora Europ. Algarum, Sect. I. p. 104.
 " 21. *Chthonoblastus repens*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. I. p. 132.
 " 22. *Lyngbya muralis*, AGARDII. HARVEY, Nereis Boreali-Americana, pt. III. p. 104.

In the text after the "*Habitat*," a name is quoted as the authority therefor; if such a name be in brackets, it signifies that the specimens were simply collected by such individual, but that the identification was made by some one else; when there is not a name *uninclosed* in brackets, it is meant that the identification was made by the author of this memoir.

Since the present memoir has gone to press, I have received from the author a copy of "Algæ Rhodiaceæ. A list of Rhode Island Algæ, collected and prepared by Stephen T. Olney, in the years 1846-1848, now distributed from his own herbarium."

In the introduction to this list, Mr. Olney says: "Of the fresh-water species, I have few for distribution. These were obtained mainly in the environs of this city, and were placed in twenty-seven small vials in Goadsby's solution, and sent to Prof. Harvey, who submitted them to the judgment of the most learned English botanist in this particular department, G. H. K. Thwaites, Esq., then of Bristol, England. The large number of species found in this collection, in so limited a range, and collected within a very short period, is surprising, and shows what more persistent collections will develop. I have not time to collate the numerous publications of the lamented Prof. Bailey, or I might have made the list of this portion of Rhode Island plants more complete."

The chlorosperms of this list are as follows:—

Porphyra vulgaris, AG.-HARV. Ner Bor. Am. 3. 53. Newport.

Bangia fuscopurpurea, LYNGB.-HARV. Ner. Bor. Am. 3. 54. Southern Rhode Island.

- Enteromorpha intestinalis*, LYNGB.-HARV. Ner. Bor. Am. 3. 56. Providence to Newport.
Enteromorpha compressa, GREV.-HARV. Ner. Bor. Am. 3. 56. Southern Rhode Island.
Enteromorpha clathrata, GREV.-HARV. Ner. Bor. Am. 3. 56. Newport.
Ulva latissima, L.-HARV. Ner. Bor. Am. 3. 59. Providence.
Ulva lactuca, L.-HARV. Ner. Bor. Am. 3. 60. Providence.
Tetraspora lacunosa, CHAUV.-HARV. Ner. Bor. Am. 3. 61. *T. perforata*, BAILEY Mss. Providence.
Tetraspora lubrica, AG. Providence.
Batrachospermum pulcherrimum, HASS. Providence.
Batrachospermum moniliforme, ROTH.-HARV. Ner. Bor. Am. 3. 63. Providence.
Chaetophora endivæfolia, AG.-HARV. Ner. Bor. Am. 3. 69. Providence.
Draparndia glomerata, AG.-HARV. Ner. Bor. Am. 3. 72. Providence.
Stigeoclonium minutum, KÜTZ. Providence.
Cladophora rupestris, L.-HARV. Ner. Bor. Am. 3. 74. Newport.
Cladophora glaucescens, GRIFF.-HARV. Ner. Bor. Am. 3. 77. Rhode Island.
Cladophora refracta, ROTH.-HARV. Ner. Bor. Am. 3. 79. Southern Rhode Island.
Cladophora Rudolphiana, AG.-HARV. Ner. Bor. Am. 3. 80. Providence.
Cladophora gracilis, GRIFF.-HARV. Ner. Bor. Am. 3. 81. Little Compton.
Cladophora fracta, HARV. Ner. Bor. Am. 3. 82. Rhode Island, Bailey.
Chaetomorpha ærea, DILLW.-HAR. Ner. Bor. Am. 3. 86. Newport, etc.
Chaetomorpha Olneyi, HARV. Ner. Bor. Am. 3. 86. Little Compton.
Chaetomorpha longiarticulata, HARV. Ner. Bor. Am. 3. 86. Little Compton.
var. crassior, HARV. Ner. Bor. Am. 3. 86. Little Compton.
Chaetomorpha sutoria, BERK.-HARV. Ner. Bor. Am. 3. 87. Newport.
Zygnema malformatum, HASS. 1. 147. Providence.
Zygnema catenæforme, HASS. 1. 147. Providence.
Zygnema Thwaitesii, OLNEY, n. s. Near *Z. subcentricosum*, Providence.
Zygnema longatum, HASS. 1. 151. Providence.
Zygnema striata, OLNEY, n. s. "Cells evidently striated," Thwaites. Providence.
Tyndaridea bicornis? HASS. 1. 162. Providence.
Tyndaridea insignis? HASS. 1. 163. Providence.
Mesocarpus parvulus, HASS. 1. 169. Providence.
Mougeotia genuflexa, AG.-HASS. 1. 173. Providence.
Vesiculifera concatenata, HASS. 1. 201. Providence.
Vesiculifera æqualis, HASS. 1. 205. Providence.
Vesiculifera bombycina, HASS. 1. 208. Providence.
Vesiculifera Candollii, HASS. 1. 208. Providence.
Bulbochæte Thwaitesii, OLNEY, n. s. Providence.
Lyngbya majuscula, HARV. Bor. Am. 3. 101. Providence.
Sphæroplea virescens, BERK. Providence.
Sphæroplea punctalis, BERK. Providence.
Tolypothrix distorta, KÜTZ.-HASS. 1. 240.
Calothrix confervicola, AG.-HARV. Ner. Bor. Am. 3. 105. Providence.
Calothrix scopulorum, AG.-HARV. Ner. Bor. Am. 3. 105. Providence.
Hyalotheca dissiliens, BREV.-RALFS. Des. 51. (*Glocoprium*.) Providence.
Hyalotheca mucosa, EHRH.-RALFS. Des. 53. Providence.
Didymoprium Grevillii, KÜTZ.-RALFS. Des. 61. Rhode Island, Bailey.
Didymoprium Borreri, RALFS. Des. 58. Rhode Island, Bailey.
Desmidium Swartzii, AG.-RALFS. Des. 61. Throughout United States, Bailey.
Aptogonum Baileyi, RALFS. Des. 209. Worden's Pond, Rhode Island, Bailey.
Micrasterias rotata, RALFS. Des. 71. Providence.
Micrasterias radiosa, AG.-RALFS. Des. 72. Maine to Virginia, Bailey.
Micrasterias furcata, RALFS. Des. 73. Worden's Pond, Rhode Island, Bailey.
Micrasterias Crux-Melilensis, RALFS. Des. 73. Maine to Virginia, Bailey.
Micrasterias truncata, BREV.-RALFS. Des. 75. United States, Bailey.

- Micrasterias foliacea*, BAILEY-RALFS. Desm. 210. Worden's Pond, Rhode Island, Bailey.
Micrasterias Bayleyi, RALFS. Desm. 211. Rhode Island, Bailey.
Euastrum oblongum, RALFS. Des. 80. Rhode Island, Bailey.
Euastrum crassum, KÜTZ.-RALFS. Des. 81. Rhode Island, Bailey.
Euastrum ansatum, EHRH.-RALFS. Des. 85. *E. binale* KNTZ. Providence.
Euastrum elegans, KÜTZ.-RALFS. Des. 89. Providence.
Euastrum binale, RALFS. Desm. 91. Providence.
Cosmarium cucumis, CORDA-RALFS. Desm. 93. United States, Bailey.
Cosmarium bioculatum, RALFS. Des. 95. Providence.
Cosmarium Meneghini, BREB.-RALFS. Des. 96. United States, Bailey.
Cosmarium crenatum, RALFS. Des. 96. Providence.
Cosmarium amœnum, BREB.-RALFS. Des. 102. Providence.
Cosmarium ornatum, RALFS. Des. 104. Providence.
Cosmarium connatum, BREB.-RALFS. Des. 108. Providence.
Cosmarium Cucurbita, RALFS. Des. 109. Providence.
Cosmarium grandituberculatum, OLNEY, n. s. ; "near *C. cucumis*, but with large tubercles on the frond." Providence.
Staurastrum orbiculare, RALFS. Des. 125. Providence.
Staurastrum hirsutum, RALFS. Des. 127. Providence.
Staurastrum Hystrix, RALFS. Des. 128. Providence.
Staurastrum gracile, RALFS. Des. 136. Providence.
Staurastrum tetracerum, RALFS. Des. 137. United States, Bailey.
Staurastrum cyrtocerum, BREB.-RALFS. Des. 139. Providence.
Tetmemoras Brébissoni, RALFS. Des. 145. Providence.
Tetmemoras granulatum, RALFS. Des. 146. Providence.
Penium margaritaceum, BREB.-RALFS. Des. (*Closterium* EHR.) Providence.
Penium Digitus, BREB.-RALFS. Des. 151. (*Closterium lamellosum*.)
Docidium nodulosum, BREB.-RALFS. Des. 155. Maine to Virginia, Bailey.
Docidium Baculum, BREB.-RALFS. Des. 158. United States. Bailey.
Docidium nodosum, BAILEY-RALFS. Des. 218. United States, Bailey.
Docidium constrictum, BAILEY-RALFS. Des. 218. Worden's Pond, Bailey.
Docidium verrucosum, BAILEY-RALFS. Des. 218. Rhode Island, Bailey.
Docidium verticillatum, BAILEY-RALFS. Des. 218. Worden's Pond, Bailey.
Closterium Lunula, EHRH.-RALFS. Des. 163. New England, Bailey.
Closterium moniliferum, EHRH.-RALFS. Des. 163. New England, Bailey.
Closterium striolatum, EHRH.-RALFS. Des. 173. New England, Bailey.
Closterium cuspidatum, BAILEY-RALFS. Des. 219. Worden's Pond, Bailey.
Pediastrum tetras, RALFS. Des. 182. New England, Bailey.
Pediastrum heptactis, RALFS. Des. 183. Providence.
Pediastrum Boryanum, MENEGH.-RALFS. Des. 187. Maine to Mexico, Bailey.
Pediastrum ellipticum, HASS.-RALFS. Des. 188. Maine to Virginia, Bailey.
Scenedesmus quadricauda, BREB.-RALFS. Des. 190. Maine to Virginia, Bailey.
Scenedesmus obtusus, MEYEN.-RALFS. Des. 193. Maine to Virginia, Bailey.

INTRODUCTION.

ALTHOUGH beset with difficulties in the outset, no branch of natural science offers more attractions, when once the study is fairly entered upon, than the fresh-water algæ. The enthusiasm of the student will soon be kindled by the variety and beauty of their forms and wonderful life processes, and be kept alive by their abundance and accessibility at all seasons of the year; for unlike other plants, the winter with them is not a period of counterfeited death, but all seasons, spring, summer, autumn, and winter alike, have their own peculiar species. They have been found in healthy life in the middle of an icicle, and in the heated waters of the boiling spring; they are the last of life alike in the eternal snow of the mountain summit and the superheated basin of the lowland geyser.

In their investigation, too, the physiologist can come nearer than in almost any other study to life in its simplest forms, watching its processes, measuring its forces, and approximating to its mysteries. Sometimes, when my microscope has revealed a new world of restless activity and beauty, and some scene of especial interest, as the impregnation of an œdogonium, has presented itself to me, I confess the enthusiastic pleasure produced has been tempered with a feeling of awe.

To any on whom through the want of a definite pursuit the hours hang heavy, to the physiologist who desires to know cell-life, to any student of nature, I can commend most heartily this study as one well worthy of any pains that may be spent on it.

An aquarium will often, in the winter time, give origin to numerous interesting forms, but it is not a necessity to the fresh-water algologist; besides his microscope and its appliances, all that he absolutely needs is a few glass jars or bottles and the fields and meadows of his neighborhood.

The great drawback to the investigation of these plants has been the want of accessible books upon them. In the English language there is no general work of value, and the various original memoirs are separated so far and wide in the Continental and English journals, as to be of but little use to most American readers. The *Flora Europæum Algarum Aquæ Dulcis et Submarinæ*, of Prof. Rabenhorst, has done much to facilitate the study, and its cheapness brings it within the reach of all. It merely gives, however, brief diagnoses of the various species, but with the present memoir will, I trust, suffice for the American student, at least until he is very far advanced in his researches.

A certain amount of experience and knowledge of the subject greatly facilitates the collection of these plants, but scarcely so much as in other departments of cryptogamic botany, since most of the species are so small that the most experienced algologist does not know how great the reward of the day's toil may be until he places its results under the object glass of his compound microscope. In order to aid those desirous of collecting and studying these plants, I do not think I can do better than give the following hints as to when and where to find, and how to preserve them.

There are three or four distinct classes of localities, in each of which a different set of forms may be looked for. These are: stagnant ditches and pools; springs, rivulets, large rivers, and other bodies of pure water; dripping rocks in ravines, &c.; trunks of old trees, boards, branches and twigs of living trees, and other localities.

In regard to the first—*stagnant waters*—in these the most conspicuous forms are oscillatorie and zygнемасеæ. The oscillatorie may almost always be recognized at once, by their forming dense, slimy strata, floating or attached, generally with very fine rays extending from the mass like a long, delicate fringe. The stratum is rarely of a bright green color, but is mostly dark; dull greenish, blackish, purplish, blue, &c. The oscillatorie are equally valuable as specimens at all times and seasons, as their fruit is not known, and the characters defining the species do not depend upon the sexual organs. The zygнемасеs are the bright green, evidently filamentous, slimy masses, which float on ditches, or lie in them, entangled amongst the water plants, sticks, twigs, &c. They are only of scientific value when in fruit, as it is only at such times that they can be determined. Excepting in the case of one or two very large forms, it is impossible to tell with the naked eye with certainty whether a zygнемасе is in fruit or not; but there are one or two practical points, the remembrance of which will very greatly enhance the probable yield of an afternoon's search. In the first place, the fruiting season is in the spring and early summer, the latter part of March, May, and June being the months when the collector will be best repaid for looking for this family. Again, when these plants are fruiting they lose their bright green color and become dingy, often yellowish and very dirty looking—just such specimens as the tyro would pass by. The fine, bright, green, handsome masses of these algæ are rarely worth carrying home. After all, however, much must be left to chance; the best way is to gather small quantities from numerous localities, keeping them separate until they can be examined.

Adhering to the various larger plants, to floating matters, twigs, stones, &c., in ditches, will often be found filamentous algæ, which make fine filmy fringes around the stems, or on the edges of the leaves; or perchance one may meet with rivularie or nostoes, &c., forming little green or brownish balls, or indefinite protuberances attached to small stems and leaves. These latter forms are to be looked for especially late in the season, and whenever seen should be secured.

In the latter part of summer, there is often a brownish, gelatinous scum to be seen floating on ditches. Portions of this should be preserved, as it frequently contains interesting nostoes and other plants.

In regard to large *rivers*, the time of year in which I have been most successful in such localities is the latter summer months. *Springs* and small bodies of clear water may be searched with a hope of reward at any time of the year when they are not actually frozen up. I have found some exceedingly beautiful and rare algæ in such places as early as March, and in open seasons they may be collected even earlier than this. The desmids are most abundant in the spring, and possibly most beautiful then. They, however, rarely conjugate at that time, and the most valuable specimens are therefore to be obtained later—during the summer and autumn months; at least, so it is said; and the experience I have had with this family seems to confirm it. *Rivulets* should be watched especially in early spring, and during the summer months.

From the time when the weather first grows cool in the autumn, on until the cold weather has fairly set in, and the reign of ice and snow commences, is the period during which the algæ hunter should search carefully all wet, *dripping rocks*, for specimens. Amongst the stems of wet mosses—in dark, damp crevices, and little grottos beneath shelving rocks—is the algæ harvest to be reaped at this season. Nostocs, palmellas, conjugating desmids, sirosiphons, various unicellular algæ, then flourish in such localities. My experience has been, that late in the autumn, ravines, railroad cuttings, rocky river-banks, &c., reward time and labor better than any other localities.

The vaucherias, which grow frequently on wet ground, as well as submerged, fruit in the early spring and summer in this latitude, and are therefore to be collected at such times, since they are only worth preserving when in fruit.

In regard to algæ which grow on trees, I have found but a single species, and do not think they are at all abundant in this latitude. Farther south, if one may judge by Professor Ravenel's collections, they are the most abundant forms.

Although perhaps of but little interest to the distant collector, yet for the sake of those living nearer, I will occupy a few lines with an account of the places around Philadelphia which will best repay a search for fresh-water algæ. As is well known, below the city, there is what is known as the "Neck," a perfectly level extent of ground lying in the fork between the rapidly approaching rivers, Schuylkill and Delaware. This is traversed by numerous large ditches, and, especially just beyond the city confines, has yielded to me an abundant harvest. My favorite route is by the Fifth Street cars to their terminus, then across the country a little to the east of south until the large stone barn, known as "Girard's Barn," is reached. A large ditch lies here on each side of the road, which is to be followed until it crosses the Pennsylvania Railroad, then along this to the west, until the continuation of Tenth Street crosses it. Here the ditches cease, and the steps are to be turned homeward. From Girard's barn to the crossing just alluded to, ditches great and small lie all along and about the route, ditches which have often most abundantly rewarded my search, and enabled me to return home richly laden. The best season for collecting here is from March to July, and again in October, when some of the nostocs may be looked for.

Crossing the river Delaware to the low country below and above the city of Camden, the collector will find himself in a region similar to that just described,

and like it cut up by numerous ditches, in which are pretty much the same forms as in the "Neck." But by taking the Camden and Atlantic cars for twenty to forty miles into New Jersey to what is known as the "Pines," he will get into a very different country; low, marshy, sandy grounds, with innumerable pools, and streams whose dark waters, amber-colored from the hemlock roots over which they pass, flow sluggishly along. I have been somewhat disappointed in my collections in such localities. Fresh-water algæ do not appear to flourish in infusion of hemlock, and consequently the streams are very bare of low vegetable life. On the other hand, in pools in the more open places, my search has been repaid by finding some very curious and interesting forms, which apparently are peculiar.

North of Philadelphia are several places, which at certain seasons will richly reward the microscopist. Along the Delaware River, there is a similar country and flora to that of the "Neck." But back from the river things are quite different. The North Pennsylvania Railroad passes near Cheltenham Hills, some eight miles or so from the city, through some deep rock cuttings, which are kept constantly dripping by numerous minute springs bursting from between the strata. At the proper season, these will yield an abundant harvest. Besides these, there is also a stream of water with ponds running along by the road, which should be looked into. I have seldom had more fruitful trips than some made very early in the spring to this locality; but then it was in little pools in the woods, and especially in a wooded marsh or meadow to the left of the road, some distance beyond the station, that I found the most interesting forms.

The Schuylkill River and its banks have afforded materials for many hours of pleasant work. In the river itself a few very interesting forms have been found; but it is especially along its high banks that the harvest has been gathered.

The dripping rocks and little wood pools in the City Park are well worth visiting; but the best locality is the western bank, along the Reading Railroad, above Manayunk, between it and the upper end of Flat Rock tunnel. Down near the river, at the lower end of the latter, will be found a number of beautiful, shaded rocky pools, which, in the late summer, are full of *Chactophora* and other algæ. Along the west rocks of the river side of the bluff, through which the tunnel passes, are to be found, late in the fall, numerous algæ. It is here that the *Palmella Jessenii* grows in such abundance.

West of the city, in Delaware and Chester Counties, is a well wooded and watered, hilly country, in which, here and there, numerous fresh-water algæ may be picked up.

As to the preservation of the algæ—most of the submerged species are spoiled by drying. Studies of them should always, when practicable, be made whilst fresh. Circumstances, however, will often prevent this, and I have found that they may be preserved for a certain period, say three or four months, without very much change, in a strong solution of acetate of alumina.

An even better preservative, however, and one much more easily obtained, is carbolic acid, for I have studied desmids with great satisfaction, which had been preserved for five or six years in a watery solution of this substance. In regard to the strength of the solution I have no fixed rule. Always simply shaking up

a few drops of the acid with the water, until the latter is very decidedly impregnated with it, as indicated by the senses of smell and taste.

Almost all species of algæ which are firm and semi-cartilaginous, or almost woody in consistency, are best preserved by simply drying them, and keeping them in the ordinary manner for small plants. The fresh-water algæ which bear this treatment well belong to the *Phycocchromophyceæ*, such as the *Nostocs*, *Scytonema*, &c., the true confervas not enduring such treatment at all. When dried plants are to be studied, fragments of them should be soaked for a few minutes in warm, or for a longer time in cold water.

The only satisfactory way that algæ can be finally prepared for the cabinet is by mounting them whole or in portions, according to size, for the microscope. Of the best methods of doing this, the present is hardly the time to speak; but a word as to the way of cleaning them will not be out of place. Many of them, especially the larger filamentous ones, may be washed by holding them fast upon an ordinary microscope slide, with a bent needle or a pair of forceps, and allowing water to flow or slop over them freely, whilst they are rubbed with a stiffish camel's-hair pencil or brush. In other cases, the best plan is to put a mass of the specimens in a bottle half full of water, and shake the whole violently; drawing off the water from the plants in some way, and repeating the process with fresh additions of water, until the plants are well scoured. At first sight, this process would seem exceedingly rough, and liable to spoil the specimens, but I have never seen bad results from it, at least when practised with judgment. The water seems so to envelop and protect the little plants that they are not injured.

After all, in many instances it appears impossible to clean these algæ without utterly ruining and destroying them—the dirt often seeming to be almost an integral portion of them; so that he who despises and rejects mounted specimens, simply because they are dirty and unsightly, will often reject that which, scientifically speaking, is most valuable and attractive.

In finally mounting these plants, the only proper way is to place them in some preservative solution within a cell on a slide. After trial of solution of acetate of alumina and various other preservative fluids, I have settled upon a very weak solution of carbolic acid, as the best possible liquid to mount these plants in. Acetate of alumina would be very satisfactory were it not for the very great tendency of the solution to deposit minute granules, and thus spoil the specimens. As every one knows, the great difficulty in preserving microscopic objects in the moist way is the perverse tendency of the cells to leak, and consequently slowly to allow entrance to the air and spoil the specimen.

As I have frequently found to my great chagrin, the fact that a slide has remained unchanged for six months, or even a year, is no guarantee that it will remain so indefinitely. It becomes, therefore, exceedingly important to find some way of putting up microscopic objects that can be relied on for their preservation. Where carbolated glycerine jelly or Canada balsam can be used, the solid coating which they form around the specimens constitutes the best known protection. Except in the case of the diatoms, however, these substances so shrivel and distort the fresh-water algæ immersed in them as to utterly ruin them. I lost so many specimens

by the old ways of mounting, that, becoming disheartened, I gave up all idea of making a permanent cabinet, until a new cement, invented by Dr. J. G. Hunt, of this city, was brought to my notice. This is prepared as follows:—

“Take damar gum, any quantity, and dissolve it in benzole; the solution may be hastened by heat. After obtaining a solution just thick enough to drop readily from the brush, add enough of the finest dry oxide of zinc—previously triturated in a mortar with a small quantity of benzole—until the solution becomes white when thoroughly stirred. If not too much zinc has been added, the solution will drop quickly from the brush, flow readily, and dry quickly enough for convenient work. It will adhere, if worked properly, when the cell-cover is pressed down, even when glycerine is used for the preservative medium. Keep in an alcohol-lamp bottle with a tight lid, and secure the brush for applying the cement in the lid of the bottle.”

Its advantages lie in the circumstance, that the glass cover can be placed upon the ring of it whilst still fresh and soft, and that in drying, it adheres to both cover and slide, so as to form a joint between them of the width of the ring of cement, and not, as with asphaltum, gold size, &c., simply at the edge and upon the outside of the cover. It is readily to be seen how much less liability to leakage must result from this. The method of mounting with it is as follows: A ring of any desired size is made, by means of an ordinary Shadbolt's turn-table, upon a slide, which is then placed to one side to dry. When required for use, the specimen, cover, &c., being all prepared and ready, the slide is again placed upon the turn-table and a new ring of cement put directly upon the old one. The specimen is immediately placed within the cell thus formed, and the requisite quantity of the carbolated water placed upon it. The cover, which must be large enough to entirely or nearly cover the cement ring, is now picked up with the forceps, the under side being moistened by the breath to prevent adhesion of air-bubbles, and placed carefully in position. It is now to be carefully and equably pressed down with some force. By this, any superfluous water is squeezed out and the cover is forced down into the cement which rises as a little ring around its edge. The pressure is best made with a stiff needle, at first on the centre and then upon the edges of the cover, which may finally be made slowly to revolve underneath the needle point. The slide may then be put aside to dry; or, better, an outside ring of the cement thrown over its edge in the usual manner. Where a deep cell is required, several coats of the cement should be placed one over the other, each being allowed to dry in turn. If time be an object, and only a shallow cell be necessary, the first ring of cement may be dispensed with, and the whole mounting of the specimen be done in a few minutes. Even with this cement and the utmost care in mounting, the cabinet should be occasionally inspected, for there will always be some slides into which air will penetrate. When such are found, efforts may be made to stop the leak by new rings of cement overlaid upon the old, but very often entire remounting of the specimen is the only satisfactory cure.

The classification which I have adopted in this memoir is that of Professor Rabenhorst. I have finally selected it, not as being absolutely natural, but as convenient, and as rarely doing much violence to the natural relations of the various species.

Our knowledge of the life-history of the algæ must make very many advances before the true system can be developed, and abstinence from adding to the present numerous classifications is an exhibition of self-control not very common.

There are, however, certain great groups, which are already plainly foreshadowed, and which no doubt will be prominent points in the perfected classification. Amongst these are the *Conjugatæ*, or those plants in which sexual reproduction occurs by the union of two similar cells. In the present paper all the plants of this family described are together, since the diatoms are not noticed; but in Rabenhorst's work the latter plants are very widely separated from their fellows, and this seems to me the weak point of the Professor's system.

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FRESH-WATER ALGÆ OF THE UNITED STATES.

CLASS **PHYCOCHROMOPHYCEÆ.**

Plantæ uni- vel multicellulares, in aqua vigentes vel extra aquam in mucro matricali nidulantes, plerumque familias per cellularum generationes successivas ortas formantes.

Cytioderma non siliceum, combustibile.

Cytoplasm phycochromate coloratum, nucleo destitutum, granulis amylaceis plerumque nullis.

Propagatio divisione vegetativa, gonidiis immobilibus vel sporis tranquillis.

Unicellular or *multicellular* plants living in water, or incased in a maternal jelly out of it, mostly in families formed from successive generations of cells.

Cytioderm not siliceous, combustible.

Cytoplasm an endochrome, brown, olivaceous, fuscous, &c., destitute of nucleus, mostly without starch granules.

Propagation by vegetative division, by immovable gonidia or tranquil spores.

The phycochroms are plants at the very bottom of the scale, distinguished by the simplicity of their structure and the color of their protoplasm, which, instead of being of the beautiful green that marks chlorophyll, is fuscous, or yellowish, bluish, brownish, or sometimes particolored, and rarely greenish, but of a shade very distinct from the chlorophyll green, more lurid, bluish or yellowish, or olivaceous in its hue. The nucleus appears to be always wanting. The cell wall is oftentimes distinct and sharply defined, but in many instances it is not so, the walls of different cells being fused together into a common jelly in which they are imbedded. In a large suborder the wall is replaced by a sheath, which in some genera surrounds cells with distinct walls, in others, cells without distinct walls, and in still others, a long cylindrical mass of endochrome, which may be looked upon as a single cell.

Many of the phycochroms are unicellular plants in the strictest sense of the word, but more often the cells are conjoined, so as to form little families, each cell of which is in a sense a distinct individual capable of separate life, yet the whole bound together into a composite individual. Rarely the phycochrom is a multi-

cellular plant in the stricter use of the term. Increase takes place by the multiplication of cells by division, and also by the formation of enlarged thick-walled cells, to which the name of spores has been given, although it is entirely uncertain whether they are or are not the result of sexual action. There are numerous peculiar forms of cell multiplication by division occurring in these plants, the discussion of which will be found scattered through the remarks on the various families and genera.

The method of reproduction, and in fact the life history in general, of the phycochroms, is still involved in such mystery, that I am not aware that absolute sexual generation has been demonstrated in any of them. This being the case, it is not to be wondered at that many have conjectured as possible, and some have roundly asserted as true, that the phycochroms are merely stages in the life history of higher plants; that they are not species, and, consequently, that any attempt at describing such is little more than a busy idleness. In regard to some of them it has certainly been rendered very probable that they are merely fixed stages of higher plants. On the other hand, in the great bulk of the forms, no proof whatever has been given that they are such. They all certainly have fixed, definite characters, capable of being expressed and compared, so that the different forms can be defined, recognized, and distinguished. If, therefore, future discoveries should degrade them as subordinate forms, names will still be required, and definitions still be necessary to distinguish them one from the other, so long as they are common objects to the microscopist.

If *Nostoc commune*, for example, were proven to be a peculiar state or development of *Polytricum commune*, I conceive it would be still known as *Nostoc commune*. But, as previously stated, no proof whatever has as yet been furnished for the vast majority of the plants of this family, to show that they bear any such relation to higher plants; and until some such proof is forthcoming, certainly the only scientific way to act, is to treat them as distinct species.

ORDER **Cystiphoræ.**

Plantæ unicellulares. Cellulæ singulæ vel plures in familias consociatæ.

Unicellular plants. Cells single or consociated in families.

In this order the cells are oblong, cylindrical, spherical, or angular. They are sometimes single, or more commonly are united by a common jelly into families, which sometimes are surrounded by distinct coats. The mucus or jelly, in which the cells are imbedded, is mostly, but not always, colorless, and varies in firmness from semifluid to cartilaginous. The division of the cells may take place either in one, two, or three directions or planes.

FAMILY CHROOCOCCACEÆ.

Character idem ac ordine.

Characters those of the order.

Genus CHROOCOCCUS, NÆGELI.

Cellulæ globosæ ovales vel a pressione mutua plus minus angulosæ, solitariæ vel in familias consociatæ, liberæ (a vesica matricali non involutæ); cytiodermate achromatico, homogæneo, sæpe in mucos plus minus firmo confluyente; cytoplasmate æruginoso vel pallide cæruleo-viridi, non rare luteolo vel aurantiaco, interdum purpurascente. Generationum successivarum divisio alternatim ad directiones tres.

Syn.—*Protococcus*, AG. et KtZ., &c., ex parte. *Pleurococcus*, MENGH.

Globulinæ et *Protosphæriæ*, TURPIN, ex part.

Cells globose, oval, or from mutual pressure more or less angular, solitary, or consociated in free families (not involved in a maternal vesicle); Cytiderm achromatic, homogeneous, often confluent into a more or less firm mucus; cytoplasm æruginous or pale bluish-green, not rarely yellowish or orange, sometimes purplish. Successive generations arising by alternate division in three directions.

C. refractus, WOOD.

C. cellulis in familias solidas arcte consociatis, plerumque subquadratis, sæpius triangularibus, rare angulosis; familiis sæpius lobatis; cytiodermate tenui, vix visibile, achroo; cytoplasmate subtiliter granulato, subfusco vel subluteo vel olivaceo, valde refringente.

Diam.—Cell $\frac{1}{8000}$ "— $\frac{1}{3000}$ ", rare in cellulis singulis $\frac{1}{2000}$ "; famil. $\frac{1}{1500}$ "— $\frac{1}{700}$ ".

Syn.—*C. refractus*, WOOD, Prodr. Amer. Philos. Soc., 1869, 122.

Hab.—In rupibus irroratis prope Philadelphia.

Cells closely associated together into solid families, mostly subquadrate, very often triangular, rarely multiangular; families often lobed; cytioderm thin, scarcely perceptible, transparent; cytoplasm finely granular, brownish, olivaceous, or yellowish, highly refractive.

Remarks.—The color of this species varies from a marked almost fuscous brown to a light yellowish-brown, the lighter tints being the most common. The cells are remarkable for their powerful refraction of the light, resembling often oil as seen under the microscope, especially if they be the least out of the focus. They are very closely joined together to form the families, many of which are composed only of four cells. Often, however, a large number of the cells are fused together into a large, irregular, more or less lobate family, and these sometimes are closely joined together into great irregular masses. I have occasionally seen large single cells with very thick coats, whose protoplasm was evidently undergoing division. Are such a sort of resting spore? The color of the protoplasm varies. Perhaps the more common hue is a sort of clay tint. Bluish-olive and a very faint yellowish-brown are not rarely seen. The species grows abundantly on the wet rocks along the Reading Railroad between Manayunk and the Flat Rock tunnel.

Fig. 5, pl. 5, represents different forms of this species; those marked *a*, magnified 750 diameters; *b*, 470 diameters; *c*, 950 diameters.

C. multicoloratus, WOOD.

C. in strato mucoso inter algas varias sparsus; cellulis singulis et sphaericis, vel 2-4 (rare 8) aut angulis aut semisphaericis aut abnormibus in familias oblongas consociatis; cytiodermate crasso, hyalino, haud lamelloso; tegumentis plerumque nullis, interdum subnullis; cytoplasmate plerumque homogæneo, interdum subtiliter granulato, vel luteo-viride vel cæruleo-viride vel luteo vel subnigro, vel brunneo, vel saturate aurantiaco, sæpe ostro tincto.

Diam.—Cell., sing. sine tegm., $\frac{7}{3000}$ " cum teg. $\frac{2}{1500}$ "; cell. in famil. sing. $\frac{2}{4500}$ "— $\frac{4}{1500}$ ".
Fam. long. $\frac{7}{4500}$ "— $\frac{1}{4000}$ "; lat. $\frac{8}{4500}$ "— $\frac{7}{4500}$ ".

Syn.—*C. multicoloratus*, WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, 122.

Hab.—in rupibus humidis prope Philadelphia.

C. occurring scattered in a mucous stratum with other algæ; cells spherical and single, or else angular semi-spherical or irregular and associated together in oblong families of from 2-4 (rarely 8); inner coat thick, hyaline, not lamellate; outer coat generally wanting, sometimes indistinctly present; endochrome mostly homogeneous, sometimes minutely granular, either a yellowish-green or bluish-green, or yellowish or brown, or blackish, sometimes tinged with bright lake.

The cells of this species do not appear to have any tendency to unite to form large masses or fronds. On the contrary they are generally very distinct. Their color varies very much, in a larger number of instances they were a decided yellowish-green, tinged at some point or other with a beautiful lake. When several cells are formed by division out of one cell, a similar division of the very thick surrounding gelatinous coat follows separating them finally entirely one from the other. I have seen a single cell which appears to be an encysted form of this, of which I give a drawing.

Fig. 6, pl. 5, represents different forms of this species magnified 260 diameters.

C. thermophilus, WOOD.

C. cellulis singulis aut geminis vel quadrigeminis et in familias consociatis, oblongis vel subglobosis, interdum angulosis, haud stratum mucosum formantibus; tegumento crassissimo, achroo, haud lamelloso, homogæneo; cytoplasmate viride, interdum subtiliter granulato, interdum homogæneo.

Diam.—Cellulæ singulæ sine tegumento longitudo maxima $\frac{1}{1600}$ "', latitudo maxima $\frac{1}{2300}$ "'.

Syn.—*C. thermophilus*, WOOD, American Journal Science and Arts, 1869.

Hab.—Benton Springs, Owen Co., California (Mrs. Parz.).

Cells single, geminate, or quadrigeminate and consociated into families, oblong or subglobose, sometimes angular, not forming a mucous stratum; tegument very thick, transparent, not lamellate, homogeneous; cytoplasm green, sometimes minutely granulate, sometimes homogeneous.

Remarks.—Remarks upon this species will be found under the head of *Nostoc calidarium*, WOOD.

Genus GLOEOCAPSA, KtZ.

"Cellulæ sphericæ aut singulæ aut numerosæ in familias consociatæ; singulæ tegumento vesiculiforme (cytiodermate tumido) inclusæ, post divisionem spontaneam in cellulas duas filiales factam utraque tegumento se induit, dum ambæ tegumento matricali involutæ remanent; cellularum harum filialium iterum in duas cellulas divisione continuo repetita, tegumentum ataviæ restat et sese extendens familiam totam circumvelat. Cytioderma crassum, sæpe crassissimum, cellulæ lumen crassitie æquans vel superans, achromaticum vel coloratum, plerumque lamellosum; lamellæ vel strata non raro discedentia. Cytoplasma æruginosum, cæruleo-viride, chalybeum, rufescens, luteo-fuscum, &c. Cellularum divisio directione ad tres dimensiones alternante. Cellulæ generationum ultimarum minores quam priorum sunt." (Rab.)

Syn.—*Globulina et Bichatia*, TURPIN, ex part.

Gloeocapsa, KtZ., ex part.

Microcystis, MENEGH., ex part.

Cells spherical, either single or associated in numbers into families; the single cell included a vesiculiform tegument (the tumid cytioderm); this cell then undergoing division into two daughter-

cells, each of which has a distinct tegument, the whole being surrounded with that of the old mother-cell. This process of division is then repeated again and again, the original cell-wall remaining and surrounding the family thus formed. Cytoderm thick, often very thick, equalling, or exceeding in diameter the cavity of the cell, achromatic or colored, mostly lamellated, lamellæ or strata not rarely separating. Cytoplasm of various colors, æruginous, bluish-green, chalybeate, reddish, yellowish-fuscous, &c. Division of the cells occurring in three directions. The last generation of cells smaller than the earlier ones.

G. sparsa, WOOD.

G. in strato mucoso sociis algis variis sparsa; cellulis sphaericis, vel oblongis vel ovatis, 2-8 in familias consociatis; familiis subglobosis vel subovatis, interdum numeroso-aggregatis; tegumentis internis aureofuscis, firmis, rarissime coloris expertibus, homogeneis, vel lamellosis; tegumentis externis achromaticis, rare subachromaticis, plerumque vix visibilibus; cytoplasmate homogeneo.

Diam.—Max. cell. oblong. sine tegum. long., $\frac{1}{3000}''$; lat., $\frac{1}{7500}''$; cell. glob., sine tegum., $\frac{1}{5000}''$; cum tegum., $\frac{1}{1875}''$; fam., $\frac{1}{750}''$.

Syn.—*G. sparsa*, WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, 123.

Hab.—In rupibus irroratis prope Philadelphia.

G. scattered in a mucous stratum composed of various algæ; cells spherical, or oblong, or ovate, associated together in families of from 2-8; families subglobose or subovate, sometimes aggregated together in large numbers; inner tegument yellowish-brown, firm, rarely colorless, homogeneous or lamellate; external tegument achromatic, rarely subachromatic, generally scarcely visible.

Remarks.—This species was found in a rather firm, grumous or gelatinous coating of a light brown color, growing on the rocks at Fairmount Water Works, chiefly composed of a very minute nostochaceous plant, but contained numerous other algæ. The color of the tegument is yellowish-brown, sometimes with some red in it, sometimes with something of a greenish tint. This inner colored coat is not generally more than once or twice lamellate, often it is not at all so. This species seems somewhat allied to *G. styophila*, but differs slightly in the form of the cell, and more especially in not having a distinct thallus, and in the families being small and containing but few cells.

Fig. 7, pl. 8, represents this species, magnified 750 diameters.

Genus CÆLOSPHÆRIUM, NÆGELI.

Thallus parvus, e cellulis minimis in familias periphericas consociatis vel in stratum periphericum simplex et in muco tegumentis celerrime confluentibus formato nidulantibus compositus. Cellularum divisio, initio generationum serierum, in omnem fit directionem, tum denique alternatim ad superficiem sphaericæ utramque directionem.

Thallus small, composed of very small cells consociated into peripheral families, or in a simple peripheral layer, inclosed in their quickly confluent teguments. Division of the cells at first in all directions, afterwards only in each direction on the surface of the sphere.

C. dubium, GRUN.?

C. thallo microscopico, subgloboso vel enorme, natante, congregato; cellulis globosis aut subglobosis; cytoplasmate pallide æruginæo, subtiliter granulato.

Diam.—Cell. plerumque $\frac{1}{5000}'' = .00016''$; rare $\frac{1}{4000}'' = .00025''$; fam. $\frac{1}{2000}''$ — $\frac{1}{1200}'' = .00083''$ — $.0033''$.

Hab.—In aquis stagnis, prope Philadelphia.

Thallus microscopic, subglobose or irregular, floating, aggregated in great numbers; cells globose or subglobose; cytoplasm finely granulate, pale aruginous green.

Remarks.—I found this beautiful little plant forming a dense scum on a stagnant brick-pond, below the city, in the month of July. The scum was of the "color of pea-soup," and so thick was it, that I think a quart of the plants might have been readily gathered. The fronds were of various sizes, and many of them were apparently undergoing division—some of them seemed to have little fronds in their interior. They were composed of an exceedingly transparent firm jelly, in which the cells were placed, often so as to leave the central parts of the frond empty, merely forming a sort of filament-like layer around the edge. Rarely they were in such numbers as to be crowded together over the whole surface of the frond. In some of the younger fronds the cells formed a little ball within the jelly, instead of being scattered through its outer portion. I have seen some large single cells three or four times the size of the ordinary frond cell, swimming amongst the plants, of which they are apparently the reproductive gonidia. Their cell-coats are very firm and thick. The fronds themselves are often closely aggregated together into little masses, and I think it probable that there is a state of the plant, in which the jelly becomes softened and the fronds more or less fused together in protococcus-like masses. This plant appears to be the same as the European *C. dubium*, but differs from the description in the fronds not attaining to anything like the size. It is very probable, however, that this depends upon age or circumstances of growth, and that American plants may be found as large as the European.

Genus MERISMOPEDIA, MEYEN.

Cellulæ globosæ, aut oblongæ, aut ovales, tegumentis confluentibus, 4, 8, 16, 32, 64, 128 in familias tabulatas, unistratas consociatæ. Thallus planus, tenuis, plus minus quadratus, in aqua libere natans. Cellularum divisio in planitie utramque directionem.

Cells globose, oblong, or oval, joined together by their confluent coats into tabular families of 4, 8, 16, 32, 64, 128. Thallus, a more or less quadrate plane, swimming free in the water. Division of the cells occurring in all directions in the one plane.

M. nova, WOOD.

M. thallo membranaceo, distincte limitato, cellulis numerosissimis composito; cellulis ovalibus, arcte approximatis, 16 in familias consociatis, dilute cæruleo-viridibus, interdum medio constrictis; thalli marginibus rectis, integris.

Syn.—*M. nova*, WOOD, Prodr. Amer. Philos. Soc., 1869, 123.

Diam.—Cell. ad. $\frac{1}{30000}$ " = 0.0025".

Hab.—In flumine Schuylkill, prope Philadelphia.

Thallus membranaceous, distinctly limited, composed of very numerous cells; cells oval, closely approximated, consociate in families of 16, light bluish-green, sometimes constricted in the middle; margin of the thallus straight and entire.

Remarks.—The only specimens I have ever seen of this species were found growing in the Schuylkill River adherent to, or entangled in, a lot of filamentous algæ.

The frond is very sharply defined, and, under a low power, is of a uniform bluish-green tint. The cells are associated in primary families of 16, of a number of which the thallus is composed. The species appears to be most closely allied to *M. mediterranea*, Næg., from which it differs very essentially in the size of the fronds, and perhaps even more closely to *M. glauca*, the only character separating it from which is the straight margin. I have myself some doubts whether it ought not to be considered as merely a form of *M. glauca*.

Fig. 8, pl. 8, represents this species, magnified 400 diameters.

M. convoluta, BREB.

M. thallo membranaceo, oculis nudis visibili, plus minus convoluto; familiis e cellulis geminis et in subfamilias dispositis, 256 compositis, interdum familiis duabus in familia gemina conjunctis; cellulis sphaericis aut oblongis; cytoplasmate homogæneo, viridi.

Diam.—Cell. $\frac{1}{8000}'' = 0.00017''$; fam. long. $\frac{1}{160}'' = .06''$; lat. $\frac{1}{250}'' = 0.04''$.

Hab.—In aquis quietis prope Philadelphia.

Thallus membranous, visible to the naked eye, more or less folded; families composed of 256 geminate cells, arranged in subfamilies, sometimes two of these families conjoined with a composite family; cells spherical or oblong; cytoplasm homogeneous, green.

Remarks.—When my *Prodromus* was published, the only specimens of this plant which I had seen were contained in a mounted slide given me by my friend Dr. J. Gibbons Hunt, of this city. Since then I have found it growing in a very shallow, quiet, but fresh, sweet pool at Spring Mills, making a distinct green layer upon the mud many feet in extent. Of course, there were millions of specimens in this layer. The fronds are irregular in shape, often somewhat ovate, sometimes subquadrate, variously torn, and not rarely somewhat lobate. Their edges are frequently very sharply defined and rendered firm and prominent by several rows of cells being crowded closely together along them. The cells in the body of the frond are arranged in large parallelogrammatic families, composed of 256 cells. There are 16 cells on each side, the families being parallelogrammatic rather than square, owing to the oblong shape of the cells. This cell family is composed of four subfamilies, each containing 64 cells. These are again subdivisible into four more or less distinct groups of 16 cells each. The cells are, finally, generally closely geminate, each pair being very distinctly separated from its neighbors. In certain stages of growth, as immediately after a general division of the cells, two of the large cell-families spoken of are often temporarily joined together to form a huge family of 512 cells, but soon separate one from the other.

ORDER **Nematogeneæ.**

Plantæ multicellulares vel pseudo-multicellulares. Cellulæ filum (trichoma) formantes et plerumque vagina tubulosa homogænea vel lamellosa inclusæ. Trichomata aut simplicia aut ramificata.

Plants multicellular or pseudo-multicellular. Cells forming a filament, and generally included in a tubular lamellate or homogeneous sheath. Filaments either simple or branched.

FAMILY OSCILLARIACEÆ.

Trichomata simplicia haud vero multicellularia, sed distincte articulata, plerumque vaginata, motionibus variis prædita.

Filaments simple, not strictly multicellular, but distinctly articulate, mostly vaginate, moving in various ways.

Genus OSCILLARIA, Bosc.

Trichomata simplicia, plerumque distincte articulata, rigida, recte vel parum curvata, rarius circinata vel spiraliter convoluta, plerumque læte colorata, motu triplici prædita, in muco matricali nidulantia vel vaginula tubulosa angustissima utroque fine aperta inclusa; articuli fronte disciformes. (R.)

Filaments simple, mostly distinctly articulate, rigid, straight, or somewhat curved, very rarely circinate or spirally convolute, capable of three motions, floating in a maternal jelly, and shut up in a fine tubular sheath, open at both ends; joints from the front disciform.

The oscillaria are very peculiar plants, which flourish almost in every situation in which fresh water is to be found. The purest springs are not always free from their presence, although they occur most abundantly in stagnant pools and ditches, where animal or vegetable matters are undergoing decay. When viewed in mass, floating upon some foul pool, few objects in the vegetable world are better calculated to excite disgust. A dark, slimy scum reeking with its putrescent surroundings, they seem to offer nothing of pleasure or interest. But, when brought home to the table of the microscopist and placed beneath his object-glass, they startle the observer by the wonders of their life-history. Living rods, writhing, twisting, bending, curling, creeping, gliding hither and thither; incessant, apparently causeless, motion, occurring too in what to most minds is the very type of fixity and passivity—a plant. No marvel, then, that they are so famous.

The structure of an oscillatoria is about as simple as it can be. An outside colorless cellulose sheath, which is probably in the uninjured filament closed at the end, although, as seen by the microscope, violence and age have often torn it open. Within is a long rod of variously colored endochrome, distinctly articulated by, at great or less intervals, breaks in the color, which appear as dark lines under a low power, but, under a higher objective, are revealed as narrow linear portions of protoplasm lighter and more transparent than the rest. Frequently at the joints there is a marked tendency to separation between the successive articles, and a very decided contraction of the endochrome on each side, so as to leave a little gutter, or dividing trench. The endochrome is sometimes homogeneous, sometimes contains numerous granules, which are, however, never amyloid in their nature.

The color of the endochrome varies very greatly in the different species. Slate color, blue, greenish, olivaceous, are among the most common hues. According to Dr. Ferdinand Cohn (Botan. Zeitung, 1867, p. 38; Sitzung, 13th Dec. 1866, der Schlesischen Gesellschaft für vaterländische Cultur), the coloring matter of the oscillatoria consists of true chlorophyll, and a substance which he calls *Phycocyan*, but which he states to be different from *Phykokyan* of Kützing, the *Phychochrom* of Nägeli, and also from *Phycocyan* of the latter authority. The chlorophyll is,

of course, soluble in ether and alcohol but not in water; but the *Phycocyan* (Cohn) is insoluble in alcohol and ether, but soluble in water after the death of the oscillatoria. It is precipitated out of its solution by acids, alcohol, and metallic salts, as a blue jelly, but potash and ammonia throw it down as in a colorless, gelatinous mass. I have myself frequently noticed that oscillatoria after death will yield a bluish coloring matter to water, but thought that such coloring matter was the result of a partial decomposition, and I think that Professor Cohn has by no means established as a fact that his *Phycocyan* exists in the oscillatoria during life.

As to the method of reproduction of these plants, we are as yet almost entirely in the dark. Individuals do multiply by the breaking up of the internal endochrome into masses or sections through a separation at the joints. These little masses frequently grow immediately into new individuals. Sometimes, however, they roll themselves into a ball, but whether they then have the power of coating themselves with a protective wall and passing into a sort of resting spore or not, I cannot say.

The specific characters of the oscillatoria are derived from the color, form, mode, and place of growth, &c., of the large common mass, its thickness, consistency, the absence or presence of radii, &c. Descending to the individual filament, the characters are drawn from the size, the color, the length of the articulations, and the shape of the uninjured ends. Thus, it is to be noted, whether the latter are gradually narrowed (attenuated), or preserve their size to the very point, whether they are acutish or obtuse, rounded or truncate, whether they are straight or constantly curled. The activity and modes of motion are also to be remarked. Some species merely glide across the field of the microscope, some are constantly curling and uncurling at their ends, some bending to and fro almost like a pendulum, some are very sluggish, others very active and restless.

After all, however, it must be confessed that the specific characters are very unsatisfactory, much more so than in any other phycochroms which I have studied.

A very large number of European forms have been described, some few of which I have been able to recognize. I have also ventured to name a few forms apparently distinct, but have refrained from going farther into their specific study, because I have found it so unenticing, and my time has been so limited.

Professor Bailey, in Silliman's Journal, N. S., vol. iii., states that he has identified a few species of this family, although with great hesitation and doubt. At the time he wrote there were really no known grounds upon which specific unity could be predicated in these plants, and I therefore think that his identifications are of but little value, although holding the most profound respect for his abilities as a naturalist. The list he gives is as follows:—

O. tenuissima, Ag. Warm Springs of Washita.

O. tenuis, Ag. Providence, Rhode Island. West Point, New York. Culpepper County, Virginia.

O. decorticans, Gener. Common everywhere on pumps, &c.

O. muscorum, Ag. West Point, New York.

O. nigra, Vauch. West Point, New York.

O. corium, Ag.

O. chlorina, Kützing.

O. internum in strato sordide viridi natante, interdum in aqua diffusa; trichomatibus rectis, vivide moventibus, vel articulatis et cum cytoplasmate granulato, vel inarticulatis et cum cytoplasmate haud granulato; cytoplasmate hyalino, interdum coloris fere expertibus, interdum dilutissime viride; apiculo haud attenuato, obtuse rotundato, recto; articulis diametro subaequalibus.

Diam.— $\frac{1}{7000}''$ — $\frac{1}{9000}''$ = .00014''—0.0001''.

Hab.—In stagnis prope Philadelphia.

Sometimes swimming on the water as a dirty-greenish stratum, sometimes diffused in the water; filaments straight, actively moving, either articulated and having the cytoplasm filled with blackish granules, or else neither articulate nor granulate, cytoplasm hyaline, almost colorless, or with a faint greenish tint; ends of the filaments not attenuate, straight, obtusely rounded; joints about equal to the diameter.

Remarks.—I found this species in the month of August, 1869, in one of the stagnant brick-ponds below the city. It occurred as a sort of floating scum, or else diffused through the water, which was then opaque and greenish. It resembled so a protococcus in gross appearance that I did not think of its being an oscillatoria until I placed it under the microscope. The filaments are almost colorless, and, in most instances, are very distinctly granulate and articulate. The dissepi-ments are in such cases clear and transparent, perfectly free from granules. This form is very close to the descriptions of the European *O. chlorina*, Ktz., but differs somewhat from descriptions, chiefly in habit of growth. The filaments, when in mass, are often seen to be curved under the restraining force of the glass cover, but when free I think always straighten themselves.

Fig. 1, pl. 1, represents a single filament, magnified 750 diameters.

O. Fröhlichii, Ktz.?

O. strato indefinito, tenue, viride; trichomatibus late viridibus, subrectis, vivide oscillantibus, ad genicula nonnihil pellucidis et leviter contractis et rarissime granulatis; articulis diametro 2, 3, 4 plo brevioribus; cytoplasmate obscure aut distincte minutissime granulato; apiculo haud attenuato, late rotundato.

Diam.— $\frac{1}{2000}''$ — $\frac{1}{1500}''$ = 0.00066''—0.0004.

Hab.—In flumine Schuylkill.

Stratum indefinite, thin, green; filaments bright green, straightish, vividly oscillating, somewhat pellucid at the joints, where they are slightly contracted and very rarely granulate; articles 2, 3, 4 times shorter than the diameter, cytoplasm obscurely or distinctly very minutely granulate; apex not attenuate, broadly rounded.

Remarks.—I found this species growing upon the bottoms of the shallows in the Schuylkill River and its larger tributaries, forming a somewhat badly defined stratum, rather, indeed, a coating on the mud than a definite stratum. The motion is exceedingly active, the filaments bending and gliding, and their apices constantly curling and extending in all directions. The apices are very blunt. The filaments are not often seen woven and twisted together into a mass composed simply of themselves, but are stuck together loosely, each filament remaining straightish, with numerous little masses of mud between them. I have not been

able to identify the species positively, but have referred it with doubt to *O. Fröhlichii*.

Fig. 2, pl. 1, represents the end of a filament.

***O. nigra*, VAUCH.**

O. strato plus minus compacto, amplo, plerumque natante, atro-viride, cum radiis longis; trichomatibus plerumque flexuosis; apice obtuse rotundato; articulis diametro $\frac{3}{4}$ plo brevioribus; dissepimentis distincte granulatis; cytoplasmate pallide casio.

Diam.— $\frac{1}{3500}$ ''— $\frac{1}{4000}$ ''.

Hab.—In fossis stagnis prope Philadelphia.

Stratum more or less compact, ample, broad, mostly floating, blackish-green, with long radii; filaments mostly flexuous; apices obtusely rounded; joints $\frac{3}{4}$ shorter than broad; dissepiments distinctly granulate; cytoplasm pale-grayish.

Remarks—This species is found in thick, rather loose strata, floating, especially when old, on stagnant waters, or adhering to plants, &c., or the muddy shores and bottom of ditches, foul aquaria, &c. The color of the stratum is a very dark blackish-green, with a peculiar, glossy, repulsive appearance. The single filaments are of a pale-bluish neutral tint, sometimes a little greenish, very much curved and entangled, or more rarely straightish. Their motion is active. The measurements do not quite equal those given by European authorities, but otherwise the plant agrees well with their descriptions.

Fig. 3a, pl. 1, represents the mass of the plant as seen with the naked eye; fig. 3b, shows a number of filaments slightly magnified; fig. 1e, a broken portion of a filament magnified 260 diameters, with the sheath projecting beyond the endochrome; fig. 1f, the end of a filament still more highly magnified.

***O. limosa*, AGARDH.**

O. trichomatibus subrigidis et subrectis, vivide oscillantibus, cæruleo-viridibus, in stratum mucosum late saturate viride et modice longe radians et natans collectis et intertextis, distincte articulatis; articulis diametro subæqualibus, interdum duplo brevioribus (post divisionem), ad genicula distincte constrictis; dissepimentis haud granulatis; apiculo obtuso, haud attenuato, interdum recto, interdum curvato; cytoplasmate granulato.

Diam.— $\frac{1}{6000}$ ''

Hab.—In stagnis prope Camden, New Jersey.

Filaments straightish and somewhat rigid, vividly oscillating, bluish-green, interwoven into a bluish-green, floating stratum, with moderately long radii, distinctly articulate; articles about equal to the diameter, or after division one-half shorter, at the joints distinctly constricted; dissepiments not granulate; apices obtuse, not attenuate, sometimes straight, sometimes curved; cytoplasm granulate.

Remarks.—I have found this species floating on foul ditches near Kaighn's Point, New Jersey, in the month of May. The color of the stratum is a very pure deep-green; the single filaments vary from a rather bright deep-green to a pale blue-green, according to the power under which they are seen. The apices are not at all attenuate. The constriction at the articles is scarcely visible with a lower power than $\frac{1}{2}$ th. The stratum is rather thin, with a good deal of dirt adhering to its bottom.

When grown in a bottle, the plant appears as a very thin stratum growing up the sides. The agreement of this plant with the descriptions of the European *O. limosa* is very close, so that I do not think it can be separated from it, although in *O. limosa* the dissepiments are said to be distinctly granular.

Fig. 4, pl. 1, represents a filament of the American plant magnified 1250 diameters. The color and form are closely counterfeited, but the characteristic separation of the endochrome into parts at the joints is decidedly exaggerated.

***O. neglecta*, WOOD.**

O. trichomatibus modice brevibus, aut dilute purpuraceo-plumbeis aut plumbeo-cinereis, plerumque rectis, aut stratum mucosum atro-purpureum haud distincte radiante formantibus, aut in strato gelatinoso haud radiante subplumbeo dispersis et cum algis aliis intermixtis, rare oscillantibus sed lente sese moventibus; articulis diametro fere 4 plo brevioribus; dissepimentis plerumque haud granulosis, rare indistincte granulosis; apiculo obtuse rotundato, interdum breviter nonnihil attenuato.

Syn.—*O. neglecta*, WOOD, Prodr. Amer. Philos. Soc., 1869, 124.

Diam.— $\frac{1}{1500}'' = .0066$.

Hab.—In stagnis prope Philadelphia.

Filaments rather short, of a dilute purplish-lead color, or leaden-gray, generally straight, either forming a mucous, blackish-purple stratum without marked rays, or diffused with other algæ in a gelatinous mass, rarely oscillating but gliding; articles about four times shorter than broad; joints for the most part not granulate, rarely indistinctly granulate; ends obtusely rounded, occasionally short, somewhat attenuate.

Remarks.—I have found this plant in the shallow ditches along the track of the Norristown Railroad above Manayunk, growing in two different ways. In the one it forms a distinct, soft, gelatinous, floating stratum of a very dark purplish color, consisting of nothing but interwoven filaments, and provided with long rays. In the other, the plant is largely mixed with diatoms and other algæ into a thick, gelatinous stratum without rays, whose color is a dirty slaty tint, which, however, is not all distinctive, and often varies as the proportion of the different constituents varies. The color of the single filaments is a slaty, almost neutral tint. The cytoplasm is remarkable for the numerous very minute spots more transparent and with less color than the surrounding parts. The ends of the filaments are often abruptly obtuse, frequently however there is a very short taper. Motion does not appear to be very active, and seems especially to be gliding, rather than a bending to and fro of filaments.

Fig. 5a, pl. 2, is an outline drawing of a filament magnified 450 diameters; 5b is a portion of a filament.

***O. imperator*, WOOD.**

O. in strato mucoso, plerumque natante, olivaceo-atro, longe radiante; trichomatibus rectis aut subrectis, tranquillibus, dilute viridibus vel saturate olivaceis, haud oscillantibus, sed ambulanti-bus; apiculis nonnihil attenuatis, late rotundatis vel subtruncatis, curvatis; articulis diametro 5-12 plo brevioribus, ad genicula indistincte contractis; cytoplasmate homogæneo, olivaceo-viride; vaginis firmis, ad genicula distincte transverse striatis.

Syn.—*O. imperator*, WOOD, Prodr. Amer. Philos. Soc., 1869, 124.

Diam.— $.002''$.

Hab.—In stagnis prope Philadelphia.

O. occurring in an olive-black, mucous stratum, mostly swimming and with long rays; filaments straight or straightish, light-green or deep-olive, tranquil, not oscillating, but moving with a gliding motion; ends somewhat attenuate, broadly rounded or subtruncate, curved; articles 5-12 times shorter than broad, slightly contracted at the joints; cytoplasm homogeneous, olive-green; sheaths firm, distinctly transversely grooved at the joints.

Remarks.—The strata of this species are often of great extent, and resemble more masses of spirogyra than of the ordinary oscillatoria. They are very loose in texture and are very slimy, whilst their edges are fringed by the long tranquil rays. In certain conditions of growth, the endochrome of the filaments is so dense as to render them very opaque and the articulations very obscure. The sheaths when emptied show the marks of the joints very distinctly; but, at times, when gorged with cytoplasm, scarcely can the sheath itself be seen. The color of the filament is also affected by the state of the protoplasm, so that it varies from a lightish-green with an olive tint to a very decided dark olive. This species seems to be closely allied to the European *O. princeps*, from which, however, it differs in its motion, which is always very slow and merely gliding, its color, the distance of the dissepiments, and the much longer curvature of the ends. It grows everywhere in the ditches around the city; when mature, generally floating upon the surface with an adherent under-stratum of dirt, but, in its earlier history, often adhering to the bottom.

Fig. 6*a*, pl. 1, is a drawing of the end of a filament; fig. 6*b*, represents a small fragment of a filament, showing the tendency to take a roundish or barrel shape; much of the endochrome has been squeezed out by the injury which has broken the filaments.

Genus CITHONBLASTUS, Ktz.

Phormidii trichomata fasciatim congesta et vagina communi mucosa apice clausa vel aperta inclusa. Tales fasciculi numerosi in stratum (quasi thallum) gelatinosum, passim amoso-divisum aggregati. Vaginæ communes achromaticæ, sæpe lamellosæ, plus minus ampliatae, rarius indistincte et subnullæ, evacuatae, plerumque valde intumescentes. Trichomata Phormidii modo oscillantia, articulata et vaginata, rigida, recta vel parum curvula, in fasciculos funiformes plus minus dense contorta, apice soluta et divaricata. Cellulas propagatorias observare mihi contigit. (R.)

Filaments fasciately placed together and included in a common mucous sheath with open or shut apex. A number of these fasciculi aggregated in a gelatinous stratum (pseudothallus), which is gelatinous, and here and there ramosely divaricate. Common sheath colorless, often lamellate, more or less enlarged, rarely indistinct and nearly wanting, when empty mostly markedly intumescent. Filaments oscillating like to those of Phormidium, articulate and vaginate, rigid, straight, or a little curved, more or less densely entangled into cord-like fasciculi, with the apex dissolved and dis-severed.

Ch. repens, Ktz.

Ch. terrestris, strato plus minus expanso, saturate ærugineo-chalybeo aut olivaceo-fuscescente, mucoso-membranaceo; trichomatibus æqualibus in fasciculos filiformes, sæpe valde elongatos, e vaginæ communis apertura penicillatim exsertos congestis; articulis diametro æqualibus dissepimentis granulatis, apiculo obtuse recto. (R.)

Species, mihi ignota.

Hab.—Common on damp earth. West Point, New York; Bingham, Massachusetts; Providence, Rhode Island; Baily, Silliman's Journ., N. S., vol. iii.

Terrestrial, stratum more or less expanded, deep æruginous chalybeate, or olivaceous fuscous, mucous membranaceous; filaments equal, in filiform fasciculi, which are often much elongate and penicillately exerted from the open common sheath; joints as long as broad, the dissepi-ments granulate; the apex obtuse, straight.

Genus LYNGBYA, AGARDH.

Trichomata inarticulata vel breve articulata, cellulis perdurantibus instructa. Vaginæ sæpe coloratæ, crassæ, sæpe lamellonæ.

Filaments not articulate, or shortly so, furnished with heterocysts. Sheaths often colored, thick, often lamellate.

“*L. muralis*, AG.

Filaments somewhat rigid, thickish, tortuous, very long, interwoven in a bright, grass-green stratum; annuli strongly defined. *Ag. Syst.*, p. 74; *Harv. Man. Ed.*, p. 160; *Conf. muralis. Dillw.*, tab. 7, E. Bot. t. 1554. β . aquatica.

Hab.—Var. β . in pools of fresh water, Whalefish Island, Davis Straits. Dr. Lyall.

The specimens are mixed with turfy soil. Except in the submerged habitat, this agrees with the ordinary form. Intermixed with threads of the usual size and structure are others covering in pairs, as in *L. copulata*, Harv., which is obviously only a state of this widely dispersed species. I have not received specimens of the ordinary *L. muralis* from America; but no doubt it is common on damp walls, &c., as in Europe generally.”

I have never identified this species, and have simply copied Harvey's account of it from the *Nereis Boreali Americana*, pt. III. p. 104.

L. bicolor, WOOD.

L. trichomatibus simplicibus, in cæspites nigro-virides vel cæruleo-virides dense intricatis, varie curvatis, plerumque inarticulatis, interdum breviter articulatis et ad genicula contractis; cytioplasmate dilute cæruleo-viride, plerumque copiose granulato, sæpe interrupto; cellulis perdurantibus cylindricis, sæpe elongatis, saturate brunneis, sparsissimis; vaginis firmis, achrois, in trichomata matura modice crassis.

Syn.—*L. bicolor*, WOOD, Prodr. Amer. Philos. Soc., 1869, 124.

Diam.— $\frac{1}{17}$ to $\frac{1}{10}$ ”.

Hab.—In flumine Schuylkill prope Philadelphia.

L. with the filaments closely interwoven into a blackish or bluish-green mat; filaments variously curved, simple, mostly inarticulate, sometimes shortly articulate with the joints contracted; endochrome light bluish-green, mostly very granulate, often interrupted; heterocysts cylindrical, often elongate, deep brown, very few; sheaths firm, transparent, in old filaments moderately thick.

Remarks.—This species is abundant in the shallow water of the Schuylkill River, near Spring Mills, where it forms dark waving tufts a half inch or more in height, which are adherent either to the bottom of the stream or to some firm support, such as large growing plants, sticks fixed in the mud, &c. When examined with the microscope, these tufts are seen to be composed of innumerable, very long, motionless, greatly curved filaments. They do not seem to be attached to their support, but in the denser parts are woven into a very thick mat, which apparently adheres *en masse* to the fixed body. These filaments are very rarely articulate,

but, when they are, the joints are shorter than broad. The endochrome is mostly very granulate; sometimes, however, it is much more homogeneous. The sheaths in the old filaments are rather thick, and frequently partially empty; the exterior of such sheaths has often a rough, ragged look. The larger cells are very few in number. They are elongated cylinders with concave ends. I have found this plant in the Schuylkill River, just above Fairmount dam, in a younger state, and apparently without heterocysts. The threads near their ends had their endochrome distinctly articulate, like an *oscillatoria*, but elsewhere the protoplasm was continuous. It often contains numerous large granules resembling minute starch grains, which however fail to exhibit the reaction with iodine.

Fig. 7, *a*, pl. 1, represents a portion of the filament slightly magnified; fig. 7, *b*, a heterocyst from the same specimen more magnified; fig. 7, *c* and *d*, are drawings from another specimen from the same locality, each magnified 800 diameters; fig. 8, pl. 1, represents the form alluded to in the text as having been found in the Schuylkill River just above the dam.

FAMILY NOSTOCHACEÆ.

Trichomata simplicia, e cellulis distinctis composita, interdum vaginata, articulata, in gelatina immersa, cellulis perdurantibus, et interdum sporis porro instructa.

Filaments simple, composed of distinct cells, sometimes vaginate, imbedded in jelly; furnished with heterocysts and sometimes with spores also.

Remarks.—The nostochaceæ are plants of simple construction, consisting of a more or less firm jelly in which are imbedded serpentine filaments, composed of numerous cells. These cells are mostly more or less globose, especially in the true nostocs, so that the filament has a moniliform aspect. They have not distinct walls, or at least any that can be distinctly seen by ordinary powers of the microscope, and are sometimes closely connected, sometimes rather widely separated. No nuclei are usually discernible; I have, however, seen in some instances central spots, which were possibly of that nature. The filaments themselves are of various length, almost always tortuous, sometimes widely separated, sometimes closely interwoven. The gelatinous portions of the fronds are of various consistence—sometimes semifluid, sometimes very firm, almost cartilaginous.

The order is divisible into two families—the *Nostocs* proper and the *Spermosireæ*.

In the former, the outer portion of the frond is condensed and firm, forming a sort of outer coat or epidermis, which is sometimes quite distinct, but in other instances can scarcely be said to exist.

In the filaments of a true nostoc are placed at irregular intervals cells, which are mostly larger than the others, and have thick, distinct walls. These cells contain very little or no chlorophyllous protoplasm. They are often, but by no means always, provided with numerous exceedingly attenuated, hair-like processes, or quiescent cilia. These bodies were supposed by Kützing to have some sexual value, and received from him the name of *Spermatia*. But, as their functions are entirely unknown, the name of heterocysts, first applied by M. Allman, is preferable. They are the “connecting cells” of Thwaites. No one has as yet demon-

strated the existence of anything indicating sexuality in the nostocs proper, or shown any body at all worthy to be looked upon as a spore.

Their ordinary method of reproduction is simply a slight modification of that of growth. If a fragment of an actively growing nostoc is placed under the microscope, the filaments of it are seen to be irregular and distorted, thicker in one place than another, the cells misshapen, and sometimes apparently lumped and fused together. The formation of new filaments is taking place in such cases by the simultaneous growth and longitudinal segmentation of the cells of the old, and this may occur through the whole or in only a portion of the length of the latter. (Pl. 2, fig. 10.)

The filament of a nostoc is, in other words, capable of a double growth or development, the result in one instance being increase in its length, in the other the production of a new form like itself. The first of these is brought about by a transverse division of the cells, so that out of each single cell two are formed, placed end to end, each daughter-cell at first only half the size of their parent, but soon attaining to its full stature. In the other case great increase in the size of the cell occurs almost consentaneously with a longitudinal or lateral segmentation, the cell dividing in the direction of its length, instead of transversely, so as to form two cells lying side by side instead of end to end. The misshapen filaments alluded to simply represent different stages of this change, which goes on until two perfect filaments lie side by side, to be finally more or less widely separated by the jelly which they secrete around themselves.

This process of growth continues until the plant has arrived at its mature size, when it ceases. During this time the inner portion of the frond has been becoming more and more liquid, and finally the outer epidermis bursts and the thoroughly softened inner portion is discharged. In this way, innumerable filaments are set free, which are endowed with a power of motion similar to, but much less active than, the gliding of the oscillatoria, by means of which they are diffused in the water. Scattered in this way, carried hither and thither by currents, each minute thread, fixing itself to some object, at last becomes the centre from which a new plant is formed in a manner similar to that already described.

In the second division of the *Nostochaceæ*, the jelly is always much less firm than in the true nostocs, and is not condensed in the outer portions. The fronds are therefore soft, almost diffuent, and entirely shapeless. The filaments themselves also differ from those of the true nostocs. There are no fixed differences in the vegetable cells or heterocysts, however, although the former are apt to become more cylindrical and the filament consequently less moniliform. It is especially in the possession of distinct reproductive sporangial cells that the differences are to be found. These are much larger than the ordinary cells, from which, in their first appearance, they are not distinguishable; but, when the frond has attained a certain age, the spore-cells begin to enlarge both in diameter and length, and finally assume a form and size apparently fixed within narrow bounds for each species, and surround themselves with distinct, often quite thick coats. It is very possible that the production of new individuals may take place by a detachment of portions of the frond and subsequent growth, as described in the *Nostocs* proper,

but increase of the species does certainly occur by means of these so-called spores. The growth of the plant takes place in the same way as in the true nostocs. The filaments increase in length by transverse division and consequent multiplication of the cells, whilst new filaments are formed by the consentaneous longitudinal division of all the cells of a filament.

The spores of a *Cylindrospermum* have the power of germinating after prolonged desiccation, they having been successfully cultivated even from specimens long preserved in the herbarium. Their development has been carefully and successfully studied by M. Thuret. According to this authority the first change consists in an elongation of the spore, which ruptures the wall of the sporangium, pushing a portion of it before it. Directly after this the spore undergoes division, so that out of it is formed a little torulose filament, composed of four or five cells. Growth takes place at both ends, but more rapidly at the free one. The new cells formed are smaller than those which arise directly from the spore, but, finally, all the articles assimilate. The wall of the sporangium remains attached for a long time to the end of the filament forming a little cap to it. The heterocysts, according to Thuret, at first are indistinguishable from the ordinary cells, but after awhile the granules in them begin to disappear, the color to pale, the outer wall to become apparent and grow thicker, until at last a perfect "connecting cell" is educed. I have, myself, carefully watched the early development of the spores of a *cylindrospermum*, and can confirm, in all essential particulars, the description Thuret has given of the process. Fig. 10, pl. 2, represents a partially formed filament, to which the empty sporangium is still attached.

As no sexual reproduction has as yet been shown to exist among the *Nostochaceæ*, it is very evident that their whole life-history is not comprised within the changes which have been detailed. It has long been known that the gonidia of many lichens have the power of independent existence, *i. e.* that when they are discharged from their thallus they can continue to live and multiply, if circumstances favor them, without giving origin to a new thallus. This, and the great similarity of structure between the nostocs and the lichen genus *Collema*, has suggested a possibly close relation between the two. The first observer, I believe, who asserted that they were different stages of the same plant was Dr. Hermann Itzigsohn.

His observations are, however, rendered of so little value by his own statements that it is not necessary to review them here. Thus, he says, that after seven years' observation he had yet to see a true one called algæ, that the *Desmidiæ* are, at least, two-celled, &c. &c. The most weighty observations upon this subject are those of Professor Julius Sachs and of J. Baranetzky—the former published in the *Botanische Zeitung* for 1855, the other in the Bulletin of the St. Petersburg Academy for 1867.

Professor Sachs states that he watched a whole bed of *Nostoc commune* developing into *Collema bulbosum*. He says that the peculiar Collemoid threads first appeared as little lateral offshoots or prolongations from the cells of the nostoc filament, and rapidly developed into well-formed collemoid filaments. Every possible stage from the typical nostoc to the typical collema was seen repeatedly.

The development of the distinguishing threads of the collema out of the ordinary nostoc-cell has never been confirmed by any other observer; but it seems to me that it must be at least provisionally accepted, although De Bary expresses some doubt of it. (*Morphol. und Physiol. der Pilze, Flechten, &c.*, p. 290.)

The researches of M. Baranetzky were directed to the developing of a nostoc out of a collema. Hicks and other observers had previously stated that they had seen this, but none of them had given sufficient details as to the method of their observations, to be fully convincing.

M. Baranetzky placed sections of actively growing fronds of *Collema pulposum*, Ach, upon smooth, damp earth, using all proper precautions to prevent external influence. After some days the sections became less transparent and intensely green from the crowding of the gonidia, which were now arranged in curved rows closely rolled together into balls. Upon the upper surface of the section appeared little gelatinous balls or warts, which contained gonidia in rows, and gradually developed typical nostoc forms, whilst on the edges of the sections appeared little colorless wart-like masses of jelly, in which, after some time, appeared gonidia, some of which developed into the typical nostoc form, others into true collemoid plants.

Mr. Baranetzky further states that he watched the body of the section gradually change by the continual growth and increase of the rows of gonidia, before alluded to, and by the disappearance of the collemoid threads, until at last the whole mass of the tissue of the lichen had been converted into a true nostoc, which was finally identified as *Nostoc vesicarium*, D. C.¹

I have no observations of my own to offer upon this subject; but think enough has been done to show not only that the nostocs proper have very close relations with the collemoid lichens, but that they are probably a peculiar phase in their life-history. This being the case, it may seem a perfectly superfluous work to indicate species amongst the nostocs. To any one who has given much study to the fresh-water algæ, the reply to this will immediately suggest itself; namely, that in the present state of the science it seems impossible to avoid it; they are so commonly thrust at one by collectors, amateurs, &c., are so distinct, are so often the subject of tongue and pen, that they must have a name. The idea that attaches to the term species is at present not a very definite one; that there are, however, amongst the nostocs fixed forms, which do not change into one another, and can readily be distinguished, I have no doubt. Such forms are herein described. If they be only life stages of lichens, I have no doubt that it will finally be found that each so-called species of nostoc has its own peculiar so-called species of lichen, from which it alone springs, and into which alone it can develop. It seems to me, then, that as yet no cause for abandoning the specific names of the

¹ In order to aid any one desirous of going over this subject more thoroughly, a list of papers is appended:—

Ventenab und Cassini *Opuscula Phytolog.*, 1817, vol. ii. p. 361.

Dr. Hermann Itzigsohn. *Botanische Zeitung*, 1854, p. 521.

Prof. Julius Sachs. *Botanische Zeitung*, 1855, p. 1.

Bayrholder. *Botanische Zeitung*, 1857.

Hicks. *Journal of Microscopical Science*, 1861, p. 90.

Baranetzky. *Bulletin de la Société des Sciences Nat.*, St. Petersburg, vol. xii. p. 418.

nostocs has been shown, but only reason to study also their relations with the various collema.

In regard to the *Spermosireæ*, there is as yet no direct proof whatever connecting them with lichens. It is very possible that they are not so closely related to the true nostocs as is generally believed, so that the probabilities of their being lichens are at present so remote, that for the systematist to refuse to take note of their distinct forms, seems to me most unwarrantable.

SUBFAMILY NOSTOCEÆ.

Thallus peridermate plus minus distincto instructus, sporis destitutus.

Thallus provided with a more or less distinct integument, and destitute of spores.

Genus NOSTOC, VAUCHER, (1803.)

Thallus gelatinosus, varic coloratus, aut globosus vel subglobosus aut foliaceo-membranaceus et irregulariter expansus, sæpe bullatus. Trichomata plus minus moniliformia. Cellulæ perdurantes exacte sphaericæ vel rare oblongæ.

Thallus gelatinous, variously colored, either globose or subglobose, or foliaceously membranous and indefinitely expanded, often a bulla. Filaments more or less moniliforme. Heterocysts exactly spherical or rarely oblong.

a. *Thallus globosus vel subglobosus, vel disciformis.*

Thallus globose, subglobose or discoid.

N. Austinii, WOOD, (sp. nov.)

N. subglobosum, parvum, plerumque magnitudine ovorum piscium, rare ad 2'', fuscescente, vel nigrescente, interdum durum interdum submolle, superficie sæpe corrugata; trichomatibus varie curvatis, dense intricatis vel distantibus et laxissime intricatis, viridibus, fusciscentibus, subplumbeis vel luteo-brunneis, in thallis minoribus sæpe distincte vaginatis, in thallis majoribus haud vel indistincte vaginatis; articulis maturis globosis, sæpe didymis, crasse granulatis; cellulis perdurantibus articularum diametro æqualibus vel paulo majoribus, globosis, interjectis vel terminalibus, plerumque sparsis.

Diam.—Cell. Veg. $\frac{2}{75000}$ "— $\frac{4}{120000}$ " = .0026"—.00033"; cell. perdurant, .00033".

Hab.—in rupibus irroratis, New Jersey. (Austin.)

Subglobose, small, mostly the size of fish-eggs, but reaching the diameter of nearly two lines, fuscous or blackish, sometimes very hard, sometimes much softer; surface often corrugated; filaments variously curved, densely intricate or distantly and loosely interwoven, greenish, fuscous, subplumbeis or yellowish-brown, in the smaller fronds often distinctly vaginate, in the larger indistinctly or not all vaginate; mature joints globose; often didymous, coarsely granulate; heterocysts equal to the diameter of the other joints or a little larger, globose interspersed or terminal.

Remarks.—The fronds of this distinct species vary greatly in appearance; the larger of them are often almost colorless, and, when viewed with the microscope, are seen to be composed of a transparent colorless jelly, with remarkably large filaments scattered through it. These filaments are generally without sheaths, though occasionally a sheath can be faintly traced. The smaller fronds are much firmer than the larger and are more decidedly colored. Some of them are entirely opaque, looking simply black when viewed by transmitted light under the microscope. In these the filaments are densely crowded together, often misshapen and

provided with distinct broad brownish sheaths: every gradation exists between these forms and the first described fronds. The heterocysts are quite uniform in size, agreeing in diameter with the largest vegetative cells, they are always single. This species is most nearly allied to *N. ichthyoon*, RABENH.; from which it is separated by the differences in the sheaths, the greater size of the filaments, and the single heterocysts. It gives me great pleasure to dedicate the species to Mr. Austin, by whom it was collected near Gloucester, New Jersey, growing amidst mosses on rocks.

N. pruniforme, (ROTH,) Agb.

N. magnum, gregarium, noncohærens, globosum, magnitudine pisi, pruni majoris et ultra, olivaceum vel saturate ærugineum, ætate provecta fusco-nigrescens, haud raro cavum, lævissimum, intus aquosum, peridermate coriaceo subæchroo; trichomatibus subæqualibus, hic illic tumidis, laxè intricatis; articulis globosis, plerumque compressis, sæpe didymis, arcte connexis; cellulis perdurantibus articulis duplo majoribus, plerumque terminalibus, rarius interjectis. R. Species mihi ignota.

Diam.—Artic. 0.00024"—0.0003"; cell. perdur. 0.0003—0.00045". (R.)

Syn.—*N. pruniforme*, (ROTH,) AG. RABENHORST, Flora Europ. Algarum, Sect. II. p. 168.

Hab.—Maine. Leidy.

Large, gregarious, not cohering, globose, varying from the size of a pea to a large plum, or even beyond this, olivaceous or deep ærugineous, in old age blackish fuscous, often hollow, very smooth, within watery, periderm coriaceous, somewhat transparent; filaments subequal, here and there swollen, laxly intricate; articles globose, mostly compressed, often twofold, closely connected; heterocysts twice the size of the vegetative cells, mostly terminal, rarely interspersed.

Remarks.—I have never found this species; but some years since some specimens, sent to the Academy of Natural Sciences of Philadelphia from Maine, were identified by Professor Joseph Leidy as belonging to it.

N. verrucosum, (LINN.) VAUCH.

N. magnum, gregarium, bipollicare et ultra, subglobosum, sæpe lobatum, verruculosum, irroratum, initio solidum, postremo cavum, vesiciforme, saturate brunneo-viride; peridermate membranaceo-coriaceo, olivaceo-fuscescente; trichomatibus varie curvatis, centralibus parcioribus et laxissime intricatis, periphericis densius intricatis; articulis oblongis, rare globosis, arcte connexis, crasse granulatis; cellulis perdurantibus interstitialibus vel terminalibus, sphericis, articulorum diametro duplo majoribus.

Diam.—Cell. vegetativ. .000166"; cell. perdurant. .000233".

Syn.—*N. verrucosum*, (LINN.) VAUCH. RABENHORST, Flora Europ. Algarum, Sect. II. p. 176.

Hab.—In fonte, Centre County, Pennsylvania.

Large, subglobose, often lobed, warty; gregarious, two inches in diameter, growing under water, fixed, in the beginning solid, afterwards hollow, bladder-shaped; periderm membranaceous, coriaceous, olivaceous-fuscous; filaments variously curved, centrally fewer, and laxly intricate, towards the outside much more close; articles oblong, rarely globose, closely connected, coarsely granulate; heterocysts interstitial or terminal, spherical, twice the size of the other joints.

Remarks.—In the summer of 1869, I found a nostoc growing in great abundance in a very cold, large, limestone spring in Centre County, Pennsylvania, which I

have referred to *N. verrucosum* with some little hesitation. Some of the fronds were smoothish, others very decidedly warty. My specimens are old plants, which have become hollow by the discharge of their internal contents. It is possibly on this account I have not been able to verify the minute description given by Professor Rabenhorst. As the latter may not be accessible to some of those who consult these pages, I append the latter part of it, which differs from that given by myself from the American plants.

“Trichomatibus flexuoso-curvatis, quasi triplici ordine; centralibus parciorebus, laxissime implicatis, apices versus plus minus attenuatis, articulis oblongis, subdistantibus, periphericis densius sæpe densissime intricatis, basi haud raro cellulis biseriatis, articulis globosis, arcte connexis, extremis (nonnisi in thallo vetusto occurrunt) subflagelliformibus, articulis oblongis, cylindraceis sphaericisque simul immixtis, distantibus; cellulis perdurantibus sphaericis interjectis terminalibusque, nonnunquam pluribus simul seriatis articulorum diametro duplo triplove majoribus.”

According to Professor Harvey (*Nereis Bor. Amer.*, part iii. p. 114), this species has been collected by Dr. Lyall in pools of fresh water, Isle of Disco, and at Beechey Island, Arctic Regions; also by Mr. Fendler at Sante Fé, New Mexico.

N. alpinum, Ktz.

N. rupestre, immersum; thallo suborbiculare, erecto, membranaceo, ad $\frac{1}{4}$ — $\frac{3}{4}$ unciam lato, ad lineas duas vel tres crasso, tenaci, saturate olivaceo-fusco, lævi, sæpe rugoso-plicato, cum margine integro et plerumque incrassato; trichomatibus varie curvatis, laxè vel nonnihil dense implicatis; articulis fuscis vel dilute ærugineis plerumque globosis, sæpe subtiliter granulatis, arcte connexis; cellulis perdurantibus sphaericis plerumque articulorum diametro paulo majoribus, interdum subæqualibus, interjectis vel terminalibus.

Diam.—Artic. vegetativ. .00016"—00023"; cell. perd. .00026.

Syn.—“*N. alpinum*, Ktz. *Phycol. General.*, p. 206, No. 10.” RABENHORST, *Flora Europ. Algarum*, vol. ii. p. 174.

“*N. Sutherlandi*, DICKIE.” HARVEY, *Nereis Boreali Americana*, part iii. p. 114.

“*N. cristatum*, BAILEY.” HARVEY, *Nereis Boreali Americana*, part iii., 1857, p. 114.

Growing attached by its margin to the rocks in running water; thallus suborbicular, erect, membranaceous $\frac{1}{4}$ — $\frac{3}{4}$ an inch high and 1—3 lines thick, very tenacious, deep olive-green, smooth, often rugosely plicate especially at the base, with the margin entire, rounded, and mostly thickened; filaments variously curved, laxly or somewhat densely interwoven; articles fuscous or greenish, mostly globose, often finely granulate, closely connected; heterocysts spherical, generally a little larger than the ordinary cells, sometimes about equal to them, interspersed and terminal.

Remarks.—This interesting little plant was found in the mountain rivulets near West Point, New York, by the late Prof. Bailey, and received from him the specific name *cristatum*, first published in Harvey’s work on the North American Algæ. I have myself seen it growing in very great abundance in rapid mountain streams in the central portions of this State. It is doubtless, therefore, an inhabitant of the whole Alleghany range. In the low country, east or west of these mountains and their outlying hills, I do not know of its having been found. I have very recently received specimens of a nostoc from Sereno Watson, Esq., undoubtedly belonging to this species, which were collected by himself, in cold streams in the Clover Mountains, Nevada, at an altitude of 11,000 feet. Under the name of *M.*

Sutherlandii a nostoc has been described by Mr. Dickie, which was collected in the neighborhood of Baffin's Bay, and must be referred to this species, although the description given of it is very imperfect. Again, *N. alpinum*, KtZ., appears to be in all respects similar to the North American forms. So that this cosmopolitan little plant seems only to ask for a cold shelter, and it flourishes. The Alps, the Alleghanies, the Rocky Mountains, and the cold North are its homes. To those who believe in a single centre for a species, the suggestion that it has spread across the globe, through the arctic regions, and followed our mountain chains southward, will of course present itself.

As I have seen it, the plant is very abundant where it grows, five, six, twelve, or more of the little fronds adhering to a single pebble. The frond is generally longer than broad, the margin sometimes sinuous but never, as I have seen it, lobate or incised. It appears finally to burst and discharge its inner portion, whilst the outer cortical portion, now a little vesicle containing a globule of air, is set free and floats down the stream.

N. depressum, Wood, (sp. nov.)

N. enormiter suborbiculare, minutum, gregarium et interdum aggregatum muscos immersos adhaerens, magnitudine seminis sinapeos vel parvius, durum, elasticum, subnigris; peridermate firme, achroo; trichomatibus plerumque laxè intricatis, haud vaginatis; articulis globosis, plerumque modice arcuè connexis, rare distantibus; cellulis perdurantibus globosis, ceteris paulo majoribus.

Diam.—Artic. veget. max. .0002"; cell. perdurant. max .00029.

Hab.—In rivulis, New Jersey (Prof. Austin).

Irregularly suborbicular, gregarious and sometimes aggregated, elastic, blackish, about the size of a mustard-seed, or smaller, adhering to immersed mosses; periderm firm, translucent; filaments not vaginate, mostly loosely interwoven; joints globose, generally rather closely connected, rarely distant; heterocysts rather larger than the other.

Remarks.—This plant was found by Prof. Austin attached to a brook-moss (*Dichelyma*), growing in a rapid rivulet in Northern New Jersey.

The minute fronds sometimes are so thin and spread out as to be almost foliaceous. The species I take to be most nearly allied to *N. lichenoides* of Europe, from which it is, however, apparently distinct. In the American plant the filaments and heterocysts are a little larger, and the frequent elliptical cells of the European plant are wanting. The frond also apparently does not grow so large as the European, and is further distinguished by its flat, discoid form. In many of the specimens examined the filaments are very thick, irregular, and contorted, the cells being fused together. In other instances, a filament will be plainly double, and every grade between these conditions is present. This is plainly owing to a process of growth, to the cells enlarging and dividing laterally so as to form new filaments.

N. sphaericum, (POIRET,) VAUCH.

N. globosum, interdum oblongum vel ovale, gregarium, sæpius aggregatum, raro tamen confluyente, durum, elasticum (in ætate provecta intus molle et subaquosum?), olivaceum, magnitudine seminis sinapeos, ad cerasi parvi; peridermate firmo, pellucido; trichomatibus intricatis, luteolis, aut prasinatis aut dilute cæruleis; articulis plerumque subquadratis, interdum transverse

subovalibus, arcte connexis; cytoplasmate granulato; cellulis perdurantibus interjectis terminalibusque, sphaericis.

Diam.—Artic. diam. long. $\frac{1}{34000}$ " = .000125"; transv. $\frac{1}{36000}$ " = .00017"; cell. perdurant. $\frac{1}{34000}$ " = .00029".

Syn.—*N. sphaericum* (POIRET) VAUCH. RABENHORST, Flora Europ. Algarum, Sect. II. p. 167.

Hab.—In fontibus, prope Philadelphia.

Globose, sometimes oblong or oval, gregarious, but rarely confluent, hard, elastic (in advanced age within soft and watery?), olivaceous, varying from the size of a mustard-seed to that of a small cherry; periderm firm, pellucid; filaments intricate, yellowish, greenish or bluish; articles mostly subquadrate, sometimes transversely suboval, closely connected; cytoplasm granular; heterocysts interspersed or terminal, spherical.

Remarks.—The specimens from which the above diagnosis was prepared were found at Spring Mills, adhering to mosses and twigs in the water. The fronds were remarkable for their firmness and elasticity. The color was a dull, rather greenish, olive; that of the filaments varied from a decided greenish to a marked yellowish, or sometimes an almost silvery bluish tint. The heterocysts were rather few in number, and were either terminal or interstitial, sometimes they were without, sometimes with evident endochrome. The length of the general articulations varied a good deal, it was, however, mostly less than their breadth, which seems quite constant. When kept in water in the house, this species softens, and the periderm as it were peels off, allowing the interior to disperse itself as it gradually becomes more and more diffuent. Most of the fronds afforded ample evidence of their method of growth by the presence of filaments in every stage of division.

Fig. 10, pl. 2, represents filaments of this species.

***N. cæruleum*, LYNGB.**

N. minimum, sæpe microscopicum, enormiter globosum vel subglobosum, affixum, gregarium, sejunctum vel aggregatum; trichomatibus valde inæqualibus; articulis elongato-cylindraceis, vel acute ellipticis, vel perfecte ellipticis, vel globosis, vel subglobosis, vel subquadrangulis, sejunctis et nonnihil distantibus vel arcte connectis aut confluentibus; cellulis perdurantibus globosis, passim interjectis terminalibusque, ceteris duplo vel subduplo majoribus.

Diam.—Cell. perdurant, .000303; cell. vegetat. plerumque .00012—000166"; rarius .0001—.00021.

Syn.—*N. cæruleum*, LYNGB. RABENHORST, Flora Europ. Algarum, Sect. II. p. 167.

Hab.—Inter muscos, New Jersey (Prof. Austin).

Very small, often microscopic, irregularly globose or subglobose, affixed, gregarious, separate or aggregated; filaments very unequal; articles elongate-cylindrical, or acutely elliptical or perfectly so, or subglobose, or globose, or subquadrangular, separate and somewhat distant or closely connected or confluent; heterocysts globose, interspersed or terminal, double or about double the size of the other cells.

Remarks.—I am indebted to Mr. Austin for specimens of this species collected by him in Northern New Jersey. The fronds grow attached to moss and are very minute, the largest I have seen being not more than half a line in diameter. The filaments are remarkable for their inequality, which is often very perceptible in different parts of the same filament. I have referred my specimens to *N. cæruleum*—

the only differences between them and the European plant are that they are not so large, and do not agree in color, many of them being browner; but these are certainly insufficient grounds for separating them. Prof. Rabenhorst speaks of observing the contents of heterocysts dividing up so as to form a little colony of cells, which finally break through the maternal wall. I have only studied mounted specimens, but have seen very clearly heterocysts in which this process was taking place.

N. punctatum, WOOD, (sp. nov.)

N. terrestre; thallo expanso orbiculare vel nonnihil irregulare, tenuissimo, ærugineo, parvo membranaceo, pellucidulo; trichomatibus laxè intricatis, variegatis, articulis globosis vel sæpius ellipticis, plerumque medio pellucidulis, laxè connexis; cellulis perdurantibus terminalibus vel interjectis.

Diam.—Cell. vegetat. $\frac{1}{20000}$ " = .000166; cell. perdur. $\frac{1}{20000}$ " = .00033.

Hab.—In terrestre, New Jersey, (Prof. Austin.)

Terrestrial; thallus expanded, irregular or orbicular, very thin, æruginous, small, membranous, pellucid; filaments loosely interwoven, variously curved, joints globose or often elliptical, mostly pellucid in the centre, loosely connected; heterocysts terminal or interspersed.

Remarks.—Mr. Austin has kindly sent me the only specimens I have seen of this species; they are labelled "Damp Ground, Sept." The fronds, which are often aggregated, are very small and exceedingly thin, especially in their central portions, where they are quite translucent; in form they are often circular, sometimes quadrangular, sometimes quite irregular. As to size, most of them are not more than two lines in diameter, some three, or possibly five lines. The margins are often reflexed and thickened, especially in the smallest fronds. Two kinds of filaments are visible; 1st, those which I take to be in a perfected quiescent state; 2d, those which are in active growth. The former are composed of globose, or more commonly elliptical joints, which are remarkable for the possession of a central translucent, almost colorless spot, the endochrome apparently being arranged in a ring around the outer part of the cell. This is, however, occasionally wanting. The filaments, which are in active growth, are very irregular in form, often much broader than the others; their cells very irregular and sometimes fused together into one mass. The measurements given in the diagnosis were taken from the filaments of the first kind.

b. Thallus indefinite expansus.

Thallus indefinitely expanded.

N. Cesatii, BALS.

N. terrestre; thallo longe lateque expanso, gelatinoso-membranaceo, viridi-flavescente; trichomatibus flexuoso-curvatis, sublaxè implicatis, pallide ærugineis; articulis sphericis, laxè vel arctius connexis; cellulis perdurantibus sphericis, et interjectis et terminalibus.

Diam.—Artic. .00016—.0002; cell. perdur.—.00033".

Syn.—*N. Cesatii*, BALS. RABENHORST, Flora Europ. Sect. II. p. 175.

Hab.—In terrestre, Kansas (Prof. Parry); Texas (Prof. Ravenel).

Terrestrial; thallus broadly and indefinitely expanded, gelatinous-membranaceous, yellowish-green; filaments flexuously curved, rather laxly implicate, pale-greenish; articles spherical, laxly or more closely connected; heterocysts spherical, both interstitial and terminalibus.

Remarks.—This plant was sent to me by Dr. C. C. Parry, from whose letter the following is extracted: "I send enclosed specimens of a singular land *Alga?* which I met with in this vicinity; lightly attached to bare patches of soil interspersed with buffalo grass. In the adjoining bluffs are cretaceous shales full of seams and layers of selenite, from the decomposition of which the bottom soil becomes strongly impregnated with various saline matters. The present season has been characterized by unusual quantities of rain, causing extensive floods over what is usually a dry, arid district."

The agreement between the mature forms is essentially perfect. There can be scarcely any doubt as to the identification, although I have not seen the American plant in its young state. The fronds appeared to be 1—2 lines in thickness, with its surface smooth, or sometimes with close subparallel ridges or wrinkles.

According to Rabenhorst, the young European *N. cesatii* is in the beginning globose, and pale golden-yellow; soon, however, bursting and spreading out into an indefinitely expanded thallus.

Among the algæ collected by Prof. Ravenel in Texas is a *Nostoc*, labelled "On Mud Flats, Cedar Bayou, Harris Co.," which comes so close to *N. cesatii*, that I think it must be referred to it. It differs only in being more olivaceous, somewhat firmer and in the size of the heterocysts—the largest of the latter which I have examined, attaining the size only of .00027". The largest vegetative cells are .00017 in diameter.

***N. calcicola?* Ag.**

N. thallo irregulariter expanso, enormiter sublobato, tenue, membranaceo, cartilagineo, elastico, pellucido, aut laete viride, vel brunneo, vel dilute viride, irregulariter undulato plicato vel bullatoo; peridermate plerumque subnullo; trichomatibus cum filis leptothricoides ramosis intermixtis, flexuosis, plerumque distantibus, rarissime e cellulis biseriatis compositis; cellulis subglobosis, oblongis, ovalibus, cum ceteris ellipticis intermixtis, plerumque laxè connexis; cellulis perdurantibus spæræicis, interjectis et terminalibus.

Diam.—Art. $\frac{7}{1000}$ "— $\frac{1}{10000}$ " = .0001"4—.0001"; cell. perdur. $\frac{3}{10000}$ "— $\frac{1}{5000}$ " = .0003"—.0002".

Syn.—*N. calcicola*, Ag. RABENHORST, Flora Europ., Sect. II. p. 174.

Hab.—In rupibus, Georgia. (Prof. Ravenel.)

Thallus irregularly expanded, membranaceous, thin, cartilaginous, elastic, pellucid, bright green, pale green or brownish, thin, irregularly undulately plicate or bullate; periderm mostly scarcely distinguishable; filaments intermixed with branched leptothrix filaments, flexuous, mostly distant, very rarely composed of biseriata cells; cells subglobose, oblong, oval, intermingled with elliptical ones, mostly loosely connected; heterocysts spherical, interspersed or terminal.

Remarks.—This species is one of those sent me by Dr. Billings. It was collected near Catoosa Springs, Georgia, by Prof. H. W. Ravenel. In the dried state it is of a dirty olive-green, and very much wrinkled and irregular on its surface. The largest specimens are about an inch long. There is no very distinct periderm, although in some places the filaments are placed more closely together on the outer portions of the frond. This plant seems to agree with the descriptions of the

European *S. calcicola*, from which it differs somewhat, however, in having its heterocysts both terminal and among the cells, and also somewhat in their size.

N. calidarium, WOOD.

N. thallo maximo, indefinite expanso, aut membranacco-coriaceo vel membranaceo-gelatinoso vel membranaceo, aut late viridi vel sordide olivaceo-viridi vel olivaceo-brunneo, irregulariter profunde laciniato-sinuato, ultimo eleganter laciniato; trichomatibus inæqualibus, interdum flexuoso-curvatis, plerumque subrectis et arcte conjunctis, in formis duabus occurrentibus: forma altera parva, viridi, articulis cylindricis, eum cellulis perdurantibus hic illic interjectis, vaginis interdum obsoletis, sæpius diffluentibus; forma altera maxima, articulis globosis vel oblongis, aurantiaco-brunnea, cellulis perdurantibus ab articulis ceteris haud diversis.

Diam.—Formæ primæ articuli maximi $\frac{1}{100000}$ unc.; cellulæ perdurantis $\frac{1}{80000}$ unc. Formæ secundæ articuli long. $\frac{1}{20000}$ to $\frac{1}{30000}$ unc., lat. $\frac{1}{80000}$ to $\frac{1}{60000}$, articuli globosi $\frac{1}{35000}$ to $\frac{1}{40000}$ unc.

Syn.—*N. calidarium*, WOOD, American Journal of Science and Arts, 1869.

Hab.—"Benton Springs, Owen's Valley, California" (Mrs. Partz).

Thallus very large, indefinitely expanded, either membrano-coriaceous or membrano-gelatinous or membranaceous, either bright green or dirty olive-green or olive-brown, irregularly profoundly laciniately sinuate, finally elegantly lacinate; filaments unequal, sometimes flexuously curved, but mostly straightish and closely conjoined, occurring in two forms; the one small, green, with cylindrical joints, the heterocysts scattered here and there, the sheaths sometimes absent, often diffluent; the other form very large, with globose or oblong articles, orange-brown, the heterocysts not different from the other cells.

Remarks.—Numerous specimens of this species were received from Mrs. Partz, who collected them in Benton's Spring, a thermal water situated in the extreme northern point of Owen's Valley, California, sixty miles southwest from the town of Aurora. The following extract from a letter of Mrs. Partz describes the place and mode of their growth more minutely.

"I send you a few samples of the singular vegetation developed in the hot springs of our valley. These springs rise from the earth in an area of about eighty square feet, which forms a basin or pond that pours its hot waters into a narrow creek. In the basin are produced the first forms, partly at a temperature of 124°—135° Fahr. Gradually in the creek and to a distance of 100 yards from the springs are developed, at a temperature of 110°—120° Fahr., the Algæ, some growing to a length of over two feet, and looking like bunches of waving hair of the most beautiful green. Below 100° Fahr., these plants cease to grow, and give way to a slimy fungus growth, though likewise of a beautiful green, which, finally, as the temperature of the water decreases, also disappears. They are very difficult to preserve, being of so soft and pulpy a nature as not to bear the least handling, and must be carried in their native hot water to the house, very few at a time, and floated upon paper. After being taken from the water and allowed to cool they become a black pulpy mass. But more strange than the vegetable are the animal organizations, whose germs, probably through modifications of successive generations, have finally become indigenous to these strange precincts. Mr. Partz and myself saw in the clear water of the basin a very sprightly spider-like creature running nimbly over the ground, where the water was 124° Fahr., and on another occasion dipped out two tiny red worms."

In regard to the temperatures given, and the observation as to the presence of animal life in the thermal waters, Mr. William Gabb, of the State Geological Survey, states that he has visited the locality, knows Mrs. Partz very well, and that whatever she says may be relied on as accurate.

The color of the dried specimen varies from a very elegant bluish-green to dirty-greenish and fuscous-brown. After somewhat prolonged soaking in hot water, the specimens regained apparently their original form and dimensions, and were found to be in very good condition for microscopical study.

The plant in its earliest stages appears to consist simply of cylindrical filaments, which are so small that they are resolved with some difficulty into the component cells by a first-class one-fifth objective. Fronds composed entirely of filaments of this description were received. Some of these were marked as "first forms," and as having grown in water at a temperature of 160° Fahr. Probably these were collected immediately over the spot where the heated water bubbled up. At this temperature, if the collection made is to be relied on as the means of judging, the plant does not perfect itself. To the naked eye these "first forms" were simply membranous expansions, of a vivid green color and indefinite size and shape, scarcely as thick as writing-paper, with their edges very deeply cut and running out into a long, waving, hair-like fringe. Other specimens, which grew at a much lower temperature, exactly simulated those just described, both in general appearance and microscopical characters. These, I believe, were the immature plant.

The matured fronds, as obtained by the method of soaking above described, were "gelatinous membranous," of a dirty-greenish or fuscous-brown at their bases, and bright green at their marginal portions, where they were deeply incised and finally split up into innumerable hair-like processes. Proximally they were one, or even two, lines in thickness, distally they were scarcely as thick as tissue paper. Their bases were especially gelatinous, sometimes somewhat translucent, and under the microscope were found to have in them only a few distant filaments.

Two sets of filaments were very readily distinguished in the adult plant. The most abundant of these, and that especially found in the distal portions of the fronds, were composed of uniform cylindrical cells, often enclosed in a gelatinous sheath. The diameter of such filaments varies greatly; in the larger the sheaths are generally apparent, in the smaller they are frequently indistinguishable.

In certain places these filaments are more or less parallel side by side, and are glued together in a sort of membrane. It is only in these cylindrical filaments that I have been able to detect heterocysts, which are not very different from the other cells; they are about one-third or one-half broader, and are not vesicular, but have contents similar to those of the other cells. In one instance only was I able to detect hairs upon these heterocysts.

The larger filaments are found especially near the base and in the other older portions of the frond. Their cells are generally irregularly elliptical or globose, rarely are they cylindrical. They are mostly of an orange-brown color; and there exists a particular gelatinous coating to each cell rather than a common gelatinous

sheath to the filament. These larger threads are apparently produced from the smaller filaments by a process of growth.

Near the base and in the under portions of the fronds, these filaments are scattered in the homogeneous jelly in which they run infinitely diverse courses. In the upper portions of the frond, and at some little distance from the base, the adjoining cells are very close to one another, and pursue more or less parallel courses, with enough firm jelly between to unite them into a sort of membrane.

This plant certainly belongs to the *Nostochaceæ*, and seems a sort of connecting link between the genera *Hormosiphon* of Kützing and *Nostoc*.

The best algologists now refuse to recognize the former group as generically distinct; and the characters presented by this plant seem to corroborate that view.

Adherent to, and often more or less imbedded in, the fronds of the *Nostoc*, were scattered frustules of several species of diatoms, none of which was I able to identify. In some of the fronds there were numerous unicellular Algæ, all of them representatives of a single species belonging to the genus *Chroococcus*, Nægeli. This genus contains the very lowest known organisms—simple cells without nuclei, multiplying, as far as known, only by cell-division. These cells are found single or associated in small families; and in certain species these families are united to form a sort of indeterminate gelatinous stratum. In these species the families are composed of but very few cells, surrounded by a very large, more or less globular or elliptical mass of transparent, firm jelly. The species is very closely allied to *Chroococcus turgidus*, var. *thermalis*, Rabenh., from which it differs in the outer jelly not being lamellated.

The technical description of this plant will be found in the proper place.

Fig. 2 *a*, pl. 2, represents the most mature and largest filament; Fig. 2 *b*, a small filament from the same frond, each magnified 800 diameters. Fig. 2 *c*, represents portions of the upper surfaces of fronds.

N. comminutum, Ktz.

N. thallo indefinite expanso, gelatinoso, natante, modo viride, plerumque sordide ferrugineo; trichomatibus flexuosis, plerumque subdense intricatis; articulis globosis (ante divisionem factam subcylindricis), subtiliter granulatis, interdum læte viridibus, plerumque ferrugineis aut luteo-fuscescentibus aut fuscis; cellulis perdurantibus globosis, articularum diametro duplo majoribus, interjectis aut terminalibus.

Diam.—Artic. $\frac{1}{3000}$ ''; cell. perdur. $\frac{1}{3000}$ ''.

Syn.—*N. comminutum*, Ktz. RABENHORST, Flora Europ. Algarum, Sect. II. p. 179.

Hab.—In fossis natante, prope Philadelphia.

Thallus indefinitely expanded, gelatinous, floating, mostly sordidly ferruginous, sometimes greenish; trichomata flexuous, mostly subdensely intricate; joints globose (before division subcylindrical), minutely granulate, sometimes bright green, sometimes ferruginous, yellowish-fuscous, or fuscous; heterocysts globose, about twice as long as ordinary joints, both interspersed and terminal.

Remarks.—This species is to be found floating on the surface of the ditches below the city in the latter part of August and September, forming a repulsive, ferruginous, slimy scum. The periderm is not very apparent, and indeed the sepa-

rate fronds are not distinct. The filaments are very long, mostly closely intricate, very much curved; in some places they are more sparse. Their color is mostly a sort of yellowish ferruginous-green, sometimes they are, decidedly, almost purely ferruginous, more rarely a bright green. This plant agrees pretty well with the descriptions of the European *Nostoc comminutum*, and I believe is the same species; if, however, *N. lacustre* of Kützing is distinct from *N. comminutum*, this is also; but I incline to the opinion that they are all different forms of one plant.

Fig. 3, pl. 2, represents a single filament magnified 800 diameters.

N. commune, VAUCH.

N. terrestre, thallo irregulariter expanso, difformi, undulato-plicato, tremulo, intus aquose gelatinoso, ætate provecata plerumque excavato, peridermate subcoriaceo firmo, olivaceo, luteofuscescente vel luteo-fusco cincto; trichomatibus flexuoso-curvatis, pallide ærugineis, laxè implicatis, æqualibus vel subæqualibus, haud raro a basi ad medium usque cellulis biseriatis compositis; articulis sphaericis vel e mutua pressione subquadrangularibus, laxè connexis, passim distantibus, puncto centrali turbato præditis; cellulis perdurantibus globosis, articulorum diametro duplo majoribus, interstitialibus terminalibusque.

Diam.—Cell. vegetat. .00012"—.00016"; cell. perdurant. .00025"—.00033".

Syn.—*N. commune*, VAUCH. RABENHORST, Flora Europ. Algarum, Sect. II. p. 175.

Hab.—In terrestre, New Jersey. (Austin.) "Rio Bravo. Schott." Harvey.

Terrestrial; thallus irregularly expanded, shapeless, undulate-plicate, tremulous, within of the consistence of thin jelly, in advanced age mostly hollow; periderm subcoriaceous, firm, olivaceous, yellowish-fuscous; filaments flexuously curved, pale green, laxly implicate, equal or subequal, not rarely composed of a double series of cells from their base to their middle; articles spherical or subquadrangular from mutual pressure, loosely connected, here and there distant, furnished with a central spot; heterocysts globose, twice as large as the vegetative articles, interstitial and terminal.

Remarks.—The only specimens I have seen of this species are very old ones, which have burst and discharged their central portions. I have consequently preferred to copy the diagnosis of Prof. Rabenhorst. My specimens agree pretty closely with it. The filaments, and also the single cells, are closer together than his words would seem to indicate. My measurements of the heterocysts, as given above, are larger than those of Prof. Rabenhorst. They agree, however, with his text, which his own measurements do not. I am indebted to Prof. Austin for specimens of this species, which he collected in Northern New Jersey. According to Professor Harvey this plant was collected by Dr. Schott along the Rio Bravo, where it is common on dry flats after rains.

SUBFAMILY SPERMOSIREÆ.

Thallus sine peridermate, interdum nullus. Trichomata sporis instructa.

Thallus without any periderm, sometimes absent. Filaments furnished with spores.

Genus ANABÆNA, BORY.

Trichomata moniliformia, evaginata; sporis sphaericis, aureis vel aureo-fuscis, plerumque singulis, cum cellulis vegetativis vel perdurantibus conjunctis.

Filaments moniliform, without sheaths; spores spherical, yellow or yellowish-fuscous, mostly single, variously placed as to the heterocysts and ordinary cells.

Remarks.—The characters which I have given are somewhat different and less exacting than those of Prof. Rabenhorst, otherwise our American species would hardly be covered by the diagnosis. Professor Harvey in his *Phycologia Britannica* states that A. Jussieu had preoccupied the name, *Anabæna*, by applying it to a genus of *Euphorbiaceæ*. The date of Bory's name is, however, 1823, whilst that of Jussieu is 1824. Hence, it is the latter which must be changed.

A. gelatinosa, WOOD.

A. thallo mucoso gelatinoso, indefinite expanso, dilutissime brunneo, nonnihil pellucido; trichomatibus haud vaginatis, leviter flexuoso-curvatis, nonnihil distantibus, haud intricatis, aut dilute aureis aut dilute cæruleo-viridibus; articulis globosis, homogeneis; cellulis perdurantibus articularum diametro fere æqualibus, globosis, vel rare oblongis; sporis terminalibus, singulis, globosis (fusco-brunneis?).

Syn.—*A. gelatinosa*, WOOD, Prodrômus, Proc. Amer. Philos. Soc., 1869, 126.

Hab.—Prope Philadelphia.

Thallus gelatinous, mucous, indefinitely expanded, somewhat pellucid, with a brownish tinge; filaments not vaginate, somewhat curved, rather distant, not intricate, either a light golden-yellow or light bluish-green; joints globose, homogeneous; heterocysts about equal to the filaments in diameter, globose or rarely oblong; spores terminal, globose.

Remarks.—The color of the shapeless mass of jelly of which the frond is composed is a light brown with, in places, a decided reddish or flesh-colored tint. The heterocysts are either interstitial or terminal, no hairs were detected on them; they are mostly globose and only occasionally are they oblong.

Fig. 4, pl. 2, represents a filament of this species magnified 750 diameters; the color of the endochrome of the large spore was possibly due to its being dead.

A. flos aquæ, (LYNGB.) KTZ.

A libere natans, submembranacea, æruginea; trichomatibus plus minus curvatis, sæpius circinnatis; articulis sphaericis vel e mutua pressione modo ellipticis modo oblongo-quadratis; cellulis perdurantibus ellipticis singulis vel geminis; cytoplasmate pallide ærugineo granulato turbato; sporis exacte globosis aureo-fulvis lucidis, singulis interjectis, articularum diametro subduplo majoribus. R. Species mihi ignota.

Diam.—Artic. 0.00017"—0.00025"; diam. long cell. perd. 0.00048"—0.00053"; spor. 0.00032"—0.0004".

Syn.—*A. flos aquæ*, (LYNGB.) KTZ. RABENHORST, Flora Europ. Algarum, Sect. II. p. 182.

Hab.—"Round Pond, West Point, New York." Prof. Bailey. Silliman's Journal, N. S., vol. iii. 18

Swimming free, submembranaceous, æruginous; filaments more or less curved, very often circinnate; articles spherical, or, from mutual pressure, elliptical or oblong quadrate; heterocysts elliptical, single or geminate; cytoplasm pale æruginous, granulate; spores exactly globular, golden-fulvous, bright, singly interspersed, nearly twice the diameter of the joints.

A. gigantea, WOOD.

A. thallo nullo, trichomatibus singulis et numeroso-consociatis, natantibus, rectis, in ætate juveni spiraliter convolutis; articulis plerumque subglobosis, arcte connexis, granulosis; cel-

Julis perdurantibus interjectis, articulis vegetativis subæqualibus, utroque polo punctiforme incrassatis, subsphæricis; sporis subsphæricis.

Syn.—*A. gigantea*, Wood, Prodr. Amer. Philos. Soc., 1869, 145.

Hab.—In stagnis natante, prope Philadelphia.

Diam.—Artic. vegetat. max. $\frac{1}{24000}$. Heterocysts $\frac{1}{20000}$ = .0005. Spor. lat. $\frac{1}{12000}$ = Long. $\frac{1}{10000}$ = .001.

Thallus wanting; filaments occurring floating singly on water or in great numbers, straight, but in the young state often spirally convolute; articles mostly subglobose, closely connected, granular, heterocysts subspherical, interstitial, a very little larger than the vegetative cells, thickened at each end in a punctiform manner; spore subspherical.

Remarks.—This plant was found by myself, late in the summer, floating upon a brick-pond below the city, forming a part of a thick, dirty-green, "pea-soup colored," almost pulverulent scum. The filaments, though occasionally in great numbers, were never, that I saw, joined together by any jelly so as to form a frond.

Fig. 5, pl. 3, represents a short filament of this species magnified 750 diameters.

Genus CYLINDROSPERMUM, KtZ.

Sporæ ante cellulam terminalem ortæ.

Spore developing from the next to the terminal cell.

C. minutum, Wood.

C. trichomatibus dilute ærugineis, plerumque flexuoso-curvatis et intricatis, interdum subrectis; articulis cylindricis, ad genicula plus minus constrictis, homogeneis vel granulatis; cellulis perdurantibus terminalibus, hirsutis, globosis; sporis ellipticis, diametro 2—3 plo longioribus, subtilissime granulatis.

Syn.—*C. minutum*, Wood, Prodr. Amer. Philos. Soc., 1869, 126.

Diam.—Artic. $\frac{1}{30000}$ "; spor. long. $\frac{1}{18300}$ "; transv. $\frac{1}{40000}$ ".

Hab.—In stagnis prope Philadelphia.

Filaments light æruginous-green, generally curved and intricate, sometimes straightish; articles cylindrical, more or less constricted at the joints, homogeneous or granulate; heterocysts terminal, hirsute, globose; spores elliptical, 2—3 times longer than broad, very minutely granulate.

Remarks.—This species was found by myself at Spring Garden, New Jersey. With a number of other algæ it formed a ferruginous-brown gelatinous mass, growing in a deep, shaded, very stagnant pool. In most instances the filaments were closely interwoven, and sometimes formed minute greenish balls, just large enough to be visible to the unassisted eye. In other instances they were mixed up with various algæ in little indefinite masses. There is apparently a stage in the life of the plant, when it consists of a single filament enclosed in a little capsule, for mixed in with the rest of the gelatinous scum were little microscopic, subglobose masses, with a firm outer periderm and a single filament coiled up in the centre. The color of the filaments was generally a faint bluish-green, sometimes, however, with a yellowish tint. The spores were decidedly yellowish.

Fig. 6, pl. 2, represents a fragment of a filament with the spore magnified 800 diameters.

C. flexuosum, (AG.) RABENH.

C. strato gelatinoso, saturate viride, indefinite expanso; trichomatibus æqualibus, pallide vel saturate caruleo-viridibus, plerumque valde flexuosis et intricatis, sæpius circinatim vel fasciatim convolutis, interdum subrectis, et fasciatim contextis; articulis oblongis, ad genicula plus minus contractis, homogeneis vel granulatis, distinctis; cellulis perdurantibus terminalibus, subglobosis, rare hirsutis, nonnunquam in trichomatis utroque fine; sporis oblongo-cylindricis, diametro 2—3 plo longioribus, distincte granulatis.

Diam.—Spor. $\frac{5}{120000}$ " = .000416"; cell. veget. $\frac{1}{60000}$ " = .000166".

Syn.—*C. flexuosum*, (AG.) RABENHORST, Flora Europ. Algarum, Sect. II. p. 188.

Hab.—In locis irroratis, prope Philadelphia.

Stratum gelatinous, deep green, indefinitely expanded; filaments equal, pale or deep bluish-green, mostly very flexuous and interwoven, often circinnately or fasciately convolute; sometimes straightish and in bundles; articles oblong, more or less contracted at the joints, homogeneous or granulate, distinct; heterocysts terminal, subglobose, rarely hirsute, sometimes at both ends of the filament; spores oblong-cylindrical, 2 or 3 times longer than broad, distinctly granulate.

Remarks.—The color of the filaments in young specimens is deeper than in the older, which, however, grew in a much darker locality. The young spores are a yellowish-green, afterwards they are of a sort of yellowish reddish-brown. In one instance two spores were seen closely conjoined together at the end of a filament. In some filaments one or more heterocysts occur interstitially. Often one or more filaments will be seen coiled together like a rope. On the banks of the Schuylkill River I have found this species in two localities in the latter part of September. In the one instance it grew along the Reading Railroad, just above the Flat Rock tunnel, in a dark little grotto, formed by shelving rocks. In the other case, it was on wet ground by a horse-trough very near the west end of the upper bridge at Manayunk.

Fig. 1a, pl. 3, represents a filament, magnified 450 diameters.

Fig. 1b, a portion of a filament, magnified 800 diameters.

C. macrospermum, Ktz.

C. trichomatibus curvatis vel subrectis, pallide ærugineis; articulis cylindricis vel subcylindricis (in forma Europæa "globosis vel ellipticis"), ad genicula plus minus constrictis, passim confluentibus; cellulis terminalibus plerumque ellipticis vel ovatis, diametro paulo vel subduplo longioribus; sporis elliptico-oblongis vel oblongo-cylindraceis, viridibus (in formam Europæam maturam "saturate fuscis"), subtiliter granulosis, diametro duplo longioribus.

Diam.—Trich. cell. transv. $\frac{1}{30000}$ " = .00003"; spor. .00046"—.00054".

Syn.—*C. macrospermum*, Ktz. RABENHORST, Flora Europ. Algarum, Sect. II. p. 186.

Hab.—In rivulis, South Carolina. (Prof. Ravenel.)

Filaments curved or straightish, pale æruginous; articles cylindrical or subcylindrical (in European species "globose or elliptical"), more or less constricted at the joints, here and there confluent; terminal cells mostly elliptical or ovate, a little longer or about twice as long as broad; spores elliptical-oblong or oblong cylindrical, greenish (in mature European specimens deep fuscous), finely granular, about twice as long as broad.

Remarks.—I have received this species from Professor Ravenel, who collected it near Aiken, South Carolina, in the month of September; with it was the follow-

ing note: "In bottom of shallow, slowly running streams, adhering to ground or fallen leaves, &c., gelatinous green." The specimens agree well with the description of the European form, except that I have never seen the joints globose or elliptical, but *always* cylindrical, as they are said to be *sometimes* in the typical specimens. The color of the spores also is not "fuscous," but that probably depends upon their not being fully mature.

Fig. 7, pl. 2, represents the spore of this species with the neighboring heterocyst, magnified 750 diameters.

C. comatum, WOOD (sp. nov.)

C. terrestre, stratum gelatinosum æruginosum interdum brunneo tinctum, formans; trichomatibus flexuosis, intricatis, haud spiralibus, æqualibus; articulis breve cylindraceis, diametro æqualibus ad plus duplo longioribus, plerumque sejunctis, pallide æruginis, obscure granulatis; cellulis terminalibus subglobosis; sporis oblongo-cylindricis, diametro fere duplo longioribus, granulatis, luteo-brunneis; membrana crassa, distincte granulata.

Diam.—Spor. transv. $\frac{5}{12000}$ " = .00042". Long. $\frac{11}{2000}$ " = .00092". Artic. .0001".

Hab.—In terra uda; Niagara, Canada.

Growing on the ground, forming a gelatinous stratum of an æruginous color, sometimes tinged on edges with brown; filaments flexuous, equal, intricate, not spiral; joints shortly cylindrical, equal to or more than twice as long as the diameter, mostly separated, pale æruginous, obscurely granulate, terminal cells subglobose; spores oblong-cylindrical, about twice as long as broad, granulate, yellowish-brown; membrane thick, distinctly granulate.

Remarks.—I found this *Cylindrospermum* growing upon the ground in the marshes which border the Niagara River just above the Canadian Falls. It formed a bright, æruginous, gelatinous, but firmish, almost membranous, stratum.

The filaments are often quite long, and are composed of short, cylindrical cells, mostly placed rather far apart. The terminal cells are remarkable for being abundantly provided with long, flexible, hair-like processes, upon the ends of which are minute lobular bodies (cells?). These appendages are so minute as to make it difficult to determine their structure, and although I have studied them with a $\frac{1}{25}$ th immersion lens, giving a power of nearly 2500 diameters, there are some points about them still undetermined. I do not know whether they or the little globules are hollow or not. I do feel pretty certain, however, that the little globules are distinct bodies, and that they finally drop off, leaving the naked hair behind. Is it possible that they have any sexual significance? The spore-wall is thick, and under a high power is seen to be distinctly granulate. The granules are of course small, but in the perfected spore can plainly be seen with an eighth objective projecting out from the margin.

Fig. 8, pl. 2, represents the spore-end of a filament, magnified 1375 diameters.

Genus DOLICHOSPERMUM, THWAITES.

Sporæ ellipticæ, oblongæ vel cylindraceæ, inter cellulas vegetativas ortæ, sæpe in seriebus connexæ, a cellulis perdurantibus disjunctæ.

Spores elliptical, oblong, or cylindrical, occurring amidst the vegetative cells, often connected in series, separated from the heterocysts.

Syn.—*Sphærozyga*, (AUCTORES, *partim.*)

Dolichospermum, THWAITE'S MSS. Mr. J. RALFS on the *Nostochineæ*, Ann. Mag. Nat. Hist. 1850, p. 335.

Remarks.—This genus differs from *Sphærozyga* in that the spores have no relation, in regard to position, with the heterocysts. Professor Rabenhorst, in his Flora, does not acknowledge it; but it is very evident that he has neither seen the original paper of Mr. Ralfs, nor the species upon which the genus was founded, for he mentions none of the latter, either as good species or synonyms, and the memoir itself is not included in his bibliographical list. The generic characters given by myself are essentially those of the original description, with the exception that the filaments in the latter are said to be aggregated into a stratum, which is not true of the American forms herein described.

D. subrigidum, WOOD.

S. natans; trichomatibus singulis, rectis aut subrectis, minimis, dilute viridibus; articulis cylindraceis aut subglobosis, distinctis; sporis cylindraceis, in medio gradatim nonnihil constrictis, singulis aut duplicis, sine cellulis perdurantibus inter se; cellulis perdurantibus breve cylindraceis, singulis, distinctis.

Syn.—*Sphærozyga subrigidum*, WOOD, Prodr. Amer. Philos. Soc., 1869, p. 123.

Diam.—Cell. veg. trans. $\frac{1}{80000}$ " = .00016"; spor. transv. $\frac{1}{43000}$ "— $\frac{1}{45000}$ " = .00023"—.00022"; long. $\frac{1}{15000}$ " = .00066"; cell. perd. transv. $\frac{1}{45000}$ " = .00022".

Hab.—In stagnis prope Philadelphia.

S. Floating; filaments single, straight or straightish, very small, light green; articles cylindrical or subglobose, distinct; spores single or double, in the middle gradually a little constricted, not having a heterocyst between them; heterocysts shortly cylindrical, single, distinct.

Remarks.—I have found this species growing in the scum floating upon the ditches below the city. The filaments are always, as I have seen them, scattered. They seem always to be nearly straight, or entirely so, and indeed preserve their straightness so constantly as to suggest the name given the species. The spores are very distinct, and all that I have seen were greenish, cylindrical, and constricted in the middle, so that their sides are concave. Their position does not seem to be uniform, any further than that they are amongst the ordinary cells. The heterocysts are large, almost equalling the spores in diameter; I have never detected hairs on them. This species appears to be most nearly allied to *D. Thwaitesii* of Ralfs, from which it differs in not forming a stratum, and in the great proportionate diameter of the heterocysts. I have never seen any measurements of *D. Thwaitesii*.

Fig. 2, pl. 3, is a filament, magnified 975 diameters.

D. polysperma, (KtZ.)

S. trichomatibus plerumque subsolitariis, sed interdum consociatis et intricatis, dilute cæruleo-viridibus, subrectis aut varie curvatis et flexuosis; articulis aut subsphæricis aut breve cylindricis; cellulis perdurantibus globosis aut latissime ellipticis, articulorum diametro paulo vel duplo majoribus; sporis plus minus elongatis, cylindraceis—in ætate immatura, sparse granulatis, dilute cæruleo-viridibus, et cum membrana haud distincta,—in ætate matura dense granulatis et cum membrana suberassa.

Diam.—Artic. $\frac{1}{80000}$ " = .00016"; spor. $\frac{1}{150000}$ "— $\frac{1}{150000}$ " = .00026"—.00033".

Syn.—*S. Carmichaelii*, HARVEY, Phycol. Britannica, T. cxiii.

S. polysperma, (KtZ.) RABENHORST, Flora Europ. Algarum, Sect. II. p. 192.

Hab.—In stagnis, New Jersey.

S. filaments mostly subsolitary, but sometimes associated and interwoven together, light bluish-green, straightish, or variously curved and flexuous; articles either subspherical or shortly cylindrical; heterocysts globose or very broadly elliptic, a little larger to twice as large in diameter as the ordinary joints; spores more or less elongate, cylindrical, in the uncertain condition sparsely granulate, light bluish-green, with the membrane not distinct, in the mature state densely granulate, and with a thickish membrane.

Remarks.—I found this species growing in a brownish jelly, with various other algæ, in a pool east of Camden, New Jersey. The filaments were mostly scattered, but in some places numbers of them were collected in little masses. In some filaments almost all the cells were developed into spores, so that a single thread contained ten or even more spores. In by far the larger number of such cases there was between each pair of spores a heterocyst; sometimes, however, the latter was wanting, and the spores would be attached to one another.

My specimens differ somewhat from the European form, but are too close to separate from them. They equally resemble, however, *S. Carmichaelii*. Indeed, I cannot see any sufficient reason for separating the species. *S. Carmichaelii* is, to be sure, a salt-water plant. I have, however, received specimens collected by Dr. Lewis, near Stonington, which I believe grew in salt water, and which agree in every respect with my fresh-water specimens.

Fig. 3, pl. 3, represents a portion of a filament, magnified 750 diameters.

FAMILY RIVULARIACEÆ.

Thallus gelatinosus, mollis vel induratus, vel crustaceus, interdum calce impletus, subglobosus vel amorphus. Trichomata ad oscillarium morem articulata, vaginata, sed interdum ætate protracta cum vaginis in gelatinam matricalem confluentibus et haud visibilibus, simplicia vel pseudoramosa, superne attenuata, sæpius in apicem piliformem longe producta, parallela vel radiatim disposita, cellula basale hyalina globosa et interdum cellulis interstitialibus instructa. Sporæ (*manubria*, KtZ.), singulæ plerumque inter cellulam perdurantem basilarem et cellulas vegetativas positæ, sæpe permagnæ, cylindricæ, plerumque pachydermaticæ.

Vegetatio terminalis. Propogatio sporis tranquillis.

Thallus gelatinous, soft, or indurated, or crustaceous, sometimes filled with lime, subglobose or amorphous. Trichomata articulated like an oscillatoria, vaginate, but sometimes, when old, with the sheaths confluent in the maternal jelly and not visible, simple or pseudoramosa, attenuated above, often with the apex prolonged into a long hair, parallel or radiately disposed, furnished with globose hyaline, thick-walled basal cells, and sometimes with interstitial cells. Spores cylindrical, generally placed between the basal and vegetative cells, often very large, mostly with thick coats.

Vegetation tranquil. Propagation by means of tranquil spores.

Remarks.—In the *Rivulariaceæ* the thallus is always small; but is most generally in the various species somewhat definite in form and size. Its consistency in our North American forms varies from that of an exceedingly soft, formless jelly to that of a gristly mass. The maternal jelly is usually colorless, sometimes brownish or yellowish. There is never any condensation of the outer portion of the

frond into a periderm. The filaments commonly radiate from the centre to the circumference; sometimes, especially in the softer fronds, they are simply parallel with one another. The sheaths vary in their breadth, firmness, and distinctness. These little plants grow chiefly in the water; some species are said to live in the air in exceedingly damp places, but I have not as yet met with any such. They appear to prefer cold climates, although I have received specimens from South Carolina. With us, I have only found them in the late autumn and winter months. As to their life-history very little appears to be known; I have not been able to make any observations myself upon this point, nor to obtain access to the papers¹ by De Bary, almost the only sources of such information, and therefore pass by the subject.

Genus NOSTOCHOPSIS, WOOD.

Trichomata ramosa cum cellulis perdurantibus aut in lateribus sessilibus aut in ramulorum brevissimorum apicibus dispositis. Vaginæ nullæ. Thallus definitus.

Syn.—*Nostochopsis*, WOOD, Prodrômus, Proc. Amer. Philos. Soc., 1869.

Thallus definite; filament branched; heterocysts sessile upon the sides of the filaments, or raised upon the apices of short branches; sheaths none.

Remarks.—The curious plant upon which this genus is founded has the habit of a nostoc. The outer portion of the frond is condensed, so as to give the appearance of a periderm; but there is, in reality, no true periderm. The consistence of the thallus is that of a firm, gelatinous mass. The trichomata or filaments radiate from the inner part of the frond towards the outer surface, but many of them take their origin in the outer portions of the thallus. In most places they are distinctly articulated, and, indeed, the joints being swollen and moniliform, in some parts they almost seem to be composed of globular cells, resembling somewhat the filaments of a nostoc; on the other hand, in certain portions they are not at all articulated, and this for long distances. No sheaths are anywhere visible. The heterocysts are, strangely enough, never placed in the continuity of the filaments. Sometimes they are sessile immediately upon the latter, sometimes they are raised upon very short branches. They are globose, with rather thick walls. Possibly, however, I am mistaken in believing these bodies to be heterocysts, for they may be rather of the nature of spores, as is somewhat indicated by their thick walls, and often apparently dense contents. Their round shape, and the absence of anything else representing heterocysts, has induced me, however, so to consider them. In my Prodrômus I placed this plant provisionally amongst the nostocs; but the radiation of the filaments from within outwards, and especially their being branched, on second thought seem to me to indicate a closer relation with the *Rivulariaceæ*. The genus appears to be a sort of connecting link between the two families.

¹ "Flora," 1863.

N. lobatus, WOOD.

N. thallo vivide viride aut luteo-viride, cavo, enormiter lobato, natante, modice magno, firmo, gelatinoso; trichomatibus plerumque longis, flexuosis, dilute viridibus, plerumque articulatis, partim inarticulatis, cylindricis aut sub-moniliformibus, sparse granulatis.

Diam.—Trichom. $\frac{1}{150000}'' = .00006''$ — $\frac{1}{75000}'' = .00013''$; cell perdum. $\frac{1}{37500}'' = .00026''$.

Syn.—*N. lobatus*, WOOD, Prodr. Amer. Philos. Soc., 1869.

Hab.—In Schuylkill Flumine, prope Philadelphia.

Thallus bright green or yellowish-green, hollow, irregularly lobed, floating, moderately large, firm, gelatinous; filaments mostly long, flexuous, dilute green, mostly articulate, partly inarticulate, cylindrical or somewhat moniliform, sparsely granulate.

Remarks.—I found this plant floating upon the Schuylkill River just above Manayunk. The hollow frond was buoyed up by a bubble of gas contained within it. It was an irregular, flattened, somewhat globose mass, of a bright green color and about half an inch in diameter. It seems very probable that in its earlier condition, it was a solid attached frond. The long slender filaments are often very tortuous, but run a pretty direct general course towards the outer surface.

Fig. 6 *a*, pl. 3, represents a section of the frond slightly magnified; *a*, *b*, *c*, portions of filaments magnified 800 diameters.

Genus GLOIOTRICHIA, J. AG. (1842.)

Trichomata e planitie orta pseudoramosa, distincte vaginata; vaginæ amplæ, basi plerumque saccatæ, transverse undulato-plicatæ, plus minus constrictæ, apice apertæ, non laciniatæ. Sporæ magnæ cylindricæ.

Filaments springing from a plane, pseudoramosæ, distinctly vaginate; sheath ample, mostly saccate at the base, transversely undulately plicate, more or less constricted, open at the apex, not lacinate. Spores large, cylindrical.

Remarks.—This genus was, I believe, first indicated by Professor Agardh in his *Algæ Maris Mediterranei et Adriatici*, a work to which I have not access. On account of this, and also because I have not seen any of the typical species of the genus, I have preferred simply copying the generic characters given by Professor Rabenhorst. If my understanding of “e planitie orta” is correct, I do not think it true. Professor Rabenhorst’s own figure of *Rivularia* shows that the filaments do not all arise on one plane; although he asserts the character equally for that genus. In our American species the filaments do not all arise on one plane, nor can they be spoken of as “pseudoramosa.”

G. incrustata, WOOD.

G. globosa vel subovalis, firma, solida, ad pisi minimi magnitudinem, dilute viridis, crystallophora; trichomatibus rectis aut leviter curvatis, in pilum productis, viridibus aut flavescensibus, sæpe infra læte viridibus sed supra flavescensibus, haud ordinatim articulatis; articulis inferioribus in trichomatibus maturis brevibus, plerumque compressis; pilo apicale recto aut leviter curvato, plerumque indistincte articulato, sæpe interrupto; vaginis amplis, achrois, saccatis, interdum valde constrictis; sporis cylindricis, sæpe curvatis, diametro ad 9 plo longioribus; cellulis perdurantibus sphaericis.

Diam.—Trichom. cum vag. $7\frac{7}{1000}''$ — $7\frac{9}{1000}''$; sporis max. $7\frac{3}{1000}''$ — $7\frac{4}{1000}''$; cell. perd. $1\frac{7}{1000}''$.

Syn.—*G. incrustata*, Wood, Prodr. Amer. Philos. Soc., 1869, p. 128.

Hab.—Schuylkill River, plantas aquaticas adhærens.

FROND globose or suboval, firm, solid, about the size of a very small pea, light green, crystal bearing; filaments straight or slightly curved, produced into long hairs, green or yellowish, sometimes bright green in their proximal portions but yellowish above, not regularly articulate; lower articles in the mature filament short, and generally compressed; apical seta straight or slightly curved, mostly indistinctly articulate, frequently interrupted; sheath ample, transparent, saccate, sometimes strongly constricted; spores cylindrical, frequently curved, about 9 times as long as broad.

Remarks.—I found this species growing attached to some little plants, either in the Schuylkill near Spring Mills, or else in the spring itself, I do not know which. The roundish fronds varied in size from a mustard-seed to that of a half-grown pea. They were of a decided green color, but appeared grayish from the amount of carbonate of lime in and upon them. The larger balls, when cut in two, were distinctly separable into a central and cortical part. The former was more gelatinous and contained fewer of the filaments than the latter. The filaments mostly arose in sets together, *i. e.* there were one or more zones or planes in which the bases of the filaments were placed together. This, however, was not strictly the case, as there were almost always some scattered trichomata. The matured filaments are very distinct. Their sheaths are very large, and often saccate, with wavy, loose-looking margins; sometimes they are suddenly transversely constricted, once or more in their length; sometimes they look as if a tight spiral band were wound around; sometimes they are entirely free from any constrictions. These sheaths are open above, appearing as though they had been melted away. The spore is long and cylindrical, and is highly granular. The endochrome is generally articulated below, the joints are often so nearly globular in the lower portions as to give a moniliform appearance; sometimes the articles are compressed. The upper portion of the trichoma is frequently interrupted, and if at all articulated is very irregularly and indistinctly so. The younger filaments have their endochrome variously and irregularly interrupted. The basal cells are globular. I believe the formation of new filaments and the consequent growth of the frond take place by distal portions of the projecting endochrome separating from the parent filament, then forming a basal cell, and lastly a sheath. (See Plate 00.) The carbonate of lime does not exist as a definite incrustation, but in the form of semi-crystalline masses scattered through the frond. This species seems to come closer to *G. boryana* than any described species, from the description of which it differs, in the color of thallus, in the latter being always solid (at least so I have found it late in the fall, when the spores were fully perfected), in its habit of inclosing crystals of carbonate of lime, in the curved spores; and, doubtless, a comparison of the specimens would show still more important differences.

Fig. 4 *a*, pl. 3, represents a section of a frond moderately magnified; fig. 4 *b*, the basal end of a filament magnified 460 diameters; fig. 8 *c*, filaments magnified 260 diameters.

G. angulosa, (ROTH.) J. AGH.

G. globoso-angulosa, cava, viridi-fuscescens, ad cerasi magnitudinem; trichomatibus strictis, torulosis, superne leviter flexuosis, passim interruptis; articulis inferioribus plus minus compressis, diametro duplo triplove longioribus; vaginis amplis, achrois hic illic leviter constrictis; sporis plus minus elongatis, oblongo-ovatis vel ellipsoideo-cylindricis, diametro 3–6–10 plo longioribus, ærugineo-fuscescentibus, nonnunquam leviter curvatis, cytoplasmate subtiliter granuloso, turbato. (R.) *Species mihi ignota*.

Diam.—Cell. perd. 0.00036"—0.0005". Spor. max. 0.00059". (R.)

Syn.—*G. angulosa*, (ROTH.), J. AGARDH., RABENHORST, Flora Europ. Algarum, Sect. II. p. 201.

Hab.—Hudson River prope West Point. (Bailey.)

Globose angular, hollow, greenish-fuscous, attaining the size of a cherry; filaments strict, torulose, above somewhat flexuose, here and there interrupted; inferior joints more or less compressed, 2–3 times longer than their diameter; sheath ample, colorless, here and there slightly constricted; spores more or less elongate, oblong-ovate or ellipsoidal-cylindrical, 3–6–10 times longer than the diameter, æruginous-fuscous, sometimes slightly curved, cytoplasm very minutely granulate.

Genus RIVULARIA, (ROTH.) AGH.

Thallus et trichomata eadem quæ Gloiotricha, sed vaginæ arctissimæ, sæpe in gelatinam matrixalem confluentes, quasi nullæ.

Thallus and filaments similar to those of Gloiotricha, but the sheaths very close, often confluent in the gelatinous matrix and apparently wanting.

Remarks.—The characters given above are those of Professor Rabenhorst. Flora Europ. Algarum, Sect. II. p. 206

R. cartilaginea, WOOD.

R. subglobosa, parva, cartilaginea, saturate brunnea vel subatra, solitaria in plantis aquaticis:—trichomatibus maturis-sterilibus, rectis aut subrectis, cylindricis, elongatis, haud articulatis; cytoplasmate sæpe interrupto; vaginis arctis et distinctis; cellulis perdurantibus globosis, diametro subæqualibus:—trichomatibus fertilibus—rectis aut subrectis, supra spora cellulis 8–9 instructis; sporis elongatis, rectis, cylindricis; vaginis nonnihil crassis, arctis:—trichomatibus immaturis breve articulatis; vaginis subamplis.

Diam.—Trich. cum vag. $\frac{1}{3000}$ "; spor $\frac{1}{3000}$ ".

Syn.—*R. cartilaginea*, WOOD, Proc. Am. Philos. Soc., 1869, p. 128.

Hab.—In palude, Northern Michigan.

Fronde subglobose, small, cartilaginous, deep brown or blackish, solitary upon aquatic plants; mature sterile filaments, cylindrical, elongated, not articulated, their cytoplasm frequently interrupted, their sheaths close and distinct, their heterocysts globose and about equal to them in diameter; fertile filaments straight or nearly so, above the spores furnished with 8 or 9 cells; spores elongate, straight, cylindrical; sheaths rather thick, close; immature filaments shortly articulate, their sheaths rather large.

Remarks.—The frond of this species grows attached to the leaves of water-plants, and has its under side markedly flattened so that it is somewhat semi-globose. The filaments which compose the mass of the very firm frond are elongated, cylindrical, and of nearly or entirely uniform diameter throughout. The sheaths are close, distinct, rather thin, open above, and, in many instances, almost or even entirely empty. Scattered amongst such filaments are the fertile ones. These have at their base an elongated cell, in which is the long cylindrical spore, which varies

very greatly in length in the various filaments, but is almost always shorter than the cell containing it. Just beyond the spore is a series of distinct, variously shaped cells, about seven in number, which are, as I have seen them, empty. In the outer portions of the frond occur what I believe to be young filaments. These are distinguished by their rapidly decreasing in diameter towards their distal end, by their being distinctly articulated, by their basal cell not being distinctly separated as in the older filaments, and by their sheaths being more ample.

These various filaments composing the fronds do not arise from any one place, but commence at very different distances from the centre, and pursue a more or less straight course to the circumference of the frond, from which they often project.

Fig. 9, pl. 2, represents a section of the frond moderately magnified; fig. 9 *b*, is a drawing of the basal part of a filament magnified 800 diameters.

Genus ZONOTRICHIA.

Thalli pulvinato-hemisphærici, sæpe confluentes, calce prægnantes, plus minus indurati, basi plani affixa, ætate provecta plerumque excavati, intus zonati; zonis concentricis, variegatis; trichomata pseudoramosa, gracilia, inæqualia, apice hyalina et plus minus longe cuspidata vel in pilum producta. Vaginæ firmæ, homogeneæ vel longitudinaliter plicato-fibrillosæ, apice integræ vel dilatatæ et in fibrillas solutæ. Sporæ ignotæ.

Thalli pulvinately hemispherical, often confluent, impregnated with lime, and more or less indurated, fixed by the flattened base, in advanced age mostly excavated, zoned within; zones concentric variegated; filaments pseudoramose, slender, unequal, their apices hyaline and more or less cuspidate or prolonged into a hair; sheaths firm, homogeneous, or longitudinally plicately fibrillose, their apices entire or dilated and dissolved in fibrillæ. Spores unknown.

Z. mollis, WOOD (sp. nov.)

*Z. inter*dum subhæmispherica sed gregaria et in stratum nonnihil mammillosum confluens, submollis, cinerea vel griseo-carnea, parcezonata; trichomatibus longissimis, angustis, flexuosis; vaginis arctis, decoloratis, non fibrosis, firmis; trichomatibus internis articulatis, sæpe interruptis; articulis disjunctis, diametro æqualibus ad 4 plo longioribus; cellulis perdurantibus singulis globosis.

Diam.—Trich. c. v. $\frac{2}{12000}$ " = .00017". Sine vag $\frac{1}{12000}$ " = .000084".

Hab.—In saxis irroratis, "Cave of the Winds," Niagara, Wood.

Z. sometimes subsemispherical but gregarious and confluent into a somewhat mammillate, rather soft stratum, ashy or grayish flesh-colored, sparsely distinctly zoned; filaments very long, narrow, flexuous; sheaths close, colorless, not fibrillose, firm; internal filament articulated, often interrupted; joints separated, equal to 4 times longer than the diameter; heterocysts single globose.

Remarks.—Every American tourist is familiar with that most wonderful spot, the so-called "Cave of the Winds," at Niagara. It is simply a place where it is possible to go underneath a portion of the great cataract, and then round upon the rocky debris outside of it. Growing upon these rocks, eternally wet and glistening with foam and spray, I found this and the following species. The present form was much the most abundant, making a slippery, grayish, or grayish flesh-colored coating to many of the rocks, dotted here and there with the rigid, blackish fronds of

its fellow. This coating was not at all uniform, but was covered with mammillated masses, and consequently varied from two to six lines in thickness. Internally, it was striated or radiated, but not so evidently as the following species, and presented several distinct variegated zones. It was quite soft to the touch, as well as readily broken or crushed, and under the microscope was seen to contain very little lime salt. When dried it has a pronounced sebaceous appearance. The filaments composing it are remarkable for their great length, often apparently running from the bottom to the top of the frond. They are rarely if ever branched, and appear never to be furnished with any heterocysts save at their enlarged base. I have never seen any distinct hairs terminating them, their ends always appearing broken and open. They are often quite flexuous or even tortuous. The internal filament is remarkable for having its articles so distinctly separated. It is often very much interrupted, and in specimens preserved in carbolic-acid water is of an orange-brown color.

Fig. 3, pl. 4, represents a single filament magnified 260 diameters.

Z. parcezonata, Wood, (sp. nov.)

Z. nigro-iridis, enormiter semiovalis, ad 6" longa, dura, lubrica, non fragilis, calce prægnans, intus a basi distincte radiata, parce et sæpe obsolete zonata; trichomatibus modice longis, subrectis; trichomatibus internis cylindricis inarticulatis vel articulatis, et interdum moniliformibus; articulis longis et cylindricis vel brevibus et globosis; vaginis amplis, fibrillosis; cellulis perdurantibus basalibus et interjectis, his oblongis vel cylindricis, illis globosis et sæpe geminis.

Diam.—Cell. perd. basal. $\frac{1}{80000}$ " = .00017"; trichom. cum vag. $\frac{1}{40000}$ "— $\frac{3}{80000}$ " = .00025"—.00037".
Sine vag. .00006"—.00008".

Hab.—In saxis irroratis. "Cave of the Winds," Niagara.

Var.—*Z. cinerea*.

Blackish green, irregularly semioval, to 6 lines long, hard, slippery, not fragile, impregnated with lime, internally distinctly radiate, sparsely and often obsolete zoned; filaments moderately long, straightish; internal filament cylindrical, not articulated or articulated, sometimes moniliform; joints long and cylindrical, or short and subglobose; sheath ample, fibrillose; heterocysts basal and interposed in the body of the filament; the former globose, often geminate; the latter oblong or cylindrical.

Var.—Cinereous in color.

Remarks.—I found this plant growing on rocks as glossy, blackish, very hard and slippery fronds or masses, which varied in size from that of very small shot to nearly half an inch in length. The larger ones were not nearly so high as long, and presented irregular, almost bossellated upper surfaces. The filaments are often very evidently and frequently pseudoramose. The external surface of the broad sheath is covered with numerous fibrillæ, which envelop and seem sometimes to wrap it round and round. The color of the frond internally, when broken, is mostly a dark chocolate, and the surface presents a radiated appearance, with but two or three zones at most, and, in the very dark specimens, even these are not evident. No signs of spores have been found. Certain specimens which I obtained growing with the others, instead of being blackish in color, are grayish, but

agree in all other respects with their fellows. This gray color depends, I believe, upon the deposit of an immense quantity of lime salts, which in such specimens constitute by far the larger portion of the frond.

Fig. 4, pl. 4, represents a section of frond, slightly magnified.

It is either this, or the preceding species, which is referred to by Professor Bailey in Silliman's Journal, vol. iii, under the name of *Rivularia calcarea*, Sm. The present form may possibly be that plant, but not having been able to find any description sufficiently well made out to make identification possible, I have described both species as new.

Z minutula, Wood, (sp. nov.)

Z. minutissima, nigro-viridis, subglobosa, haud distincte zonata, nonnihil mollis, muscicola, calce non prægnans; trichomatibus internis, breve articulatis, distinctissime fasciculatim pseudoramosis; vaginis crassis, amplis, sæpe dilute aurantiaco-brunneis, apice plerumque coloris expertibus fissis et apertis; cellulis perdurantibus ovato-globosis.

Diam.—Trich. intern. .00012"—.00021"; cell. perd. .00025."

Hab.—In lacu, "Clear Pond," muscis affixa, Adirondack Mountains.

Very small, blackish-green, subglobose, not distinctly zoned, rather soft, growing on mosses, not impregnated with lime; internal filaments shortly articulate, very distinctly fasciculately pseudoramose; sheaths thick, ample, often pale orange-brown, with their apices mostly colorless, torn and open; heterocysts ovately globose.

Remarks.—The locality in which I found this plant is in the heart of the Adirondack wilderness. The little frond in none of my specimens is larger than a mustard-seed, and is not distinctly zoned. The plants were collected in the beginning of July, and very possibly are not fully grown, as the season of general growth opens very late in its parent lake. Very possibly, later in the year, it may be found larger and distinctly zoned. The general appearance of the plant, the character of its sheath, and the marked branching habit of the filaments have caused me to place it in this genus.

Genus **DASYACTIS**, Kütz.

Thallus gelatinosus, mollis, non zonatus. Trichomata matura sæpe haud vaginata. Sporæ nullæ.

Thallus gelatinous, soft, homogeneous, not zoned. Mature filaments often not vaginate. Spores absent.

D mollis, Wood.

D. parva, ad magnitudinem pisi minimi, enormiter subglobosa, mollis, gelatinosa, dilute viridis; trichomatibus plerumque subrectis, partim distincte, partim indistincte articulatis; vaginis, in trichomatibus maturis haud visibilibus, in trichomatibus juvenibus supra subamplis; cellulis perdurantibus sub-globosis, globosis, vel ellipticis, diametro duplo majoribus, plerumque singulis sed interdum bi vel triseriatis.

Diam.—Trich. $\frac{1}{8000}$ "— $\frac{1}{4000}$ "; cell. perd. $\frac{1}{1800}$ ".

Syn.—*D. mollis*, Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 128.

Hab.—In palude plantas aquaticas adhærens, Northern Michigan.

Fronde small, about the size of a small pea, irregularly subglobose, soft, gelatinous, light green, filaments generally straightish, partly distinctly, partly indistinctly articulate; sheaths in the mature filament not perceptible; in the young filaments rather large in the upper portion; heterocysts subglobose or globose or elliptic, twice as large as the filament, generally single but sometimes bi or tri-seriate.

Remarks.—I found this species growing attached to the little leaves of various minute cryptogamic and phanerogamic water-plants, in a small bog, near the mouth of Carp River, in Northern Michigan. The frond is somewhat translucent, with a slightly greenish tint, and has a soft, gelatinous consistency. The matured trichoma or filaments are more or less radiating, very long, generally nearly straight and parallel. Their joints or articles are long, mostly not very distinctly separated, and often are entirely wanting. The sheaths are entirely lost, no traces of them being perceptible. They seem to be altogether melted down into the homogeneous jelly, in which the filaments are imbedded. The basal cell is large, mostly globular, and very prominent. On the edges of the frond may frequently be seen small, evidently immature filaments, which have no distinct basal cell. Around the basal portion of these young trichoma there is a well-marked close sheath, which near the apex is wanting. In their immature filaments the joints are mostly very short, rather distinctly separated, almost globular.

Fig. 5, pl. 4.

Genus MASTIGONEMA, SCHWABE.

Trichomata articulata, sursum flagelliformia vel subulata, simplicia vel pseudoramosa (nonnunquam fasciculatim pseudoramosa), procumbentia vel erecta, in thallo indistincto cæspitose-aggregata; vaginæ arctæ et homogeneæ vel amplæ et plus minus distincte lamellosæ, apice plerumque apertæ, interdum laciniatæ.

Filaments articulate, superiorly flagelliform or subulate, simple, or falsely branched, sometimes fasciculately so, procumbent or erect, cæspitose aggregated into a sort of thallus; sheaths close and homogeneous or ample, and more or less distinctly lamellate, the apex for the most part open, sometimes lacinate.

M. fertile, WOOD, (sp. nov.)

M. cæspitosum, cum algis alteris intermixtum; trichomatibus simplicibus, elongatis, flexuoso-curvatis, apice truncatis; trichomatibus internis viridibus, sæpe interruptis, interdum distincte articulatis interdum inarticulatis; articulis diametro 3-5 plo longioribus; vaginis modice arctis, firmis, achrois, crassis, coloris expertibus, apice truncatis et apertis; sporis cylindricis, sparsis, in filamentis unico sæpe pluribus, in cellulis inclusis; cellulis perdurantibus globosis, interdum compressis trichomatis diametro fere æqualibus.

Diam.—Filam. $\frac{1}{3000}$ " = .00033"; spor. $\frac{1}{6000}$ " = .000166".

Hab.—In stagnis. Alleghany Mountains, Centre County, Pennsylvania.

Cæspitose, intermixed with other algæ; filaments simple, elongate, flexuously curved, truncate at the apex; internal filament green, often interrupted, sometimes articulated, sometimes not articulate; joints 2-3 times longer than their diameter; sheath moderately close, thick, firm, transparent, and colorless, truncate and open at the apex; spores cylindrical, scattered, each contained in a cell, frequently several in a filament; heterocysts globose, sometimes compressed, about equal in diameter to the filament.

Remarks.—I found this plant in a stagnant pool in "Bear Meadows," forming a filamentous, felted mass with *Edogonium echinatum* and other algæ. The variously curved and interlaced flexible filaments are always simple and of uniform, or nearly uniform, diameter through their whole length; excepting that in some instances there are small, local, bulbous enlargements of the sheath. Though the ends of the filaments in all the specimens I have seen are abruptly truncate, it is very possible that in the young trichoma the apex is prolonged into a long hair as in most of the *Mastigonema*. The inner filament is sometimes very distinctly articulated, often, however, it is not at all so. The sheaths are firm, not at all lamellate, and generally project beyond the inner trichoma. The spores are cylindrical, yellowish, with a pretty distinct, although very close coat. They are always inclosed in distinct cells, and are mostly several in a filament, placed at intervals in its length.

This is the first instance, at least that I know of, in which a species of this genus has been found in fruit, and it is interesting to note the resemblance of the spores to those of the more commonly fruiting rivularias. At the same time the peculiar arrangement of the spores is remarkable, and if the other species of *Mastigonema* should be found to have the more common exclusively basal arrangement of spores, I think it would afford good ground for considering *M. fertile* as the type of a new genus. Moreover, the filaments are not united into a distinct thallus, and also want the apical hair of *Mastigothrix*, so that it is very probable that they represent an undescribed genus. Until, however, the fructification of the European species is elucidated, it seems best to forbear multiplying names.

Fig. 1, pl. 4, represents a single filament of this species.

M. halos, Wood, (sp. nov.)

M. cæspitulis; trichomatibus simplicibus, in ætate matura valde elongatis et cum vaginis truncatis et apertis,—in ætate immatura modice brevibus et in setam modice longam achroam productis; trichomatibus internis breve articulatis, subtiliter granulatis continuis vel varie interruptis; vaginis firmis, modice crassis, sæpe distincte lamellosis, coloris expertibus; cellulis perdurantibus subglobosis

Diam.—Sine vag. = .0003"; cum vag. = .0005".

Hab.—In æstuario, Stonington, Conn. (Dr. F. Lewis.)

In little tufts; filaments simple, in mature state greatly elongate, and with the sheath truncate and open,—in the young condition shorter and often ending in a rather short seta; internal filament shortly articulate, minutely granular, continuous or variously interrupted; sheath firm, rather thick, often distinctly lamellated, colorless; heterocysts subglobose.

Remarks.—This species is an inhabitant of salt, or at least brackish water, having been collected in Stonington Inlet by Dr. Frank Lewis. The filaments are very long and always simple; forming apparent exceptions to this, I have seen once or twice a number of young filaments so united as to give the appearance of having been produced from one old one, and in other cases young filaments growing from the side of an old one; but I believe those are always set free so soon as they attain a certain size. In one instance there were large, globular cells, with very thick walls, produced, and lying free, in the sheath. Are these spores? They are

well shown in figure 2 *b*, pl. 5. Associated with them were a number of similar cells which had not obtained as yet the outer thick wall. The color of the filaments is in my specimens of a rich golden brown; but, as they have been preserved in carbolic acid water, I cannot speak positively as to the original tint. The heterocysts are subglobose, sometimes compressed, sometimes somewhat triangular. They about equal in diameter the internal filament.

Fig. 2, pl. 5, represents a small cluster of youngish filaments of this species.

M. sejunctum, WOOD, (sp. nov.)

M. thallo cæspitulo, molle, parasitico; trichomatibus simplicibus, plerumque inarticulatis, sed, interdum breve, interdum longe, articulatis, continuis, rarius interruptis, apice attenuatis, flavo-olivaceis aut viridibus, sparse granulatis; vaginis plerumque amplis et distinctis, hyalinis, sæpius valde undulatis, apice plerumque valde amplificatis et in fibrillas solutis; cellulis perdurantibus diametro subæqualibus; sporis nullis.

Diam.—Trichom $\frac{1}{8000}$ " = .00016"; cum vag. $\frac{1}{2000}$ " = .0005".

Hab.—In plantarum aquaticarum foliis, Carp River, Michigan.

Thallus somewhat cæspitose, soft, parasitic; filaments simple, mostly inarticulate, but sometimes shortly sometimes long articulate, continuous or more rarely interrupted, attenuate at the apex, yellowish-olive or greenish, sparsely granulate; sheaths mostly ample and distinct hyaline, often strongly undulate, the apex mostly much amplified and dissolved into fibrillæ; heterocysts about equal to the filament in diameter; spores wanting.

Remarks.—This species was found in the Carp River bog, growing on the edges of minute leaves, so as to form little prominences or thickenings of the margin. The trichomata are quite distinct from one another, and can scarcely be said to be united into a frond, although they all appear to radiate from the base, where they are consolidated into a dense mass. The sheaths are generally quite distinct, much broader than the cytoplasm, and are not sensibly dilated below. In most specimens they are very distinctly alternately dilated and contracted, or in other words, undulated. This is especially the case when the sheaths are quite wide. Above, they are rapidly and widely dilated, are distinctly fibrillose, and appear to gradually melt away. The cytoplasm is rarely articulated, and, when it is so, the joints are scarcely longer than broad, and are most generally confined to the distal end of the filament. The species appears to be most nearly allied to *M. Bauerianum*, GRUN., from which, however, it is quite distinct.

Fig. 2 *a*, pl. 4, represents this species magnified 250 diameters; fig. 2 *b*, a single filament magnified 800 diameters.

M. elongatum, WOOD.

M. initio subglobosum, postea sæpe nonnihil fusinum, nigro-viride, lubricum, firme; trichomatibus ærugineis, valde elongatis, flagelliformibus, interdum inarticulatis sed sæpius breve articulatis, interdum ad genicula valde constrictis, apice interdum truncatis sed plerumque in pilum, longum, achroum, flexuosum, productis; vaginis achrois, arcatis, sæpe apice truncatis; cellulis perdurantibus globosis vel subglobosis.

Diam — $\frac{2}{7500}$ " = .00026."

Syn.—*M. elongatum*, WOOD, Prodrômus, Proc. Amer. Philos. Soc., 1869, p. 128.

Hab.—In aquario.

Thallus at first subglobose, afterwards frequently fusiform, blackish-green, slippery, firm; filaments æruginous, very elongate, sometimes not articulated, but more generally shortly articulated, sometimes strongly contracted at the joints; apices sometimes truncate but generally produced into a long, flexuous, translucent hair; sheath transparent, close, frequently truncate at the apex; heterocysts globose or subglobose.

Remarks.—This species grew in my aquarium on some brook-moss, which I obtained from a spring above Manayunk. It forms little nodules of the size of a pin's head upon the wire-like stems, or sometimes longer fusiform masses, which are apparently produced by the coalescence of a number of the little globes. The color of these fronds, which are very firm, is a blackish-green. The filaments radiate from the base in all directions, and at the apex are tipped with a very long hair-like flexuous point, or they are truncate, apparently from the breaking off of this terminal seta. The endochrome is not unfrequently interrupted within the sheath. When it is articulated, the joints are usually about as long as broad, and frequently are distinctly separated from one another. The sheath is sometimes quite apparent and distinctly truncate and open above, but in other instances is with difficulty perceived anywhere, and above is lost in the long hyaline point. At the points of attachment of the frond the filaments are so densely crowded as almost to appear to be coalescent, though I believe they are never really so; yet it is often almost impossible to separate them one from another by pressure on the glass cover, without entirely mashing and distorting the filaments.

Fig. 1 *a*, pl. 5, represents a section of a frond of this species slightly magnified; fig. 1 *b*, a single filament magnified 460 diameters.

Genus MASTIGOTHRIX, Ktz.

Trichomata singula, plerumque sparsa, parasitica intra thallum Chætophorarum aliarumque algarum, flagelliformia, in apicem piliformem achroom hyalinum cuspidata, distincte articulata, arete vaginata, basi cellula perdurante instructa. (R.)

Filament single, mostly scattered, parasitic within the thallus of Chætophora or other algæ, flagelliform, with the apex produced in a hyaline hair, distinctly vaginate, furnished with a basal heterocyst.

Remarks.—I have simply copied the generic description of Prof. Rabenhorst, although it seems to me more than doubtful whether the place of growth is any generic character whatever. I have relied more on the long hyaline apical hair, although our American form does grow in a gelatinous palmella like jelly.

M. fibrosa, Wood.

M. dilute vel cæruleo-viridis, vel olivaceo-viridis, vel sub-æruginea, infra haud articulata, sursum sæpe breve articulata, apice in trichomata matura in setam hyalinam, distincte articulatam, longam, producta; vaginis achroois—in filamento immaturo, supra distinctis, latis, hyalinis, infra modice crassis, arctis,—in trichomata matura infra arctis, indistinctis, supra in fibrillis dissolutis, apice absentibus; cellulis perdurantibus globosis, interdum geminis.

Diam.—Trichone $\frac{1}{22500}$ ''; cell. perdur. $\frac{7}{18000}$ ''— $\frac{4}{18000}$ ''.

Syn.—*M. fibrosa*, Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 129.

Hab.—Prope Philadelphia.

Light bluish-green, or olivaceous-green, apex in the mature filament prolonged into a long, distinctly articulated hyaline seta; sheath transparent—in the immature filament distally, broad, and distinct although hyaline, below rather thick and close; in the mature filament below close, indistinct, above dissolved in fibrillæ and wanting at the apex; heterocysts globose, sometimes geminate.

Remarks.—This plant was found growing with other low algæ in a thick jelly, which clothed some wet, dripping rocks near Manayunk. In the young filaments the sheath is produced above into a broad, thick, gelatinous-looking portion, the cavity of which is often scarcely apparent. The cytoplasm in such filaments is mostly of a light bluish-green color, is granular and not very apparent. In older filaments, the trichoma above is prolonged into a long, curved hyaline point, and the sheath just below the base of this is split into a number of fibrillæ. No spores were perceived. The increase of the species appears to take place in the following manner: Near the middle of the filament a tumid swelling forms, in the centre of which appears after awhile a constriction, and this increases until at last there are shaped out the bases of two filaments. Then the heterocysts appear, and finally the two halves of the original trichoma separate—each a perfect filament. (Fig. 3, pl. 5.) Sometimes, instead of a pair of filaments being thus formed, but a single base is shaped out at the place of swelling, and the original filaments split, as it were, thus giving origin to a second trichoma, which for awhile appears as a branch of the former, but is soon detached from it. In some specimens there are two heterocysts, unless the proximal of these, which is a light orange-clay color, represents a spore.

Fig. 3, pl. 5, represents different forms of this species.

FAMILY SCYTONEMACEÆ.

Trichomata articulata, sæpe moniliformia vel submoniliformia, vaginata, pseudoramosa, cellulis limitaneis, ad pseudoramulorum basin, vel interstitialibus, plerumque pachydermaticis instructa. Vaginæ e stratis pluribus (etsi non semper distinctis) formitæ, superficie læves, corrugatæ vel exasperatæ, crustatæ, nonnumquam stratis exterioribus in fibrillas discedentibus, haud raro passim intumescens vel ocreatæ.

Vegetatio non terminalis; cellularum vegetativarum divisio ad unam directionem, initio in trichomatibus medio, postea in utroque fine sæpe alternans. Cellulæ limitaneæ ad utrumque polum locello lucido instructæ.

Propagatio gonidiis plerumque exultima generatione ortis. Gonidia plerumque numerosa seriata e vagina se exserunt tumque in singula secedunt. (R.)

Filaments equal, articulate, often moniliform or submoniliform, vaginate, pseudoramosæ, furnished with heterocysts which are either interstitial or at the base of the branches, and are mostly thick-walled. Sheaths formed of numerous strata (not always distinct), their surface smooth, corrugate, or roughened, the exterior stratum sometimes breaking up into fibrillæ, not rarely intumescens or ocreate.

Vegetation not terminal; division of the cells occurring in one direction, in the beginning in the middle of the trichoma, afterwards often alternately at each end. Heterocysts furnished with a translucent spot at each end.

Propagation mostly by gonidia arising from the last generation. Gonidia mostly numerously seriate, passing out of the sheaths and then separating one from the other.

R marks.—The *Scytonemaceæ* are simple or branched filamentous plants, which grow in water, or in the air, upon tree-trunks, rocks, fences, &c., in moist localities. A number of individuals of one or more species are almost always associated to form on the ground little mats, or in the water attached or floating masses of varying color and characters according to the species. The individual filaments are composed of two distinct parts, the inner the protoplasmic matter, the outer the cellulose sheath. The former of these is a long cylindrical mass, which is occasionally interrupted by a distinct thick-walled cell, spoken of in this memoir as the *heterocyst*, or "*cellulis perdurantibus*." The inner filament is composed of colored protoplasm, which is sometimes homogeneous, but in other cases is distinctly granular. It is most generally articulated after the manner of an *oscillatoria*, but occasionally it is continuous for a great portion of its length, and in one species, which is here described, although very possibly not belonging in the family, there are, at regular intervals, partitions running across from one side to the other of the sheath, so that the inner filament may be said to be made up of a number of cells.

The heterocysts are of various shapes, globular, compressed, cylindrical, oblong, &c. &c. They are mostly provided with a bright colorless spot at each end. Their number varies according to the species. Sometimes they are single, in other cases there are several of them arranged in series. They are placed either at the origin of the branches, or are scattered apparently without definite arrangement in the length of the filament. In the one case, they are known as "*basal*," in the other as "*interstitial*." In any species, either of these methods, or both of them, may prevail; but a certain amount of specific value attaches to the situation of the heterocysts. Their function is totally unknown, although some have imagined them to have a sexual significance and even to be spermatozoids, but there is no proof whatever of the truth of such suppositions, and it is, I think, very certain that these heterocysts are not of the nature of spores.

The sheath of the *Scytonema* is composed of one or more strata, which are often very distinct from one another, but are more often, perhaps, not so. It is opaque or translucent, and has its outer surface smooth, or tubercular, fibrillate or roughened in some way.

The specific characters in this family can best be commented upon under two heads—namely, those which are discoverable with the unaided eye, and those which the microscope alone can reveal. The points to be observed under the first of these are as follows: The place of growth of the plant, whether in the air or in the water, and, if it live in the air, to what it is attached—whether to stones, dead wood, or living trees, and it is possible that in some cases it may be found that certain species of *Scytonema* inhabit only certain species of trees. If the plant be in water, it must be noted whether it be attached or floating. Then the habit of growth must be looked at, including in this the size and thickness of the masses of filaments, whether they be flocculent, turfy, crustaceous, membranous-gelatinous, &c., their softness or rigidity, their color, as well as the arrangement in them of the filaments. To discover the latter, it will generally be necessary to use a low power of the microscope, and at the same time the mode and profusion of branching of the individual plant should be studied.

The second class of characters are those discoverable only with the higher powers. They are divisible into two sets; those afforded by the inner filament and those derived from the sheaths. In the first of these the points to be noted are, the diameter of the filament; its color, whether it be or be not articulated, and if it be the length of the joints; whether it is uniform or moniliform; whether it be homogeneous or granulate; then the heterocysts should be examined as to their size, position, arrangement, shape, number, and color.

The diameter of the sheath, its homogeneity, its color, firmness, and the condition of its outer surface are to be included in the specific study.

Genus SCYTONEMA.

Trichomata cæspitose-congregata vel fasciculata, plus minus pseudoramosa, cellulis interstitialibus instructa; vaginæ gelatinoso-membranaceæ, e stratis (interdum obsoletis) pluribus cylindraceis compositæ; cellulis perdurantibus singulis.

Filaments cæspitously-congregate or fasciculate, more or less pseudoramose; furnished with interstitial cells; sheaths gelatinous-membranaceous, composed of many cylindrical, sometimes obsolete, strata; heterocysts single.

a. Terrestres vel aquaticæ.

a. Terrestrial or aquatic.

S. simplicè, WOOD, (sp. nov.)

S. in strato modice crasso, subtomentoso, nigro-viride; trichomatibus valde elongatis, flexuoso-curvatis, parçissime pseudoramosis vel sæpe sine pseudoramulis; pseudoramulis geminis vel singulis, plerumque elongatis; trichomatibus internis modo distincte articulatis, modo inarticulatis, apice interdum brevissime articulatis, granulosis, pallide viridibus, ad genicula sæpe nodosis vel disjunctis, articulis plerumque diametro æqualibus ad 7 plo longioribus; vaginis plerumque supra truncatis et apertis, pellucidis, sæpe coloris expertibus, interdum dilute aureo-brunneis; cellulis perdurantibus cylindricis, interjectis, diametro 2-5 plo longioribus.

Diam.—Trich. cum vag. $\frac{1}{3500}$ " — $\frac{1}{1500}$ " = .0004" — .00066"; sine vag. $\frac{1}{7500}$ " — $\frac{2}{7500}$ " = .00013" — .00026".

Hab.—In lignis irroratis, South Carolina. (Ravenel.)

S. in a moderately thick, somewhat tomentose, blackish-green stratum; trichomata very elongate, flexuously curved, very sparsely branched or frequently without branches; branches geminate or single, mostly elongate; internal filament partly distinctly articulate, partly inarticulate, granular, pale-greenish, in its apex sometimes very shortly articulate, sparsely granular, often nodose or disjoined at the joints; articles mostly from equal to to 7 times longer than the diameter; sheaths thick, transparent, often colorless, sometimes pale yellowish-brown, mostly open and truncate at apex; heterocysts cylindrical, interspersed, 2-5 times longer than their diameter.

Remarks.—I am indebted to Professor Ravenel for specimens of this species. They are preserved in solution of acetate of alumina and accompanied by the following label: "Adhering to the wet sides of a wooden gutter, leading water from a spring, September 29, 1869: Aiken, South Carolina." The filaments are remarkable for the fewness of their branches. Generally, indeed, there are no

branches whatever, and I have never seen more than a single pair, or, at most, three branches to a filament. The mass of filaments is blackish-green, somewhat tomentose and quite shiny in appearance. The articles are often very long, and the internal filament is frequently in such cases enlarged into a sort of globular node at the joint. Not at all rarely there is a very decided break in the endochrome at the joints.

This species is very close to *S. Austinii*, from which, however, I think it sufficiently distinct. The points of difference are in the much firmer, much more colored and opaque, and rougher sheath of that species; in the swollen ends of the internal filament of *S. Austinii*, and its shorter articles, with the absence of nodes or distinct interruption of the endochrome at the joints. The heterocysts are also quite different in the two forms, whilst the filaments of *S. simplicis* are much the longer.

S. Austinii, Wood, (sp. nov.)

S. rupicola, strato tomentoso, cæspitoso, crasso, fusco-nigro; trichomatibus adscendentibus, curvatis, plerumque simplicibus; trichomatibus internis ærugineis vel fusciscentibus, articulatis vel inarticulatis, fine sæpe valde incrassatis; articulis diametro plerumque multo brevioribus, interdum longioribus; vaginis rubido-vel aureo-fusciscentibus, sæpe sub-opacis, firmis, indistincte lamellosis, in apice plerumque achrois et coloris fere expertibus, superficie subrugosa et hirta; cellulis pedrurantibus breviter cylindricis, vel subquadratis vel subglobosis, interdum valde compressis et diametro multo brevioribus.

Diam.—Fil. cum. vag. .0006"—.0008"; sine vag. .00016"—.0004".

Hab.—In rupibus, "Little Falls, New Jersey." (Austin.)

S. growing on rocks, stratum tomentose, and somewhat turfy, brownish-black; trichomata ascending, mostly simple, curved; internal filament æruginous or fuscous, articulate or inarticulate, often very much thickened at the ends; articles much shorter to longer than their diameter; sheaths reddish or yellowish-fuscous, at the apex colorless and transparent, firm, indistinctly lamellate; surface rough; heterocysts shortly cylindrical, subquadrate or subglobose, sometimes strongly compressed and much shorter than broad.

Remarks.—This plant occurs as a blackish stratum of one or two lines in thickness, forming a sort of miniature turfy cushion upon the rock. When examined with the hand-glass, this layer is seen to be composed of a great number of ascending curved filaments whose color, in some specimens, is a reddish-brown; in others, apparently younger, yellowish-brown. Under the compound microscope the sheaths in the older filaments are seen to be much roughened externally and irregular in outline. The young sheaths are smooth. The filaments are mostly simple, since I have not seen more than a half dozen having even a single branch.

The heterocysts are scattered at irregular intervals, and are remarkably irregular in form—sometimes much shorter than broad, sometimes several times as long. As the ends of the filaments are approached the internal filament suddenly swells out and increases sometimes to twice the diameter it has in the central part of the filament. In the filament proper it rarely attains a diameter of more than .0003", and is commonly about .00025", whereas at the ends it very generally approaches the maximum .00042".

S. immersum, Wood, (sp. nov.)

S. immersum cum algis alteris intermixtum et plantas aquaticas adhærens; trichomatibus elongatis; pseudoramulis plus minus distantibus, plerumque geminis, et e basi divergenter adscendentibus, brevibus aut elongatis; trichomatibus internis læte ærugineis, interdum distincte articulatis, interdum inarticulatis, apice obtuse rotundato, ærugineo; articulis diametro subæqualibus vel brevioribus; vaginis amplis, hyalinis, coloris expertibus; cellulis perdurantibus distinctis, singulis, interjectis, subcylindricis, diametro interdum fere duplo brevioribus, interdum duplo longioribus.

Diam.—Sin. vag. $\frac{5}{12000}'' = .000415''$. Cum vag. $\frac{9}{12000}'' = .00075''$.

Hab.—In aquis quietis, Cumberland County, New Jersey.

S. immersum, intermixed with other algæ and adhering to aquatic plants; filaments elongate; branches mostly geminate, more or less distant, short or elongate; internal filaments bright æruginous, sometimes distinctly articulate, at others not so, apex obtusely rounded æruginous; joints about equal to the diameter or shorter; sheath ample hyaline, colorless; heterocysts distinct, single, interjected, subcylindrical, sometimes about half as long as broad, sometimes nearly twice as long.

Remarks.—I found this plant in September, 1869, in Shepherd's Mill Pond, near Greenwich, Cumberland County, New Jersey, forming, with other algæ, a flocculent, greenish-black, slimy coating to the stems and finely dissected leaves of *Ranunculus aquatilis*. The branches are very few in number in most specimens, and when they are more plentiful are apt to be short and abortive. Their apices do not differ materially from their other portions.

Fig. 9, pl. 2 *a*, represents a portion of a filament of this specimen magnified 750 diameters; fig. 2 *b* a whole filament magnified 260 diameters.

S. Nagelii, Ktz. (?)

S. cæspitoso-floccosum, bryophilum, nigro-viride; trichomatibus, plerumque sparse pseudoramosis, pseudoramulisque elongatis et intricatis; trichomatibus internis breviter articulatis, sæpe interruptis, sæpe nonnihil moniliformibus, viridibus aut in ætate protracta brunneis; articulis sæpe sejunctis, diametro plerumque brevioribus, subtiliter granulatis; pseudoramulis plerumque singulis; vaginis modice arctis, interdum subamplis, haud distincte lamellosis, modice crassis, hyalinis, coloris expertibus aut in ætate protracta dilute fusco-brunneis; cellulis perdurantibus nonnihil reniformibus, plerumque nullis, basilaribus.

Diam.—Fil. cum vag. plerumque $\frac{5}{7600}''$ —max. $\frac{5}{8000}''$; sine vag. $\frac{5}{12000}''$, cell. perdurant. lab. $\frac{5}{12000}''$ —long. $\frac{3}{12000}''$.

Syn.—*S. Nagelii* (Ktz.), RABENHORST, Flora Europ. Algarum, Sect. II. p. 252.

Hab.—In fonte, prope Belvidere, Centre County, Pennsylvania.

Growing in small, blackish-green woolly mats attached to mosses; filaments mostly sparsely branched, with the branches elongate and intricate; internal filament shortly articulate, often somewhat moniliform, often interrupted, green, or, in mature state, brownish; joints often disjoined, mostly shorter than the diameter, finely granulate; branches mostly single; sheaths moderately close, sometimes ample, not distinctly lamellate, rather thick, hyaline, colorless, or, in old age, light fuscous brown; heterocysts mostly wanting.

Remarks.—I found this plant in the large spring that supplies Bellefonte with water, growing attached to mosses, so as to form little dark-green mats around

their stems and branches. These mats never exceeded an inch in length in any specimens that came under my notice. The filaments themselves are apparently not much branched and are densely interwoven. The sheaths are close, rather thick, not lamellate, of uniform diameter, except in that they are occasionally locally swollen, and are truncate and open at the end. The internal filaments are frequently much interrupted, and in the younger plants are of a deep green. The joints are in many instances much separated, and in most cases very distinct. The filaments indeed show a remarkable tendency to break up at the joints, so as to form a series of dish-like gonidia, so that the articles, or endochrome masses, may be generally described as strongly compressed spheres. In all the specimens that I have examined, I have seen but a single heterocyst. This was at the base of a branch, was somewhat reniform, and about three-fifths as long as broad. I have referred this species, doubtfully, to *S. Naegeli*, Ktz., the only account of which that I have met with, or know of, is a brief diagnosis in Rabenhorst's Flora, in which many of the essential characters are omitted.

Fig. 6, pl. 8, represents a portion of a filament of this species.

S. thermale, Ktz.

S. strato tenue, nigrescente; trichomatibus flexuoso-curvatis, intricatis, parce pseudoramosis, internis pallide ærugineis, sæpe coloris fere expertibus, passim interruptis, plerumque inarticulatis sed sæpe indistincte et interdum distincte articulatis, granulosis; articulis diametro brevioribus vel subæqualibus; pseudoramulis plerumque brevibus, geminis, in diametro trichomatibus æqualibus vel subæqualibus et interdum usque ad medium conjunctis, basi coalitis, sæpe e basi divergentibus; vaginis crassis, indistincte lamellosis, vel luteo-fuscis vel fuscis, sed passim fere coloris expertibus, plerumque vix pellucidulis, in ramulorum apice sæpe hyalinis et coloris fere expertibus; cellulis perdurantibus, subquadratis vel cylindricis, singulis, interjectis.

Diam.—Tr. cum vag. $\frac{5}{12000}$ "— $\frac{7}{12000}$ " = .00042"—.00058; sine vag. $\frac{1}{8000}$ " = .000166"— $\frac{1}{4000}$ " = .00025.

Syn.—*S. thermale*, Ktz., Rabenhorst, Flora Europ. Algarum, Sect II. p. 250.

Hab.—In terra argillacea, South Carolina. (Ravenel.)

Stratum thin, blackish; filaments flexuously curved, intricate, sparingly branched; internal filament pale-greenish, often almost colorless, here and there interrupted, mostly inarticulate, but often indistinctly and sometimes distinctly articulate, granular; joints shorter or about as long as broad; branches geminate, mostly short, equal or subequal to the filament in diameter, coalescent at the bases, rarely so even to their middle, mostly divergent from the base; sheath thick, indistinctly lamellate, yellowish-fuscous, and scarcely semitransparent, but here and there nearly colorless and pellucid, generally so in the apices of the branches; heterocysts subquadrate or cylindrical, single, interspersed.

Remarks.—I am indebted to Professor Ravenel for specimens of this species preserved in solution of acetate of alumina. The label reads, "Damp surface of hard clay, Sept. 25, 1869." The sheaths are quite thick and scarcely translucent, so that the color of the inner filament seen through them is that of themselves. Curiously enough, one of these dark sheaths will for a space lose its color and be very transparent, in such places and in the apices of the branches, the inner filament is often a decided pale-green; at other times it is almost colorless. The end of the sheaths are mostly closed, but I have seen them open, with the inner filament project-

ing. The branches are nearly always short, and divergent from their united bases. The heterocysts have frequently one of their ends rounded; and are quite numerous. This species corresponds too closely to Rabenhorst's description of *Seytonema thermale* to be separated, but it is possible a comparison of specimens might show decided differences—the description of the European form is not very full. The American plants seem to approximate most closely the *Var. intextum*. I have seen a single branch given off only in one instance.

Fig. 1, pl. 6, represents a filament of this species magnified 260 diameters; fig. 1 b, the outline of a heterocyst magnified 750 diameters.

S. Myochrous, Ag.

S. strato tenui, pannoso-tomentoso, obscure fusco (nonnunquam subsericeo); trichomatibus validissimis, fuscis, lucidis, leniter curvatis, adscendentibus, internis ærugineis, apice (articul. term. 5–6) rubellis, distincte articulatis; pseudoramulis plerumque geminis, sæpe longissimis flaccido-erectis, trichomate dimidio circiter tenuioribus; trichomatis vaginis crassis, distincte lamellosis, firmis, pulchre luteo-fuscis, superficie lævissimis, ramulorum semper pallidioribus (luteis, rarius achrois), apice sæpe achrois, clausis et obtuso-rotundatis; cellulis perdurantibus oblongis vel subcylindricis, achrois, trichomatis interni diametro subæqualibus. (R.) *Strato* obscure olivaceo, trichomatibus parce pseudoramosis, ad $\frac{1}{8}$ ''' crassis; pseudorannulis singulis, vaginis achrois vel luteolis; vag. trich. luteo-fuscis. (R.) Species mihi ignota.

Diam.—Trichom 0.0011"—0.0014"; ramulorum ad 0.00068". (R.)

Syn.—*S. Myochrous*, AGARDH; *Var. Contextum*, CARMICHAEL. RABENHORST, *Flora Europ. Algarum*, Sect. II. p. 254.

Hab.—"Foot of Crow's-nest, West Point." Bailey. *Silliman's Journal*, N. S. vol. iii.

Strato thin, pannosely tomentose, obscurely fuscous (sometimes somewhat silky); filaments very strong, fuscous, bright, slightly curved, ascending; the internal æruginous, distinctly articulate with the apex (terminal 5–6 joints) reddish; branches mostly geminate, often very long, flaccidly erect, about one-half thinner than the filament; sheath of the filament thick, distinctly lamellate, firm, beautifully yellowish-fuscous, surface very smooth; sheath of the branches always paler (luteous or rarely colorless) with the apex colorless, short and obtusely rounded; heterocysts about equal in diameter to the internal filament. *Stratum* obscurely olivaceous, filaments sparsely branched, about $\frac{1}{8}$ ''' thick; branches single, with the sheaths transparent or yellowish; sheath of the trichoma luteo-fuscous.

S. calotrichoides. KÜTZING(?).

S. cæspitosum, mucosum, plerumque cum algis variis intermixtum; trichomatibus plus minus curvatis; pseudoramulis plerumque geminis, varie curvatis, simplicibus, elongatis; trichomatibus internis modo distincte articulatis, modo inarticulatis, interdum moniliformibus, luteo-viridibus vel ærugineis, granulosis; articulis plerumque diametro brevioribus sed interdum permulto longioribus, haud rare vel subglobosis vel valde compressis; cellulis perdurantibus singulis, subcylindricis; vaginis plerumque pellucidulis, distincte lamellosis, in trichomatibus plerumque rubido-vel luteo brunneis sed interdum coloris expertibus, in pseudoramulis hyalinis, coloris expertibus vel dilutissime luteis vel dilute luteo-brunneis.

Diam.—Cum vag. max. $\frac{3}{4000}$ " = .00075"; plerumque $\frac{2}{4000}$ " = .00045"; sine vag. $\frac{1}{7500}$ " — $\frac{1}{4000}$ "; pseudoram. $\frac{1}{2000}$ " = .0005".

Syn.—*S. calotrichoides*, Ktz. RABENHORST, *Flora Europ. Algarum*, Sect. II. p. 252.

Hab.—South Carolina. (Ravenel.)

Cæspitose, mucous, mostly intermixed with various algæ; filaments more or less curved; branches mostly in pairs, elongate, simple, variously curved; internal filament partly distinctly articulate, partly not articulate, sometimes moniliform, yellowish-green or æruginous, granular; joints mostly shorter than the diameter, sometimes much longer, sometimes subglobose or strongly compressed; heterocysts single, subcylindrical; sheaths distinctly lamellate, mostly reddish or yellowish-brown, but sometimes colorless, in branches hyaline, colorless, or with a very faint yellowish tint, or sometimes brownish.

Remarks.—The specimens, from which the above description was drawn up, were sent me by Professor Ravenel from South Carolina. The extremities of the sheaths are either closed, or open. The branches are almost always in pairs, and sometimes three or four are given off together, but this is not common. They are often nearly or quite colorless; the main filament is generally a sort of brown—sometimes quite bright from the predominance of the yellow hue. Although my specimens do not precisely agree with the descriptions of the European *S. catotrichoides*, yet the disagreement does not seem sufficient or sufficiently constant to separate specifically the two forms; the most important of the differences is in the coloration of the sheaths and heterocysts, which in the American plant are commonly, but not universally, respectively brownish and greenish.

The label, which Professor Ravenel has attached to some of the specimens, reads, “In wet, boggy places, on rotten pine boards, Sept. 25, 1869.”

Fig. 2, pl. 6, represents a filament of this plant magnified 250 diameters.

S. cataracta, Wood.

S. rupicola, cæspitosum, fusco-atrum, longe et late expansum; trichomatibus flexuosis, flexilibus, fere 0.25" longibus, vage pseudoramosissimis, superficie lævibus; pseudoramis elongatis, singulis, rarissime geminis, liberis, interdum fuscis, sæpius hyalinis, apice plerumque truncatis et rare nonnihil attenuatis et sæpe barbais sed haud rubellis; trichomatibus internis æruginis, tenuissimis, plerumque distincte articulatis; articulis diametro plerumque brevioribus, sed interdum longioribus, sæpe sejunctis, sæpe subglobois; vaginis crassis et firmis; cellulis perdurantibus et basilaribus et interjectis, singulis, rarissime geminis.

Diam.—Trich. cum vag. plerumque .00045"; max. .0011"; sine vag. max. .00013".

Syn.—*S. cataracta*, Wood, Prodr. Am. Phil. Soc., p. 129, 1869.

Hab.—In flumine Niagara prope cataractam.

S. forming on rocks an extended turf-like stratum of a brownish-black color; filaments flexuous, flexible, almost 0.25" long, irregularly branched, their surface smooth; branches elongate, single, rarely in pairs, free, sometimes fuscous, frequently hyaline, their apices generally truncate, rarely somewhat attenuate, frequently provided with enlargements, never reddish; cytoplasm æruginous, very thin, generally distinctly articulate; articles mostly shorter than broad, but sometimes longer, frequently disjoined, often subglobose; sheaths thick and firm; heterocysts both basal and interjected, single, extremely rarely geminate.

Remarks.—This species grows abundantly in the Niagara River, on the rocks below the great cataract. It is really in little tufts, but these are in many cases placed so closely as to form a broad turf-like coating to the stones. Often, however, the tufts are in smaller patches, and are of sufficient length to wave with the eddies and currents in the water. The branches are almost always given off

singly since I have examined some hundreds of specimens, and have only in one instance detected them in pairs. The apices of the branches, and indeed of the main filaments, are beautifully colorless and hyaline, and not unfrequently a branch will have this hyaline sheath for a long distance. The extreme ends are mostly truncate and open, and, often near them, the sheaths will have marked swellings; a condition which, for want of a better term, I have spoken of as being *barbate*. Sometimes near the end of the filament the diameter of the sheath will be suddenly lessened. The large cells are both interstitial and placed at the bases of the branches; they are more or less oblong or quadrangular, sometimes being scarcely longer than broad, but in other cases several times longer. At their position there is very generally a sort of globular enlargement of the filament. The sheath is sometimes very obscurely lamellate. The color of the older filaments is a dark, almost chocolate-brown. This is apparently the species referred to by Professor Bailey as being *Scytonema ocellatum* of Harvey, in Silliman's Journal, vol. iii. N. S., although that plant, according to Professor Rabenhorst, belongs to the genus *Sirosiphon*.

Fig. 1 *a*, pl. 7, represents a portion of a filament, magnified 280 diameters; fig. 1 *b*, a whole filament slightly magnified.

S. dubium, Wood (sp. nov.)

S. immersum, in floccis mucoso-tomentosis olivaceo-nigris plantas aquaticas adhærens, vel in strato mucoso et nonnihil tomentoso dispositum; trichomatibus valde elongatis et arete intricatis, varie curvatis, plerumque sparse pseudoramosis; pseudoramulis plerumque singulis, et plus minus distantibus et modice brevibus, vel interdum brevissimis et abortivis et nonnihil confertis; trichomatibus internis sæpe in pseudocellulis distinctis contentis, interdum continuis et indistincte articulatis vel inarticulatis, plerumque dilute cæruleo-viridibus sed interdum læte ærugineis, subtiliter granulatis; vaginis arctis plerumque modice crassis et firmis, hyalinis, coloris expertibus; cellulis perdurantibus cylindricis, diametro 2-6 plo longioribus.

Diam.—Cum vag. $\frac{3}{12000}$ "— $\frac{5}{12000}$ " = .00025"—.0004".

Hab.—In aquis quietis, Cumberland County, New Jersey.

Immersed, adhering to water plants in olive-black tomentose flocculent masses, or arranged in a mucous and somewhat tomentose stratum; trichomata very long and closely interwoven, variously curved, mostly sparsely branched; branches generally single, more or less distant, and moderately short, sometimes very short, abortive, and somewhat crowded; internal filament often contained in distinct cell-like apartments, sometimes continuous and indistinctly articulate, or not at all articulate, finely granulate, mostly a pale bluish-green, sometimes a bright ærugineous color; sheath close, mostly rather thick and firm, hyaline colorless; heterocysts cylindrical, 2-6 times longer than broad.

Remarks.—I found this plant, September, 1869, in Shepherd's Mill Pond, near Greenwich, Cumberland County, New Jersey. It formed dark, ugly, somewhat slimy, tomentose flocculi adhering to, and binding together, the finely-dissected leaves of *Ranunculus aquatilis*. The filaments are very long, slender, and sparsely branched. The branches are given off at right angles, or nearly so, but are frequently sharply bent just above their origin. They are often, but not always, rather short. The most remarkable character that the plant possesses is that in many filaments there are very distinct regular partitions stretching across from

side to side, so that the interior is divided, as it were, into successive cell-like chambers, in which the colored protoplasm is contained. This character seems almost to separate the plant from the genus *Scytonema*, but I have deemed it insufficient grounds for indicating a new genus. Since writing the preceding remarks, I have received specimens of this species from Professor Ravenel, who collected them in South Carolina, near the town of Aiken. They agree in all respects, except that they form a dark, mucous, somewhat tomentose coating to pieces of wood.

Fig. 3 *a* represents the outline of a series of the cells alluded to, magnified 750 diameters, and figs. 3 *b* and 3 *c*, portions of filaments magnified 460 diameters.

b. Arboricolæ.

b. Growing on trees.

S. cortex, Wood.

S. minutissimum, stratum tenue submembranaceum formante; trichomatibus sparse pseudoramulosis, pseudoramulisque repentibus et plus minus concretis, viridibus aut dilute fuscis, varie curvatis, haud rigidis; cytoplasmate viride, articulato, rare distincte granuloso; articulis diametro longioribus aut brevioribus; vaginis arctis, nonnihil tenuibus, plerumque coloris expertibus, sed interdum dilute fuscis; cellulis perdurantibus et singulis et geminis, et basilibus et interjectis, globosis vel subglobosis.

Diam.—Trich. cum vag. $7\frac{2}{5}00''$ — $7\frac{3}{5}00''$.

Syn.—*Scytonema cortex*, Wood, Prodromus, Proc. Am. Philos. Soc., 1869, p. 130.

Hab.—South Carolina.

S. very minute, forming a thin, submembranaceous stratum; filaments sparsely branched, together with the branches, creeping and more or less concreted together by their sides, green or light brown, variously curved, not rigid; cytoplasm (internal filament) articulate, rarely distinctly granulate; joints longer or shorter than broad; sheaths close, rather thin, transparent, generally colorless but sometimes light brown; heterocysts globular or subglobular, single or in pairs, basal or otherwise.

Remarks.—I have specimens of this species collected in South Carolina by Professor Ravenel, who found it growing on the bark of *Platanus occidentalis*. The thin, almost membranous stratum which it forms, is of a dark olive-black, and has to the eye a sort of minutely warty appearance. The filaments are so involved and so adherent, one to the other, that I have not been able to separate any length of them, nor are the branches distinguishable from the main filaments. The sheaths are rather thin, and often not very apparent.

Fig. 4, pl. 6, represents this species.

S. Ravenelii, Wood.

S. lignicola, breve cæspitosum, viride-nigrum; trichomatibus plerumque repentibus, vel fusco-olivaceis vel aureo-fuscis, modice pseudoramosis; ramis ascendentibus, rigidis, flexuosis rare pseudoramulosis, vel fusco-olivaceis vel aureo-fuscis, rarissime cum apicibus subachrois; trichomatibus internis coloris expertibus, granulosis, sæpe vagina erumpentibus, plerumque articulatis; articulis diametro longioribus aut brevioribus; vaginis arctis, crassibus, fusco-olivaceis vel aureo-fuscis, plerumque supra truncatis et apertis, superficie nonnunquam irregularibus; cellulis perdurantibus subquadratis vel subglobosis singulis aut rare geminis, interjectis; in stato juvene trichomatibus internis ærugineis, vaginis tenuibus.

Diam.—Trich. cum vag. $7\frac{2}{5}00''$ — $7\frac{6}{5}00''$; ram cum vag. $7\frac{4}{5}00''$ — $7\frac{6}{5}00''$; trich. sine vag. $2000''$ —2.0005."

Syn.—*S. Ravenelii*, Wood, Prodr. Am. Philos. Soc., 1869, p. 130.

Hab.—In cortice, South Carolina.

S. Forming little turfy spots of a greenish color, on bark; filaments mostly creeping, either brownish-olive or yellowish-brown, moderately branched; branches ascending, rigid, flexuous, very rarely provided with secondary branchlets, either brownish-olive or yellowish-brown, rarely subtransparent at the apex; cytoplasm colorless, granular, often extending out beyond the sheaths, generally articulate; joints longer or shorter than broad; sheaths close, thick, brownish-olive or yellowish-brown, for the most part truncate at their ends and open, their surface sometimes irregular; heterocysts subquadrate, single, interstitial.

Remarks.—I am indebted to Prof. H. W. Ravenel for specimens of this very distinct species. Some of these are labelled as having grown on the twigs of a celtis in South Carolina, other specimens are on the bark of a willow. The branches, which are mostly shortish, simple, and variously curved, are sent up in great numbers by the creeping stems, and, like the stems themselves, are mostly free, but not unfrequently are closely adherent by their edges.

The internal trichoma or cytoplasm, owing to the great thickness of the sheaths, is not very apparent within these latter, but not unfrequently projects for a distance beyond them, when it is seen to be colorless, very granular, and mostly, but not always, distinctly articulated. In the young plant the filaments are bright-green, often not more than $\frac{1}{3500}$ of an inch in thickness, and have the sheath very thin, or may be almost imperceptible. It affords me great pleasure to dedicate this species to Professor Ravenel, not as an acknowledgment merely of his aid in my studies of this hitherto neglected branch of the North American Flora, but rather of the great services he has rendered science in some of its kindred branches.

Fig. 4, pl. 5, represents the end of a filament of this species magnified some 450 diameters.

Genus TOLYPOTHRIX, Kütz.

Trichoma scytonemacea cum cellulis perdurantibus seriatis.

Filament similar to that of *scytonema*, but with the heterocysts seriate.

T. distorta, (MÜLLER) KÜTZ.

T. cæspitosa-floccosa, læte et pulchre viridis; trichomatibus intertextis, læte viridibus, modo distincte articulatis modo inarticulatis; articulis diametro brevioribus sæpe aut sub-nullis aut nullis; pseudoramulis singulis; vaginis aretis, homogeneis, vitreis; cellulis perdurantibus basilaribus et interdum interjectis, pachydermaticis, plerumque in parallelogrammæ enormis forma, plerumque 4-seriatis, subachrois, interdum sparsissime granulatis.

Diam.— $\frac{1}{1800}$ "— $\frac{1}{3000}$ ".

Syn.—*T. distorta*, (MÜLLER) KÜTZ. RABENHORST, Flora Europ., Algarum, Sect. II. p. 275.

Hab.—In aquario, Philadelphia, Wood. Rhode Island (Olney) Thwaites. Warden's Pond, Rhode Island; Reservoir Pond, West Point; Fourth Lake, Madison, Wisconsin, Bailey.

Flocculent cæspitose, bright, beautiful green; filaments interwoven, bright green, partly distinctly articulate, partly continuous; articles shorter than long, often very indistinct, sometimes absent; branches single; sheaths close, homogeneous, glassy; heterocysts basilar, 9 May, 1872.

sometimes interspersed, thick-walled, mostly irregularly parallelogrammatic, mostly 4-seriate, semitransparent, sometimes very sparsely granulate.

Remarks.—This species grew spontaneously in the aquarium of my friend Dr. Frické, to whom I am indebted for specimens of it, forming little, bright-green balls adherent to the various aquatic plants. It approaches so very closely the European *T. distorta*, that I have considered it as a mere variety of it, although it differs in having the heterocysts mostly arranged in fours, and also apparently in their shape—they being in our plant mostly parallelogrammatic.

Fig. 1 *a*, pl. 8, represents a section of heterocysts magnified 800 diameters; fig. 1 *b*, a portion of filament magnified 800 diameters.

Genus PETALONEMA, BERK. (1833.)

Scytonematis trichomata vaginis crassissimis e stratis numerosissimis brevioribus, infundibuliformi dilatatis, imbricatis et plerumque dilutissime coloratis compositis. (R.)

Syn.—*Arthrosiphon*, KtZ. (1845.)

“Filaments stratified, decumbent, free, simple, or branched. Tube or sheath very wide, flattened, longitudinally and transversely striate and crenulate at the edge; endochrome olivaceous annulated, here and there interrupted by a heterocyst. Branches issuing in pairs, formed by the division and protrusion of the endochrome of the original filament.

“When placed under the microscope the filaments present the appearance of a cylindrical central column, containing annulated, olive-colored endochrome, and a wide wing-like border at each side of the column. This border or sheath is obliquely striate, the striæ running in an arch from the margin toward the centre, where they become parallel, and are then continued longitudinally downward along the medullary column, till lost in the density. The margin of the wing is closely crenulate and in age transversely striate at the crenatures as though jointed. Such is the apparent structure; the real structure seems to be, that an annulated central filament is inclosed within a number of compressed, trumpet-mouthed gelatinomembranaceous tubular sheaths, one arising within the other, and successively developed as the growth proceeds. These sheaths, thus concentrically arranged, are indicated by arching longitudinal striæ; and the mouths of the younger sheaths, projecting slightly beyond those of the older, form the crenatures of the margin.” HARVEY.

P. alatum, BERK.

A. pulvinato-crustaceus, rupicola, varie coloratus; trichomatibus internis ærugineis, curvatis, parce pseudoramosis, modo continuis, modo torulosis, submoniliformibus, apice plerumque paulum incrassatis, sæpe roseolis, rotundatis; articulis distinctis, granulosis, diametro subæqualibus vel paulo brevioribus; vaginis stratis internis, aureis vel aureo-fuscentibus, externis achrois, vitreis; cellulis perdurantibus interjectis et ad pseudoramulorum basin, plerumque solitariis, subglobosis vel oblongis, dilute fuscis. (R.) *Species mihi ignota.*

Diam.—Trich. intern. 0.00016"—0.00032"; vag. 0.00377". (R.)

Syn.—*Arthrosiphon alatus*, (GREV.) RABENH. Flora Europ. Algarum, Sect. II. p. 265.

Petalonema alatum, BERKELEY. HARVEY, Nereis Boreis Americana, part iii. p. 99, Smithsonian Contributions, 1846.

Hab.—“On dripping rocks under Biddle Stairs, Niagara Falls.” (Harvey.)

“This forms strata of a dark chestnut-brown color and of indefinite extent on the surface of rocks or soil exposed to the constant drip of water. The filaments are decumbent, lying without order in the gelatinous matrix in which they are developed, and which forms the

groundwork of the stratum. They appear to be unattached to the soil, and each filament may be about half an inch in length; but they are commonly found broken off at the inferior end, or the lower part decays whilst the upper continues to grow. They are slightly curved, in serpent-like fashion, never quite straight; at first they are simple, but now and then emit lateral branches, which issue at considerable angles and generally in pairs. When a filament is about to branch, a rupture takes place in the side of the sheath, and the endochrome issues in two portions, one connected with the upper, the other with the lower half of the filament; these form the nuclei or medullary portion of two new branches and become duly invested with a membranous sheath, and gradually put on the aspect of the adult filament. The endochrome is granular, dark-brown, and annulated at short intervals, the transverse rings being placed very close together in the youngest portions, and less closely in the older, where they are distant from each other about twice the diameter of the column. This annulated endochrome is interrupted at certain fixed places, where an ellipsoidal cell is formed, separating the endochrome of the lower from that of the upper portions." HARVEY.

Remarks.—I have never seen either the genus or species, and therefore am forced to copy the descriptions of both from Rabenhorst and Harvey.

FAMILY SIROSIPHONACEÆ.

Thallus ramosus, e cellulis pachydermaticis aut uni vel pluri seratis et in vagina ampla inclusis formatus, interdum cellulis perdurantibus instructus. Ramificatio vera fit cellularum vegetatarum quarundam divisione in axis longitudinalis directionem, qua ex re cellulæ duæ sororiæ gignuntur; cellula inferior in trichomatis continuitate permanet, superior divisione continua repetita in eandem directionem se ad ramum explicat.

Propagatio adhuc ignota.

Fronde branched, formed of thick-walled cells in an ample sheath, sometimes furnished with heterocysts. Cells uni- or multi-seriate. Branches formed by a longitudinal division of certain cells, so as to form two sister cells; the inferior of which remains as part of the trichoma, whilst the other, by repeated divisions, grows into a branch.

Propagation not known.

Remarks.—The *Sirosiphonaceæ* are the most complex in their organization of all the *Phycochromophyceæ*, in so far as the protoplasm within the sheaths is everywhere broken up into a number of distinct cells, each of which is provided with a thick coat or wall as well as in the circumstance of the frond having more perfect branching. The so-called pseudo-branches in the other families are more truly comparable to distinct fronds or thalli remaining attached to the parent thallus than to distinct branches, whilst among the sirosiphons the branches really belong to the original thallus. The heterocysts are much more frequently absent than present, only one of the known American species being furnished with them. The sheaths are generally not so distinctly sheaths as among the oscillatoria, &c., for, instead of being distinct tubes, they appear rather in most cases as masses of firm jelly, the outer portion of which is hardened almost into a periderm, and in the inner part of which the cells are imbedded. Their color varies from the transparent colorlessness of glass to a dark opaque-brown. Their surface is perhaps most frequently smooth, but at times is tuberculate or otherwise roughened. I have never seen anything like spores about them.

These plants grow in the majority of cases in the air, in such situation as on the face of dripping rocks, on the trunks and branches of trees, on moist ground, &c.; but some of the species are found in the water, either attached or floating. They generally form little mats of indefinite extent, but occasionally the filaments are united more closely into an almost membranaceous stratum.

The species are, I think, in most instances readily distinguished, the characters being partly discoverable with the unaided eye and partly microscopic. The points to be attended to in the first category are the size, color, form, and consistency of the mats of fronds, and the place of growth. In the second are included the general shape of the frond and its size and method of branching; the general shape, color, and size of the cells, the thickness of their walls and the method of their arrangement, both in the main thallus and the branches, also the form, &c., of the end cells of the branches; the heterocysts, their absence, or, if present, their frequency, size, shape, color, and position; the sheaths, their color and firmness, and the character of their surface.

Genus SIROSIPHON, Ktzt.

Trichomata torulosa, vaginata, plerumque ramossissima et aureo-vel olivaceo-fusca, e cellulis pachydermaticis 1-2-3 vel pluri-seriatis formata et cellulis interstitialibus (sæpe nullis) subglobosis vel oblongis coloratis instructa. Vagina plerumque crassissima, firma, pulchre aureo-fusca, lutea vel olivacea, in apicem obtusum plus minus attenuata.

Filament torulose, sheathed, mostly very much branched, yellowish, or olivaceous-fuscous, formed of thick-walled 1-2-3 or many seriate cells and furnished with interstitial cells (often wanting) which are globose or oblong and colored. Sheaths mostly very thick, firm, beautiful golden fuscous, clay-colored or olivaceous, more or less attenuate at the obtuse apex.

a. Cellula in trichomatibus plerumque in serie simplice vel duplici ordinata.

a. Cells mostly arranged in a simple or double series in the filament.

S. scytenematoides, Wood.

S. strato submembranaceo, nigro-viride, sæpe interrupto, cum superficie inæquale; trichomatibus sæpe arete intricatis, flexuosis aut varie curvatis, haud rigidis, plerumque vix ramosis; cellulis uniseriatis, interdum interruptis, arctis, irregulare quadrangulis, diametro subæqualibus aut 1-3 plo brevioribus, haud distincte granulatis, cæruleo-viridibus; vaginis amplis, haud distincte lamellosis, superficie enormiter corrugatis et hirtis, plerumque coloris expertibus sed interdum dilute brunneis.

Diam.—Sine vag. max. $\frac{5}{7500}$ " = .00066"; cum vag. max. $\frac{10}{7500}$ " = .0013".

Syn.—*S. scytenematoides*, WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 134.

Hab.—South Carolina. (Ravenel.)

S. In a submembranaceous, blackish-green, frequently interrupted stratum, with an uneven surface; filaments often closely intricate, flexuous or variously curved, not rigid, mostly sparsely branched; cells uni-seriate, sometimes interrupted, close, irregularly quadrangular, about equal in length to their diameter, or about 1-3 times shorter, not distinctly granulate, bluish-green; sheaths ample, not distinctly lamellate, their surface rough and corrugate, transparent, mostly colorless, sometimes light-brown.

Remarks.—This species was collected in South Carolina by Prof. Ravenel, who found it in the month of February growing on the limbs of *Myrica cerifera*. The

blackish-green layer, which it makes upon the bark is very peculiar, being almost membranaceous, and especially in the dried state, presenting a rough, somewhat warty surface. The trichomata have the sheath more distinctly in the form of a hollow cylinder, or, in other words, more plainly a sheath, than any other species I have seen of the genus; the cells are also without any apparent walls, and are placed very closely together, so that the whole filament looks very like a *scytonema*.

Fig. 1, pl. 9, represents a portion of a frond magnified 260 diameters.

S. pellucidulus, WOOD.

S. immersus; trichomatibus ramosissimis, solitariis vel subsolitariis; ramis plerumque unilateralibus, ramulosis; ramulorum apicibus late rotundatis, haud attenuatis; cellulis in seriebus simplicibus dispositis, in trichomatibus nonnihil rotundatis, in ramulis sæpe angularibus, plerumque compressis, diametro æqualibus—4 plo brevioribus; terminalibus cylindricis et obscure articulatis; cellulis interstitialibus nullis; vaginis arctis, hyalinis, haud lamellosis; cytoplasmate ærugineo vel brunneo, minute granulato.

Diam.—Trich. cum vag. $\frac{8}{500}$ " = .00106"; sine vag. = .0008".

Syn.—*S. pellucidulus*, WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 133.

Hab.—In stagnis, prope Hibernia, Florida. (M. W. Canby.)

S. immersed; filaments very much branched, solitary or subsolitary; branches mostly unilateral, branched; apices of the branches not attenuate, broadly rounded; cells disposed in a simple series, in the trichoma somewhat rounded, in the branches frequently angular, mostly compressed, from equal to 4 times shorter than the diameter; terminal cell cylindrical, obscurely articulate; interstitial cells none; sheath close, hyaline, not lamellate; cytoplasm æruginous or brown, minutely granulate.

Remarks.—This species was collected by Mr. William Canby in a little marsh pool near Hibernia, Florida. The branches are given off in abundance, mostly in a unilateral manner, are often very long, and about equal in diameter to the main filament, and give origin to numerous branchlets. The sheaths are very transparent and very close. I have never seen them in any way lamellate or fibrous, or of any color. The cells do not have very apparent walls. In the main filament and branches they are globose, or, more commonly, very much compressed, but in the newer branches, and sometimes in the older, they are very angular. The few cells near the end of the branches are so shaped as to remind one of the phalanges of the fingers. The last cell is cylindrical and has a number of cells indicated in it. The color of the young cells varies from a deep bluish-green to a ferruginous-brown—that of the older from a light bluish-green to ferruginous-brown.

Fig. 2 *a*, and 2 *b*, pl. 8, represent portions of filaments of this species.

S. compactus, (AG.) KTZ.

S. strato expanso, tomentoso, fusco-nigro; trichomatibus elongatis ramulisque adscendentibus, apice interdum paullum attenuatis sed sæpe clavatis, obtusis; trichomatibus internis e cellularum serie simplici formatis, et plerumque moniliformibus; cellulis diametro subæqualibus vel brevioribus, subglobosis vel subquadratis, sæpe compressis; cytoplasmate dilute caruleo-viride, subtiliter granulatis; cellulis apicalibus cylindricis et oscillarum modo, sæpe indistincte,

articulatis; vaginis firmis, aureo-vel rubido-fuscis, in ramulis sæpe subluteis, haud distincte lamellosis; cellulis perdurantibus plerumque modice numerosis, singulis, subglobosis, sæpe valde compressis, dilute fuscentibus.

Diam.—Plerumque $\frac{10}{12000}$ "— $\frac{13}{12000}$ " = .0008"—.001"; max. $\frac{4}{3000}$ " = .0013"; cell. perdurant. $\frac{7}{12000}$ " = .00058".

Syn.—*Scytonema compactum*, AGARDH. Syst. p. 38, N. 3. HARVEY'S Manual, p. 154.

Hassalia compacta, HASSAL, Fresh-water Algæ, p. 232, t. lxxviii. f. 3.

Sirosiphon compactus, (AG.) KTZ. RABENHORST, Flora Algarum, Sect. II. p. 287.

Hab.—In rupibus calcareis, New Jersey. (Austin.) Prope Salem, Mass. (Russel.)

Stratum expanded, tomentose, fuscous-black; filaments and branches ascending, with their obtuse ends sometimes slightly attenuate but often clavate; internal filaments composed of a single series of cells, mostly moniliform; cells shorter than or nearly as long as broad, subglobose or subquadrate, often compressed; apical cell cylindrical and articulate somewhat like an oscillatoria; cytoplasm light bluish-green, finely granulate; sheath firm, reddish or yellowish-brown, yellowish in the branches and near the ends; heterocysts mostly rather numerous, single, subglobose, brownish.

Remarks.—The specimens from which the above description was drawn up were received from Messrs. Austin and Russell, and have been considered as identical with the European *S. compactus*, although not in absolute agreement with the descriptions thereof. The most important of the differences are in the matter of size, the measurements given by Prof. Rabenhorst not equalling those attained to by the American plant.

The differences, however, do not seem sufficient to separate the forms, and, in the absence of European specimens, the two have been considered one species. The sheaths in the older portions of the filaments are nearly opaque, but in the branches and younger portions they are quite translucent. The heterocysts sometimes are truncate at one end. The internal cells are rarely arranged in a double series, such arrangement is, however, much more common in the specimens received from near Salem, than in those found in Northern New Jersey. Mr. Russell's specimens are labelled as growing on shaded and moist rocks in patches two or three inches wide.

Fig. 3 *a*, pl. 8, represents the end of a filament of this magnified 150 diameters; 3 *b*, a fragment magnified 250 diameters; 3 *c*, a heterocyst magnified 860 diameters.

S. Crameri, BRÜGG.

S. cæspitibus, tomentosus, spatioso expansis, fusco-nigris; trichomatibus vage ramosis; ramis plerumque singulis, sæpe elongatis, sæpe clavatis; cellulis internis uniseriatis, diametro subæqualibus vel brevioribus, interdum subglobosis, sæpe subquadratis, in ætate provecta sæpe e pressione mutua valde compressis et transverse oblongis, aureo-fulvis vel in ætate juvene interdum ærugineis; cellulis terminalibus in massam subcylindricam coalescentibus; cellulis perdurantibus nullis; vaginis aureo-fuscis in ætate provecta plus minus subopacis et distincte lamellosis, in ætate juvene plus minus pellucidis et sæpe coloris expertibus.

Diam.—Trich. cum vag. max plerumque .002"; interdum .00225"; ram. .0015"—.0025"; trich. sine vag. .00083".

Syn.—*S. Crameri*, BRÜGG. RABENHORST, Flora Europ. Algar., Sect. II. p. 288.

Hab.—In rupibus irroratis inter muscis minutis. Mount Tahawus (*vulgo* Mount Marcy),¹ alt. 5000 feet.

Forming a blackish, widely expanded, tomentose turfy covering to rocks; filament with scattered branches; branches mostly single, often elongate and clavate; cells uniseriate, about equal, or shorter than long, sometimes subglobose, often subquadrate; in advanced age often strongly compressed and transversely oblong from mutual pressure, yellowish, or sometimes, when young, greenish; the apical cells coalescent into an irregularly cylindrical mass; heterocysts wanting; sheaths yellowish-brown; at maturity more or less subopaque, and distinctly lamellate; in youth more or less transparent, and sometimes colorless.

Remarks.—Near the top of Mount Tahawus, in the Adirondack Mountains, there is, at an altitude of about five thousand feet, a steep slope of bare rock, the bed of an old landslide, over portions of which water is continually dripping. In such places the plant under consideration flourishes, forming with some very minute mosses a blackish, turfy coating to the rock of many feet, or even yards, in extent. The specimens agree well with the descriptions of the European plant, which also grows at about the same altitude as the American. They have, however, one peculiarity not noted in description of the European form, namely, that oftentimes the sheath of a branch widens out until it is actually much larger than the main filament. The color of the cells in the European form is said to be æruginous; but I conceive this depends somewhat upon the age of the specimens and is scarcely of primary value. The only other difference worth noticing is that my measurements exceed somewhat those given of the European plant. I do not think, however, there is any good ground for separating the forms as distinct species.

The finding of an Alpine plant growing on a mountain half way across the world from its first discovered home, at practically the same altitude, is a matter worth noting as a fact in Botanical Geography.

S. neglectus, WOOD.

S. immersus; trichomatibus subsolitariis, longis usque ad lineas quatuor, cylindricis, ramossissimis; ramulis singulis; cytoplasmate interdum æruginæo, plerumque aureo-brunneo; cellulis uniseriatis rarissime biseriatis, subglobois, interdum sejunctis sed plerumque arcte connectis et moniliformibus, modo confluentibus, haud distincte pachydermaticis; cellulis terminalibus elongato-cylindricis, sæpe nonnihil oscillatorium modo articulatis; cellulis interstitialibus nullis; vaginis interdum brunneis, plerumque coloris expertibus.

Diam.—Trichom. cum vag. $\frac{1}{570}$ " = .0017"; sine vag. $\frac{1}{1000}$ " = .001".

Syn.—*S. neglectus*, Wood, Prodr. Amer. Philos. Soc., 1869, p. 133.

Hab.—In stagnis, New Jersey.

S. immersus, subsolitary, attaining a length of 4 lines, cylindrical, very much branched; branches single; cytoplasm æruginous, mostly yellowish-brown; cells uniseriate, very rarely biseriata, subglobose, sometimes separate but more frequently closely united and moniliform; terminal cell an elongated cylinder, often articulate somewhat like an oscillatoria; interstitial cells wanting; sheaths transparent, sometimes brown, mostly colorless.

¹ "Tahawus," cloud splitter. The Indian names of the American mountains ought to be retained, in spite of the fact that some vulgar land surveyor has defiled the Adirondacks with the names of politicians, through whose influence he hoped for patronage.

Remarks.—This plant was found in a very stagnant pool, forming, with various other species of algæ, a gelatinous, æruginous-brown stratum, through which the single plants were thickly scattered, without anywhere forming the major portion of the mass. The plants themselves are large enough to be distinguished by the unaided eye. Under the microscope the sheaths are seen to be exceedingly transparent and colorless, except in the older part of the filament, where they are often dark brown and opaque; but even in such case, the edges are translucent and lighter colored.

The internal cells or globose masses rarely have distinct coats, and even when such were apparent, as in the older portions of the plants, there appeared to be a communication between the cells. The original main stem is rather short, shorter often than numerous branches into which it breaks up. Very often the apices of the branches are colorless and entirely empty, consisting simply of sheath; often, however, they are occupied by a cylinder of protoplasm, which is sometimes articulated more or less distinctly like an oscillatoria.

Fig. 4, pl. 8, represents a fragment of a filament with a small branch.

S. lignicola, WOOD.

S. strato expanso, tomentosio, atro; trichomatibus ramosissimis, arcte intertextis; ramulis abbreviatis vel elongatis, subrectis aut varie curvatis, apicibus obtuse rotundatis vel subacuminatis; trichomatium et ramulorum cellulis uni-vel biseriatis, rare in trichomatibus maturis multiseriatis, plerumque pachydermaticis, dilute vel saturate æruginis, enormibus, plerumque homogeneis; cellulis terminalibus in trichomatibus immaturis elongatis, cylindricis, sæpius nonnihil oscillatorium modo articulatis, granulosis; vaginis sat amplis, haud achrois, vel luteo-brunneis vel fuscentibus vel ferrugineis.

Diam.—Trich. cum vag. max. $\frac{1}{1500}'' = .00066''$.

Syn.—*S. lignicola*, WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 133.

Hab.—South Carolina. (Ravenel.)

Occurring in an expanded, tomentose, black stratum; filaments very much branched, closely interwoven, branches abbreviate or elongate, nearly straight or variously curved, their apices obtusely rounded or subacuminate; cells 1-2 seriate, mostly thick-walled, light or deep æruginous, irregular, mostly homogeneous; terminal cells elongate, cylindrical, frequently articulate somewhat like an oscillatoria, granulate; sheaths somewhat ample, not transparent, light bright, fuscous or ferruginous.

Remarks.—I have seen dried specimens only of this plant, which were collected by Prof. H. W. Ravenel, in South Carolina. It is said to grow on old boards, and appears to be a very distinct species. There are frequently two or three very short, stubby branches arising together. The apices of the filaments and branches are in some cases filled with endochrome to the end, and are broadly rounded at the apex. In other cases the sheath of the filament extends a distance beyond the endochrome, and is finally rapidly diminished to a point. The cells within the filaments are of various shapes, sometimes globular, sometimes quadrangular, more often irregular. The original specimens from which this description was written were collected in April. I do not know whether they grew immersed, or merely on boards exposed to the weather. I have since received

specimens collected in the month of August, which grew on boards over which spring water was constantly running. These specimens agree perfectly with the others, except that the filaments are larger and the elongated apical cell is wanting; differences which I believe to be due to the specimens collected in August being older than those first received.

Fig. 2 *a* and 2 *b*, pl. 9, were taken from the types, whilst fig. 2 *c*, pl. 9, from the August specimens.

a. Cellulæ plerumque in serie duplici vel multiplici.

a. Cells generally in double series, or multiple series.

S. argillaceus, WOOD, (sp. nov.)

S. strato tenui, expanso, subnigro, submembranaceo; trichomatibus brevibus, dense intricatis et sæpe nonnihil concretis, ramosis, irregularibus; pseudoramulis brevibus, varie curvatis, nonnihil rigidis, plerumque ascendentibus, apice nonnihil attenuatis; cellulis subglobosis, sæpe compressis, plerumque in serie simplici sed interdum in serie duplici, vel rare multiplici; cellulis apicalibus valde elongatis, cylindricis, scytonemæ trichomatibus internis similibus; vaginis crassis, firmis, in trichomatibus maturis saturate rubido-brunneis, in ramulis sæpe luteo-brunneis et in apice hyalinis et fere coloris expertibus; cellulis perdurantibus nullis.

Diam.— $\frac{5}{6000}$ " = .000833".

Hab.—In palude argillacea, South Carolina. (Ravenel.)

Stratum thin, expanded, blackish, submembranaceous; filaments short, densely intricate, and frequently somewhat concreted, giving origin to numerous branches, irregular; branches short, variously curved, somewhat rigid, mostly ascending, apex somewhat attenuate; cells subglobose, often compressed, mostly in simple series, sometimes in double, rarely even in multiple; apical cells elongate, cylindrical, resembling the inner filament of a scytonema; sheath thick, firm, in the mature filament deep reddish-brown, in the branches yellowish-brown, at the apices of the branches nearly colorless and transparent; heterocysts absent.

Remarks.—I am indebted to Prof. Ravenel for this plant, which was found by him on a moist clay bank near Aiken, South Carolina, August, 1869. It forms a thin, somewhat membranous, dark stratum, the filaments of which are so closely united that it is almost impossible to tease them apart with needles. Neighboring filaments are often united at the edges so as to form distinct bundles, and even the branches are sometimes concreted, although, generally, as seen under the microscope, they project from the mass in all directions. The surface of the filaments is mostly rough and ragged with fibrillæ and membranous projections. In the older filaments the cells are often entirely absent. They are mostly single, but sometimes multiple in the filaments; in the branches they are often partially double. The ends of the older branches are often broken and empty, whilst those of the younger are rounded. The color of the cells, as I have seen it, does not strikingly differ from that of the sheaths.

Fig. 3 *a*, pl. 9, represents a portion of an old frond magnified 460 diameters, and fig. 3 *b*, the end of a younger branch. No. 79. Collection of Ravenel, Aug. 1869.

S. guttula, WOOD.

S. in maculis subnigris, parvis, tenuibus, plerumque rotundatis, interdum enormibus, dispositus; trichomatibus arete intertextis, ramossissimis, rigidis, inæqualibus, subcylindricis, nonnihil

contortis; ramulis abbreviatis vel nonnihil elongatis, apice obtuse rotundatis; ramulorum et trichomatum cellulis tri-multiseriatis, plerumque pachydermaticis, ferrugineo-fuscis, enormiter globosis, homogeneis; cellulis apicalibus interdum breve cylindricis, haud articulatis; vaginis sat amplis, luteo-brunneis vel dilute ferrugineo-brunneis.

Diam.—Max. trich. cum vag. $\frac{1}{7500}'' = .0013''$.

Syn.—*S. guttula*, Wood, Prodr. Amer. Philos. Soc., 1869, p. 132.

Hab.—South Carolina, in *Taxodium distichum*. (Prof. Ravenel.)

Arranged in small, thin, black spots, which are generally round, but sometimes irregular: filaments closely interwoven, very much branched, rigid, unequal, subcylindrical, somewhat contorted; branches abbreviate or somewhat elongate, apex obtusely rounded; cells of the trichoma and branches 3 to many seriate, mostly with thick coats, ferruginous-fuscous, irregularly globose, homogeneous; apical cells sometimes shortly cylindrical, not articulate, sheaths ample, yellowish-brown.

Remarks.—This species was found growing on the bark of *Taxodium distichum*, by Prof. H. W. Ravenel, in South Carolina, and by him given to Dr. Billings, U. S. A., to whom I am indebted for specimens. It forms on the bark minute roundish, blackish, dot-like spots of about a line in diameter, or sometimes, apparently, by the coalescence of two or more of these spots, larger irregular patches. The habit of the plant is a rigid one. The main stem is often irregular in size, variously bent and rebent, and mostly gives off a number of branches, which frequently nearly equal the main filament in size, and like it are bent in various directions. They also frequently give origin to numerous short branches. In some instances, there is a distinct apical cell, which is cylindrical, but only two or three times longer than broad; in many cases, however, this cylinder being wanting, the ordinary cells extend to the extreme apex.

Fig. 4 *a*, pl. 8, represents a filament, and fig. 4 *b*, the end of a branch magnified 460 diameters.

***S. acervatus*, Wood.**

S. in guttulis minutissimis, suberustaceis, nigris, in strato subcontinuo sæpe aggregatis; trichomatibus parvis et brevibus, rigidis, admodum inæqualibus, prostratis, tuberculis, arcte et dense ramosissimis, viridibus aut aureis aut brunneis; ramulis brevibus, plerumque haud ramulosis, erectis aut ascendentibus, sæpe abbreviatis et papilliformibus, obtusis, sæpe lateraliter connatis; cellularum serie in trichomatibus multiplici in ramulis plerumque simplici; cellulis subglobosis vel subangularibus, viridibus, haud distincte granulosis, in ramulorum apice sæpe breve cylindricis et interdum obsolete articulatis; vaginis aureis, nonnihil hyalinis.

Diam.—Trich. max. $\frac{1}{7500}''$; ram. $\frac{3}{7500}'' - \frac{4}{7500}''$.

Syn.—*S. acervatus*, Wood, Prodr. Amer. Philos. Soc., 1869, p. 132.

Hab.—South Carolina, in cortice (*Ilex opaca*). (Prof. H. W. Ravenel.)

Arranged in drops, which are very minute, suberustaceous, black, and frequently aggregate into a subcontinuous stratum; filaments small and short, prostrate, rigid, somewhat unequal, tuberculate, densely and closely branched, green or golden or brown; branches short, for the most part not branched, erect or ascending, frequently abbreviate, and papilliform, obtuse; series of cell multiple in trichoma, mostly simple in the branches; cells subglobose or subangular, green, not distinctly granulate, in the apices of the branches frequently shortly cylindrical and sometimes obsolete articulate; sheaths golden, somewhat hyaline.

Remarks.—This species was found in winter by Prof. H. W. Ravenel in South Carolina, growing upon the bark of *Ilex opaca*, forming minute, firm, crustaceous, roundish dots or masses, much smaller than a mustard-seed, but in some cases so closely aggregated as almost to make a continuous stratum. When one of these dots is placed under the microscope, the branches are seen presenting their ends upon all sides, reminding one of some varieties of coral, and between these are blackish matters, which prevent the whole dot from being seen. These branches are frequently placed very close to one another, and cohere by their edges so as to make a sort of membrane or a solid mass. The filaments themselves are mostly obscured in the dense mass of branches which clothe them. This species seems to be closely allied to *S. coralloides*, and I am not certain whether it is distinct or not. It is certainly very much smaller.

S. pulvinatus, BRÉB.

S. pulvinatus, humectatus, saturate olivaceo-niger, ad tres lineas crassus; trichomatibus crassissimis, ramosissimis, fusciscentibus, enormiter curvatis; ramulis polymorphis pro ætate crassitie magnitudineque variis, apice plerumque obtuse rotundatis; trichomatum cellularum serie multiplici, ramulorum 2-4 plicis; vaginis crassis, luteo-fuscis ad saturate-fuscis, vel pellucidis vel non pellucidis, interdum rugoso-tuberculis.

Diam.—Trich. cum. vag. max. .0042".

Syn.—*S. pulvinatus*, (BRÉB.) RABENHORST, Flora Europ. Algar., Sect. II. p. 290.

Hab.—In rupibus prope Philadelphia. Wood.

In moist, deep olive-black cushion-like masses of two or three lines thick; filaments very thick, much branched, brownish, irregularly curved; branches polymorphous, varying in thickness and size, mostly with their apices obtuse; cells of the filament many seriate, of the branches two to four seriate; sheaths thick, yellowish-fuscous to deep fuscous, pellucid or opaque, sometimes rugose-tuberculate.

Remarks.—I have received specimens of this species found by Mr. Austin in Northern New Jersey, growing on the exposed face of rocks.

The size attained to exceeds that given by Mr. Rabenhorst for the European form. The color of the cytoplasm varies from an almost verdigris-green to fuscous.

Besides these specimens, Dr. I. Gibbons Hunt has given me fresh ones of a *Sirosiphon* which he found growing on the face of dripping rocks along the Wissahickon Creek, near this city. These are much smaller in every way than their more northern brethren, and differ in other respects, I think, sufficiently for a distinct variety. The filaments and branches are much flatter than in Mr. Austin's specimens. I append a description.

(*Var. parvus.*)

S. trichomatibus in cæspite saturate olivaceo-nigro arcte intertextis; trichomatibus crassissimis, enormiter ramosissimis, luteo-fusciscentibus, varie curvatis; ramulis polymorphis, apice plerumque obtuse rotundatis; trichomatum cellularum serie multiplici, ramulorum 1-4 plicis; cytoplasmate granulato, plerumque saturate fusciscente, interdum læte viride; vaginis crassis, dilute luteo-fusciscentibus, interdum achrois.

Diam.—Trichom. cum. vag. max. $\frac{1}{32}$ " = .03"

Filaments closely interwoven into a deep olive-black turfy mass, very thick, irregularly and frequently branched, yellowish-fuscous, variously curved; branches polymorphous, their apices mostly obtusely rounded; series of cell in filament multifold, in branches 1-4 fold; cytoplasm granulate, mostly deep brown, sometimes bright green; sheaths thick, light yellowish-brown, sometimes transparent.

Remarks.—The fronds are very irregular in form and size, much branched, and so closely interwoven that they mostly cannot be separated without breaking. The branches are sometimes short and stumpy, sometimes they are very long. The color of the cells approaches somewhat to a chocolate, at times with a little red in it so as to give something of a mahogany tint. The walls of the cells are mostly very thick, but they are often lost in the general mass of the frond. In the branches, the cells are often so closely crowded as to almost obliterate their walls. In a few specimens I have found the cells to be of a bright green color, instead of that just mentioned. The exact meaning of this I do not know; it would scarcely seem to indicate immaturity, for I have found it in the oldest portion of large fronds, whose other parts were of the normal color.

Fig. 1, pl. 10 represents a filament of this variety magnified 160 diameters.

I have received from Prof. Ravenel certain dried algæ, labelled *Stigonema Ravenelli*, BERKELEY, which appear to me to belong to this genus. In what place Berkeley described them, if ever, I do not know, nor why he placed them in the genus *Stigonema*. The following is a description of the species:—

S. strato sub-nigro; trichomatibus arcu intertextis, ramossissimis, enormibus, varie curvatis; ramulis brevibus et sublongis, varie curvatis, latis, apice nonnihil attenuatis et obtusis; trichomatum et ramulorum cellulis arcatis, enormibus, in serie duo-multipli enormiter dispositis; cytoplasmate homogeneo, late viride; vaginis aureis, lucidis.

Diam.—Max. trich. cum vag. $\frac{1}{500}$ "

When dried blackish; filaments closely interwoven, very much branched, irregular, and variously curved; branches short or largish, variously curved, broad, their apices somewhat attenuated and obtuse; cells of the filament and its branches very close, irregular, irregularly arranged in a twofold or multiple series; endochrome homogeneous, bright green; sheath yellow, semitranslucent.

Remarks.—This plant was collected by Prof. Ravenel on the now famous Look-out Mountain. It is of a thick, bushy habit, and appears to form turf-like mats of a line or two in thickness and of a blackish color. The filaments throw off in all directions very numerous branches, some of which are short and stumpy, others quite long, and are themselves the parents of numerous secondary branches. The longer branches often rival the main filament in size, and like it vary continually, in being irregularly expanded and contracted. There is never a long, articulated cell, not even in the apices of the branches. The apices are often somewhat attenuated, and are always more or less obtuse. The cells are of a bright green color, are very irregular in form, and are often very irregularly arranged in rows of from two to five, both on the main filament and branches. The base of the filament often gives origin to several small, cylindrical, root-like processes.

Fig. 4 *a*, pl. 9, represents a frond of this plant magnified 125 diameters; fig. 4 *b*, a fragment magnified 460 diameters.

Professor Bailey, in *American Journal of Sciences*, vol. iii., new series, states that he has found two species of the genus *Stigonema*, namely, *St. atrovirens*, AG. and *St. mammillosum*, AG.; the former growing on wet rocks at Indian Falls, Putnam County, New York; the latter at Round Pond, near West Point. I have no personal knowledge of the genus, but, according to authorities, it belongs to the lichens rather than the algæ, apothecia having been detected in various species.

CLASS CHLOROPHYLLACEÆ.¹

Plantulæ aquaticæ vel aëreæ, uni-, bi-, vel multicellulares, aut singulæ aut consociatæ, familias formantes.

Vegetatio terminalis vel non terminalis.

Ramificatio aut nulla aut vera, sed cellularum non divisione, potius proliferatione.

Cytioderma non siliceum, combustibile, sæpius e stratis successivis compositum, substantiam gelatinosam plerumque liquidam exsudans.

Cytoplasma chlorophyllosum, chlorophylli loco nonnunquam erythrino vel substantia oleosa coccinea, carnea aut rufescente coloratum, nucleo (centrali vel laterali) plerumque præditum, granulis amylaceis rarissime carens.

Multiplicatio fit cellularum divisione vegetativa. Fœcundatio plerumque sexualis.

Propagatio fit aut oosporis vel zygosporis aut gonidiis tranquillis vel agilibus.

Aquatic or *aërial* uni-, bi-, or multicellular plants occurring singly, or consociated in families.

Vegetation terminal or not so.

Branches either wanting, or if present, true branches, although formed rather by a process of proliferation than division of the cells.

Cytioderm not siliceous, combustible, often composed of successive strata.

Cytoplasma chlorophyllous, sometimes colored by an oily crimson, flesh-colored or yellowish-red substance, in the place of the chlorophyl, generally furnished with a nucleus (either lateral or central), very rarely without starch granules. Growth occurring by the division of the cells. Fecundation generally sexual.

Propagation taking place by oospores or zygosporis, or by tranquil or motile gonidias.

¹ The description of this Class and Order is that of Prof. Rabenhorst.

ORDER **Coccolphyceæ.**

Algæ unicellulares. Cellulæ aut singulæ (plerumque perfecte segregatæ) aut plures in familias consociatæ, tegumentis involutæ vel nudæ, aut ramificatione aut vegetatione terminali destitutæ. Propagatio fit aut cellularum divisione aut zoogonidiis.

Unicellular algæ. Cells either single (mostly entirely segregate), or mostly consociated in families, walled or clothed with teguments, destitute of branches or terminal vegetation. Propagation by means of zoospores, or by the division of the cells.

FAMILY PALMELLACEÆ.

Algæ unicellulares sensu latiori. Cellulæ aut singulæ aut numerosæ, familias constituentes, in muco matricali plus minus firmo, stratum gelatinosum amorphum, sæpius figuratum, tubulosum (Hormospora) varie divisum et perforatum (Tetraspora), quasi ramificatum (Hydrurus) formante nidulantes, vel nullo (Raphidium, Dactylococcus). Cytioderma plerumque tenue, sæpius tegumento gelatinoso aut homogeneo aut lamelloso præditum. Cytoplasmata homogenea, ætate provecata plerumque distincte granulosa, viride, aut rubescens aut fuscescens, vesicula chlorophyllosa semper instructum (excepto Raphidio).

Multiplicatio fit cellularum divisione vegetativa, propagatio gonidiis ex ultima cellularum generatione transitoria cytoplasmatis divisione varia ortis. Gonidia tegumentis liberata, polo antico ciliis vulgo binis plerumque instructa et alacriter circumvagantia. (R.)

Algæ unicellular in a broad sense. Cells either single or numerous, constituting families, imbedded in a jelly to form a gelatinous stratum which is amorphous or shaped, as tubular (Hormospora), variously divided and perforate (Tetraspora), falsely branched (Hydrurus), or sometimes is wanting (Raphidium, Dactylococcus). Cytioderm mostly thin, often furnished with a gelatinous or homogeneous or lamellate tegument. Cytoplasm homogeneous, mostly at maturity distinctly granular, green-reddish or fuscous, always furnished with a chlorophyllous vesicle (except Raphidium).

Multiplication taking place by a vegetative division of the cells, propagation by transitory gonidia arising by various divisions of the protoplasm from the last vegetative generation. Gonidia without integument, mostly furnished with two cilia at the anterior end, and moving about actively.

Genus **PLEUROCOCCUS**, MENGH. (RABENH.)

Cellulæ globosæ vel e mutua pressione angulosæ, plerumque nucleo instructæ, tum singulæ tum in familias consociatæ. Cytioderma firmum, sæpe crassum, læve, hyalinum; cytoplasmata homogenea viride vel oleosum rubrum. Multiplicatio cellularum vegetativarum divisione in directionem ad omnes dimensiones alternantem. Propagatio fit gonidiis intra sporangia ortis.

Cells globose or angular from mutual pressure, mostly furnished with a nucleus, sometimes single, sometimes aggregated into families. Cytioderm firm, often thick, smooth, hyaline; cytoplasm homogeneous-green or oleaginous-red. Multiplication occurring by a vegetative division of the cells alternately in three directions. Propagation by means of gonidia, formed within sporangia.

P. seriatus, WOOD, (sp. nov.)

P. corticolus, strata pulverula, rubido-brunnea, nonnihil crustacea formans; cellulis enormiter subglobosis, vel ovalibus, late aurantiacis, interdum viride tinctis, haud distincte nucleatis, in seriebus singulis rectis vel curvatis conjunctis; tegumentis crassis, haud lamellosis, coloris expertibus.

Diam.— $\frac{4}{7666}$ "— $\frac{2}{7666}$ " = .00053"—.0012".

Hab.—In palude. New Jersey. (Austin.)

Growing on bark, forming a reddish-brown, somewhat crustaceous powdery mass; cells irregularly subglobose, or oval, bright orange, sometimes tinged with green, not evidently nucleated, conjoined in single straight or curved series; tegument thick, lamellate, or not so, colorless.

Remarks.—I am indebted to Mr. Austin for specimens of this little plant, which he found growing in a swamp near Closter, Northern New Jersey, on a young pin oak. It forms a sort of crustaceous powder, with little aggregations here and there, of a dull reddish-brown color. When these little masses are broken up, they are found to be composed of little series of very closely joined cells, generally a half dozen to a dozen in the row. I believe that at certain states of their growth these cells are green, as many of them have a very decided green tint on their edges, and I have seen one or two of them quite green.

Fig. 2, pl. 10, represents this species magnified 460 diameters.

P. pulvereus, WOOD, (sp. nov.)

P. cellulis minimis, cæruleo-viridibus, enormiter subglobosis, vel angulosis, in familias numerosas consociatis; familiis e cellulis numerosissimis et dense confertis compositis, irregularibus, interdum confluentibus, plerumque pseudotegumentis hyalinis involutis, in strato pulvereo læte viridi aggregatis.

Diam.— $\frac{1}{24000}$ "— $\frac{3}{24000}$ " = .00004"—.00013".

Hab.—In fonte. "Boiling Springs," prope Bellefonte, Pennsylvania.

Cells very small, bluish-green, irregularly subglobose, oval, or angular, associated in numerous families; families composed of very numerous and densely crowded cells, irregular, sometimes confluent, mostly surrounded by a false hyaline tegument, aggregated into a bright green pulverulent stratum.

Remarks.—In Centre County, Pennsylvania, two miles from Bellefonte, there is a very large and beautiful limestone spring, which is a favorite roadside watering place, and is laid down on the maps as "Boiling Springs." Forming a stratum over most of the bottom of this spring is the little plant here described. The stratum is in places nearly an inch in thickness, and when lifted by the hand is found to be dry and crumbly, instead of mucous and tenacious. Under the microscope it is seen to be composed of vast numbers of irregular masses or families of cells imbedded in a firm jelly, which projects so as to form a sort of transparent coat to the whole mass; this cast I have spoken of in the description as a false tegument. The cells themselves are exceedingly small and furnished with an excentric point, which is probably a nucleus.

Genus PALMELLA.

Cellulæ globosæ vel ovals vel oblongæ, tegumentis plus minus crassis in mucum gelatinosum, sæpius mox confluentibus involutæ, thallum difforme efficientes. Cellularum divisio directione in omnes dimensiones alternante.

Cells globose, oval, or oblong, surrounded with a more or less thick integument generally very soon confluent into a firm or soft jelly. Thallus shapeless. Division of the cells alternately in all directions.

P. Jesenii, WOOD.

P. thallo indefinite expanso, initio dilute aut læte viride, molle, pellucidulo; ætate provecta firmo, tuberculoso, saturate olivaceo-viride; cellulis globosis vel ellipticis,—in thalli ætate immaturo, plerumque singulis aut geminis, sæpe distantibus,—in ætate provecta sæpe in familias connexis, plerumque confertis; tegumentis in thalli ætate immaturo plerumque diffluentibus, ætate provecta plerumque distinctis.

Diam.—Cell. glob. max. $\frac{1}{35000}$ " = .00028"; cell. oblong. long. max. $\frac{1}{25000}$ " = .0004".

Syn.—*P. Jeseni*, Wood, Prodromus, Proc. Am. Philos. Soc., 1869, p. 134.

Hab.—In rupibus irroratis, prope Philadelphia.

Thallus indefinitely expanded, in the beginning soft and pellucid, afterwards firm, tubercular, deep olive-green; cells globose or elliptical; in the immature thallus, single or geminate, frequently scattered; in the mature thallus often closely conjoined into families, mostly crowded; in the young thallus the teguments of the cells are mostly diffluent, afterwards distinct.

Remarks.—This little plant was found along the banks of the Schuylkill River, just above Flat Rock tunnel, near Manayunk, forming in the early winter a gelatinous mass of two to three lines in thickness, irregularly and interruptedly spread over the face of wet, dripping rocks. In what appeared to be the younger portions, the jelly was often quite soft and almost colorless, and had the cells scattered rather sparsely and distantly through it. The cells were but partially filled with chlorophyl, the vacuole left containing often numerous granules, and had distinct walls, being, as it were, merely immersed in the general maternal jelly. In the older fronds the texture is more firm, the color a deep green, and the bright green cells are mostly surrounded by a thick, very distinct tegument. They are also largely arranged in little families of two, four, or even eight cells, surrounded by a common integument. The oldest fronds are of a deep olive, almost blackish color, markedly tuberculate upon their upper surface and very firm in texture. They are surrounded by very distinct, firm, dark brown coats (a simple coat often involving two or more cells), and arranged in groups or families. As shown by the microscope in the superficial portion of such fronds, the jelly is of a yellowish-fuscous color, and the cells are themselves of a dark brown tint. The number of cells in the individual families varies from two to a dozen or more. Even in these old, firm fronds, the interior portions are frequently composed of greenish cells, without any distinct teguments or coat. In such cases the cells are mostly oblong or elliptical, and very much crowded together. This species appears to come closest to *P. Brébissonii*, Ktz., from which it differs, however, in its habit of growth and the size of its cells.

Fig. 3 *a*, pl. 10, represents a fragment of the upper surface of an old frond magnified 750 diameters; fig. 3 *b*, when taken from the inner jelly of similar fronds.

P. dura, Wood, (sp. nov.)

P. thallo enormiter subglobose, enormiter minute lobato vel verrucoso, cæruleo-nigro, nonnihil crustaceo, minuto; cellulis acutissime confertis, plerumque enormiter oblongis, sæpe in seriebus irregulare dispositis, cæruleo-viridibus vel luteo-brunneis; tegumentis haud distinctis; sporis globosis vel ovalibus.

Diam.—Cell. $\frac{1}{12000}$ " = .00008"— $\frac{1}{6000}$ " = .00016"; spor. $\frac{7}{12000}$ " = .00058"— $\frac{6}{7500}$ " = .0008"

Hab.—In fonte prope Philadelphia.

Thallus irregularly subglobose, irregularly minutely lobate or warty, bluish-black, somewhat crustaceous, minute; cells densely crowded, mostly irregularly oblong, often arranged irregularly in series, bluish-green or yellowish-brown; coats not apparent; spores globose or oval.

Remarks.—I found this plant growing in the large spring at Spring Mills in March or April. The fronds were in the form of little blackish balls attached to the stems of mosses in the water. They varied in size from the minutest speck, scarcely visible to the naked eye, up to ten lines in diameter; they are globose, very firm and hard, and the larger look almost as if they were aggregations of smaller ones. They are gregarious. The spores are mostly borne on the edges of the frond, sometimes they appear to be imbedded in its substance. At first they are of an intense bluish-green, but afterwards they appear to be yellowish-brown. None of the cells, as I have seen them, have their contents granulate.

Fig. 5 *a*, pl. 10, represents a section of a frond magnified 460 diameters; fig. 5 *b*, a section of the edge of an old frond, developing spores.

***P. hyalina*, LYNGB.**

“Fronds from a quarter of an inch to an inch in diameter, somewhat globose, but at length frequently more or less elongated into an ovate or even cylindrical form. Substance gelatinous and very tender, of a pellucid, watery appearance. Granules numerous, globose, green. The fronds are produced at first on rocks and stones at the bottom of streams, and afterwards become disengaged and float on the surface.”

Remarks.—Professor Bailey states that he has found this species from Rhode Island to Wisconsin. Whether it is identical with the *P. hyalina* of Brébisson, or not, I cannot say.

Genus PAGEROGALA,¹ WOOD.

Thallus solidus, gelatinosus, indefinitus, exalbidus, nonnihil pellucidulus, nodulis dense aggregatis et sæpe confluentibus formatus. Cellulæ globosæ, confertæ, in familias consociatæ. Familiæ tegumentis tenuibus et membranaceis involutæ, in nodulorum centro positæ.

Thallus solid, indefinite, gelatinous, whitish, somewhat pellucid, composed of closely aggregated nodules which are often indistinct. Cells globose, crowded in families. Families surrounded by a thin membranaceous coat and placed in the centre of the gelatinous nodule.

Remarks.—This curious plant was found by myself floating as indefinite masses of milk-white jelly on a mountain spring near Bear Meadow, Centre County, Pennsylvania. The largest of these gelatinous masses was six inches long. On taking them out of the water they were seen to be composed of somewhat irregular nodules, which in some portions of the mass were very distinct one from the other, but in other parts were confluent into an almost uniform jelly. When the nodules were separated it was discovered that each contained a membranous very delicate sack of a pale green color, which the microscope showed to be really a cell family. Their interior was hollow, or at least only partially filled with a transparent fluid, and they contained all round their exterior portion a layer of round, closely placed cells. In some instances the outer membrane was ruptured, and the sac only contained a few cells, which could often be seen to be moving freely in the inner liquid. The sac membrane is thin and delicate, colorless, and marked with curious, regular wrinkles or folds. In those portions of the common gelatinous mass, where the nodules were lost, I could not find any of these sacs.

¹ Παγερος, frozen; γαλα, milk.

No opportunity was afforded to study the development of this plant; but there can be but little doubt that the globular, thickish-walled cells are finally discharged by a rupture of the membrane and escape from the softening jelly into the water, each to be a possible starting point for a new frond.

I have given this curious plant the name of *Pagerogala*, from its milky whiteness. Floating in the water it offered so close a resemblance to the spawn of frogs, though more opaque, that my companion, a most excellent naturalist, insisted, until its true nature was absolutely demonstrated, that I was simply wasting my time collecting the spawn of an amphibian.

P. stellio, (sp. nov.)

Diam.—Frond $\frac{1}{2}$ inch; cells $\frac{4}{3000}$ "— $\frac{1}{2000}$ ".

Genus TETRASPORA, LINK.

Thallus gelatinosus, membranaceus vel submembranaceus, initio saccato-clausus, ætate proveciori vel postea explanatus. Cellulæ globosæ (vel anguloso-rotundatæ) plus minus distantes sed in familias magnas unistratas consociatæ; tegumentis crassis in mucum homogeneous cito diffluentibus. Cellularum divisio in planitie duarum directiones alternans.

Propagatio fit gonidiis mobilibus.

Thallus gelatinous, membranous or submembranous, in the beginning a short sack, afterward expanded. Cells globose, or angularly so, more or less distant but consociated in a single stratum into large families. Tegument thick, very rapidly diffluent into a homogeneous mucus. Division occurring in two directions in the one plane.

Propagation by means of zoospores.

T. lubrica ? (ROTH) AG.

T. thallo gelatinoso-membranaceo, lubrico, dilutissime viride, tubuloso sed sæpe postea explanato, simplice vel ramoso, undulato-sinuoso, sæpe læunis munerosis perforato; cellulis globosis vel ellipticis, læte viridibus, interdum singulis sed plerumque quaternis vel geminis, locello achroo hyalino parietali sæpe præditis; cytodermate tenuissimo, haud distincte visibile.

Diam.—Cell. $\frac{1}{2000}$ "— $\frac{1}{4000}$ " = 0.00025"—0.0005".

Syn.—*T. perforata*, HARVEY. BAILEY, Silliman's Journal, N. S. vol. iii.

T. lubrica, (ROTH) AG. RABENHORST, Flora Europ. Algarum, Sect III. p. 41.

Hab.—Northern Atlantic States.

Thallus gelatinoso-membranaceus, slippery, very dilute green, tubular, but often finally expanded, simple or branched, undulately-sinuate, often perforated with numerous holes; cells globose or elliptical, bright green, sometimes single but mostly in pairs or fours, furnished with a parietal transparent hyaline space; cytoderm very thin, not distinctly visible.

Remarks.—This little plant is very common around this city, growing usually in limpid, quiet water, such as springs, little rushy pools, and clean ditches. The frond is a translucent, light green or scarcely greenish, very slippery jelly, with the edges often very markedly undulate. It is very rarely simple, but on the contrary is often very much and very irregularly branched, frequently indeed consisting of several broad portions united by narrow necks. It is an irregular sack, generally profusely perforate, and often with large imperfect portions. I think it finally in many instances becomes expanded and open. It is sometimes found lying on the bottom, but more frequently floats on the surface of the water. The breadth of the frond varies from two or three lines to an inch. The length often reaches several inches. The cells are mostly globular; but, immediately after

division, they are elliptical. They are of a bright green color and almost always have a conspicuous rounded granule within them; sometimes, but not commonly, at one end there is a hyaline space or vesicle, similar to that seen in zoospores. I have watched the production of zoospores in a plant gathered late in November. The outer wall of the cell is always so thin as to be scarcely perceptible, and when the zoospore is beginning to move, it looks as though the whole cell were rocking, the thin outer coating being lost to sight. After a considerable period of vain effort the zoospore escapes from the thick gelatinous mass which surrounds it. It is biciliated, roundish, and furnished with a hyaline space at the end.

I have observed a *Tetraspora* growing in rapidly running water, which some would no doubt consider distinct, but which seems to me rather a variety. The saccate frond was of a very vivid green, erect, buoyed up by an air-bubble contained in its upper end. Its shape was that of a long sack widened very much above, and below constricted into a fine point, by which it was firmly attached. In some instances it attained a length of seven or eight inches. In all other respects these plants agreed with the others found in quiet water.

The species of this genus are to me not at all well-defined in any work which I have had access to. The plant now under consideration abounds everywhere in this neighborhood, and is without doubt the one identified by Prof. Bailey as *T. gelatinosa* (Vauch), of which, however, he afterwards states that Prof. Harvey, to whom he had sent specimens, writes that it is a distinct species, and proposes to call it *perforata*. In my Prodrômus I referred the plant to *T. lubrica* (Roth). My reasons for doing this were that the size of the cells corresponds very closely with the measurements of that species as given by Prof. Rabenhorst, and the absence of anything that seemed to me definite in the descriptions of the two species. Moreover, if the possession of a parietal hyaline spot be not simply an accident of growth, it would indicate that the plant belongs to *P. lubrica*. I do not think, however, that any importance is to be attached to this, as the vacuole is often absent, and, although Prof. Rabenhorst makes no mention of it, is, in all probability, present in certain states or stages of *T. gelatinosa*. My own conviction is, at present, that *T. gelatinosa* and *T. lubrica* are very probably synonyms. If they be distinct, the plant from which the above description was taken is referrible to *T. perforata* (Harvey), which, if not new, is a form of *T. lubrica* rather than *T. gelatinosa*. If *T. lubrica* and *T. gelatinosa* be united, no grounds are left for sustaining the separateness of *T. perforata*.

Whilst botanizing in a primeval glade and forest, known as Bear Meadows, in this State, I came across a spring, covered with a *Tetraspora*, which appears to represent the *T. gelatinosa* type. It formed great masses half an inch in thickness, at first attached, afterwards floating and covering the surface of the pool for several feet each way. When young these masses were elongated and were formed of numerous lobes attached often by very slender pedicles, and having their margins thickened and undulated so as to give a beautiful waved appearance to the light green mass. Under the microscope the structure was similar to that of the other form, except that the cells varied more and attained a greater size. Their diameters ranged from $\frac{1}{375000}$ " = 0.00027" to $\frac{1}{150000}$ " = 0.00066".

I have also received from Prof. Ravenel specimens of a *Tetraspora*, which may be the young of a variety of this species, but which is very possibly distinct. If the specimens are adult, it certainly is. They consist of numerous little fronds not more than a third of an inch in length, often composed of several subcylindrical arms, as it were, radiating from a central portion, and attaining a length of a third of an inch or so. These fronds are irregularly perforate, and are composed of cells agreeing perfectly in form, size, and arrangement with the more ordinary forms of *T. lubrica*.

T. bullosa, (ROTH) AG.

T. thallo membranaceo-saccato, obovato, sinuoso-buloso, unciam usque palmam longo, postea explanato, dilacerato, saturate viridi, plus minus verrucoso; cellulis subsphericis (post divisionem factam hemisphericis vel angulosis) geminis vel quaternis, confertis, granulosis. (R.) *Species mihi ignota*.

Diam.—Cell. ante divis. 0.00032"—0.00049"; post divis. 0.00022"—0.00029". (R.)

Syn.—*T. bullosa*, (ROTH) AG. RABENHORST, Flora Europ. Algarum, Sect. III. p. 39.

Hab.—"Salem, North Carolina. Schweinitz, Newburgh, New York." Bailey, Silliman's Journal, New Series, vol. iii.

Thallus membranaceous saccate, obovate, sinuately-bullose, from one to six inches in length, afterwards expanded, torn, deep green, more or less verrucose; cells subspherical (after division hemispherical or angular) in twos or fours, crowded, granular.

Genus DICTYOSPHERIUM, NÆG.

Thallus gelatinosus plus minus liquidus, libere natans, sæpe quasi nullus. Cellulæ vesicula chlorophyllosa unica et locello achroo parietali præditæ, tegumentis crassis in gelatinam homogeneam confluentibus involutæ, filis propriis subtilibus dichotome divisis, e familiarum centro ad peripheriam radiantibus connexæ. Cellularum divisio ad omnes directiones.

Propagatio fit gonidiis mobilibus

Thallus gelatinous, more or less liquid, swimming free, often almost wanting. Cells furnished with a single chlorophyllous vesicle and a lateral transparent spot, surrounded with thick coats, which are confluent into a homogeneous jelly and united by very fine filaments, which are dichotomously divided and radiate from the centre to the peripheral families. Division of the cells occurring in all directions.

Propagation by motile gonidia.

D. pulchellum, WOOD, (sp. nov.)

D. thallo subgloboso vel subovale, interdum subnullo, interdum indistincte lobato; cellulis globosis plerumque sparsis sed interdum nonnihil confertis.

Diam.—Cell. $\frac{1}{4000}$ " = 0.00025"; thalle plerumque $\frac{1}{300}$ " = 0.0033"; interdum $\frac{1}{185}$ " = 0.0054."

Hab.—In stagnis prope Philadelphia.

Thallus subglobose or suboval, sometimes indistinctly lobate, sometimes almost wanting; cells globose, mostly scattered, but sometimes rather crowded.

Remarks.—I found this little plant, one August day, floating, in company with *Closterium acerosum*, in a brick-pond below the city. The little fronds are mostly roundish, or longer than broad, with a distinct outline, sometimes, however, the constituent jelly seems to fade into the surrounding water. There is never a distinct

outer coat. The lateral transparent spot in the cells is mostly very evident, sometimes it is wanting, however. Occasionally there is a very distinct blackish "eye spot." The threads which join the cells are very delicate, and I have never been able to absolutely demonstrate their meeting in the centre of the frond, although I believe they do so. In mounted specimens, even when preserved in carbolic acid water, they disappear after a time. I have never seen zoospores or any other reproductive bodies.

Genus RHAPHIDIUM, KtZ.

Cellulæ fusiformes vel cylindraceæ, utrinque (plerumque) sensim sensimque cuspidatæ vel acuminatæ, rarius obtusatæ, rectæ vel varie curvatæ, singulæ, geminæ vel fasciculatim aggregatæ, medio decussatim vel radiatim conjunctæ, rarius binæ sub polis lateraliter connexæ, ceterum liberæ. Cytioderma tenue, læve. Cyttoplasm viride, subtiliter granulosum, locello pallidiori vel achroo, centrali, rarius laterali, præditum. Cellularum divisio ad unam directionem. (R.)

Cells fusiform or cylindrical, generally very gradually cuspidate or acuminate at the ends, rarely obtuse, straight or variously curved, single, geminate, or fasciculately aggregate, decussate in the centre or radiately conjoined, rarely two laterally united at the end, other cells free. Cytoderm thin, smooth. Cytoplasm green, very finely granular, furnished with a central or rarely lateral transparent vacuole. Division of the cells occurring only in one direction.

R. polymorphum, FRESEN.

R. cellulis rectis vel varie curvatis, singulis, vel 2-4-8-16 fasciculatim collocatis, gracilibus, sæpe gracillimis, nonnunquam medio paulum turgidis, subventricosis, nonnunquam paulum constrictis, apices versis, sensim attenuatis, acutissimis.

Diam.— $\frac{1}{7600}$ " = .00013".

Syn.—*R. polymorphum*, FRESEN., RABENHORST, Flora Europ. Algarum, Sect. III. p. 44.

Hab.—Prope Philadelphia, Wood.

Cells straight or variously curved, single or 2-4-8-16 fasciculately joined together, slender, often exceedingly so; sometimes slightly turgid in the centre, subventricose, sometimes slightly constricted; the apices gradually attenuate, very acute.

Var. falcatum.

Cellulis fusiformibus, gracilibus, utroque fine acutissime cuspidatis, curvatis vel semilunaribus, 4-16 fasciculatim congregatis.

Syn.—*Ankistrodesmus falcatus*. (CORDA.) RABENHORST, Flora Europ. Algarum, Sect. III. p. 45.

Hab.—South Carolina, Georgia, Florida, Rhode Island. (Bailey.)

Cells fusiform, slender, at each end very acutely cuspidate, curved or semilunar, 4-16 fasciculately congregate

Remark.—Fig. 3, pl. 7, represents different forms of *R. polymorphum*.

FAMILY PROTOCOCCACEÆ.

Algæ unicellulares sensu strictissimo, chlorophyllosæ, et vegetatione terminali et ramificatione vera carentes, sine cellularum generatione vegetativa. Vivunt aut singulæ, segregatæ aut in familiarum consociatæ. Harum familiarum cellulæ numero aut indefinitæ semper se augentes (tum sensu vero familiarum nomen ferunt), aut definitæ, se non augentes (quæ cœnobium dicuntur).

Propagatio fit gonidiis, quæ intra cellulam matricalem cytiogenesi libera oriuntur et duplcis indolis sunt; altera majora, quæ macrogonidia, altera minora quæ microgonidia dicuntur; illa oblonga, polo antico plerumque rostelliformi-producta, pallidiora, ciliis vibratoris prædita, polo postico truncato-rotundata, obscure viridia, individuum propagant; hæc forma similis, itidem mobilia, brevi postea in statum quiescentem transeunt, druique in sporas perdurantes (*Hypnosporas*, BRAUN) transmuntur. (R.)

Unicellular algæ, in the strictest sense of the word, chlorophyllous, without terminal growth or true branching, without a vegetative generation of cells. They live either single, segregate, or consociated into families. The cells of these families, either indefinitely increasing in number (then families in the true sense of the term), or of definite number (then forming a cœnobium).

Propagation by means of gonidia arising within the mother-cell by free cell-formation; gonidia of two kinds; the one larger, macrogonidia—the other smaller, microgonidia; the former oblong, mostly produced into a pale biciliate beak anteriorly, rounded and greenish at their hinder end, developing into the individual plant; the microgonidia similar to these and also motile, but passing after a short time into a quiescent state, and at last into resting spores or hypnosporas.

Genus PROTOCOCCUS, Ag. 1824.

Cellulæ spheroidæ, segregatæ, cytiodermate tenui, hyalino, absque tegumentis, libere natantes vel extra aquam in stratum tenue pulvereum cumulata. Cytoplasmia initio homogœnum, denique granulosum, viride vel rubellum.

Spheroidal cells, segregate, cytoderm thin, hyaline, without integument, swimming free or collected out of water into a thin pulverulent stratum. Cytoplasm in the beginning homogeneous, finally granular, green, or reddish.

Remarks.—I have introduced this genus as given by Professor Rabenhorst in his *Flora Europæa Algarum* for the purpose of describing a little plant, upon which I have made some observations. As the notes were originally drawn up as a description of a species, I leave them in that form. I believe it has never before been described.

Protococcus, (sp. nov. ?)

P. aquaticus; cellulis globosis vel angulis, viridibus in stratum pulvereum cumulatis vel in familiis arete conjunctis; cytiodermate plerumque distincto; sporis rotundatis, tegumentis duobus vel tribus protectis; tegumentis externis, crassibus; zoogonidiis ovalibus, vel subrotundatis, vel subellipticis, ciliis duobus instructis.

Diam.—Max. spor. perdurant. $\frac{7}{1600}$ " = .00093"; microg. $\frac{4}{7500}$ " = .00053".

Aquatic; cells green, globose or angular, accumulated in a green pulverulent stratum, often closely united into families; cytoderm mostly not distinct; resting spores round with two or three thick coats; zoospores oval or roundish, or somewhat elliptical, furnished with two cilia.

Remarks.—I found this species growing in a spring near Hestonville, West Philadelphia, in the month of March. The large winter spores are round, with thick coats. Except in one instance, in which the color was a decided reddish-brown, all that I have seen have been green. How they are produced I do not know. The history of their development into the plant appears to be as follows: The first change is the rupture of their outer thick coat (fig. 4 *b*, pl. 7) from which the spore finally escapes still clothed with a coat of moderate thickness. The green contents next divide into a number of oval bodies (fig. 6 *b*, pl. 7) which

grow, and, at the same time, separate from one another. Whilst these changes have been taking place the spore coat has been becoming gelatinous and enlarging, so that it continues to enclose its progeny. In this way a family of oval cells is formed (fig. 4 *b*, pl. 7). So far, I think, is positive. The next step I have never actually seen, but believe to be the escape of these oval bodies as zoospores (fig. 4 *c*, pl. 7) which are of very various sizes and are elliptical, globose, or oval. They have a tolerably well-marked bright vacuole at their beak, and after swimming about actively for a time finally settle down, lose their cilia, and undergo division. They seem often to cluster together before thus becoming quiescent, so as to make little colonies (fig. 5, pl. 7).

Genus CHLOROCOCCUM, FRIES.

Cellulæ sphæroideæ, aut singulæ, liberæ, vesicula chlorophyllosa et locello laterali pallidiori cavo? instructæ, limbo hyalino et tegumentis sæpe amplissimis cinctæ, aut plures in stratum vel acervulos cumulatae.

Propagatio fit zoogonidiis cytoplasmatis divisione ortis, e cytiodermatis abaviæ (intellige tegumentum extremum) rupturis excedentibus.

Cells spheroidal, either single, free, furnished with a chlorophyllous vesicle and a paler lateral (hollow?) spot, with a hyaline nimbus and surrounded by a wide coat; mostly accumulated together into strata or little heaps. Propagation by means of zoospores, which are formed by a division of cytoplasm and escape from their general tegument (the cytioderm of the original cell).

Remarks.—But a few weeks after the commencement of my study of fresh-water algæ, a friend, a young microscopist, asked me to look at his aquarium, as the water of it had become stagnant, opaque, and green. On examining a little of the water with the microscope it was found to be full of what I now know to have been either one of the forms already described under this genus, or else one undescribed, but still embraced within its limits. There were two sets of bodies, the one motile the other at rest. The motile forms (Fig. 5, pl. 3) were globular or pyriform, and generally contained a large, roundish, green, distinct mass. They were of course provided with cilia, although at that time I was not able to demonstrate their presence. These bodies, even when moving, appeared to have a distinct wall. After a time they settled down and assumed the quiescent state. The outer coat now rapidly enlarged so as to leave a considerable space between it and the green endochrome, which rapidly underwent division, forming two or more new cells which were still surrounded by the enlarged maternal coat. The number of daughter-cells enclosed in the parent cell varied. A considerable quantity of the water was allowed to stand in a glass jar, exposed to the light. In a very few days all the motile forms had disappeared. The contents of the vessel were allowed slowly to evaporate. The jar being tall and narrow it was some weeks now before this process was completed, before which consummation *hamatococcus* forms were abundantly developed.

Instead of being green, and surrounded by a distant, almost sac-like wall, the cells had acquired a dark brownish-red color, were very opaque, and were protected by a thick wall, whose surface was quite rough. Unfortunately, I did not measure either the active gonidia or their progeny, the quiet cells, but I found the general

diameter of these hæmatococcus cells to be one twelve-hundredth of an inch (.00083").

MM. Fammitzin and Boranetzky, in a recent paper ("Zur Entwicklungsgeschichte der Gonidien und Zoosporienbildung der Flechten," Mem. de L'Académie Impériale des Sciences de St. Petersburg, 1868, Annals and Mag. Nat. History, Feb. 1869), state as the result of direct observation that this genus of algæ, so called, is really a stage in the life history of the gonidia of lichens. These gentlemen took thin slices of lichen thalli containing gonidia, and placed them upon pieces of fir and linden bark, which had been previously boiled to kill any plants that might be growing on them. These were then put in a glass jar inserted over a vessel containing water, in such way that they would be constantly exposed to a very damp atmosphere, and at the same time communication with the external air would be impossible. In another set of experiments, pieces of the lichens were allowed to lie for a long time in water, until the component filaments were decomposed into a gelatinous mass, in which the still green vigorous gonidia were imbedded. These pap-like (*breiige*) masses were then washed with pure water and smeared upon pieces of linden bark. The results obtained were identical in the two cases. The gonidia were at first provided each with a distinct nucleus and a well-marked lateral vacuole, and resembled closely the first form of cystococcus. The next change was a division of their contents into a large number of roundish masses, with the disappearance both of the vacuole and of the central nucleus. The cell-membranes were next ruptured, and the endochrome, protruding through the opening, formed a little ball sitting upon the parent cell. In doing this it doubled in size, so that the part without was as large as the part within, although the latter still filled the cell. The contents finally escaped, but were yet surrounded by a very thin membrane, which soon, however, ruptured, and freed the biciliated zoospores into which the endochrome had in the mean time resolved itself. These zoospores remained a long time in the motile state, but finally settled down, dropping their cilia, and became little round cells, which grew to three or four times their original size. Further development was not made out.

Certain of the gonidia, belonging to a lichen of the genus *Phycia*, failed to produce zoospores, but their endochrome, divided so as to form a number of quiescent cells, which either ruptured very early the original cell-membrane and became free in the water, or else remained bound together by it into a family for a longer period. In these researches MM. Fammitzin and Boranetzky employed lichens of three genera, namely *Phycia*, *Cladonia*, and *Evernia*, and claim, as above stated, that their investigations prove that they developed the algæ genus *Cystococcus* of Nægeli (*Chlorococcum*, Fries), from the gonidia.

Genus POLYEDRIUM, NÆGELI, (1849.)

Cellulæ singulæ, segregatæ, libere natantes, compressæ, 3-4-8 angulares, angulis plus minus productæ, nonnunquam radiatim elongatæ, aut integræ aut bifidæ, plerumque armatæ, a latere oblongo-ellipticæ, utroque polo rotundatæ vel subtruncatæ. Cytioderma tenue, læve. Massa chlorophyllacea plerumque granulosa, per cellulæ lumen æqualiter distributa, nonnunquam guttulis oleosis rubris 1-4 mixta.

Propagatio adhuc ignota. (R.) *Genus mihi ignotum.*

Cells single, segregate, swimming free, compressed, 3-4-8-angled, more or less produced as to their angles, sometimes radiately elongate, either entire or bifid; mostly armed, oblong-elliptical when viewed laterally, at each end rounded or subtruncate. Cytioderm thin, smooth; chlorophyl mostly granular, equally distributed through the cell, sometimes mixed with reddish oil-drops.

Propagation unknown.

Remarks.—This genus was described by Nægeli in his “Gattungen Einzelliger Algen,” and, although I have never seen any specimen of it, it claims a place here, because one species has been found in this country by Prof. Bailey.

P. enorme, (RALFS) DE BARY.

P. tetraëdicum, angulis productis achrois profunde bilobis, nonnunquam repetito-bilobis, lobis mucronatis. (R.)

Diam.—0.0011"—0.0016". (R.)

Syn.—*P. enorme*, (RALFS) DE BARY. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 62.
Staurastrum enorme, RALFS, *British Desmidiæ*.

Hab.—Florida. Bailey.

“Fronde irregular or quadrate, spinous; end view three or four-lobed; lobes broad, more or less emarginate or bifid, and terminated by spines, which are either simple or branched. Sometimes the front view differs but little from the end one, usually, however, there is a slight constriction or sinus at the junction of the segments, but I have never observed any difference in the endochrome at that part. The spines, which are almost confined to the angles, are irregular, some simple and some branched. The end view has three or four broad and very irregular lobes; these are spinous and more or less emarginate, and frequently one lobe is much broader and more spinous than the others. The spines on such lobe form two groups, separated by the notch; they vary much in size and are either simple and subulate, or else forked; sometimes the forked spines are again divided at the apex.”—*Ralfs' British Desmidiæ*, p. 141.

Genus SCENEDESMUS, MEYEN.

Cellulæ polymorphæ, utroque polo æquales vel inæquales, sæpe in cornu spiniforme productæ, in ætate perfecto 2-16 aut in seriem simplicem aut parenchymatice arcte conjunctæ et cœnobium constituentes; cytioplasmate initio homogæneo, postea granuloso, vesicula chlorophyllosa centrali vel sublaterali et sæpe locello achroo laterali instructo.

Propagatio fit cytioplasmatis divisione succedanea, unde gonidia oriuntur, quæ intra cellulam matricalem jam in cœnobium planum sese conjungunt et membranæ matricalis ruptura vel dissolutione prodeunt.

Cells polymorphous, equal or unequal at the ends, often produced into a spine-like horn, in the perfect state 2-16 closely conjoined, either as a simple series or in a parenchyma-like manner so as to form a cœnobium. Cytoplasm in the beginning homogeneous, afterwards granular, furnished with a central or sublateral chlorophyllous vesicle, and often with a lateral transparent spot.

Propagation occurring as a succedaneum to the division in the cells, whence arise gonidia, which, already within the mother-cell, join themselves into a cœnobium, and are finally set free by the rupture and dissolution of the maternal cell-wall.

Remarks.—According to Unger, in the genus *Scenedesmus* the cells never exist singly, but always in families.

Two of the species here described as representatives of the genus certainly do not conform to this, for I have frequently seen them both separate and in cœnobia

or families. The latter were exactly like those of the European forms, at least in one of the two species, and I do not therefore think it justifiable to indicate a new genus. Moreover, I have certainly seen single cells, belonging to a species which agrees precisely in its characters with a European form, save only in the occasional existence of these single cells.

I have never studied the method of propagation, but it is said to occur by the division of the cytoplasm of a large cell into a minute cœnobium composed of two or more cells, which remains for some time within the walls of the mother-cell, but is finally set free by the solution of the latter.

The cells are mostly much longer than broad, cylindrical, elliptical, or oval, but in one species herein described they are habitually globular.

a. Cellulæ inermes.

a. Cells unarmed.

S. obtusus, MEYEN.

S. cellulis oblongis vel ovatis, utroque polo obtusis, 4-6-8 modo arcte modo laxè in seriem simplicem aut rectam aut duplicem obliquam conjunctis, diametro 3-5 plo longioribus. (R.)

Diam.—Transv. max. 0.00023"—0.00028". (R.)

Syn.—*S. obtusus*, MEYEN. RABENHORST, Flora Europ., Algarum, Sect. III. p. 63.

Hab.—Georgia: Rhode Island, Bailey.

Cells oblong or ovate, obtuse at each end, 4-6-8, partly closely partly laxly conjoined into a simple series either straight or oblique and double, 3-5 times longer than broad.

Remark.—I have never met with this species.

S. acutus, MEYEN.

S. cellulis fusiformibus, vel ovato-fusiformibus vel ovatis, utrinque acutis sed inermibus, interdum singulis sed plerumque in seriem aut simplicem rectam aut duplicem inordinate alternantem dispositis, arcte concretis, diametro 2-4 (6?) -plo longioribus.

Diam.—Trans. vag. max. ? .00016".

Syn.—*S. acutus*, MEYEN. RABENHORST, Flora Europ. Algarum, Sect. III. p. 64.

Hab.—Prope Philadelphia, Wood. Rhode Island, Bailey.

Cells fusiform, or ovate-fusiform or ovate, acute at each end but unarmed; sometimes single but mostly conjoined into a single straight series or into an irregularly alternate double series, 2-4 times longer than broad.

Remarks.—This species is common around Philadelphia. Our specimens agree very well with the descriptions and figures of the European, excepting that occasionally a cell is single, and that none which I have measured have attained the size given by Prof. Rabenhorst as the maximum, namely, 0.00023". According to Rabenhorst, *S. obliquus*, Ktz., is only a variety of *S. acutus*, Meyen. It has been found by Prof. Bailey in South Carolina, Georgia, and Rhode Island.

b. Cellulæ armatæ.

b. Cells armed.

S. polymorphus, WOOD.

S. cellulis fusiformibus, aut ovalibus aut ellipticis aut globosis, singulis aut 2-8 conjunctis, plerumque utroque polo aculeo unico, interdum aculeis duobus, instructis: apicibus obtusis, acutis, vel acutissimis; aculeis gracillimis, rectis, modice elongatis, inclinatis.

Diam.— $\frac{1}{2500}$ "— $\frac{1}{7500}$ "; plerumque $\frac{1}{4000}$ ".

Syn.—*S. polymorphus*, WOOD, Prodrômus, Proc. Am. Philos. Soc., 1869, p. 135.

Hab.—In aquis quietis prope Camden, New Jersey.

S. cells fusiform, or oval, or elliptic, or globose, single or 2-7 conjoined, furnished in most cases with a single spine, sometimes 2, at each end; ends obtuse, acute, or very acute; spines exceedingly slender and acute, straight, moderately long, inclined.

Remarks.—This plant was found in a quiet pool, filling the water in such numbers as to make it opaque and very green. The color of the cells, as first obtained, under the microscope, was a vivid green, but, the water containing them having been placed in a dish, during the slow desiccation which followed the color of the cells changed to a golden yellow.

Fig. 1, pl. 11, represents different forms of this species magnified 450 diameters.

S. quadricauda, (TURPIN) BRÉB.

S. cellulis oblongo-cylindricis, utroque polo obtuse rotundatis, 2-4-8 arctissime conjunctis, ordine aut simplici recto aut duplice alternante, omnibus rectis, medianis inermibus vel his illisve apice uno alterove aculeo curvato instructis, extimis utroque apice sæpius item dorso armatis.

Diam.—0.00035"—0.00039"; long. 0.00091".

Syn.—*S. quadricauda*, (TURPIN) BRÉB. RABENHORST, Flora Europ. Algar., Sect. III. p. 65.

Hab.—Rhode Island, Bailey. Pennsylvania, Wood.

Cells oblong-cylindrical, obtusely rounded at each end, 2-4-8 very closely conjoined either in a single straight series or a double alternating one, all straight, the median unarmed or some of them with the apex furnished with a curved spine, the external with both apices and sometimes the dorsum thus armed.

Remark.—Fig. 2, pl. 11, represents this species magnified 750 diameters.

S. rotundatus, WOOD, (sp. nov.)

S. cellulis globosis vel subglobosis, spinulis longissimis, rectis, gracillimis, acutissimis, 3-6 armatis, aut singulis aut geminis aut 3-4 arcte duplice conjunctis.

Diam.— $\frac{1}{6000}$ " to $\frac{1}{3000}$ ".

Hab.—In aquis quietis prope Philadelphia. (Dr. Chapman.)

Cells globose or subglobose, armed with three to five very long, slender, acute, straight spines, single or in pairs, or three to four closely conjoined in a twofold rank.

Remarks.—The cells of this species are globular, and, when more than two, they are arranged in two rows placed at right angles one to the other. The contents of the cells are markedly granular, and the endochrome a bluish-green, and from the surface of the walls project outwards, very long and fine, rigid hair-like spines.

It seems scarcely correct to place this plant in the genus *Scenesdesmus*, but I do

not know any other genus to which it is more closely allied, and do not feel disposed to indicate a new one for it.

Fig. 3, pl. 11, represents a cell-family magnified 250 diameters.

Genus HYDRODICTYON, ROTH. (1800.)

Cellulæ oblongo-cylindricæ, in cœnobium reticulato-saccatum connexæ, omnes fertiles; aliæ procreant macrogonidia, quæ jam intra cellulam matricalem in cœnobium filiale se connectunt; aliæ microgonidia, quæ multo minora, cellulæ matricalis membranam perrumpunt, polo antico ciliis vibratoriiis binis et puncto rubro laterali prædita sunt, brevi postea in globulos protococcoideos tranquillos transformata sporas perdurantes efficiunt.

Cells oblong-cylindrical, joined into a reticulated saccate cœnobium, all fertile; some producing macrogonidia, which join themselves into a cœnobium within the parent cell; the others producing microgonidia, which are furnished with two vibratile cilia and a lateral red spot, and which, escaping from the parent cell, are, after a brief period of motile life, transformed into protococcoid thick-walled spores.

Remarks.—The genus *Hydrodictyon* comprises, as far as known, but a single species, which is common to North America and Europe. It grows in great abundance in the neighborhood of Philadelphia, especially in the ditches and stagnant brick-ponds in the low grounds below the city known as the "Neck." There it very frequently forms floating masses several inches in thickness and many feet in extent, so that with the aid of a rake it could be gathered by the bushel. When thus in mass the color is very generally dingy and yellowish, although the fronds, when in active vegetative life, are mostly of a bright, beautiful green. The plant is in greatest profusion in June and July, after which time it gradually disappears, until in the autumn it is scarcely to be found, but early in the spring it reappears. The very young fronds are minute, oval, cylindrical, filmy-looking, closed nets, with the meshes not appreciable to the eye; when growth takes place, the fronds enlarge until finally they form beautiful cylindrical nets two to six inches in length, with their meshes very distinct and their ends closed. In the bright sunlight they, of course, by virtue of the life-functions of their chlorophyl, liberate oxygen, which being set free in the interior of the net, and its exit barred by the fine meshes, collects as a bubble in one end of the cylinder and buoys it up, so that, the heavier end sinking, the net is suspended, as it were, vertically in the water. I know of few things of the kind more beautiful than a jar of limpid water with masses of these little nets hanging from the surface like curtains of sheen in the bright sunlight. A few cells collected in the fall or early spring, if put into a preserving-jar and the water occasionally changed, will multiply, and in a little while become a source of frequent pleasure to the watcher.

As the fronds increase in size they are always in some way or other broken up, so that, instead of being closed cylinders, they appear as simple open networks of less or greater extent. The extreme length to which the frond attains is, I think, very rarely over twelve inches, with meshes of about a third of an inch in length. The construction of the frond is always the same. It is composed of cylindrical cells united end to end in such a way as to form polygonal, and mostly pentagonal

meshes, the size of which varies with the age of the plant. These cells, which are closely conjoined but have no passage-ways between them, are capable of independent life, so that the hydrodictyon may be looked upon as an elaborate type of a cell-family, one in which cells are conjoined in accordance with a definite plan, so as to make a body of definite shape and size, yet in which each cell is an independent being, drawing nothing from its neighbors. The cells themselves are cylindrical, with a thickish cellulose wall, and have no nuclei. Their chlorophyllous protoplasm is granular, and is placed in the exterior portion of the cell, forming thus, within the outer wall, a hollow cylinder, in which are imbedded starch granules, and whose interior is occupied with watery contents. The hydrodictyon cell, when once formed, is capable of growth, but not of going through the usual process of cell multiplication by division, so that the adult frond is composed of just as many and indeed the same cells, as it had in its earliest infancy.

No true sexual reproduction has as yet been discovered in the water-nets. There have been described, however, two forms or methods in which the species multiplies, both of them occurring by means of motile zoosporoid bodies. In the one case these develop immediately into the new plant, whilst in the other before doing so they pass through a resting stage. Of the life-history of the latter, the *microgonidia*, I have no personal knowledge.

The investigation of the production and development of the *macrogonidia*, however, has occupied considerable of the time devoted by myself to the microscope, and I have seen large numbers of specimens in almost all the stages of development. I have never been able to detect, however, any decided motion in the *macrogonidia*.

They are formed in the protoplasmic stratum, already alluded to as occupying the outer portion of the interior of the hydrodictyon cell. The first alteration in this, presaging their formation, is a disappearance of the starch granules, and a loss of the beautiful, transparent green color. Shortly after this, even before all traces of the starch-grain are gone, there appear in the protoplasm numerous bright spots placed at regular intervals; these are the centres of development around which the new bodies are to form. As the process goes on, the chlorophyll granules draw more and more closely around these points, and at the same time the mass becomes more and more opaque, dull, and yellowish-brown in color. This condensation continues until at last the little masses are resolved into dark hexagonal or polygonal plates, distinctly separated by light, sharply defined lines. In some, the original bright central spot is still perceptible, but in others it is entirely obscured by the dark crowded chlorophyll. The separation of these plates now becomes more and more positive, and they begin to become convex, then lenticular, and are at last converted into free, oval, or globular bodies. When these are fully formed, they are said to exhibit a peculiar trembling motion, mutually crowding and pushing one another, compared by M. Braun to the restless, uneasy movement seen in a dense crowd of people in which no one is able to leave his place. Whilst the process just described has been going on, the outer cellulose wall of the hydrodictyon cell has been undergoing changes, becoming thicker and softer and more

and more capable of solution, and by the time the gonidia are formed it is enlarged and cracked, so that room is afforded them to separate a little distance from one another within the parent cell. Now the movements are said to become more active—a trembling jerking which has been compared to the ebullition of boiling water. There is, however, with this a very slight change of space, and in a very short time the gonidia arrange themselves so as to form a little net within the parent cell, a miniature in all important particulars of the adult hydrodictyon. The primary cell-wall now becomes more and more gelatinous, and soon undergoes complete solution, so that the new frond is set free in its native element. As previously stated, in my investigations I have never seen the peculiar motion above described, the newly formed gonidia simply separating and arranging themselves without my being able to perceive any motion, or exactly how they fell into position.

It is evident that when the species is multiplied in the way just described, the birth of the new frond is consentaneous with the death of the old cell. But when the hydrodictyon disappear in the fall, it is months before they reappear in the spring. It is, therefore, evident there must be some other method of reproduction. This slow development of new fronds takes place, according to Pringsheim, by means of little motile bodies which he calls "*Dauerschwärmer*," which has been translated into English *chronisporos* (*statosporos*, Hicks). M. Braun stated already some years since that sometimes, instead of the hydrodictyon producing the ordinary reproductive bodies (*macrogonidia*), there are formed in the cells much smaller and more active bodies, the *microgonidia*. The changes which occur in the production of these are very similar to those already described as happening when the *macrogonidia* are formed. When the *chronisporos* are once formed, however, they, instead of uniting together escape in a free distinct condition into the water. They are now small ovate bodies, with a large anterior transparent space, to which are attached a pair of cilia, and their life and history, according to Pringsheim, is as follows: For a few hours they move about very actively in the water, and then, dropping their cilia, and acquiring an outer cellulose wall, pass into a quiescent stage, in which they closely resemble *protococcus granules*. They are capable of living in this state for a long time, if kept in water. They can also endure desiccation if the light be excluded during the process, but, if it be present, they wither and die, and cannot be revived.

After a longer or shorter period, but never shorter than three months, according to Pringsheim, they recommence their life, provided they be in water. For four or five months after this the chief change consists simply in an increase in size. The dark-green protoplasm is arranged around the exterior of the cell, within are the more fluid colorless contents, the whole body still looking like a *protococcus* cell. After a size of about $\frac{1}{40}$ mm. is attained, the endochrome divides successively into several portions. The external layers of the surrounding wall now give way in some spot and allow the inner layers to protrude and form a sort of hernial sac, into which the several endochrome masses soon pass, at the same time assuming the well-known characters of true zoospores. From two to five of these

bodies are thus produced out of each original microgonidium. They are large, ovate, biciliate, and, generally, soon escaping from the hernial sac, move about actively in the water for a few minutes. Sometimes, however, they settle down within the generative utricle. In either case, after a little time, they become motionless, lose their cilia, and develop into polyhedral cells, which are structurally remarkable for having their angles prolonged into long horn-like appendages. Under favorable circumstances, at the end of a few days, the bright green endochrome of these undergoes similar changes to those described as presaging the production of the microgonidia, and is finally formed into zoospores, which, in from twenty to forty minutes, unite, within the polyhedron or large cell, into a *Hydrodictyon*, which is finally set free by a solution of the cellulose coat of the polyhedron. The network thus formed differs in no essential way from that which arises in the better known way, except that it is composed of much fewer cells. It is generally a closed sac; but when the polyhedron, out of which it is developed, is small, it is sometimes merely an open network. Its after-history appears to be identical with that of the ordinary hydrodictyon frond.

H. utriculatum, ROTH.

Species unica.

Syn.—*H. utriculatum*, ROTH. RABENHORST, Flora Europ. Algarum, Sect. III. p. 66.

Hab.—In aquis quietis. West Point, Bailey. Wechawken, (Mr. Walters.) "Waterholes between Van Horn's Mills and Mueote on the Mexican boundary, Dr. Bigelow. Pennsylvania, Wood.

Genus PEDIASTRUM, MEYEN, (1829.)

Cœnobium planum, disciforme, libere nataus, e cellularum strato unico, rarius centro entro duplicato, continuo vel perforato formatum. Cellulæ polygoniæ, periphericæ sæpe bilobæ, lobis cuneatis et simplicibus et bidentatis, nonnunquam in cornua productis.

Cœnobium plain, discoid, swimming free, formed of cells in a single, rarely in the centre double stratum, which is continuous or perforate; cells polygonal, the peripheral often bilobed, the lobes cuneate, either simple or bidentate, sometimes produced into a horn.

Remarks.—The cœnobium or cell-family, or colony, in the genus *Pediastrum* is always discoid, and has generally a more or less truly circular outline. The cells are mostly in a single stratum, but in some species there are two, more or less, complete strata superimposed one upon the other. The arrangement of the cells in this stratum, or these strata, varies greatly, as does also their number. They are mostly more or less polyhedral, and often have their margins scooped out or their angles prolonged. This may occur in such a way that the projecting point of one cell fits into the hollow in its neighbor, and the cœnobium be rendered entire, or, no such relation existing between the parts of adjacent cells, the cœnobium may be perforated with regular or irregular openings. The outer or marginal cells are often deeply notched externally, and frequently are prolonged into acute or obtuse lobe-like processes. The walls of the cells are, in adult specimens, quite thick. The contents consist of chlorophyl, protoplasm, starch granules, &c.

There are generally one or more hyaline spaces, besides a distinct chlorophyll vesicle, but no distinct nuclei.

At certain periods of their existence the Pediastrums produce both macrogonidia and *microgonidia*. The life-history of the former is very similar to that of the same bodies in the water-nets. The ultimate fate of the microgonidia has not as yet been determined, but in all probability they go through cycles of change similar to those seen in the lives of the corresponding bodies in the *Hydrodictyon utriculatum*. I have not had an opportunity of watching the development of either of these reproductive forms, but, according to MM. Braun, Pringsheim, &c., their life-history, as far as known, is as follows: In most cases, all the cells of a pediastrum produce their macrogonidia simultaneously, or within a very short period of time, so that the cœnobium will be left emptied of its contents as a mere shell, the outer skeleton of its former self. When a cell is about to give birth to these reproductive bodies, the endochrome divides into two parts; each of which then undergoes a similar binary division. This is repeated once, twice, thrice, or oftener, until the endochrome is divided into 8-16-32-64 gonidial masses, the number of which, generally, but not always, corresponds to the number of cells in the colony, to which the parent-cell belongs. After the division of the endochrome is completed, a slit occurs in the outer strata of the wall of the mother-cell through which a hernial protrusion of the inmost stratum occurs. The protruded part now rapidly enlarges until at last there is formed a sort of hourglass-shaped sac, one portion of which is within, the other part without, the old parent-cell. Whilst this has been going on a portion of the gonidia have escaped from the parent-cell into the outer free portion of the sac, and each end of the hourglass, therefore, contains some of them. The sac with its contents now gradually withdraws itself more and more from the parent-cell until at last it lies a free globose vesicle in the water. The gonidia occupy the centre, and M. Braun states, that, although he has never been able to demonstrate any cilia upon them, yet they have an active swarming motion. At first, they are irregularly heaped together in the nearly filled sac; but the latter rapidly enlarges and elongates, and the gonidia in a little while arrange themselves in a flat, tabular group within it, and cease to move. Then the several individuals of this group begin to develop, becoming emarginate and assuming the form of the parent-cell, until, finally, they have all grown into the shape which is peculiar to the adult cells of the species, and after a few hours have closely cohered to form a young cœnobium.

The microgonidia are formed in a very similar way by the dividing of the endochrome, the cracking of the outer membrane, and the protrusion and final escape of the inner. They are, however, much smaller and more numerous than the macrogonidia. When the parent vesicle first escapes into the water, they are crowded in its centre, and are nearly globose. As it enlarges, however, they elongate more and more, and finally become distinctly bi- or, more rarely, uni-ciliate. The cilia are much longer than the body, and are attached to the smaller end, which is prolonged into a pointed, transparent beak, about equal to the green portion in length. The microgonidia now become more and more restless, they, moving about very actively, and after awhile bursting the parent sac, escape into the water.

What becomes of them after this, as has been stated, is a mere matter of conjecture. M. Braun¹ and others have described unicellular forms of several of the multicellular species of *Pediastrum*, and Pringsheim suggests that these are really polyhedrons developed out of these microgonidia, as is seen in the water-nets. This, of course, may or may not be the case.

P. Boryanum, (TURPIN) MENGH.

P. cœnobia orbiculari, oblongo vel elliptico, magnitudine vario, continuo, læte viridi, e cellulis 4-8-16-32-64 (rarissime 128) composito (cellularum strato simplici, nonnunquam medio duplicato); cellulis periphericis plus minus profunde emarginatis vel bilobis, lobis cornutis, cornibus achrois hyalinis, abbreviatis vel elongatis, teretibus, obtusis vel subobtusis, interdum capitellato-incrassatis, centralibus arctissime conerctis, polygonis (4-6 angularibus), in antica parte modo angulo prominulo modo plane truncatis, modo leviter repandis, omnium membrana decussatim punctata. (R.)

Diam.—Transv. cell 0.000795"; rarius 0.00088"—0.00094". (R.)

Syn.—*P. Boryanum*, (TURPIN,) MENGHINI. RABENHORST, Flora Europ. Algarum, Sect. III. p. 74.

Hab.—Georgia, Florida, Rhode Island, Bailey; Pennsylvania, Wood.

Cells arranged in one or more circles round one or two central cells; the inner variable, generally concave at one side, the outer tapering into two long subulate points, the notch narrow.

L. 1-2083" to 1-1633"; B. 1-2733" to 1-2222". (Archer.)

P. Selenæa, Ktz.

P. cœnobia orbiculari, integro, e cellulis 8-16 (rarius 31 = 1 + 5 + 10 + 15, Ktz.) formato; cellulis periphericis angustis, lunatis, acute lobatis, disci cellulis leviter excisis, centrali unica 5-angulari, omnium membrana firma, subcrassa, ætate provecata rubescente. (R.)

Diam.—Cœnobia 0.00124"—0.0035"; cell. (distantiæ interlobos) 0.00026"—0.00069". (R.)

Syn.—*P. Selenæa*, Ktz. RABENHORST, Flora Europ. Algarum, Sect. III. p. 73.

Hab.—Rhode Island, Bailey.

Cells crescent-shaped, arranged in one or more circles round one or two central ones, connecting medium colored. (A.)²

P. pertusum, Ktz.

P. cœnobia orbiculari, lacunis pertuso, magnitudine vario, e cellulis plerumque 1 + 5 + 10 + 15 (in formis quibusdam ad 64) composito; cellulis periphericis basi tantum laxè connexis, ad medium usque bilobis, lobis rectis, in cornua hyalina modo subacuta modo obtusa vel truncata plus minus productis, centralibus plus minus exacte quadrangularibus, et in antica parte et utrinque emarginatis, omnibus lævibus, locellis pallioribus finis instructis. (R.)

Diam.—Transv. cell. perfecte evolut. circiter 0.00065"—0.00089". (R.)

Syn.—*P. pertusum*, Ktz. RABENHORST, Flora Europ. Algarum, Sect. III. p. 75.

Cells arranged in circles round one or two central ones; inner cells quadrangular, sides concave and leaving angular vacant intervals; the outer cells with square bases, externally triangularly notched, the subdivisions tapering to an acute point. L. 1-2266"; B. 1-3268". (A.)

P. constrictum, HASSAL.

P. cœnobia orbiculari vel suborbiculari, læte viridi, continuo, lævi?, e cellulis 16 (ad 1 + 5 + 10) vel 32 (ad 1 + 6 + 10 + 15) formato; cellulis periphericis irregulariter bilobis, sinu

¹ The best exposition of this genus is to be found in Braun's Unicellular Algæ.

² The letter A used here signifies that the description is copied from Mr. Archer in Prichard's Infusoria.

angusto, lobis inæqualibus, basi plerumque constrictis, in cornua subcrassa obtusa productis, centralibus polygonis, in antica parte repandis. (R.)

Syn.—*P. ellipticum*, HASSAL. RABENHORST, Flora Europ. Algarum, Sect. III, p. 77.

Hab.—South Carolina, Georgia, Rhode Island, Bailey.

Cells varying in number and arrangement; outer cells suddenly contracted into two short, cylindrical, obtuse processes. L. 1-1754" to 1-906"; B. 1-1515" to 1-1020".

β, Processes of the lobes truncately emarginate. (A.)

P. Ehrenbergii, (CORDA) BRAUN.

P. cœnobia et orbiculari et oblongo, perfecte clauso, e cellulis 8 vel 16 composito et quadrato, e cellulis 4, late cuneatis, profunde lobatis, exacte cruciatim dispositis formato; cellulis periphericis cuneatis a basi truncata ad apicem usque concretis, profunde bilobis sinu angusto, lobis sæpe oblique truncatis, plus minus sinuato-excisus, angulis interioribus ad duplum longioribus, omnibus acutis vel breviter appendiculatis; cellulis centralibus aut singulis aut pluribus (2-5-6 v. 8), omnibus flavo-viridibus, polygonis, uno latere repandis vel profunde incisus. (R.)

Syn.—*P. Ehrenbergii*, (CORDA.) BRAUN. RABENHORST, Flora Europ. Algarum, Sect. III. p. 77.

Hab.—South Carolina, Georgia, Florida, Rhode Island, Bailey.

Fronde minute; cells eight (seven disposed in a single series round a central one), bilobed, angular. L. 1-2900"; B. 1-2500". (A.)

P. simplex, MEYEN.

P. cellulis periphericis ovato-cuspidatis, 8-10-16 basi tantum concretis, circulum simplicem constituentibus, centralibus sæpe nullis. (R.)

Syn.—*P. simplex*, MEYEN. RABENHORST, Flora Europ. Algarum, Sect. III. p. 71.

Monactinus octonarius, BAILEY, Smithsonian Contributions.

Hab.—South Carolina, Rhode Island, Bailey.

Var.—duodenarius.

Cœnobia clathrato, cellulis periphericis 12, centralibus 4, regulariter cruciatum dispositis. (R.)

Syn.—*Monactinus duodenarius*, BAILEY.

Inner cells four, somewhat triangular, enclosing a central, quadrate vacant interval, and four broadly lanceolate vacant intervals between them and the outer series, to which they are united by their terminal angles, outer cells twelve, subovate, truncate below, much attenuated, acuminate. (A.)

FAMILY VOLVOCINEÆ.

Cœnobia mobilia, globosa, subglobosa vel quadrangulo-tabulata, e cellulis viridibus cilia bina agilia gerentibus, intus vesica duplici contractibili præditis composita, membrana (tegumento, chlamyde) communi achroa hyalina plus minus ampliata involuta.

Propagatio aut sexualis, monoica vel dioica (adhuc in paucis tantum generibus probata); cellulis cœnobia aut omnibus aut quibusdam genus masculinum vel feminum exhibentibus, illis in fasciculos spermatozoideorum (antheridia), has in oosporas episporio inclusas, non mobiles commutatis, aut non sexualis, gonidiis agilibus, (et macrogonidiis et microgonidiis—etiam zoogonidia vocantur). Macro- et microgonidia (cellulæ primordiales) cytioplasmatis divisione simultanea et multiplici orta; priora numero definita (2-4-8-16, &c.), majora oblonga vel rotundata, polo antico plus minus rostri-formi producta, ciliis binis per vesicæ membranam exsertis, puncto (ocello Ehrberg. stigma) sanguineo centrali vel parietali et locellis (vacuolis) sæpe binis contractilibus instructa; ultima numero indefinita, multo minora, pallide vel sordide viridia vel luteola, apice ciliis instructa, plerumque jam intra cellulam matricalem vivide vacillantia, postea membranæ ruptura libere erumpentia, examinantia. (R.)

Cœnobium mobile, globose, subglobose or in square tables, composed of green cells which have two motile cilia and a double contractile vesicle. The common tegument surrounding the cœnobium hyaline, and more or less amplified.

Propagation either sexual or non-sexual. The sexual monœcious or diœcious; either all or some of the cells of the cœnobium exhibiting male and female characters. The male cells containing spermatozoids, the female finally converted into a quiet oospore. Non-sexual propagation taking place by means of motile gonidia (both macrogonidia and microgonidia, by some called zoogonidia). Macro- and micro-gonidia arising by the simultaneous and repeated division of the cytoplasm; the first definite in number (2-4-8-16, &c), the larger, oblong or rounded, with the anterior end more or less rostellate, with two cilia exerted through the membrane of the vesicle, furnished with a central or parietal red spot, and often with two contractile vacuoles; the microgonidia indefinite in number, much the smaller, pale or dirty green or luteolous, furnished at the apex with cilia, mostly even within the mother-cell, moving rapidly, and finally escaping on the rupture of the membrane.

Genus CHLAMYPOCOCCUS, A. BRAUN.

Cellulæ globosæ, vel subglobosæ (4-8 in cœnobium fugacissimum conjunctæ), cytiodermate subcrasso firmo, cytioplasmate granuloso, fusco-rubro vel puniceo (in evolutionis gradibus quibusdam in colorem viridem mutato). Macrogonidia 2-4-8, rotundata, polo antico rostriformi producta, duo cilia longissima gerentia, nucleo centrali rubro, globulis amylaceis 4-6, non semper visibilibus instructa, tegumento amplissimo hyalino plerumque ovoideo vestita. Microgonidia multo minora, numerosa, luteola vel sordide viridia, apice rubella, ciliis binis instructa, intra tegumentum matricali alacriter vacillantia, denique membranæ ruptura elabentia. (R.)

Cells globose, or subglobose (4-8 conjoined in a very fugitive cœnobium), cytioderm thickish, firm, cytoplasm granular, brownish-red or puniceous, in certain stages of evolution changed into green. Macrogonidia 2-4-8, rounded, the frond end bearing very long cilia, furnished with a central reddish nuclei and with four to six, not always perceptible, starch granules, clothed with a very ample, hyaline, mostly ovoidal tegument. Microgonidia much the smaller, numerous, luteolous or sordid green, the apex reddish, furnished with two cilia, moving actively within the maternal tegument, and at last escaping by the rupture of the membrane.

Ch. nivalis (BAUER, AG.). A. BRAUN.

Ch. globulis, 0.004"—0.00135". (R.)

Hab.—In nive æterna, Greenland. Rocky Mountains.

Syn.—*Ch. nivalis* (BAUR, AG.). A. BRAUN. RABENHORST, Flora Europ. Algarum, Sect. III. p. 97.

Globules, 0.004"—0.00135" in diameter.

Remarks.—I have never seen any good specimens of this plant, merely some cells mounted in Canada balsam, and therefore ruined for scientific study, which had been collected by Dr. Kane in one of his Arctic voyages. I have also had some indications of plants in a little parcel sent me by Mr. Sereno Watson, who informs me he has seen the red snow very abundant in the higher peaks of the Rocky Mountains. It is a matter of presumption rather than determination, therefore, that the species is identical with the European.

Genus VOLVOX, EHRB.

Cœnobium exacte sphæricum, continuo rotatum et agitatum, globum cavum quasi fingens, e cellulis numerossissimis æquali distantia peripherice dispositis, gelatina matricali connexis, puncto rubro laterali, locellis (vacuolis) binis contractilibus necnon ciliis binis longe exsertis instructis, vesica communi hyalina circumcinctis compositum.

Propagatio duplex ist, aut non sexualis aut sexualis; illa fit cellulis quibusdam certa distantia intumescens, multipartitis, in cœnobium filialia intra cœnobium matricale evolutis, postea libere erumpentibus; hæc cellulis masculis multipartitis in fasciculos spermatozoideorum mobilium, contractilium, pyriformium, ciliis binis instructorum, postea liberorum evolutis; cellulis femineis intumescens, non divisis, sed post fecundationem in oosporas immobiles episporio duplici circumdatas postremo rubras evolutis. (R.)

Cœnobium exactly spherical, continually rotating and agitated, looking like a hollow globe, composed of very numerous cells, which are arranged on the periphery at equal distances, and are connected by the maternal jelly, and surrounded by a common hyaline bladder; they are also furnished with a lateral red point, with two contractile vacuoles, as well as two long exserted cilia.

The propagation is both sexual and non-sexual. In the latter, certain distant cells enlarge greatly, divide into numerous parts, and evolve within the parent cœnobium daughter-cœnobium, which are finally set free. In the sexual propagation certain molecular cells undergo a multipartite division into fasciculi of spermatozoids, which are motile, contractile, pyriform, and furnished with two cilia; the feminine cells are enlarged, and do not undergo division, but after fecundation develop into immovable oospores, which are finally red, and are surrounded by a double episporium or coat.

V. globator, (LINN.) EHRL.

V. cœnobiis majoribus ad $\frac{1}{3}$ ''' , cellulis numerosissimis (ad 12,000); cœnobiis filialibus semper octo intra matricale fructificatione non sexuali evolutis; fructificatione dioica; cœnobiis masculis fasciculos spermatozoideorum numerosos rubescentes foventibus (= *Sphærosira volvox*, Ehrb.); cœnobiis femineis cellulas sexuales (oogonia) 20-40 post fecundationem in totidem oosporas globosas rubras episporio hyalino stellato circumdatas foventibus (= *Volvox stellatus*, Ehrb.). (R.)

Syn.—*V. globator*, (LINNÉ,) EHRL. RABENHORST, Flora Europ. Algarum, Sect. III. p. 97.

Hab.—In stagnis. United States.

Larger cœnobium, about $\frac{1}{3}$ '' in diameter, composed of very numerous (about 12,000) daughter-cœnobium, always 8 within the maternal one, evolved without sexuality; fructification diœcious; male cœnobium giving origin to numerous reddish spermatozoids (= *Sphærosphæra Volvox*, Ehrb.); female cœnobium, giving origin to from 20-40 sexual cells, which, after fecundation, develop into the same number of globose red oospores surrounded by a stellate hyaline episporium.

Remarks.—Some of my friends tell me they have found this species abundantly around Philadelphia. I have not been so fortunate, and have seen but a few scattered specimens, which have afforded no opportunity of studying their development and life-history.

ORDER **Zygothyceæ.**

Algæ aut uni- aut pseudomulti-cellulares, sine vegetatione terminali et ramificatione vera. Cellulæ singulæ aut geminatæ aut seriatim conjunctæ. Multiplicatio fit cellularum divisione in unam directionem.

Propagatio fit zygosporis conjugatione cellularum similium binarum ortis.

Algæ either uni- or pseudomulti-cellular, without terminal growth or true branches. Cells segregate or geminate, or arranged in a single row. Multiplication taking place by a division of the cells in one direction.

Propagation by zygosporis, formed by the conjugation of two similar cells.

FAMILY **DESMIDIACEÆ.**

Algæ unicellulares, sine ramificatione vel vegetatione terminali. Cellulæ forma admodum varia, plerumque in medio plus minus profunde constrictæ et in duas semicellulas symmetricas divisæ.

liberæ vel in fascias filiformes aut tæniiformes arcte conjunctæ aut in mucro matricali nidulantes et in familias indefinitas consociatæ. Cytioderma non siliceum, plus minus firmum, læve aut varie asperatum (striatum, costatum, aculeatum, &c.). Massa chlorophyllacea in laminales axiles vel parietales, sæpe e centro radiantes, distributa.

Propagatio non sexualis per divisione transversa in eandem directionem repetita; sexualis per zygosporas, quæ per cellularum binarum conjugationem oriuntur.

Unicellular algæ, without branches or terminal growth. Cells of very various forms, mostly more or less profoundly constricted in the middle and divided into two symmetrical semicells, free or conjoined in filiform or tæniiform fascia, or involved in the maternal jelly so as to form indefinite families. Cytioderm not siliceous, more or less firm, smooth, or variously roughened (striate, costate, aculeate, &c.) Chlorophyl masses in axillary or parietal lamina, which often radiate from the centre.

Non-sexual propagation by repeated transverse division in one direction; sexual by zygosporas which are formed by the conjugation of two cells.

Remarks.—Of all the fresh-water algæ, with the exception of the diatoms, this family has attracted most attention, owing, not only to the beauty and variety of its forms, but also to their universal presence and abundance, and the ease with which their most wonderful life-histories are observed. They are exclusively, as far as known, denizens of fresh-water, and preferably that which is pure and limpid. Although Mr. Ralfs states that they never grow in stagnant water, I have often found them in great abundance in such, yet never in that which was actually putrid. The same authority is also too sweeping, at least as far as this country is concerned, in stating they are never found in woods, although they are really most abundant in the open country. My experience has taught me to look for them in brick-ponds, small mountain lakes, springy fens, ditches, and, in the fall, growing among mosses and in the thick jelly composed of unicellular algæ on the face of dripping rocks, or, to sum up in a word, they dwell in quiet, shallow waters, for I have never found them in rapidly moving or very deep water.

The single cell, of which a desmid is composed, is mostly divided into two very marked similar portions, the exact counterparts one of the other, which by some have been asserted to be distinct cells. Their close union and connection, and their inherent oneness are, however, so apparent that it is needless here to spend time in demonstrating that they really are halves of one individual cell. They contain together all the parts found in the typical vegetable cell; an outer cellulose wall, chlorophyllous protoplasm, a nucleus, starch granules and semiliquid contents. The cell-wall, or cytioderm, as it is called in this memoir, varies in thickness and firmness. During life it is mostly, if not always, colorless; but in certain species in the dead empty frond is of a reddish-yellow. The markings upon it are various, and are not infrequently altogether absent; they are such as fine or coarse punctations, granulations of various size, striæ, furrows or elevated ribs, tubercles, obtuse or sharp simple or forked spines, hair-like processes, umbonations, &c. &c. These markings are within narrow limits constant in each species, and more or less peculiar, so that they afford valuable characters to the systematist. The cytioderm itself is mostly composed of cellulose free from appreciable inorganic matters, but in certain species contains a large amount of silex. Thus,

according to De Barry, if *Closterium lunula* be carefully burnt upon a slide, a perfect hyaline silex cast of the cells is left.

The chlorophyl is variously placed in the cell, sometimes it is arranged in lamina, sometimes in spirals, sometimes in the form of radii from a central mass. These different methods afford good generic characters, and will be dwelt upon more in detail under the various genera. The color of the chlorophyl during active life is a vivid green, which, as the vital forces lessen, changes to a faded yellowish tint.

Nägeli and others affirm that there is always a central nucleus in the desmid, and probably do so with truth, although in many instances I have found it impossible to demonstrate its presence from the size and opaqueness of the frond, crowded with endochrome, &c. In a large number of cases, however, it is very apparent.

As ordinarily viewed under the microscope the two most striking peculiarities presented by these little plants are the motion of the whole desmid in the water and the various movements exhibited within the fronds. The general movement is most apparent in the larger species, which exist free and distinct in the water, especially in the boat-shape closteria. It mostly consists of a steady, stately, slow onward movement, with sometimes backward oscillations. By virtue of it, desmids in a bottle will often congregate in such positions as are most exposed to light. There have been various theories advanced as to the cause of this motion. Ehrenberg believed that he had found foot-like processes protruding from the end of the frond and giving the motile power. Others, such as Rev. Mr. Osborne and Mr. Jabez Hogg, have attributed the movements to the presence of cilia, but I think have failed so entirely to establish this that their views are more than problematical. That the motion is due to vital actions, taking place especially under the action of light, is as much as can be at present affirmed with any certainty, though it is probable that the immediate agents are endosmotic currents of gas or water.

The movements of the contents within the cells are chiefly of two kinds. Taking *Closterium lunula* as an illustrative example, there will be found on examination with an $\frac{1}{3}$ th objective, a narrow, very transparent, and therefore often not very apparent layer or zone lying immediately within the cell-wall, between it and the endochrome, and dipping inward in the middle of the frond so as to communicate with the nucleus. In this zone are protoplasm, watery fluid, and scattered granules. In the ends of the fronds the different portions of this zone, meeting and widening, fill up the whole of the cavity, and within the space thus occupied by them, there is a globular, sharply defined, still more transparent vacuole. This, some have thought to be a closed sac, with a distinct wall, but it seems really to be a vacuole lying in the midst of the inner protoplasm, which with a few granules occupies more or less completely the transparent zone already described. Sometimes the chlorophyl encroaches upon this zone at the ends so as to more or less completely surround the vacuole, within which are always found watery fluid and granules. In the protoplasmic zone and its vacuole active movements are probably always present during active life. Streams of protoplasm appear to be constantly passing to and fro between the nucleus and the ends of

the cell along the outer zone, and granules can be always seen passing backwards and forwards with an unsteady motion.

When the streams of protoplasm are setting very actively from the centre towards one end, there will often be an accumulation of the protoplasm there, and a consequent decided lessening in the size of the vacuole, which will again expand as the return currents arouse themselves. Within the vacuoles are seen more or less numerous smaller or larger granules in active busy motion, swarming over and about one another with an unsteady hurrying to and fro.

A form of motion, similar in appearance to this, but probably of different significance, is seen in most desmids when in an unhealthy feeble condition. I have seen it most marked in *Cosmarium margaritaceum*. In such fronds the endochrome has lost its deep green color, and become shrunken, and lying within it is a great space containing myriads of minute blackish particles swarming about actively. This peculiar state and appearance is by no means confined to the desmids, for I have seen it very highly developed both in species of *Spirogyra* and *Œdogonium*. It appears to be connected with decay. Is it possible that these minute particles are foreign to the plant, vibrionic in nature?

In regard to the nature of the movements seen within a healthy desmid, some have viewed them as exceedingly mysterious, the result of the presence of cilia, &c.; but these views have been so thoroughly exploded that it is scarcely necessary even to mention them here. The movements are, in truth, precisely parallel to the so-called cyclosis of the higher plants. Protoplasmic germinal matter, wherever it exists, be it in animal or vegetable, has as one of its distinguishing characters the power of active, spontaneous, apparently causeless movements, and it is simply the carrying out of this power or attribute which has attracted so much attention in the desmids, because it is in them so readily seen.

There are, in this family, two distinct methods in which the species are multiplied one with, the other without, the intervention of anything like sexuality. The non-sexual method of increase is really a modification of an ordinary vegetative process, a peculiar cell multiplication by division. In such fronds as those of the genus *Cosmarium*, which are composed of two evident halves connected by a longer or shorter isthmus, the first step in the process is an elongation of this neck. In a very short time there appears around the centre of this a constriction, and I believe an actual rupture of the outer coat. By this time a new wall has formed inside each half of the isthmus, and stretches also across its cavity, forming with its fellow a double partition wall, separating the two halves of the old frond. Rapid growth of the newly formed parts now takes place, the central ends become more and more bulging as they enlarge, and in a little time two miniature lobules have shaped themselves at the position of the old isthmus. These are at first small, colorless, and destitute of all markings, looking, as Mr. Ralfs says, like condensed gelatine. They, however, rapidly increase in size and firmness, their contents assuming a green color and their walls taking on the peculiar markings of the species. At last, the parts thus formed having assumed the shape and appearance of the original lobules, the two fronds, which have been developed out of one, separate, mostly before the new semicells have acquired their full size.

What part the nucleus has in the process just described I have never actually demonstrated, but have little doubt but that it undergoes a division in the very commencement, so that the new nucleus of each secondary frond is formed out of one-half of the old one.

In proportion as the form of the desmid becomes simpler, so do the peculiarities of its cell multiplication become less. In those species which are simple cylindrical cells, there appears to be nothing peculiar in the method of dividing, which, however, always takes place through the centre of the cell, and subsequent growth occurs, generally, only in the newly formed part.

True sexual reproduction apparently does not take place as freely in this family as the former process, for whilst I have seen hundreds of cells undergoing the latter, it has not been my good fortune to meet with conjugating specimens on more than two or three occasions.

The process has, however, been studied very closely by De Bary, Braun, Hofmeister, and others, and appears to consist generally in a rupture of the outer wall of two cells and the protrusion of delicate processes from an inner, often newly formed coat, with subsequent union of these, and consequently of the two cells, and afterwards a condensation of the contents in the enlarged connecting passage. The connecting passage between the fronds is really a sporangium in which the spore is perfected, the contents of the cells finally condensing it into a firm globe and secreting around themselves a thick coat.

The after-history of this spore has been very successfully studied by M. Hofmeister, whose observations were made upon *Cosmarium tetraophthalmum*, which he watched conjugating and forming a sort of resting spore which was perfected early in the month of July. This was composed of a thick outer coat and green endochrome lying within as a distinct ball, nowhere in contact with the investing membranes. In three weeks' time this chlorophyllous protoplasm had divided into ellipsoidal masses, or primordial cells, which soon surrounded themselves with cellulose walls and became distinct free cells in the granular fluid which filled the cavity of the original spore. In August, each of these masses was divided into two and in the month of September the process was repeated, so that out of the original endochrome eight strongly flattened primordial cells were produced. Division in some specimens ceased here, and in others took place once more, so that by the following spring all of the living *Sporangia* contained eight or sixteen green daughter-cells, each of them discoid in outline with a strongly marked central notch. These daughter-cells were finally set free by the solution of the spore wall, as *Cosmaria* of minute size, but agreeing in all other characters with the specific form to which they belonged.

According to Braun, in the larger, more or less lunate *Closteria*, conjugation occurs in the following method: Two fronds approach one another in such a way that they lie back to back. In the middle of each of them, there then appears an annular line or trench reaching through the cell wall, and accompanied by a distinct separation of the endochrome into two halves. Whilst these changes have been progressing there has also formed a new double wall at the position of the trench, so that out of the two *Closteria* two pairs of separate equal cells have been

formed. Near to the larger or central end of each of these now appears a pouting transparent nipple-like process. The corresponding opposing processes enlarging and meeting coalesce, so that the upper half of one closterium, in the form of a daughter-cell, is finally united with the upper half of the other closterium, and the two lower halves are also joined together. Thus from a single pair of fronds arise two conjugating pairs of cells, and finally two sporangia, in each of which a spore is perfected.

This process does not seem, however, to be universal amidst the *Closteria*, for in many, if not all, of the smaller species, a pair of fronds produces a single sporangium.

In the genus *Palmogloca*, in which I have had an opportunity to study the development of the spores, the process closely simulates that seen in certain of the *Spirogyra*. The contents of the cells first became broken up and confused, and almost simultaneously the nucleus disappeared (fig. 4, pl. 11) the cells became swollen at one side and slightly bent backward so as to form jutting processes, which meeting grew together, became confluent and developed into a sporangium much larger than either of the parent cells. Into this sporangium the contents of the latter passed and soon became converted into a thick-walled spore (fig. 00, pl. 00) often completely filling the cavity, and apparently with its wall adherent to that of the latter.

Genus PALMOGLŒA, Ktz. (1843).

Cellulæ oblongæ, ellipticæ vel cylindricæ, utroque polo rotundatæ, medio non constrictæ, plerumque in mucō gelatinoso nidulantes, liberæ, singulæ vel in familias consociatæ, lamina chlorophyllacea axili vel excentrica, ætate provecta medio constricta, denique divisa præditæ. (R.)

Syn.—*Mesotæmium*, NÆGELI.

Cell oblong, elliptical or cylindrical, rounded at each end, not constricted in the middle, mostly swimming in a gelatinous mucus, free, single or associated in families, chlorophyl lamina axillary or excentric, in the early state constricted, and at length divided in the middle

Remarks.—The above diagnosis of the genus is that given by Prof. Rabenhorst, and agrees essentially with that of De Bary, Nægeli, &c. In the species herein described however, the axillary lamina of chlorophyl were not so pronounced, for the green coloring matter seemed often to surround the cavity of the cell, and in other specimens was broken up and diffused through it.

P. clepsydra, Wood.

P. saxicola et bryophila, in gelatina achroa interdum dilute viride nidulans; cellulis cylindricis, cum polis obtuse truncato-rotundatis, diametro 2-3 plo longioribus; lamina chlorophyllacea axili, plerumque indistincte, sæpe nulla; plasmate dilute viride; nucleo plerumque distincto; zygosporis subfuscis aut subglobosis aut enormiter in clepsydræ forma; membrana externa enormiter excavata et sulcata.

Diam.— $7\frac{13}{500}$ "

Syn.—*P. clepsydra*, Wood, Prodr. Amer. Philosophical Soc. 1869.

Hab.—In rupibus et in muscis irroratis ad Cheltenham Hills, prope Philadelphia.

14 June, 1872.

P. living on rocks and mosses, swimming in a transparent, sometimes light-green jelly; cells obtusely truncated, rounded at the ends, 2-3 times longer than broad; chlorophyl lamina axillary, mostly indistinct, often wanting; endochrome light-green; nucleus generally distinct; zygospore subfuscous, either globose or of an irregular form, somewhat resembling that of an hour-glass; external coat irregularly excavated and sulcate.

Remarks.—This species was found along the North Pennsylvania Railroad, near Cheltenham Hills, growing amid mosses on the rocky jettings over which the water was dripping. It occurs as a rather firm, transparent jelly, mostly of a light greenish tint, in which the cells are often placed quite thickly. They are cylindrical, mostly straight, but sometimes slightly curved, and often completely filled with a light greenish endochrome. The central lamina is irregular, and mostly not at all pronounced. In some cells the endochrome is much broken up, so that the interior is filled with little green masses with light spaces between them. In these cells the nucleus is generally not perceptible, whilst in the others it is very well marked. The zygospore is often globular, sometimes it is irregularly elliptical, with a constriction in the centre, so as to give it somewhat of an hour-glass shape. The outer coat mostly fits pretty closely on the inner contents, and is very often distinctly marked with little pits, some round, some irregular in shape; in other cases, instead of being thus pitted, the spores seem to be marked with deep curved furrows.

Fig. 4, pl. 11, represents this plant in different stages of growth. (See Explanation of Plates.)

Genus PENIUM, BRÉB. (1848.)

Cellulæ cylindricæ vel fusiformes, rectæ, utroque polo rotundatæ vel truncato-rotundatæ (nec emarginatæ nec excisæ), medio sæpius constrictæ. Lamina chlorophyllacea axillis, ex transverso conspecta radiatim-divergens, radii sæpe furcati, granula amylacea plerumque longitudinaliter seriata includens. Individua in aqua libere natantia, singula, sparsa vel in massa gelatinosa consociata. Cellulæ membrana laevis vel granulata, achroa vel fuscescens vel rubicunda, sæpius longitudinaliter striata. (R.)

Syn.—*Netrium*, NÆGELI.

Cylindrocystis, MENGH.

Closterium, partim, EHRENBURG.

Cells cylindrical or fusiform, straight, rounded at each end, or truncately rounded (not emarginate or excised), medianly often constricted. Chlorophyl lamina axillary, when seen transversely radiately divergent, arms often forked, and containing starch granules, mostly longitudinally striate. Individuals swimming free in the water, scattered and single, or associated in gelatinous masses. Cell membranes smooth or granulate, transparent or fuscous or reddish, often longitudinally striate.

a. Lamina chlorophyllacea peripherice lobata vel radiatim expansa.

a. Chlorophyl lamina, lobate on the periphery or radiately expanded.

P. Digitus, (EHRB.) BRÉB.

P. cellulis ovato-cylindricis, diametro 3-5 plo longioribus, utroque polo parum attenuatis, subtruncato-rotundatis; laminis chlorophyllaceis peripherice lobatis, medio interruptis.

Diam.— $\frac{1}{7} \frac{3}{66}'' = .00173''$ — $\frac{2}{7} \frac{2}{66}'' = .0029''$.

Syn.—*P. Digitus*, (EHRB.) BRÉB. RABENHORST, *Flora Europ. Algar.*, Sect. III. p. 118.

Cells ovately cylindrical, 3-5 times as long as broad, at each end slightly attenuate, subtruncately rounded; chlorophyl lamina lobate on the periphery, interrupted in the middle.

Remarks.—This species is probably widely diffused through the temperate portions of North America. I have found it abundantly near Philadelphia, as well as among the Alleghanies, and have received specimens from Dr. Lewis, collected in Saco Lake, Northern New York; Prof. Bailey also notes it as occurring in Georgia. There is one form of it which resembles somewhat in outline the modern coffin, one end being much broader and much more rapidly narrowed than the other. There is no distinct vacuole at the end, at least in any specimen I remember to have seen, although frequently large numbers of moving granules can be detected in that portion of the frond.

Fig. 6, pl. 20, represents the outline of a frond of this species.

P. lamellosum, BRÉB.

P. cellulis oblongo- vel fusiformi-cylindricis, diametro 5-6 plo longioribus, medio sæpe leviter constrictis, utroque polo magis attenuatis, obtuso rotundatis. (R.)

Diam.—0.0023"—0.0029". (R.)

Syn.—*P. lamellosum*, BRÉB. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 118.

Hab.—Rhode Island. (Olney) Thwaites.

Cells oblong or fusiform cylindrical, 5-6 times longer than broad, often slightly constricted in the centre, more attenuate at the ends, obtusely rounded.

Remarks.—I have never recognized this species, but it is one of those sent over by Mr. Olney, and identified by Prof. Thwaites.

b. Lamina chlorophyllacea integerrima.

b. Chlorophyl lamina entire.

* *Cellulæ in medio plus minus constrictæ.*

* *Cells more or less constricted in the middle.*

P. margaritaceum, EHRB.

P. elongato cylindricum, diametro 8-9 plo longius, medio plerumque leviter constrictum, utroque polo rotundato-truncatum; cellulæ membrana nodulis seriatis quasi margaritacea; locellus in medio (circiter) utriusque cruris corpusculis mobilibus in more Closteriorum repletus. (R.)

Diam.—0.00098"—0.0011". (R.)

Hab.—Rhode Island. (Olney) Thwaites; Florida. Bailey.

Elongate cylindrical, 8-9 times longer than broad, in the centre generally slightly constricted, at each end roundly truncate; membrane of the cells somewhat pearly with seriate granules; vacuole about in the centre of each crus, filled with moving granules, as in closterium.

Remarks.—I have not seen this desmid, but it is in Prof. Bailey's list; it was also among those sent by Mr. Olney to Prof. Thwaites.

P. minutum, CLEVE.

P. cylindricum, gracile, diametro 5-7 plo longius, læve, ad polos obtusissimos (latissime rotundatos) parum attenuatum, medio leviter constrictum. (R.) Species mihi ignota.

Diam.—0.00044"—0.00063". (R.)

Syn.—*Docidium minutum*, RALF'S British Desmid.

P. minutum, CLEVE. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 122.

Hab.—South Carolina; Florida. Bailey. Rhode Island. (S. T. Olney) Thwaites.

Fronde slender, suture not prominent; segments four to six times longer than broad, somewhat tapering, inflation obsolete, sides straight, ends entire; e. f. without puncture. L. $\frac{1}{3}\frac{1}{2}$ "; B. $\frac{1}{15}\frac{1}{8}\frac{1}{2}$ ". (Archer.)

b. *Cellulæ in medio non constrictæ.*

b. *Cells not constricted in the middle.*

P. interruptum, BRÉB.

"P. cellulis late lineari-cylindricis, diametro 5-6 plo longioribus, utroque polo subito cuneato-acutatis, apicibus obtuso-rotundatis; laminis chlorophyllaceis longitudinalibus saturate viridibus, ætate protracta fasciis transversis tribus pallidis interruptis." (R.) *Species mihi ignota.*

Diam.—0.00147"—0.00177". (R.)

Syn.—*P. interruptum*, BRÉB. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 119.

Hab.—In fossis, South Carolina, prope Grahamsville. Prof. Bailey.

Cells broadly linearly cylindrical, 5-6 times longer than broad, at each end suddenly cuneately sharpened, the apex obtusely rounded; longitudinal chlorophyll lamina deep green, in advanced age interrupted by three transverse pale fascia.

P. Jenneri, RALFS.

P. ab *P. Brebissonii* vix discernendum, cellulis cylindricis, utroque polo rotundatis, lævibus, diametro $2\frac{1}{2}$ -5 plo longioribus; zygosporis plerumque globosis, membrana fuscescente subgranulata. (R.) *Species mihi ignota.*

Diam.—0.00057"—0.0006". (R.)

Syn.—*P. Jenneri*, RALFS. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 120.

Hab.—In fossis, Florida. Prof. Bailey.

Scarcely distinguishable from *P. Brebissonii*, cells cylindrical, rounded at each end, smooth, $2\frac{1}{2}$ -5 times longer than broad; zygosporis mostly globose, membrane somewhat fuscous, subgranulate.

P. Brebissonii, (MENGH.) RALFS.

P. in massa mucosa indefinite expansa sæpe cum algis alteris intermixtis; cellulis perfecte cylindricis, interdum nonnihil curvatis sed plerumque rectis, diametro $2\frac{1}{2}$ -4 plo longioribus, utroque polo late rotundatis, in medio non constrictis; "zygosporis angularibus vel rotundatis, membrana fuscente, subtiliter granulata."

Diam.— $\frac{5}{7500}$ " = .00066".

Syn.—*P. Brebissonii*, (MENGH.) RALFS. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 120.

Hab.—In fossis, South Carolina. (Prof. Ravenel.)

In an indefinitely expanded mucous mass, intermixed with other algæ; cells perfectly cylindrical, sometimes slightly curved, but generally straight, at each end broadly rounded, not constricted in the middle; "zygosporis angular or rounded, membrane fuscous, finely granulate."

Remarks.—Among the numerous desmids which I have received from Prof. Ravenel are some which, I think, must be referred to *P. Brebissonii*, although they do not nearly equal the size of the European form, nor even the diameter given above, which is almost the lowest limit of the mature foreign plant. I believe, however, Prof. Ravenel's specimens are immature.

Mr. Ralfs' description of the conjugation is as follows: The process of the conjugation in this species differs from that in the rest of this genus; for, as in *Hyalotheca dissiliens*, the conjugation cells enter into the formation of the containing cell and are permanently attached to the sporangium, instead of being detached, as commonly happens, in the Desmids. The sporangium is at first cruciform, then quadrate, and finally orbicular.

P. closterioides, RALFS.

P. cellulis anguste lanceolatis, diametro maximo 5-6 plo longioribus, a medio in apices subtruncato-rotundatos sensim attenuatis; laminis chlorophyll. saturate viridibus, medio fascia transversa pallida interruptis. (R.) Species mihi ignota.

Diam.—0.00159"—0.00175". (R.)

Syn.—*P. closterioides*, RALFS. RABENHORST, Flora Europ. Algarum, Sect. III. p. 121.

Hab.—Prope Grahamsville, South Carolina. Prof. Bailey.

Cells narrowly lanceolate, 5-6 times longer than the greatest diameter, sensibly attenuate from the middle into the subtruncate apices; chlorophyll lamina deep green, interrupted by a median pale band.

Genus CLOSTERIUM, NITSCH.

Cellulæ interdum cylindricæ sed sæpius fusiformes et utroque polo attenuatæ, plus minus lunulatum curvatæ, in medio hæud constrictæ sed striæ transversa unica vel 2-5 impressæ. Cytioderma tenue, sat firmum, læve vel plus minus distincte striatum et interdum longitudinaliter costatum. Cyttoplasm chlorophyllosa plerumque in laminis longitudinalibus disposita, et sub cellulæ polis locello achroo, plerumque globoso et corpusculis plus minus numerosis se vivide moventibus impleto instructa.

Cells sometimes cylindrical, but more often fusiform and attenuate at each end, more or less lunately curved, in the centre not constricted but marked with from 1-5 transverse striæ. Cytioderm thin, moderately firm, smooth or more or less distinctly striate, and sometimes longitudinally costate. Chlorophyllous cytoplasm mostly arranged in longitudinal lamina, and furnished at each end with a clear space, which is mostly globose, and contains more or less numerous actively moving corpuscles.

a. Zygosporæ globosæ, rarissime angulares; cellulæ crura aut non aut minus producta.

a. Zygosporæ globose, very rarely angular; crura of the cells not at all, or only slightly, produced.

1. *Cellulæ cylindricæ, ad utrumque polum vix vel paululum attenuatæ, rectæ vel leviter curvatæ, apicibus rotundatis vel truncatis.*

1. *Cells cylindrical, not at all or but slightly attenuated at the ends, straight or slightly curved, the apex rounded or truncate.*

C. striolatum, EHRB.

C. anguste lanceolato-fusiforme, leviter arcuatum, 8-12 plo fere longius quam latum, utroque polo paulum sensimque attenuatum, apicibus truncatis sæpe fusciscentibus; membrana distinctissime striata, vacuata fusciscente; vesiculis chlorophyllaceis 5-7 (in quoque crure); locello apices versus sito, submagno, corpuscula 12-20 includente. (R.)

Diam.— $\frac{1}{8}$ "— $\frac{1}{4}$ " = 0.00152"—0.00187". (R.)

Syn.—*C. striolatum*, EHRB. RABENHORST, Flora Europ. Algarum, Sect. III. p. 125.

Hab.—In aquis quietis, Centre County, Pennsylvania. Wood. Saco Pond, New Hampshire. (Lewis).

Narrowly lanceolately-fusiform, slightly bent, 8-12 times longer than broad, sensibly attenuated

at the ends, which are truncate and often somewhat fuscous; membrane very distinctly striate, when empty somewhat fuscous; chlorophyl globules 5-7 (in each limb); vacuole placed in the bent apex, moderately large, including 12-20 corpuscles.

Remarks.—The measurements given are those of Prof. Rabenhorst. Our American forms agree well with them.

C. angustatum, KTZ.

C. gracile, sublineare, diametro 16-18 plo longius, ad polos levissime attenuatum, apicibus late truncatis; costis longitudinalibus paullulum prominulis 4-5, interstitiis circiter $\frac{1}{450}$ latis; vesiculis chlorophyllaceis in quoque cruro 6-7; locello ab apice subremoto mediocri, corpusculis 12-20 impleto. (R.)

Diam.— $\frac{1}{101}$ "— $\frac{1}{84}$ " = 0.00081"—0.0010". (R.)

Syn.—*C. angustatum*, KTZ. RABENHORST, Flora Europ. Algarum, Sect. III. p. 126.

Hab.—In aquis quietis, prope Philadelphia, Pennsylvania. Wood. Rhode Island. Bailey. New Hampshire. (Lewis)

C. slender, sublinear, 16-18 times longer than broad, very slightly attenuate at the ends, which are broadly truncate; with from 4-5 somewhat prominent longitudinal ribs, the interstices about $\frac{1}{450}$ broad; chlorophyl globules in each limb 6-7; vesicle subremote from the apex, moderate, containing from 12-20 corpuscles.

C. juncidum, RALFS.

C. elongatum, anguste lineare, diametro 20-35 plo longius, leviter arcuatum, utroque polo vix attenuatum; apicibus truncatis; cytiodermate luteolo, interdum longitudinaliter striato.

Diam.— $\frac{1}{2500}$ " = .0004".

Syn.—*C. juncidum*, RALFS. RABENHORST, Flora Europ. Algarum, Sect. III. p. 127.

Hab.—In fossis, South Carolina. (Ravenel) In lacu Saco, New Hampshire. (Lewis)

Elongate, narrowly linear, 20-35 longer than broad, slightly bent, scarcely narrowed at the ends; apices truncate; cytioderm yellowish-brown, sometimes longitudinally striate.

Remarks.—I am indebted to Prof. Ravenel for specimens of this species, by whom they were found on the slimy surface of a half dried-up ditch, associated with numerous other desmids. The specimens are all smaller than the measurements of Rabenhorst, but much larger than those given by Mr. Ralfs. None of the plants have any chlorophyl granules—a circumstance probably simply dependent upon the stage of their development. The longitudinal striæ are in none of the specimens very distinct, and in many cannot be demonstrated.

Since writing the above I have seen specimens collected by Dr. Lewis in "Saco Pond," near the Crawford House, New Hampshire.

Mr. Archer (Pritchard's *Infus.*, p. 749) lays stress upon the fronds being straight in the middle, with the ends curved downwards; but I have seen numerous specimens in which the curve was through the whole length.

Fig. 2 o, pl. 12, represents one of the specimens collected by Prof. Ravenel in South Carolina.

2. *Cellulæ cylindricæ, dorso plus minus convexæ, ventre subplanæ, nunquam ventricoso—inflatæ.*

2. Cells cylindrical, with the dorsum more or less convex, the belly straightish, never ventricosely inflated.

C. Lunula, (MÜLLER) EHRB.

C. permagnum, sublæve (striæ subtilissimæ vel indistinctæ), semilunare, dorso alte convexum, ventre subplanum, apicibus attenuatis rotundatis; vesiculis chlorophyllaceis numerosis sparsis; locello distincto subapicali corpuscula numerosa includente. (R.)

Diam.— $\frac{1}{27}$ "— $\frac{1}{26}$ " = 0.00032"—0.0045". (R.)

Syn.—*C. Lunula*, (MÜLLER,) EHRB. RABENHORST, Flora Europ. Algarum, Sect. III. p. 127.

Hab.—South Carolina, Georgia, Florida. Prof. Bailey. Pennsylvania. Wood.

Very large, smoothish (striæ very fine or indistinct), semilunar, dorsum strongly convex, belly straightish, the ends attenuate and rounded; chlorophyl globules numerous, scattered; vesicle distinct, subapical, including numerous corpuscles.

C. acerosum, (SCHRANK) EHRB.

(*Var. nov. maximum.*)

C. lineare-fusiforme, sub-rectum aut leve curvatum, utroque fine sensim et paullulum attenuatum, diametro 15–24 plo longiore; apicibus angustissime truncatis, achrois; membrana hand striata; vesiculis chlorophyllaceis 11–14 in quoque crure, in serie axilli simplici collocatis; locello apicali parvo, corpuscula numerosa includente; zygosporis globosis.

Diam.—Transv. max. $\frac{13}{7500}$ " = .0017"; zygosp. $\frac{2}{7500}$ " = .0027".

Syn.—*C. acerosum*, (SCHRANK) EHRB. RABENHORST, Flora Europ. Algarum, Sect. III. p. 128.

Hab.—Pennsylvania; Wood. South Carolina, Georgia, Florida; Bailey.

Linear, fusiform, straightish, or slightly curved, at each end sensibly little by little attenuate, 15–24 times longer than broad; apices narrowly truncate, transparent; membrane not striate; chlorophyl globules 11–14 in each limb placed in a simple axillary series; apical vesicle small, containing numerous corpuscles; zygosporis globose.

Remarks.—The desmid, described above, was found in New Jersey, near Camden. It differs from the typical form of *C. acerosum* in its size, proportionate length to breadth, and in not being striate. The European "*formâ major*" (RABENH.) appears, however, to exceed it in transverse diameter, and, according to some authors, certain fronds of the species are not striate, and all authorities agree that at times the striæ are exceedingly delicate. For these reasons, I think, our American form must be regarded simply as a variety. As far as can be judged from the rude figure, it is this species which Prof. Bailey identifies as *C. tenue*, KTZ., in Silliman's Journal for 1841.

Fig. 5, and 5 *a*, pl. 11, represent this species magnified 250 diameters; 5 *b* represents the sporangium with portions of the dead fertile fronds still attached.

C. areolatum, WOOD, (sp. nov.)

C. fusiforme, subrectum vel nonnihil curvatum, lateris ventralis medio sæpe paullulum concavum, diametro 9–10 plo longius, utrinque modice attenuatum; apicibus truncato-rotundatis; membrana crassa, et firma, rubido-brunnea, profunde distante striata, et minutissime sed distincte granulata vel areolata; suturis medianis distinctissimis 4–10.

Diam.—0.0024".

Hab.—In aquis puris quietis; Northumberland Co., Pennsylvania.

Fusiform, straightish, or very slightly curved, the ventral side often a little concave in the middle, 9-10 times longer than broad, moderately attenuated at each end; the apices truncately rounded; cell-membrane reddish-brown, thick and firm, distantly profoundly striate, and very minutely but distinctly granulate or areolate; median sutures very distinct, 4-10 in number.

Remarks.—I found this species growing in a quiet pool of pure water, in a wild, deeply wooded ravine, near Danville, Central Pennsylvania. It was in great abundance, forming a translucent greenish jelly, one or two gills of which might have been readily gathered. Unfortunately, I had no microscope with me and cannot, therefore, determine at all as to the arrangement of the endochrome, the carbolic acid, used as preservative, having entirely disarranged this by the time I got the fronds upon a slide. The empty frond is of a reddish-brown color. The membrane is quite thick and firm, and is marked with very prominent broad striæ or grooves. In a number of cases I have counted these and always found nine present upon one face of the frond. There are also upon the surface numerous minute markings not fairly visible with a lower power than a $\frac{4}{10}$ th objective. Under this glass they appear as minute punctations. An eighth resolves them into granules mostly of an oblong shape, arranged more or less regularly in longitudinal rows. Very generally, each side of the stria or groove has a close row of larger and more distinct granules forming a sort of border to it. In truth, the surface of the frond is covered with broad longitudinal bands of these granules, and the narrow smooth spaces between them constitute the stria spoken of. This species is very closely allied to *C. turgidum*, EHRB., agreeing pretty well with it in general outline and size. I think, however, the peculiar markings upon the membrane are sufficient to separate it, and do not doubt that if fresh specimens were at hand, differences would be found to exist also in the arrangement of the cell-contents. The turning up of the ends, generally so marked in *C. turgidum*, is mostly entirely absent in this species, rarely there is some tendency to it.

Fig. 6, pl. 11, represents in outline a frond magnified 160 diameters; Fig. 6 a, the end of an empty cell, magnified 1375 diameters; the color of this is, perhaps, a little too dark.

C. lineatum, EHRB.

C. valde elongatum, gracile, quater vicies-tricies longius quam latum, distincte striatum, e medio recto cylindrico utrinque valde attenuatum, apices versus leviter incurvum, obtusotruncatum; vesiculis chlorophyllaceis in quoque crure 20-21, in seriem unicam axilem distributis; locello parvo, ab apice remoto, corpusculis 10-12 impleto. (R.)

Diam.— $\frac{3}{2000}$ " = .0015".

Syn.—*C. lineatum*, EHRB. RABENHORST, Flora Europ. Algarum, Sect. III. p. 130.

Hab.—Pennsylvania, Wood.

Very much elongate, slender, distinctly striate, from the centre straight and cylindrical, at each end very greatly attenuate, apex bent, slightly incurved, obtusely truncate; chlorophyll globules 20-21 in each limb, placed in a simple axillary series; vacuole small, remote from the apex, containing from 10-12 corpuscles.

Remarks.—The American forms agree well with the above description; some

of them, however, are a little more curved in the central portion than it would imply.

Fig. 1, pl. 12, is a drawing of an American plant, magnified 160 diameters.

C. Cucumis, EHRB.

C. oblongum, turgidum, leviter curvum, læve, diametro 4–7 plo longius, apicibus obtusis. (R.)

Syn.—*C. Cucumis*, EHRENBURG, *Verbreit. b.* 28, IV. F. 28. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 138.

* *Hab.*—New York; Ehrenberg.

Oblong, turgid, slightly curved, smooth, 4–7 times longer than broad, the apex obtuse.

Remarks.—I have no knowledge of this species other than that in the above short description, which has been copied from Rabenhorst's works.

3. *Cellulæ semilunares, plerumque magis curvatæ quam in Sect. 1 et 2, ventre semper tumidæ, ventricosoinflatæ.*

3. *Cells semilunar, mostly more curved than in Sect. 1 and 2, with the belly always tumid, ventricosely inflated.*

C. Ehrenbergii, MENGH.

C. fusoideo-semilunare, ventre inflato, ceterum ut in *C. Lunula*. (R.)

Diam.—Lat. $\frac{1}{4} \frac{3}{100}$ " = .0029". Long. $\frac{2}{4} \frac{1}{100}$ " = .0042."

Syn.—*C. Ehrenbergii*, MENGH. RABENHORST, *Flora Europ. Sect. III. p. 131.*

Hab.—Prope Philadelphia.

"Frond large, stout, about five or six times as long as broad, lunately curved, extremities tapering; upper margin very convex, lower concave with a *conspicuous central inflation*; ends broadly rounded; large granules, numerous, scattered; fillets several; e. f. colorless, without striæ, central suture not evident. Sporangia orbicular, smooth, placed between the but slightly-connected empty conjugated fronds, the endochrome during the process of conjugation emerging from the opened apex of a short conical extension from each under side of each younger segment (or shorter cone) of each pair of recently divided fronds, the conjugating fronds being produced immediately previously by the self-division of a pair of old fronds—two sporangia being thus the ultimate produce of the two original fronds. L. $\frac{1}{8}$ " ; B. $\frac{1}{4} \frac{1}{100}$ ". Archer." Pritchard's *Infusoria*, p. 748.

Remark.—Fig. 2, pl. 12, represents a plant of this species magnified 160 diameters.

C. moniliferum, (BORY) EHRB.

C. semilunare, plus minus curvatum, diametro maximo 6–9 plo longius, ventre inflato, utroque polo sensim attenuatum, apicibus achrois obtusis, vesiculis chlorophyllaceis in serie unica longitudinali axili dispositis, in quoque crure 7–10; locello apicali submagno, corpuscula numerosa includente (corpusculum in quoque locello unicum mobile ellipsoideum, magnitudinæ lineæ partem millesimam æquans, cetera mobilia per totum corporis distributa observavit cl. Perty.) (R.) *Species mihi ignota.*

Diam.—0.0019"—0.0022". (R.)

Syn.—*C. moniliferum*, RABENHORST, *Flora Europ. Algarum, Sect. III. p. 131.*

Hab.—Georgia; Rhode Island; Bailey.

• Frond smaller than *C. Ehrenbergii*, stout, five or six times as long as broad, lunately curved, extremities tapering, upper margin convex, lower concave, with a *central inflation*, ends rounded; large granules, conspicuous, in a single longitudinal series; e. f. colorless, without striae, suture not evident. L. $\frac{1}{75}$ "— $\frac{1}{60}$ ". B. $\frac{1}{510}$ "— $\frac{1}{468}$ ". Archer." Pritchard's *Infusoria*, p. 748.

C. Leibleinii, KtZ.

C. priore minus, semilunare, magis incurvum, ventre inflato, ad utrumque polum largius attenuatum, apicibus achrois acutis; vesiculis chlorophyll. in quoque crure 5-6, in serie simplici axillari dispositis; locello magno, apices versus sito, corpuscula numerosa includente. (R.)

Diam.— $\frac{1}{75}$ $\frac{3}{00}$ ".

Syn.—*C. Leibleinii*, KÜTZING. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 132.

Hab.—Georgia; Rhode Island; Bailey. Pennsylvania; Wood.

• Frond somewhat stout, distance between the extremities six or eight times the breadth, crescent-shaped, much curved, rapidly attenuated, upper margin very convex, lower very concave, often with a slight central inflation; ends subacute; large granules, in a single series; fillets few or indistinct; e. f. somewhat straw-colored, without striae; suture evident. Sporangium orbicular." Archer.

Remark.—Fig. 6, pl. 12, represents this plant, magnified 260 diameters.

4. *Cellulæ maxime curvatæ, ventre non tumidæ.*

4. *Cells most curved, the belly not tumid.*

C. Dianæ, EHRB.

C. anguste fusiforme, semilunare, utroque polo valde attenuatum, apicibus subacutis; cytodermate achroo (vel dilutissime umbrino), striis subtilissimis medio interruptis prædito, in media parte striis transversalibus 3-5; vesiculis in quoque crure 6-7, in serie unica axili dispositis; laminis chlorophyllaceis pluribus, sæpe flexuosis; locello indistincto, corpusculis pluribus vivide mobilibus. (R.)

Diam.—Lat. $\frac{4}{75}$ $\frac{00}{00}$ " = .00053". Long. $\frac{6}{75}$ $\frac{2}{00}$ " = .00082".

Syn.—*C. Dianæ*, EHRENBERG. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 133.

Hab.—Georgia; Florida; Rhode Island; Bailey. Pennsylvania; Wood.

Frond crescent-shaped, six or eight times as long as broad, much curved, rapidly attenuated; upper margin very convex, lower very concave without a central inflation; ends subacute with a very slight emargination at the upper outer extremity; large granules in a single series; empty frond, somewhat straw-colored, or faintly reddish, without striae, suture evident. (A.)

Remarks.—Mr. Archer marks *C. Venus*, KtZ., as a doubtful synonym of this species; not having Prof. Kützing's work at hand, I do not know whether *C. Venus*, KtZ. is really the following species or not. The two forms *here known* as *C. Dianæ*, EHRB. and *C. Venus*, KtZ. are, however, I think sufficiently distinguished.

Fig. 4, pl. 12, represents this species of desmid.

C. Venus, KtZ.

C. parvum, plus minus gracile, semicirculare, octies-duodecies longius quam latum, in apices subacutos æqualiter sensimque attenuatum; cytodermate tenui, læve; laminis chlorophyllaceis oblitteratis; vesiculis in quoque crure 3-4; locello distincto corpusculis 4-6 repleto. (R.)

Diam.—.0004".

Syn.—*C. Venus*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 134.

Hab.—South Carolina. (Ravenel.)

Small, more or less slender, semicircular, eight to twelve times longer than broad, equally and very perceptibly attenuate at both apices; cytioderm thin, smooth; chlorophyllous lamina obliterated; vesicles in each crus 3–4; vacuole distinct, containing 4–6 corpuscles.

Remark.—Fig. 7, pl. 11, represents in outline a frond magnified 450 diameters.

C. parvulum, NÆG.

C. parvum, semicirculare, medio non tumidum, gracile, anguste lanceolatum, sexies-octies longius quam latum, apicibus acutis; cytiodermate tenui, lævissimo, vacuato nonnunquam luteofuscescente et subtiliter striato; vesiculis uniseriatis, in quoque crure 2–4, varius 1–7; laminis chlorophyllaceis 4–5. (R.)

Diam.—Max. 0.00026"—.00062" (R.) (.0008" W.)

Syn.—*C. parvulum*, NÆGELI. RABENHORST, Flora Europ. Algarum, Sect. III. p. 134.

Hab.—Prope Philadelphia, Wood.

Small, semicircular, not swollen in the middle, slender, narrowly lanceolate, six to eight times longer than broad, with the apices acute; cytioderm thin, very smooth, when empty somewhat yellowish-fuscescent and finely striate; vesicles uniseriate, in each crus 2–4, rarely 1–7; chlorophyllous lamina 4–5.

Remarks.—I have referred to this species a desmid which I have found about Philadelphia, and which agrees in all respects with the description of Prof. Rabenhorst except in attaining a larger size.

Fig. 5, pl. 12, represents this plant magnified 450 diameters.

C. Jennerii, RALFS.

C. cylindraceo-fusifforme, semilunare, læve, utrinque modice attenuatum, sexies-octies longius quam latum, apicibus obtuse rotundatis; vesiculis in quoque crure 5–7, in serie unica axili dispositis; laminis chlorophyllaceis 2–3; locello subæquali magno, corpusculis numerosis impleto. (R.)

Diam.—0.00057". (R.)

Syn.—*C. Jennerii*, RALFS. RABENHORST, Flora Europ. Algarum, Sect. III. p. 134.

Hab.—Rhode Island, Bailey.

Frond small, distance between the extremities six or seven times the breadth, crescent-shaped, much curved, gradually tapering (sometimes with an obscure central constriction); upper margin very convex, lower very concave without a central inflation; ends obtuse, rounded; large granules in a single series; e. f. colorless without striæ. L. $\frac{1}{2}\frac{1}{8}$ ". B. $\frac{1}{7}\frac{1}{30}$ ". Archer. Pritchard's *Infusoria*.

b. Zygosporæ plerumque quadrangulares, cellularum crura longe vel longissime producta, sæpe setiformia.

b. Zygosporæ mostly quadrangular, crura of the cells greatly produced, often setiform.

C. rostratum, EHRE.

C. corpore lanceolato-fusiforimi, utrinque valde et longe attenuato, leviter curvato, striato; cornibus setaceis singulis corpus vix æquantibus, sæpius longe brevioribus; cytiodermate dilute umbrino vel luteolo, dense striato; vesiculis uniseriatis, in quoque crure 5–6; locello oblongo, sæpius indistincto, corpusculis 12–15 vivide se moventibus. (R.)

Diam.—0.0008". (0.0009"—0.0016." R.)

Syn.—*C. rostratum*, EHRENBERG. RABENHORST, Flora Europ. Algarum, Sect. III. p. 135.

Hab.—In fossis, prope Philadelphia; Wood.

Body lanceolate-fusiform, at each end greatly and for a long distance attenuated, slightly curved, striate; crura setaceous and scarcely as long as the body and sometimes much shorter; cytioderm light or luteolous, densely striate, vesicles uniseriate, 5–6 in each crus; vacuoles oblong, often indistinct, containing from 12–15 actively moving granules.

Remark.—Fig. 3, pl. 12, is a drawing of this species, magnified 260 diameters.

C. setaceum, EHRB.

C. corpore anguste lanceolato, recto vel subrecto, distincte striato, utrinque in rostrum setaceum, levissime incurvum, obtusum, longissime porrecto; singulo rostro corpore 3–4 plo longiore; et vesiculis et locello indistinctis. (R.)

Diam.—Max. (plerumque) 0.0004"—0.00044." (R.)

Syn.—*C. setaceum*, EHRENBERG, Flora Europ. Algarum, Sect. III. p. 136.

Hab.—Stonington. (Lewis) Pennsylvania; Wood. Georgia; Florida; Providence, Rhode Island; Bailey.

Fronde very slender, from twenty to twenty-five times as long as broad, *narrow-lanceolate*; upper and lower margins nearly equally and but slightly convex; each extremity tapering into a very long and slender setaceous colorless beak, longer than the body, ultimately curved downwards, ends obtuse; e. f. colorless, *striæ close, faint, central suture solitary*. Sporangium cruciform. L. $\frac{1}{16}$ ". B. $\frac{1}{331}$ ". Archer. Pritchard's *Infusoria*.

C. Amblyonema, EHRB.

C. filiforme, cylindricum, læve, utroque fine parum attenuatum, apice rotundum. (R.)

Syn.—*C. lineatum*, EHRB. BAILEY, American Journal of Science and Arts, 1841, p. 303.

C. Amblyonema, EHRB. *Verbreit.* p. 123. RABENHORST, Flora Europ. Algarum, Sect. III. p. 138.

Hab.—West Point, New York; Providence, Rhode Island, Bailey.

Filiform, cylindrical, smooth, gradually attenuated at each end; the apex rounded.

Remarks.—I have never recognized a specimen of this species, nor have I had access to the original description of Ehrenberg.

Genus TETMEMORUS.

Cellulæ cylindricæ vel fusiformes, rectæ, medio distincte constrictæ, utroque polo anguste incisæ cytioderma sat firmum, plerumque granulatum vel punctatum.

Cells cylindrical or fusiform, straight, distinctly constricted in the middle, narrowly incised at each end. Cytioderm firm, mostly punctate or granulate.

T. Brébissonii, (MENGH.) RALFS.

T. diametro 4–6 plo longior; a fronte cylindricus, utroque polo non attenuatus sed rotundato-truncatus; a latere fusiformis et a medio in apices rotundatos sensim attenuatus; cytiodermate striato-punctato.

Diam.— $\frac{1}{7600}$ " = .0016".

Syn.—*T. Brébissonii*, MENGHEINI, RABENHORST, Flora Europ. Algarum, Sect. III. p. 139.

Hab.—In fossis, Atlantic States.

Four to six times longer than broad; from the front cylindrical, not attenuate at the truncately rounded ends; viewed laterally fusiform, attenuated from the middle to the rounded ends; cytioderm striately punctate.

Remarks.—The central constriction is more apparent in the lateral than front view. When the frond is full of endochrome the punctæ on the outer wall are not apparent, but when it is empty they are seen to be small, and closely arranged in stria-like rows. This species extends through all the Atlantic sea-board States. Prof. Bailey has found it in South Carolina and Florida, as well as in Rhode Island. I have collected it in Centre County, of this State.

Fig. 3, pl. 21, represents an empty half frond of this species; 3 a the outline of the frond.

T. granulatus, (BRÉB.) RALFS.

T. habitu Tetm. Brebissonii, sed major et cytiodermate irregulariter granulato-punctato. (R.)

Diam.— $\frac{1}{750}$ " = .0013". (.00155". R.)

Syn.—*T. granulatus*, (BRÉBISSEON.) RALFS. RABENHORST, Flora Europ. Algamum, Sect. III. p. 140.

Hab.—Prope Philadelphia; Wood. Rhode Island; (S. T. Olney.) Thwaites. South Carolina; Bailey.

Frond somewhat longer than *T. Brebissonii*, about five or six times longer than broad; in both f. v. and s. v. fusiform, the constriction a very shallow groove, ends with a hyaline lip-like projection extending beyond the notch; endochrome with a longitudinal series of large granules; e. f. punctate, the puncta scattered, except near the constriction; where they are disposed in two transverse rows. Sporangium orbicular, smooth, margin finely striated, placed between the deciduous empty fronds. L. $\frac{1}{130}$ ". B. $\frac{1}{645}$ ". Archer. Pritchard's *Infusoria*.

Remark.—Fig. 8, pl. 12, represents this species magnified 450 diameters.

T. giganteus, WOOD.

T. maximus, oblongus, diametro 3 plo longior; apicibus haud attenuatis, late rotundatis; suturis profundis, linearibus; cytiodermate irregulariter granulato-punctato.

Diam.— $\frac{2\frac{1}{2}}{7500}$ " = .0031".

Syn.—*T. giganteus*, WOOD. Proc. Acad. Nat. Sci., 1869.

Hab.—In stagnis, Centre County, Pennsylvania.

Very large, oblong. 3 times longer than broad; with the ends not attenuate but broadly rounded; suture profound, linear; cytioderm irregularly granulatedly punctate.

Remarks.—I found this beautiful desmid in a stagnant pool in Bear Meadows, Centre County, in the month of August. It is very different in its outline from its nearest ally, *T. granulatus*. The diameter is preserved uniform until near the end, where there is an alteration in the line of the margin, so as to cause some contraction, which is, however, wanting in some specimens. The ends are therefore broad and obtuse. The size is also double that of *T. granulatus*.

Fig. 7, pl. 12, represents a frond of this species magnified 260 diameters.

T. levis, (KUTZ.) RALFS.

T. Brébissonii formis similis sed parvior, 3-4 plo longior quam latus; cytiodermate plerumque levissimo, interdum indistinctissime punctato.

Diam.— $\frac{1}{1500}$ " = .00066".

Syn.—*T. levis*, KÜTZING. RALFS. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 140.

Hab.—In aquis quietis, prope Philadelphia.

Similar in form to *T. Brébissonii*, but smaller, 3-4 times longer than broad; cytioderm mostly very smooth, sometimes indistinctly punctate.

Remarks.—Prof. Rabenhorst states that the cytioderm of this species is very smooth, and Mr. Ralfs says that he has failed to detect any punctations, but also states that "Mr. Jenner and Mr. Ross assure me that they (punctæ) are scattered as in *T. granulatus*." I have no doubt of their existence in certain individuals, whilst in other cases they appear to be absent.

Genus **PLEUROTÆNIUM**, NÆGELI (1849).

Cellulæ singulæ in aqua natantes, rectæ vel subrectæ, cylindricæ vel fusiformes, valde elongatæ, utroque polo rotundatæ vel truncatæ, medio leviter constrictæ, ex transverso circulares. Cytio-plasma chlorophyllaceum in laminis longitudinalibus pluribus dispositum, et sub utroque polo locello rotundato corpusculis se vivide moventibus impleto instructum.

Cells single, swimming in water, straight or nearly so, cylindrical or fusiform, very much elongate, rounded or truncate at each end, in the end view with a circular outline. Chlorophyllous protoplasm arranged in longitudinal laminae and furnished at each end with a round vacuole containing actively moving corpuscles.

Remarks.—This genus appears to include the main portion of the species, which have been described under the name of *Docidium*; the remainder being representatives of a number of genera. I have not had access to the original description of *Docidium*, and do not know in what year it was published; but, according to De Bary, *Docidium* is much the older name ("Ueber de Conjugat.," p. 75). M. De Bary states, however, that he prefers the name of Nægeli, because that authority first defined the genus and his name expresses very clearly the character of it, as well as from the circumstance that the name *Docidium* having been made to cover a heterogeneous mass of species, its retention might cause confusion. I confess to thinking that this action of De Bary is not in accordance with the recognized laws of priority, but, in the absence of the original description, have thought best to follow it.

P. trabecula, (EHRB.) NÆGELI.

P. sæpe valde elongatum, octies vicies-longius quam latum, cylindraceum, utroque fine levissime attenuatum aut incrassatum, juxta medium constrictum sæpius bigibbum (quasi biundatum), apicibus late truncatum; cytiodermate tenui levi, achroo. (R.)

Diam.— $\frac{1}{750}$ " = .0013".

Syn.—" *Docidium Ehrenbergii*. RALFS." BAILEY, *Microscopical Observations*. Smithsonian Contributions.

Pleurotænium trabecula, (EHR.) NÆGELI. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 141.

Hab.—South Carolina, Georgia, Florida; Bailey. Pennsylvania; Wood.

(*Docidium Ehrenbergii*. RALFS.) Frond slender, *linear*; suture forming a very sharply defined rim; segments 8–12 times longer than broad, basal inflation having another smaller one above it, sides otherwise straight, parallel; ends crenate, owing to a number of emarginations on the edge of the truncate extremities, from three to five of the crenations being usually visible; e. f. punctate or rough with minute granules. Sporangium suborbicular or elliptic, or slightly angular, smooth, placed between the deciduous empty fronds. Ciliated zoospores formed by segmentation of the cell contents, and their emission effected through the opened apex of each one, two or three, especially, formed lateral tubes arising from beneath the base of one of the segments. Archer.

Remarks.—This species is quite common around Philadelphia; but I do not remember ever to have seen one with the cell-wall granulate. The smaller of the two umbonations near the centre is often wanting or exceedingly small, and the crenulations in the ends are very often obsolete.

Fig. 9, pl. 12, represents a cell of this species magnified 160 diameters.

P. Baculum, (BRÉB.) DE BARY.

P. priori simile, sed gracilius, angustius et plerumque longius, medio tantum semel constrictum; cytodermate lævi. (R.) *Species mihi ignota*.

Diam.—0.00054"—0.0009". (R.)

Syn.—*P. Baculum*, (BRÉB.) DE BARY. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 141

Hab.—Georgia; Bailey.

Frond slender, suture not prominent; segments very many times longer than broad, basal inflation very conspicuous, solitary, sides otherwise straight, very nearly parallel, large granules of the endochrome in a single series; ends entire; e. f. without puncta. L. $\frac{1}{11}$ ". B. $\frac{1}{137}$ ".

P. breve, WOOD.

P. robustum, diametro 4–8 plo longius, in medio distincte constrictum sed haud undulatum, utroque polo nonnihil attenuatum; apicibus truncatis et nonnihil rotundatis; cytodermate crassissimo, dense granulato-punctato; marginibus vel rectis, vel breve undulatis.

Diam.—.0038"—.00095".

Syn.—*P. breve*, WOOD. Proc. Acad. Nat. Sciences, 1869.

Hab.—District of Columbia. (Billings.)

Robust, 4–8 times longer than broad, distinctly constricted but not undulated in the middle, slightly attenuated towards the ends; apex truncate and somewhat rounded; cytoderm very thick, densely minutely granulate; margins either straight or shortly undulate.

Remarks.—This species was sent to me by Dr. Billings, who obtained it near Washington, D. C. The margins are sometimes straightish, but in other fronds there are three or more distinct short undulations, or rounded projections in each half margin. The cell-wall is excessively thick, especially at the end—in many cases much thicker than the drawing.

Fig. 2, pl. 21, represents an empty frond of this plant magnified 750 diameters.

P. crenulatum, (EHRB.) RABENHORST.

P. robustum, cylindræo-subclavatum, octies-duodecies longius quam latum, medio undulato-nodosum, stricture mediæ margine tumido, apicibus late truncatis, altero sæpe crenulato; cytodermate granulato-punctato. (R.) *Species mihi ignota*.

Diam.—0.0023". (R.)

Syn.—*P. crenulatum*, (EHRB.) RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 142.

Docidium nodulosum, BRÉB. RALFS. British Desmidiæ, p. 155.

Closterium trabecula, BAILEY. American Journal of Science, 1841.

Hab.—In aquis quietis, South Carolina; Georgia; Florida; Rhode Island; Bailey. Pennsylvania; New Jersey; Wood.

(*Docidium nodulosum*.) Frond very stout, the thickened sutures forming a projecting rim; segments four to six times as long as broad, scarcely attenuated, regularly inflated at intervals so as to form an undulated margin, the basal inflation the most prominent, the others, as they approach the ends, less so, where they are indistinct or wanting; ends entire; e. f. coarsely punctate. L. $\frac{1}{50}$ ". B. $\frac{1}{428}$ ". Archer. Pritchard's *Infusoria*.

Remarks.—I have found this species in "Shepherd's Mill Pond," near Greenwich, Cumberland County, New Jersey, and also in a Spring in the Philadelphia Park, near Columbia bridge.

Fig. 1, pl. 21, represents the outline of a frond of this species magnified 160 diameters.

P. clavatum, (KTZ.) DE BARY.

P. subcylindraceum, multoties (16–24) longius quam latum, ad utrumque polum sensim incrassatum, subclavatum, apicibus late truncatis; cytiodermate firmo achroo, dense et irregulariter granulato-punctato. (R.) *Species mihi ignota*.

Diam.—Max. 0.00165"—0.00147"; min. = 0.0010"—0.00092". (R.)

Syn.—*P. clavatum*, (KTZ.) DE BARY. RABENHORST, Flora Europ. Algarum, Sect. III. p. 141.

Docidium clavatum, KÜTZING. RALFS, British Desmidiæ. ARCHER. Pritchard's *Infusoria*.

Hab.—South Carolina; Georgia; Bailey.

Frond slender, suture scarcely prominent, segments eight to ten times as long as broad, slightly clavate near the ends, and ultimately somewhat attenuated, basal inflation sometimes solitary, sometimes having another slight one above it; ends entire; e. f. punctate. L. $\frac{1}{85}$ ". B. $\frac{1}{13}$ ".

P. undulatum, (BAILEY.)

D. læve, gracile cylindricum, undulatum, latitudine 18–20 plo longius, medio modice constrictum; cruribus et basi et apice truncatis et crenatis. (R.) *Species mihi ignota*.

Syn.—*Docidium undulatum*, BAILEY. Micros. Observ. p. 36.

Hab.—Florida, Bailey.

"Segments eight to ten times longer than broad, constricted six to eight times at regular intervals throughout their entire length, with the base and ends crenate, smaller than *D. nodulosum*, BRÉB., with more frequent and deeper constrictions. The same characters distinguish it from *D. nodosum* and *D. constrictum*."

P. nodosum, (BAILEY.)

D. validissimum, undulatum, spinulis sparsis hirsutum, medio valde constrictum, diametro 8–10 plo longius; cruribus e basi dilatata leviter attenuatis 4-undatis, apicibus quasi productis, latissime truncatis; locello apicali ratione parvo, rotundo, corpusculis paucis (ut videtur) repleto. (R.) (*Species mihi ignota*.)

Syn.—*Docidium nodosum*, BAILEY. Micr. Observ., pl. 1, fig. 4. RALFS, British Desmids, p. 218.

Hab.—United States; Bailey.

"Frond stout; segments with four prominent nodes separated by constrictions; end view crenate. An end view shows that each node is not a simple swelling, but really formed by whorls of tubercles. 'This species is easily recognized by the deep indentations in its outline, corresponding to the constrictions which separate the transverse rows of knob-like projections. It is one of the largest species in the genus,' Bailey." Ralfs.

P. constrictum, (BAILEY)

D. subvalidum, læve, latitudine 10–12 plo longius, medio valde constrictum, stricturæ margine non prominente; cruribus a basi tumida in apicem late truncatum non attenuatis, 4 undulatis. (R.) (*Species mihi ignota.*)

Syn.—*Docidium constrictum*, BAILEY. RALFS, British Desmids, p. 218.

Hab.—Rhode Island, Bailey.

“Fronde stout, segments with moderately deep constrictions, which separate four equal, gently curving prominences; end view entire. ‘This species is at once distinguished from *D. nodosum* by the cross section of the nodes being a simple circle instead of an indented one,’ Bailey.” Ralfs.

P. verrucosum, (BAILEY)

D. validum, granuloso-verrucosum, latitudine 10–12 plo longius, undulatum, apicibus integris truncatis. (R.) (*Species mihi ignota.*)

Syn.—*Cosmarium verrucosum*, BAILEY, Amer. Journ. Sci. and Arts, 1846.

Docidium verrucosum, RALFS, Brit. Desm. p. 218. BAILEY, Micr. Observ. p. 28.

Hab.—Rhode Island; Bailey.

“Segments, with numerous whorls of small prominences, which give the margins an undulated appearance, all the undulations are equal. ‘This is a very pretty species with a waved outline, caused by the slight projections, which are arranged in numerous transverse rings,’ Bailey.” Ralfs.

P. hirsutum, (BAILEY)

D. spinuloso-hirsutum, medio valde constrictum, diametro 10–12 plo longius; cruribus et basi et apice subdilatis, truncatis. (R.) (*Species mihi ignota.*)

Syn.—*Docidium hirsutum*, BAILEY, Micr. Observ. p. 36.

Hab.—Florida; Bailey.

“Segments many times longer than broad, slightly inflated at the base, surface hirsute. A small species resembling *D. Ehrenbergii* in its form, but strongly hirsute on its outer surface.” Bailey.

Genus **TRIPLOCERAS**, BAILEY.

Cellulæ singulæ, rectæ, valde elongatæ, processus magnorum seriebus transversis armatæ, utroque polo trilobatæ, lobis acute bidentatis.

Syn.—*Triploceras*, BAILEY, Microscopical Observations, p. 37, Smithsonian Contributions, 1850.

Cells single, straight, very much elongate, armed with transverse series of large processes, trilobate at each end, lobes acutely bidentate.

T. verticillatum, BAILEY.

T. cellalis subcylindricis, sed utroque fine leviter angustatis et nonnihil fusiformibus, modice robustis, diametro 12–20 plo longioribus; processibus lateralibus robustis, magnis, apice emarginatis.

Diam.—Cum process. $7\frac{1}{5}\frac{1}{5}\frac{0}{0}$ " = .00146"; sine process. $1\frac{7}{5}\frac{0}{0}\frac{0}{0}$ " = .00113".

Syn.—*T. verticillatum*, BAILEY. Microscopic Observations. Smithsonian Contributions, 1850.

Docidium verticillatum, RALFS, British Desmids, p. 218.

Pleurotænum verticillatum, RABENHORST, Flora Europ. Algar., Sect. III. p. 148.

Hab.—Rhode Island, New Jersey, Georgia, Florida; Bailey. Saco Lake, (Dr. Lewis) Wood.

Subcylindrical, but slightly narrowed at each end, and therefore somewhat fusiform, moderately robust, 12–20 times longer than broad; lateral processes large, robust, with their apices emarginate.

16 June, 1872.

T. gracile, BAILEY.

T. cellulis subcylindricis, utroque fine vix angustatis, gracillimis, diametro 25–30 plo longioribus; processibus lateralibus brevibus, conicis.

Diam.—Cum process. $\frac{6}{5000}$ " = .008"; sine proc. $\frac{9}{15000}$ " = .0006".

Syn.—*T. gracile*, BAILEY, Smithsonian Contributions.

Docidium pristidæ, HOBSON, Magazine Natural History, v. p. 168.

Pleurotanium gracile, RABENHORST, Flora Europ. Algar., Sect. III. p. 144.

Hab.—In iisdem cum antecedente locis.

Subcylindrical, scarcely narrowed at the ends, 25–30 times longer than broad; lateral processes, short, conical.

Genus SPIROTÆNIA, BRÉB.

Cellulæ rectæ, cylindricæ vel subfusiformes, sæpe in muco gelatinoso aggregatæ, medio haud constrictæ, utroque polo rotundatæ vel acuminatæ. Cyttoplasm chlorophyllaceum in laminis spiralibus dispositum.

Cells straight, cylindrical or subfusiform, often aggregated in a gelatinous mucus, not constricted in the middle, rounded or acuminate at each end. Chlorophyllous cytoplasm arranged in spiral lamina.

Sp. bryophila, (BRÉB.) RABENHORST.

Sp. mimina, bryophila; cellulis in gelatina matricali consociatis, oblongo-cylindricis, rectis vel subcurvatis, bis vel ter longioribus quam latis, utroque polo rotundatis; lamina chlorophyllacea singula anfractu 1–2½.

Diam.— $\frac{1}{3000}$ " = .00033" (0.00024"—.00029". R.)

Syn.—*Spirotænia bryophila*, (BRÉB.) RABENHORST, Flora Europ. Algarum, Sect. III. p. 146.

Hab.—Prope Philadelphia; Wood.

"(S. muscicola (De Bary)) Frond cylindrical two to four times as long as broad, ends rounded; endochrome a single, broad, smoothly defined, widely wound spiral band, its revolutions very few (one or two)." (A.)

Remarks.—I found this beautiful little desmid on the North Pennsylvania Railroad, near Cheltenham Hills, growing amongst some mosses which were kept constantly wet by overhanging dripping rocks. It formed little transparent masses of almost colorless jelly looking much like drops of dew. It agrees well with the descriptions of the European form, except that there were generally from 2–2½ turns of the spiral, and the cells exceed somewhat the measurements of Prof. Rabenhorst. The cells are closely placed in the jelly.

Fig. 10, pl. 12, represents some plants of this species.

Sp. condensata, (BRÉB.) RABENHORST.

Sp. cellulis cylindricis, rectis (vel leviter curvatis) octies vel decies longioribus quam latis, utroque polo rotundatis; laminis chlorophyll. singulis, anfractibus subareolis (plerumque 8–12).

Diam.—0.00075".

Syn.—*Sp. condensata*, (BRÉB.) RABENHORST, Flora Europ. Algarum, Sect. III. p. 146.

Hab.—Florida; Rhode Island; Bailey. Pennsylvania; Wood.

Frond cylindrical, two to four times as long as broad, ends rounded; endochrome a single, broad, closely wound spiral band, its revolutions numerous. L. $\frac{1}{200}$ ". Br. $\frac{1}{1048}$ ". Archer. Pritchard's *Infusoria*.

Remarks.—The only specimens that I have seen of this species were found in a spring in the Philadelphia City Park, near Columbia bridge.

Fig. 11, pl. 12, was drawn from one of these specimens.

Genus SPHÆROZOSMA, CORDA.

Cellulæ compressæ, medio transversim profunde incisæ, itaque bilobatæ, in quoque lobo massa chlorophyllosa quadriradiata nucleum amylaceum involvente præditæ, in filum planum tæniiformem literaliter isthmis conjunctæ. Zygosporæ globosæ vel ovales, glabræ. (R.)

Syn.—*Isthosira*, KTZ.

Odontellæ, spec., EHRB.

Isthmiæ, spec., MENEG.

Spondylosium, BRÉB.

Cells compressed, transversely very deeply incised in the centre and therefore bilobate, furnished in each lobe with a quadriradiate mass of chlorophyl surrounding a starch grain, conjoined laterally by isthmuses in a tæniiform fascia.

Remarks.—I have never found any species of this genus in America. Professor Bailey has, however, detected the following:—

Sph. excavatum, RALFS.

Sph. plerumque nudum (sine tubo mucoso) sph. vertebratum multo minus; cellulis diametro duplo-longioribus, medio excavato-constrictis, a latere ellipticis utroque polo rotundatis; lobis brevibus truncato-rotundatis, lævibus vel granulato-denticulatis; isthmis binis parvis verruciformibus; zygosporis plerumque ovatis. (R)

Latit. filor. 0.00047"—0 00032". (R.)

Syn.—*Sph. excavatum*, RALFS, British Desmids, p. 67.

Hab.—Florida; Georgia; South Carolina; Rhode Island; Bailey.

"Joints longer than broad, having a deep sinus on both sides and two sessile glands at each margin at their junction, very minute, seldom more than twenty-five joints in the filament, which is fragile, and finally separates into single joints; at their junction, in the front view are two minute glands or processes, situated one near each angle, and nearly invisible before the escape of the endochrome. The joints are nearly twice as long as broad and much constricted in the middle; the constriction is like an excavation or broad sinus on each side, so that the margins of the filaments appear sinuated. The endochrome is pale bluish-green with minute scattered granules. The transverse view is oblong with four sessile glands, two on each side and situated near the ends."—Ralfs' *Brit. Desm.*, p. 67.

Sph. pulchrum, BAILEY.

Sph. cellulis oblongo-quadrangularibus, diametro duplo-brevioribus, acute incis, arete connexis; lobis oblongis rectis, apice rotundatis; isthmis nullis, vagina mucosa ampla distincta. (R.)

Syn.—*S. pulchrum*, BAILEY. RALFS, British Desmid., p. 209 (Cum icone).

Hab.—West Point, New York; Princeton, New Jersey; BAILEY.

"Joints twice as broad as long, deeply incised on each side; junction margins straight, connected by short bands."

Remark.—"Prof. Bailey informs me that this species is twice as large as *Sph. vertebratum*," RALFS.

Sph. serratum, BAILEY.

Sph. cellulis diametro duplo brevioribus, profunde et acute excisis, arcte conjunctis; lobis utrinque cuspidatis, paulum conniventibus; isthmis nullis; vagina crassa. (R.)

Syn.—*Sph. serratum*, BAILEY, *Micros. Observation.* Smithsonian Contributions, 1850. Cum icone.

Hab.—South Carolina; Georgia; Florida; Bailey.

“Joints broader than long, deeply notched or divided into two transverse portions with acute projecting ends, which give a serrated outline to the chain.” Bailey.

Genus HYALOTHECA.

Cellulæ brevæ, cylindricæ, medio non profunde constrictæ, a latere disciformes, in fila confervacea sine isthmis arcte conjunctæ et vagina mucosa ampla achroa inclusæ. Massa chlorophyllosa in quaque semicellula 4–8, 5–10 radiata.

Cells short, cylindrical, not profoundly constricted in the middle, disciform in the end view, closely united without intervening isthmuses into a confervoid filament, which is inclosed in an ample mucous sheath. Chlorophyl masses in each cell 4–8, 5–10 radiates.

H. disilliens, (SMITH) BRÉB.

H. fasciis prælongis; cellulis oblongo-quadrangularibus, diametro sub-duplo brevioribus, interdum ante divisionem subæqualibus, angulis nonnihil rotundatis, plerumque medio obsolete constrictis, sæpe haud constrictis.

Diam—0.00089"—0.00098". (R.)

Syn.—*H. disilliens*, (SMITH) BRÉB. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 152.

Hab.—South Carolina; Florida; Rhode Island; BAILEY. Rhode Island (S. T. Olney), Thwaites. Pennsylvania; Wood.

Filament very long, cells oblong, quadrangular, about one-half as long as broad, sometimes before division as long as broad, angles somewhat rounded, mostly obsoletely constricted in the middle, often not constricted.

Remarks.—The specimens which I have identified as *H. disilliens*, agree with the various figures and descriptions of the European form, in every thing except that in many cases there is no constriction whatever in the centre of the cell, and when the constriction does exist, it is never so pronounced, as some of the descriptions indicated. The plant is very common about Philadelphia, growing in springs and ditches.

Fig. 12, pl. 12, represents this part of a filament of this species.

H. mucosa, (MERT.) EHRB.

H. fasciis confervaceis, minus fragilibus; cellulis quadrangularibus, diametro æqualibus vel subæqualibus, medio non constrictis, ad utrumque finem (annuliformi-bicarinatis) bidentatis. (R.) *Species mihi ignota.*

Diam.—0.00073"—0.0008". (R.)

Syn.—*Gloeoprium mucosum*, HASSAL, *Fresh Water Algæ*, p. 346.

H. mucosa, (MERT.) EHRB. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 132.

Hab.—Rhode Island; (S. T. Olney) Thwaites.

Filament scarcely fragile, mucous sheath very broad; joints about as broad as long, not constricted, but having at one of the ends a minute bidentate projection on each margin, the

adjoining ends of the next joint being similar, these projections being produced by an annular grooved rim. L. $\frac{1}{250}$ "—600". B. $\frac{1}{250}$ "— $\frac{1}{1111}$ ". (Archer)

Genus BAMBUSINA.

Cellulæ oblongo-orculiformes, in filamenta articulata nodosa dense conjunctæ, medio vitta transversa carinis duabus annuliformibus limitata instructæ, itaque superne et inferne bidentatæ, fronte circulares, supra et infra dente unico prominente. (R.)

Cells oblong-orculiform, densely united into an articulate nodose filament, surrounded by two median bands.

B. *Brébissonii*, Ktz.

B. filamentis nodoso-articulatis; cellulis diametro duplo longioribus. (R.)

Diam.—0.00077"—0.00092". (R.)

Syn.—*B. Brébissonii*, KÜTZING. RABENHORST, Flora Europ., Algarum, Sect. III. p. 152.

Hab.—South Carolina. (Ravenel) Wood. South Carolina; Georgia; Florida; Rhode Island. Bailey.

(*Didymoprium Borreri*, (Ralfs)) Joints inflated, barrel-shaped, longer than broad, without a thickened border at their junction; angles bicrenate, crenatures rounded; transverse view circular; sporangium elliptic, formed within the (for some time) persistent extensions from the conjugating joints, which do not previously break up into single joints, but couple, still united in the filament, in a confused or zigzag manner, some of the joints remaining unchanged. L. $\frac{1}{33}$ ". B. $\frac{1}{1030}$ ".

Remarks.—The specimens which I have seen agree well with the descriptions, except in regard to size; some of the cells which I measured were more than $\frac{1}{1200}$ of an inch in diameter.

Genus DIDYMOPRIUM.

Cellulæ oblongo-ellipticæ, modice compressæ, ancipites, angulis porrectis inciso-bidentatis, in filamenta articulata biconvexa et torta sine isthmo arcte conjunctæ, et in vagina mucosa inclusæ. Cytoplasm chlorophyllosa cellulæ a fronte cruciatim disposita, ejus crura e laminis duabus parietalibus divergentibus granum amylicum unicum involventibus formantur.

Cells oblong-elliptical, moderately compressed, two-edged, with the produced angles incisely-bidentate, closely united into a biconvex and twisted filament, which is inclosed in a mucoid sheath, cytoplasm so placed as to be cruciate when viewed from the front (end), each crus composed of two parietal divergent lamina, each of which contains a single starch granule.

D. *Grevillii*, Ktz.

D. cellulis oblongis diametro duplo brevioribus, saturate viridibus. (R.)

Diam. 0.0024"—0.0031". (R.)

Syn.—*D. Grevillii*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 153.

Hab.—Georgia, South Carolina; Bailey. Pennsylvania; Wood.

Sheath distinct; joints broader than long, with a thickened border at their junction; angles bidentate; teeth angular; transverse view broadly elliptic. Sporangium orbicular, formed within one of the two conjugating joints, the endochrome passing over from one by a narrow connecting tube produced between the otherwise, but little altered, broken-up single joints. (A.)

Remark.—Fig. 13, pl. 12, represents the end view of a broken filament of this species.

Genus DESMIDIUM.

Cellulæ oblongo-tabulares, medio inciso-bilobæ, lobis integris vel irregulariter dentatis, a fronte tri- vel quadrangulares, angulis obtuse rotundatis, in fila angulosa, prælonga, torta, fragiles arcte connexæ. Massa chlorophyllosa (a cellulæ fronte visa) 3-4 radiata; quisque radius e laminis duabus lateralibus divergentibus compositus. Zygosporæ globosæ vel oblongæ, glabræ.

Cells oblong-tabular, medianly incisely bilobate, with the lobes entire or irregularly dentate, as seen from the front tri- or quadrangular, and having the angles obtusely rounded, closely conjoined into an angular, fragile, twisted filament. Chlorophyl (as seen from the front) 3-4 radiate; each radius composed of two lateral divergent lamina; zygospores globose or oblong, smooth.

D. Swartzii, Ag.

D. cellulis a fronte triangularibus, diametro 2-3 plo brevioribus. (R.)

Diam.—0.00096"—0.00189". (R.)

Syn.—*D. Swartzii*, AGARDII. RABENHORST, Flora Europ. Algarum, Sect. III. p. 154.

Hab.—In aquis quietis, Atlantic States. Florida; Georgia; South Carolina; Rhode Island; Bailey. New York; Edwards. Pennsylvania; Wood.

Filament triangular, equal, with a single longitudinal waved, dark line, formed by the third angle; joints in front view somewhat quadrangular, broader than long, with two slightly angular crenatures on each lateral margin, united at the whole of their end margins by a thickened border, end view triangular; endochrome three-rayed. Archer. Pritchard's *Infusoria*.

D. quadrangulatum, Ktz.

D. quadrangulare, cellulis oblongo-quadrangularibus, diametro 2-3 plo brevioribus, lobis dentiformibus obtusis, a fronte sinuato-quadrangularibus, angulis late rotundatis, lateralibus excavatis. (R.) *Species mihi ignota*.

Diam.—0.0021"—0.0029".

Syn.—*D. quadrangulatum*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III, p. 155.

Filament quadrangular, varying in breadth from its twisting, having two longitudinal waved lines; joints in f. v. broader than long, with two somewhat rounded crenatures on each lateral margin, united by the whole of their end margins; e. v. quadrangular; endochrome four rayed. L. $\frac{1}{12\frac{1}{4}}$ ". B. $\frac{1}{8\frac{1}{2}}$ "— $\frac{1}{4\frac{1}{5}}$ ". (Archer)

D. aptogonium, BRÉB.

D. fasciis plerumque subbrevis, nudis, perforatis; cellulis quadrangularibus, inciso-bilobis, lateralibus concavis, lobis crenatis, a fronte triangularibus (nonnunquam biangularibus), centro concavo, angulis rotundatis protensis isthmum brevissimum triplicem efficientibus. (R.) *Species mihi ignota*.

Diam.—0.00089"—0.00147". (R.)

Syn.—*Aptogonium desmidium*, RALFS, British Desmids.

D. aptogonium, BRÉBISSEON. RABENHORST, Flora Europ. Algarum, Sect. III. p. 155.

Hab.—Georgia; South Carolina; Bailey.

Joints in f. v. quadrangular, broader than long, with two rounded crenatures on each lateral margin, united at the outer portion only of each end margin by mutual projections, thus producing intervening central oval foramina. Archer.

Genus APTOGONIUM, RALFS.

Cellulæ 3-4 angulares vel compressæ, non constrictæ; margine laterali planæ vel crenatæ, in fascias perforato-articulatas, angulares conjunctæ. (R.)

Cells 3-4 angular or compressed, not constricted, their lateral margins plain or crenate, conjoined into angular perforately articulate fascia.

A. Baileyi, RALFS.

" Filament not crenated; joints about equal in length and breadth.

Syn.—*Odontella? tridentata*, BAILEY. *In lit. cum iconc* (1846).

Hab.—Worden's Pond, Rhode Island; near Princeton, New Jersey, with sporangia," Bailey.

" Filament triangular; joints excavated at their junction like those of *Aptogonum desmidium*. The joints are not bicrenate, hence the margins of the filament are entire, a character which distinguishes it from that species. The end view is triangular, with rounded angles." RALFS, *British Desmidiæ*, p. 208.

Genus COSMARIUM, (CORDA)

Cellulæ oblongæ, oblongo-cylindricæ, ellipticæ, vel orbiculares, medio transverse plus minus constrictæ, utroque polo obtusæ vel rotundatæ et integræ, a vertice ellipticæ. Zygosporæ muricatæ vel verrucosæ.

Cells oblong cylindrical, elliptical or orbicular, more or less transversely constricted in the middle, obtuse or rounded, and entire at each end, viewed from the end elliptical. Zygospore warty or muricate.

1. *Cellulæ sejunctæ.*

1. *Cells separate.*

a. *Cellulæ ellipticæ, vel subellipticæ; semicellulæ medio nonventricosæ.*

a. *Cells elliptical or subelliptical; semicells medianly not ventricose.*

* *Cytodermate granuloso vel verruculoso.*

* *Cytoderm granular or warty.*

C. margaritiferum, (TURP.) MENGH.

C. paulo longius quam latum, profunde constrictum; sinu amplo, vel modice angusto, interdum intra excavato; semicellis semiorbicularibus, vel reniformibus vel nonnihil quadrangulis dorso plerumque late rotundatis; cytodermate verruculoso.

Diam.—Max. $\frac{4}{2500}$ " = .0006" (0.00073"—0.0012". R.)

Syn.—*Euastrum margaritiferum*, EHRB. BAILEY, Silliman's Journal, 1841.

Cosmarium margaritiferum (TURPIN), MENEGHINI. RABENHORST, Flora Europ. Algar., Sect. III. p. 157.

Hab.—In aquis quietis, South Carolina; Florida; Mexico; Bailey. Pennsylvania, Wood.

A little longer than broad, profoundly constricted; sinus ample or moderately narrow, sometimes widened on the inside; semicells semiorbicular, reniform or somewhat quadrangular; dorsum mostly broadly rounded; cytoderm warty.

Remarks.—I have found a form of this species growing in the vicinity of this city, which I at first was disposed to look upon as distinct, but which, in truth, grades into the typical form. In it the cells are almost quadrangular, often with their basal angles acute. The margin of the frond in *C. margaritiferum*, as it occurs with us, is sometimes distinctly serrate or, more correctly, crenulate from the presence of the granulations. The granules are larger than in *C. botrytis*, but smaller than in *C. tetraphthalmum*. When viewed laterally the semi-cells are roundish, or nearly so (according to Ralfs' elliptical), and closely connected by

a very broad neck. I have never seen the sporangia, but, according to Mr. Ralfs, they are orbicular and inclosed in a granulated cell.

Fig. 8, pl. 21, represents half of an empty frond of this species magnified 750 diameters; and fig. 21, pl. xii., a frond densely filled with living endochrome

C. Botrytis, (BORY) MENGH.

C. late ovale, profunde constrictum, diametro plerumque $1\frac{1}{4}$ —2 plo longius; sinu angusto, lineare; semicellulis nonnihil triangularibus, apice interdum truncatis, interdum late rotundatis; cytodermate minute granulato.

Diam.— $\frac{1}{5}\frac{1}{36}'' = 0.0019''$ ($0.0014''$ — $0.0023''$). (R.)

Syn.—*C. Botrytis*, (BORY) MENEGHENI. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 158.

Hab.—Pennsylvania, Wood.

C. broadly oval, profoundly constricted, $1\frac{1}{4}$ —2 times longer than broad; sinus narrow, linear; semicells somewhat triangular, with the apex sometimes truncate, sometimes broadly rounded; cytoderm minutely granulate.

Remarks.—In this species the semicells, as viewed transversely, are broadly elliptic in outline. The end view presents a longer narrower ellipse. Their outline, when seen from the front, varies remarkably from that of a very broad semi-oval to distinctly triangular with a truncate apex. The granules are small and arranged regularly, sometimes they are very obscure. I have often seen the endochrome so arranged as to leave a large pyriform central vacuole in each cell, communicating with the narrow margin between it and the cell-wall. This vacuole was apparently filled with a transparent fluid, in which were minute granules in immense numbers, in constant active motion circling among one another and passing out, into and along the marginal connecting space. According to Ralfs, the sporangia of this species are large ($\frac{1}{4}\frac{1}{35}''$), with branched spines.

Fig. 5, pl. 21, represents an empty frond of this species; 5 a, outlines of semicells to show the variations, and fig. 14, pl. 12, represents a frond crowded with endochrome, magnified 460 diameters.

C. ovale, RALFS.

C. magnum ovale, compressum, profunde constrictum, diametro subduplo longius, ambitu integerrimum vel crenatum, a vertice late ellipticum; semicellulis basi paulo latioribus quam longis, triangulo-rotundatis, disco punctatis, margine verrucis margaritaceis achrois hyalinis in series 4 ordinatis. (R.)

Diam.—Long. $0.0053''$ — $0.0067''$. Lat. plerumque $0.0041''$. (R.)

Syn.—*C. ovale*, RALFS, *British Desmidiæ*, p. 98.

Hab.—South Carolina; Rhode Island; Bailey. Cobble Mountain, Pa. (Lewis) Wood.

Frond very large, elliptic, nearly twice as long as broad, constriction very deep, linear; segments somewhat broader than long, somewhat triangular, rounded at ends, rough near the margin, with a band of large pearly granules, producing a dentate appearance, the disc punctate; e. v. elliptic. (A.)

C. Brébissonii, MENEGH.

C. paulo longius quam latum; semicellulis semicircularibus, diametro paulo longioribus, angulis inferioribus obtusis approximatis, ventre modice concavis subplanis, dorso latissime rotundatis; cytodermate muricato, muricibus conicis in ordinibus regularibus collocatis. (R.)

Diam.—Semicell, 0.0019"—0.0022". (R.)

Syn.—*C. Brébissonii*, MENEGHENI. RABENHORST, Flora Europ. Algarum, Sect. III. p. 158.

Hab.—White Mountains, New Hampshire, (Dr. F. W. Lewis) Wood.

Fronnd somewhat longer than broad, constriction deep, linear; segments semiorbicular, rough all over, with somewhat elongate conical scattered pearly granules; c. v. elliptic. (A)

Remark.—Fig. 6, pl. 21, represents an empty frond of this species, magnified 750 diameters.

C. suborbiculare, WOOD.

C. parvum, suborbiculare, paulo longius quam latum, cum margine enormiter crenato vel crenato-undulato; semicellulis a latere orbicularibus, a vertice ellipticis; sinu extrorsum angustissimo sed introrsum nonnihil excavato; cytodermate crasso, sparse verruculoso; granulis in semicellulis singulis subdistantibus et in seriebus elongatis, duabus (interdum unica) externis curvatis, et in seriebus duabus internis brevibus et rectis.

Diam.—Lat. $\frac{1.4}{12000}$ " = .0012"; lat. $\frac{1.6}{12000}$ " = .0013".

Syn.—*C. orbiculare*, WOOD, Proceed. Acad. Nat. Sc. 1870.

Hab.—In lacu "Saco," New Hampshire, (Lewis.)

Small, suborbicular, a very little longer than broad, with the margin irregularly crenate, or crenate undulate; semicells from the side orbicular, from the vertex elliptical; sinus very narrow, but within somewhat excavated; cytoderm thick, sparsely coarsely granulated; granules subdistant, in each cell arranged in one or two curved marginal series and in a central group of two or three short rows.

Remarks.—The arrangement of the granules in this desmid is peculiar, one, or sometimes two rows of large obtuse pearly granules are placed at rather wide intervals along the whole outer margin, and then in the centre of each semicell is a group of two or three, or even more short straight rows of three or four similar but rather smaller granules. The isthmus is rather broad and short; sometimes it has on it one or two granules.

Fig. 9, pl. 21, represents an empty frond of this species, magnified 750 diameters; 9 a, the outline of the end view of the same.

C. tetrophthalmum, (KTZ.) BRÉB

C. tertiam partem circa longius quam latum, profunde constrictum; sinu angusto, plerumque sublineare; ambitu obtuse crenato; semicellulis nonnihil semicircularibus, ventre subplanis, dorso rotundatis; cytodermate verruculoso; verruculis magnis, obtusis, subordinatim dispositis.

Diam.— $\frac{2.5}{12000}$ " = .0025".

Syn.—*C. tetrophthalmum*, (KÜTZING), BRÉBISSON. RABENHORST, Flora Europ. Algarum, Sect. III. p. 159.

Hab.—New Jersey; Wood.

About one-third longer than broad, deeply constricted; sinus narrow, mostly sublinear; margin obtusely crenated; semicells somewhat semicircular, belly nearly even, dorsum rounded; cytoderm warty; prominences large, obtuse, arranged somewhat regularly.

Remarks.—The only specimens I have seen, and I believe the only ones hitherto

found on the continent, were collected by myself in "Shepherd's Mill Pond," near Bridgeton, Cumberland County, New Jersey.

Fig. 7 *a*, pl. 21, represents the outline of a frond magnified 460 diameters.

C. amœnum, BRÉB.

C. medioere, oblongum cylindricum, leviter compressum, diametro duplo fere triplove longius, utroque polo rotundatum, medio profunde constrictum, sinu angusto, lineari, ambitu granulis margaritaceis aërois obsessum, a vertice ellipticum; semicellulis oblongo-rotundatis, dorso alte convexis, lateribus vero rectis parallelis, angulis inferioribus rectis et subacutis; cytiodermate granuloso-verrucoso, verrucis hyalinis in series regulares dispositis. (R.) *Species mihi ignota*.

Long. 0.0017"—0.1016"; lat. 0.00087". (R.)

Syn—*C. amœnum*, BRÉBISSON. RABENHORST, Flora Europ. Algar., Sect. III. p. 159.

Hab.—Florida; Bailey. Rhode Island (S. T. Olney); Thwaites.

Frond twice as long as broad, sides parallel, ends rounded, constriction deep, linear; segments rough with crowded obtuse papilla-like pearly granules; s. v. much compressed, about thrice as long as broad; e. v. elliptic. (A.)

** *Cytiodermate glabro*.

** *Cytioderm smooth*.

C. Cucumis, CORDA.

C. ovale ellipticum, utroque polo late rotundatum, tertiam partem vel duplo longius quam latum, profunde constrictum; sinu lineari; semicellulis angulis inferioribus rotundatis, cytiodermate glabro, haud punctato.

Diam.—Max. long. $\frac{3\frac{1}{5}}{7500}$ " = 0.0026"; lat. $\frac{1\frac{3}{5}}{7500}$ " = .0019".

Syn.—*C. Cucumis*, CORDA. RABENHORST, Flora Europ. Algarum, Sect. III. p. 161.

Hab.—South Carolina; Georgia; Florida; Bailey. Pennsylvania; Wood. Saco Lake, (Lewis.)

Oval or elliptic, at each end broadly rounded, one-third to twice as long as broad, profoundly constricted; sinus linear; semicells with their inferior angles rounded; cytioderm smooth, not punctate.

Remarks.—This species is very abundant around Philadelphia. The semicells generally each contain two large globular masses placed near the median line, which are sometimes hidden by the crowded endochrome.

Figs. 15, 15 *a*, pl. 12, represent this species with their endochrome in different conditions; 15 *b*, represents a monstrous frond, which had attempted to divide, but had not succeeded in so doing.

C. depressum, BAILEY.

"Elliptical, binate, division in the plane of the longest axis. Segments entire, nearly twice as long as broad, rounded above, very much flattened at base.

Hab.—Lakes in Florida.

This species resembles *C. bioculatum*, BRÉB.; but the segments are much closer together, and are angular, not rounded at the basal extremities." BAILEY. Microscopical Observations. Smithsonian Contributions.

C. pyramidatum, BRÉB.

C. medioere, ovale vel subovale, utroque polo truncatum, medio profunde constrictum, duplo

fere longius quam latum; semicellulis breviter pyramidatis, angulis inferioribus rotundatis, apice (dorso) modo truncatis modo rotundatis, a vertice late ellipticis; cytiodermate punctato vel subtilissime granulato. (R.)

Long. 0.0021"—0.0037". Lat. max. 0.0026".

Syn.—*C. pyramidatum*, BRÉBISSEON. RABENHORST, Flora Europ. Algarum, Sect. III. p. 162.

Hab.—Georgia; Florida; Bailey. Pennsylvania; Wood.

Fronde scarcely twice as long as broad, suboval; constriction deep, linear; segments pyramidal, rounded at basal angles, somewhat truncate at the ends, punctate; e. v. broadly elliptic. (A.)

Remark.—Fig. 14, pl. 13, is a drawing of this species.

C. bioculatum, BRÉB.

C. parviter, circiter tam longum quam latum vel paulo longius, profunde constrictum, sinu extrorsum ampliato; semicellulis diametro duplo latioribus, elliptico-prope hexagonis angulis obtuse rotundatis, integerrimis aut levissime crenulatis; cytiodermate laevi vel subtilissime punctato. (R.) *Species mihi ignota.*

Long. 0.00069". Lat. 0.00066". (R.)

Syn.—*C. bioculatum*, BRÉBISSEON. RABENHORST, Flora Europ. Algarum, Sect. III. p. 163.

Hab.—Rhode Island, (S. T. Olney) Thwaites.

Fronde minute, about as long as broad, constriction deep, producing a gaping notch at each side; segments about twice as broad as long, elliptic, smooth; s. v. compressed s. v. elliptic. Sporangium orbicular with conical spines. L. $\frac{1}{4}\frac{1}{16}$ "; B. $\frac{1}{17}\frac{1}{3}$ ". (A.)

C. Meneghenii, BRÉB.

C. parvum, tam longum quam latum, modo paulo-longius, modo paulo-brevius, profunde constrictum, sinu lineari, extrorsum non ampliato; semicellulis subquadratis, leviter sinuato-hexagonis; angulis rotundatis, cytiodermate laevi vel subtilissime punctato. (R.)

Long. $\frac{1}{8}\frac{1}{7}$ "— $\frac{1}{6}\frac{1}{5}$ " = 0.00103"—0.0013"; lat. $\frac{1}{10}\frac{1}{5}$ "— $\frac{1}{10}\frac{1}{8}$ " = 0.00081"—0.00089". (R.)

Syn.—*C. Meneghenii*, BRÉBISSEON. RABENHORST, Flora Europ. Algar., Sect. III. p. 163.

Hab.—Pennsylvania; Wood.

Fronde very minute, rather longer than broad, constriction linear; segments subquadrate, bicrenate at the sides and ends, smooth; e. v. elliptic. (A.)

Remark.—Fig. 18, pl. 12, represents a frond of this species, magnified 750 diameters.

C. crenatum, RALFS.

C. oblongum, tertiam partem circa longius quam latum, profunde constrictum, sinu lineari angusto; semicellulis e basi lata subsemicircularibus, dorso plus minus depressis vel truncatis, ambitu crenatis vel regulariter undulato crenatis, crenis 10–14; cytiodermate punctato. (R.) *Species mihi ignota.*

Long. 0.0021"—0.0023"; lat. 0.0015". (R.)

Syn.—*C. crenatum*, RALFS, British Desmidiæ, p. 96.

Hab.—Rhode Island; (S. T. Olney) Thwaites.

Fronde slightly longer than broad, constriction linear; segments semiorbicular, ends and sides broadly rounded, crenate or minutely undulate at margin; e. v. elliptic. Sporangium orbicular, spinous; spines elongate, slender, swollen at the base and divided at the apex. L. $\frac{1}{4}\frac{1}{16}$ "; B. $\frac{1}{6}\frac{1}{11}$ ".

C. undulatum, CORDA.

C. submedioere, oblongum, diametro subduplo longius, utroque polo late rotundatum, ambitu leviter sinuato-undulatum, profunde constrictum, sinu lineari extrorsum paullum ampliato; semicellulis semiorbicularibus, et dorso et lateribus late rotundatis, margine undulato-crenatis, crenis 9, sublatis; cytiodermate lævi; zygosporis sphaericis spinis elongatis, apice bi-tri-fidis obsitis. (R.) *Species mihi ignota.*

Long. 0.0024". Lat. 0.0017". (R.)

Syn.—*C. undulatum*, CORDA. RABENHORST, Flora Europ. Algarum, Sect. III. p. 165.

Hab.—South Carolina; Rhode Island; Bailey.

Fronde rather larger than that of *C. crenatum*, slightly longer than broad, constrictions linear; segments semiorbicular, ends and sides broadly rounded, crenate or minutely undulate at the margin; e. v. elliptic. Sporangium orbicular, spinous; spines elongate, slender, swollen at the base and divided at the apex. (A.)

b. Semicellulæ medio-ventricoso inflatæ.

b. Semicells medianly ventricose.

* *Cytiodermate lævi.*

* *Cytioderm smooth.*

C. sublobatum, (BRÉB.) ARCHER.

C. parvum, oblongo subquadratum, diametro subduplo longius, sinu angusto lineari; semicellulis subquadratis, e basi dilatata ad verticem sensim angustatis, angulis et inferioribus et superioribus rotundatis, dorso late truncatis lateribusque leviter sinuatis; cytiodermate levissimo. (R.)

Long. 0.00179"—0.00196". Lat. max. 0.0015"—0.00157". (R.)

Syn.—*C. sublobatum*, (BRÉBISSON) ARCHER. Pritchard's Infusoria, p. 731.

Hab.—Georgia; Florida; Rhode Island; Bailey.

Fronde scarcely twice as long as broad, oblong; constriction linear, segments subquadrate, somewhat wider at the base, lateral and end margins slightly concave, smooth and transverse vein cruciform. (A.)

** *Cytiodermate granulato.*

** *Cytioderm granulate.*

C. ornatum, RALFS.

C. parvum, plerumque tam longum quam latum; semicellulis reniformibus, diametro duplo longioribus, angulis inferioribus una cum lateribus rotundatis; dorso sub-producto late truncatis; cytiodermate granulato-verruculoso; zygosporis longe spinosis, spinis elongatis apice fureatis. (R.) *Species mihi ignota.*

Long. 0.0016"—0.0015". Lat. 0.0016". (R.)

Syn.—*C. ornatum*, RALFS, British Desmidiæ, p. 104.

Hab.—Rhode Island; (S. T. Olney) Thwaites.

Fronde in f. v. about as long as broad, constriction deep, linear; segments semiorbicular or subreniform, with a central truncate projection at the ends produced by the continuation of a central inflation, rough towards the margin and on the inflation with pearly granules; e. v. with a rounded lobe on each side. Sporangium orbicular, spinous; spines elongated, dilated at the base and slightly divided at the extremity, $\frac{1}{3}$ ". (Archer.)

C. commissurale, (BRÉB.)

C. minutum, fere dimidio latius quam longum, profundissime constrictum, sinu amplo basi excavato; semicellulis anguste reniformibus, diametro paene triplo longioribus, leviter incurvis,

angulis rotundatis, dorso truncato-rotundatis, margine crenulato-dentatis, a dorso oblongis, medio ventricosis, utroque polo plus minus tumidis; cytodermate granulato margaritifero. (R.)

Long. 0.0010"—0.0012." Lat. 0.0013"—0.0015". (R.)

Syn.—*C. commissurale*, BRÉBISSEON. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 170.

Hab.—In lacu. White Mountains, New Hampshire; (Dr. F. W. Lewis)

Fronde small, in f. v. one-third broader than long; constriction very deep, rounded; segments narrow-reniform, with a central, somewhat truncate projection, produced by the continuation of the central inflation, rough on the inflation and on the extremities, with somewhat large pearly granules, e. v. three times longer than broad, constricted between the central inflation and the rounded extremities. Sporangium as in *C. ornatum*. (A.)

Remarks.—I have seen but a single specimen of this species which differed from the typical form, in having the sinus very narrow in its outer portion, and in being shorter.

Fig. 16, pl. 13, represents the frond of this specimen, magnified 750 diameters.

C. cælatum, RALFS.

C. suborbiculare, profunde constrictum; sinu angustissimo lineari; semicellulis inciso-crenatis, angulis rotundatis, a vertice medio nonnihil inflatis; cytodermate granulato, granulis in serie regulariter circulares positus.

Diam.—Long. $\frac{2.0}{12000}$ " = .0017". Lat. $\frac{1.7}{12000}$ " = .0014".

Syn.—*C. cælatum*, RALFS, *British Desmidiæ*, p. 103.

Hab.—In stagnis prope Aiken, South Carolina. (Ravenel.)

Suborbicular, profoundly constricted; sinus very narrow, linear; semicells inciso-crenate, angles rounded, when seen from the end somewhat inflated in the middle; cytoderm granulate, granules placed in circular series.

Remarks.—This species was collected by Prof. Ravenel in a quiet ditch near Aiken, South Carolina, sparsely scattered amidst innumerable diatoms and desmids. The number of the crenations appears to vary. In the few individuals I have seen there were six end ones besides the two very broad basal ones, if the latter can be called crenatures. Ralfs gives six as the total number, and yet every one of his figures has many more. So I think the number a character of but little importance. The circular arrangement of the granules is not so positive and regular in the specimens I have seen, as is represented in the figure of Ralfs, otherwise the agreement is perfect.

C. Broomei, THWAITES.

C. subparvum, plerumque tam longum quam latum, nonnunquam paulo longius, obtuse quadrangulare, profunde constrictum, sinu angustissimo lineari; semicellulis oblongo-quadrangulis, diametro duplo longioribus, angulis et inferioribus et superioribus obtuse rotundatis, ventre subplanis, dorso latissime truncatis et sæpius leniter retusis vel plane convexis; cytodermate granulato-margaritaceo, granulis in seriebus subrectis collocatis. (R.)

Long. 0.00194"—0.0022". Lat. max. .002", thick .0015".

Syn.—*C. Broomei*, THWAITES. RALFS, *British Desmidiæ*, p. 103.

Hab.—Georgia; Bailey. Prope Philadelphia; Wood.

Fronde in f. v. about as long as broad, constriction deep, linear; segments quadrilateral, ends straight, angles rounded, rough all over with minute granules; e. v. twice as long as broad, slightly inflated at the middle and rounded at the ends. Sporangium orbicular, smooth. (A.)

Remarks.—The only specimens which I have seen were found in a brick-pond below the city in the month of June. They agree well with the descriptions, excepting in that I should describe their central inflation as pronounced. The sinuses also are ampliate or hollowed out within. The granulations are quite large, and are arranged somewhat irregularly in rows.

Fig. 15, pl. 13, is a view of the front of the frond magnified 460 diameters; fig. 10, pl. 21, the outline of the lateral view.

c. Cellulæ fusiformes, cylindricæ vel ovales, in medio leviter constrictæ.

c. Cells fusiform, cylindrical, or oval, lightly constricted in the middle.

C. Thwaitesii, RALFS.

C. medioere, diametro bi-triplo longius, fusiformi-cylindraceum, medio leviter constrictum, ambitu integerrimum, utroque polo rotundatum; semicellulis e cylindraceo subconicis, e medio in apicem sensim sensimque (sed modice) attenuatum; cytodermate lævi vel indistincte punctato. (R.) *Species mihi ignota.*

Long. 0.00267"—0.00287". Lat. max. 0.0012". (R.)

Syn.—*C. Thwaitesii*, RALFS, British Desmidiæ, p. 109.

Hab.—Florida; Bailey.

Fronde in f. v. two or three times longer than broad; constriction a very shallow groove; segments subcylindrical, with rounded ends; endochrome scattered; e. v. circular, or very slightly compressed; e. f. not punctate, or puncta very indistinct. (A.)

C. connatum, BRÉB.

C. validum, submagnum, leviter compressum, diametro duplo circa longius, subpanduriforme, plus minus constrictum, utroque polo late rotundatum, a vertice lato ellipticum; semicellulis subhemisphericis, ambitu æqualiter rotundatis, integerrimis; cytodermate punctato. (R.) *Species mihi ignota.*

Long. 0.0035". Lat. max. 0.00165"—0.0019". (R.)

Syn.—*C. connatum*, BRÉBISSEON. RALFS, British Desmidiæ, p. 108.

Hab.—Florida; Bailey.

Fronde large, in f. v. about one-half longer than broad; constriction shallow; segments about two-thirds of a circle, coarsely punctate, and with a distinct, sometimes striated, border; e. v. circular. (A.)

d. Cellulæ in familias connexæ.

d. Cells united into families.

C. Quimbyii, WOOD. (sp. nov.)

C. cellulis parvis, sub-ellipticis, medio profunde constrictis, in familias copulis hyalinis connexis; semicellulis a fronte ellipticis et diametro subduplo longioribus, a vertice ellipticis, a latere rotundatis; sinu lato; marsis chloro-phyllaceis in quaque semicellula singulis; cytodermate tenue, glabro.

Diam.—Long. $\frac{1}{1000}$ " = 0.001". Lat. a fronte $\frac{3}{4000}$ " = 0.00075"; a latere $\frac{5}{12000}$ " = 0.00042".

Hab.—In aquis puris, New Jersey.

Cells small, subelliptical, profoundly constricted in the middle, joined by translucent bands into families; semicells seen from the front elliptical, and nearly twice as long as broad, from the vertex elliptical, from the side roundish; sinus broad; chlorophyl masses single in each cell; cytioderm thin, smooth.

Remarks.—This plant was found by my friend Mr. Quimby growing in a beautiful spring above Camden, upon whose bottom it formed a gelatinous, translucent, greenish mass. The cells resemble in shape those of *C. cucumis*, although much smaller. They are joined by bands into little families, in which the original parent-cell is generally very distinct, it, or rather the two cells into which it first divides, remaining in the centre of the group. The bands are so hyaline that their edges can alone be distinctly seen, and hence the latter often look as though they were threads—there appearing to be two parallel threads, or two threads crossing one another, or a single thread, according as the band is flat, twisted, or on edge.

It gives me great pleasure to dedicate the species to my friend Mr. Quimby, by whom it was collected.

Fig. 9, pl. 1, represents one of the family groups of this plant.

Genus EUASTRUM, EHRB.

Cellulæ vel oblongæ vel ellipticæ, medio profunde incisæ, symmetricè sinuatæ, vel lobatæ, tumori-
bus inflatis circularibus (rare obsoletis) instructæ, utroque polo sinuato-emarginatæ vel inciso-bilo-
batæ, a vertice ellipticæ.

Cells either oblong or elliptic, profoundly incised in the middle, symmetrically sinuate or lobed, provided with circular inflated protuberances (which are rarely absent), at each end sinuately emarginate or incisely-bilobate, from the vertex elliptic.

A. Lobo polares in apice late sinuato-excisi.

A. Polar lobe with its apex broadly sinuately excised.

E. multilobatum, WOOD.

E. magnum, fere duplo longius quam latum, medio profunde constrictum, et cum sinu modice amplo; a latere medio ventricosum et duplo biumbonatum, ad verticem dilatatum et emarginatum; semicellulis a fronte trilobatis, lobis sinus amplissimis inter se sejunctis; lobi basale distincte late emarginato, lobo centrale obtuso, lobo polare late leviter sinuato-emarginato; semicellulis a vertice quinque lobulatis; cytiodermate lævi.

Diam.—Long. $\frac{57}{12000}'' = .00475''$. Lat. $\frac{30}{12000}'' = .0025''$.

Syn.—*E. multilobatum*, WOOD, Proc. A. N. S., 1869.

Hab.—In lacu "Saco;" New Hampshire; (Lewis.)

E. large, about twice as long as broad, in the centre profoundly constricted, with the sinus moderately large; from the lateral view somewhat enlarged and doubly biumbonate in the middle; semicells from the front trilobate, the lobes separated by very wide sinuses, the basal lobe broadly emarginate, the central lobe obtuse, the end lobe broadly and shallowly sinuately emarginate; semicells from the vertex five-lobed; cytioderm smooth.

Remarks.—The basal lobes of this beautiful desmid are distinctly five lobulate, the lateral lobules being longer and broader than the others, which, instead of being emarginate, are obtuse. The sinuses, separating lobes and lobules, are very broad, with very obtuse angles. When the desmid is viewed from two-thirds round, so as to show the anterior and posterior lobules especially, it presents an

outline in which all the sinuses are of similar form, and the central and basal lobes are about equal size; whereas, when viewed from the front, the basal lobe is much the broader. When the desmid is viewed from the side it is seen to be enlarged in the centre, and provided with two distinct umbonations each side of the comparatively narrow central sinus.

Fig. 19, pl. 12, represents the front view of a frond of this plant; fig. 5, pl. 20, the outline of a two-thirds view, and fig. 5 a, the outline of a lateral view, all magnified 450 diameters.

E. verrucosum, EHRB.

E. magnum, late ovatum, vix longius quam latum, medio profunde constrictum, sinu extrorsum dilatato; semicellulis trilobatis, lobis triangularibus, divergentibus, apice late et profunde sinuatis; a latere ovato-oblongum, sinuato-lobatum, lobis octo in apice rotundatis, polaribus singulis porrectis, lateralibus ternis; cytiodermate granulato-verrucoso. (R.) *Species mihi ignota.*

Long. 0.0036"—0039". (R.)

Syn.—*E. verrucosum*, EHRENBURG. RABENHORST, Flora Europ. Algarum, Sect. III. p. 179.

Hab.—South Carolina; Georgia; Florida; Rhode Island; Bailey.

Frond somewhat longer than broad, rough all over with conic granules; segments 3-lobed, somewhat divergent, all the lobes broad, cuneate, with a very broad, shallow, or external sinus. Empty frond; f. v. segments with one large circular basal inflation on surface, one smaller on each side, and two others on the end lobe; s. v. segments inflated at the base, narrowed into a short neck, end dilated with a central sinus; e. v. oblong, with three inflations at each side, one at each end, end lobe having 4 divergent lobelets. (A.)

E. gemmatum, BRÉB.

E. mediocre, diametro duplo longius, profunde constrictum, sinu angusto lineari, a vertice ovato-oblongum, ambitu sinuato-lobatum, lobis 8 conformibus, rotundatis; semicellulis trilobatis, basi tumoribus 3 in seriem dispositis, lobis in apice profunde emarginatis, lobulis rotundatis, lobo polari dilatato et paulum producto; cytiodermate in tumoribus et lobulis granulato-punctato, cæterum lævi. (R.) *Species mihi ignota.*

Long. 0.00224"—0.0029". Lat. 0.00157"—0.0017". (R.)

Syn.—*E. gemmatum*, BRÉBISSEON. RABENHORST, Flora Europ. Algar., Sect. III. p. 180.

Hab.—Rhode Island; Bailey.

Frond scarcely twice as long as broad; segments 3-lobed, lateral lobes horizontal, deeply emarginate, the protuberances minutely granulate; end lobe dilated, its dilatations inclined upwards, and minutely granulate; ends with a deep rounded emargination. Empty frond slightly punctate; f. v. segments with three granulate inflations near the base; tr. v. broadly elliptic, with three granulate inflations at each side and one at each end; e. v. end lobe cruciform, lobelets rounded, granulate. (A.)

E. oblongum, (GREV.) RALFS.

E. magnum, diametro duplo triplove longius, oblongum, profunde constrictum, sinu angusto, a latere oblongo-lanceolatum, utroque polo truncato leniter retusum, ambitu undulato-sinuatum; semicellulis (fronte) sinuato-quinquelobis, basi et in quoque lobo tumore instructis, lobis lateralibus in apice dilatato sinuato-retusis, inferioribus latioribus quam superior., lobo polari late cuneato in apice profunde inciso, angulis omnibus rotundatis, cytiodermate lævi; zygosporis globosis verrucosis, verrucis obtuse conicis achrois hyalinis. (R.) *Species mihi ignota.*

Long. 0.0057"—0.0065". Lat. max. 0.00346".

Syn.—*E. oblongum*, (GRÉVILLE) RALFS' British Desmidiæ, p. 80.

Hab.—Rhode Island; Bailey.

FronD rather more than twice as long as broad, smooth, oblong; segments 5-lobed; lobes nearly equal, cuneate; lateral lobes, or the basal only, with a broad, shallow, marginal concavity, all their angles rounded, terminal notch linear.

Empty frond; f. v. seg. punctate, with three large inflations, on surface near the base, two others above and two on terminal lobe; tr. v. three times as long as broad, with three sub-distant marginal inflations at each side, and one at each end, in β , broader in proportion, more elliptic, and inflations close; e. v. end lobe notched at opposite external margins. Sporangium orbicular, beset with numerous conical tubercles. (A.)

B. Lobi polares evidenter discreti et in apice anguste incisi.

B. End lobes evidently separated and narrowly incised in the centre.

E. crassum, (BRÉB.) KtZ.

E. oblongum, diametro subtriplo longius, profunde constrictum, sinu angusto lineari, e vertice subquadrangulare, utroque polo profunde excisum, angulis rotundatis; semicellulis (fronte) trilobis, basi et in quoque angulo tumore instructis, lobis lateralibus latissimis unisinuatis, lobo polari paullum prominente, in apice bifido, segmentis late rotundatis; cytiodermate distincte punctato, punctis in series transversas ordinatis. (R.)

Long. 0.0051"—0.0073". Lat. max. 0.0041". (R.)

Syn.—*E. crassum*, (BRÉBISSEON) KÜTZING RABENHORST, Flora Europ. Algarum, Sect. III. p. 181.

Hab.—United States.

FronD about twice as long as broad, subquadrilateral, smooth; segments 3-lobed; basal lobes very broad, with a very broad, shallow marginal sinus, in which there is sometimes a slight intermediate rounded projection; end lobe creneate, rounded, terminal notch linear.

Empty frond; f. v. punctate, segments with three inflations below and two above; tr. v. two or three times longer than broad, with three lobes or inflations at each side and one at each end; e. v. end lobe sinuate at opposite external margins. (A.)

E. ornatum, WOOD.

E. oblongum, diametro duplo longius, profunde constrictum, sinu angusto lineari; semicellulis a fronte trilobatis; lobis basalibus latissimis, nonnihil sinuato-emarginatis, angulis plus minus productis et rotundatis; lobo polari medio profunde lineare inciso, segmentis late rotundatis; semicellulis a latere bilobatis, lobis basalibus profunde emarginatis et cum angulis plus minus acutis; cytiodermate distincte ordinatim punctato.

Diam.— $\frac{35}{12000}$ " = .00029".

Syn.—*E. ornatum*, WOOD, Proc. A. N. S., 1869.

Hab.—Saco Lake; New Hampshire. Lewis.

E. oblong, twice as long as broad, profoundly constricted; semicells from the front trilobate; basal lobe very broad, slightly sinuately-emarginate, angles more or less produced and rounded; polar lobe medianly profoundly linearly incised, segments broadly rounded; semicells bilobate at the sides, basal lobes profoundly emarginate and with the angles more or less acute; cytioderm distinctly regularly punctate.

Remarks.—This species is close to *E. crassum*, from which it differs in the proportionate length, being only twice instead of three times as long as broad; in the size being only three-fourths as large; and especially in the peculiar lateral splitting, as it were, of the basal lobes.

Fig. 12, pl. 21, represents the front view of an empty half frond of this species, magnified 450 diameters; fig. 12 a, the side view of an empty frond.

E. affine, RALFS.

E. *E. humerosum* affine, paulo minus; semicellulæ quinquelobæ; lobi basales quales in *E. humerosum* sed tumores quatuor in seriem transversam simplicem dispositi, lobi intermedii valde abbreviati eorumque basi tumoribus duobus instructi, lobus polaris magis porrectus et in apice minus dilatatus; cytoderma subtilissime punctatum sublævæ. (R.)

Long. 0.0038"—0.0041". (R.)

Syn.—*E. affine*, RALFS, British Desmidiæ, p. 82.

Hab.—South Carolina; Georgia; Bailey.

Frond about twice as long as broad; segments 3-lobed; basal lobes slightly emarginate, having intermediate between them and the end lobe on each side a tubercle representing middle lobes, the upper margin of which is horizontal; end lobe exerted, dilated, its notch linear.

Empty frond; f. v. minutely punctate; the segments with four basal inflations, two above and two on end lobe; tr. v. elliptic, with four inflations on each side and one at each end; e. v. end lobe emarginate at opposite; e. v. end lobe emarginate at opposite external margins, producing four shallow lobulets. (A.)

E. Didelta, (TURPIN) RALFS.

E. robustum, diametro duplo longius etiam supra, in sectione transversa ellipticum, ambitu undulato-crenatum, in utroque latere crenis quaternis; semicellulis pyramidalibus, quinquelobis, tumoribus 9 in series tres alternantibus ordinatis, lobi inferioribus oblique truncato-rotundatis nonnunquam leniter retusis, intermediis subadscendentibus, rotundatis, lodo polari minus dilatato, bifido, segmentis rotundato-truncatis, comiventibus, in apice tumidis; cytodermate distincte punctato, punctis modo irregulariter sparsis modo in seriibus rectis collocatis. (R.)

Long. 0.0055". Lat. 0.00279".

Syn.—*E. Didelta*, (TURPIN) RALFS, British Desmidiæ, p. 84.

Hab.—South Carolina; Georgia; Rhode Island; Bailey. Pennsylvania; Wood.

Frond rather more than twice as long as broad; segments pyramidal, inflated at the base and again at the middle, end scarcely dilated, rounded, its notch linear.

Empty frond punctate; f. v. segments with several inflations in lines and two at the end; tr. v. elliptic with four inflations at each side and one at each end; e. v. end lobe entire at margin. Sporangium orbicular, with subulate spines. (A.)

Remark.—Fig. 13, pl. 21, represents this species.

E. ampullaceum, RALFS.

E. diametro duplo longius; semicellulis trilobis, ad basin tumidis, e basi latissima subito in lobi polaris collum attenuatis, lobi basalibus maximis integris, loco loborum intermediorum processu deutiformi, lobo polari cuneato, in apice bifido, segmentis late truncato-rotundatis; cytodermate subtiliter punctate. (R.)

Long. 0.0035"—0.0038". Lat. max. .0026"; lat. in colli (lobi polar.) 0.00085". (R.)

Syn.—*E. ampullaceum*, RALFS, British Desmidiæ, p. 83.

Hab.—South Carolina; Florida; Bailey.

Frond rather more than one-half longer than broad; segments obscurely 3-lobed, short, with broad inflated base; basal lobes not emarginate, having on each upper side a small intermediate tubercle between each and the end lobe; end lobe exerted and dilated, its notch

linear. Empty frond minutely punctate; f. v. narrow elliptic, with several inflated protuberances, ends scarcely dilated, rounded; tr. v. with four inflations at sides and one at each end. (A.)

circulare, HASSAL.

E. mediocre, diametro duplo longius; semicellulis trilobis (at non semper distincte), ad basin versus tumoribus quinis aut pluribus in series duas v. tres alternantes aut singulo centrali, quaternis semicirculariter ordinatis instructis, lobis basalibus sinuato-emarginatis subito in lobum polarem apice paulum dilatatum attenuatis; cytodermate subtiliter punctato. (R.)

Syn.—*E. circulare*, HASSAL, Fresh-Water Algæ, p. 383.

Hab.—Providence, Rhode Island; Bailey.

“Frond about twice as long as broad, tapering upwards into a neck, end not dilated, its notch an acute incision. Empty frond, segments with five basal inflations, four in a half circle around the fifth and two others at the extremity.” Archer.

(*Var. Ralfsii*)

Semicellula tumoribus minimis 11 in series tres alternantes ordinatis.

Hab.—Saco Lake, New Hampshire; (F. W. Lewis) Wood.

E. Jenneri, nobis. Frond scarcely twice as long as broad; segments 3-lobed, basal portions subquadrate, emarginate at the sides; end lobe, its notch linear. Empty frond punctate, segments with several inflations arranged in alternate lines. (Archer.)

E. insigne, HASSALL.

E. subgracile, diametro duplo-triplo longius, a vertice fere quadratum, lateribus concavis, angulis rotundatis; semicellulis basi inflatis, integris, e basi subreniformi in collum elongatum citius attenuatis, lobo polari dilatato bifido truncato; cytodermate subtiliter punctato. (R.)

Long. 0.0039"—0.0043". Lat. max. 0.00236". (R.)

Syn.—*E. insigne*, HASSALL, Fresh-Water Algæ, p. 21.

Hab.—Florida; Rhode Island; Bailey.

Frond rather more than twice as long as broad; segments inflated at base, sides entire, without lateral tubercles, and tapering into a long slender neck; end lobe dilated, its notch linear. Empty frond minutely punctate; f. v. segments with two inflations at the base; f. v. narrower, gradually tapering to the end, which is considerably dilated; projections rounded, with a sinus between; tr. v. subquadrate, slightly concave at sides, with a rounded lobe at the centre of each end; e. v. end lobe with a sinus at opposite external margins, angles thus protruded into four divergent rounded lobclets. (A.)

E. Ralfsii, RABENH.

E. mediocre, leviter compressum, medio inflatum, diametro duplo circiter longius; semicellulis pyramidalibus, e basi ventricosa in lobum polarem rectum truncatum sinuato-attenuatis; cytodermate subtiliter punctato, punctis in lineas rectas ordinatis. (R.)

Syn.—*E. ansatum*, EHR. et auctores. RABENHORST, Flora Europ. Algarum, Sect. III. p. 184.

E. Ralfsii, RABENHORST, Flora Europ. Algarum, Sect. III. p. 184.

Hab.—South Carolina; Rhode Island; Bailey. White Mountains, New Hampshire (F. W. Lewis).

“*E. ansatum, Ehrb.* Frond about twice as long as broad; segments inflated at the base, tapering upwards without sinuations into a neck, end not dilated, rounded, its notch linear. Empty

frond punctate; f. v. segments turgid on the surface, at the middle without circular inflations; tr. v. elliptic, with a single large inflation at each side; e. v. end lobe entire at the margin, its divisions circular. (A.)

Remarks.—I have seen only a very few specimens in a gathering made in Saco Lake, New Hampshire, by Dr. Lewis, which differ considerably from the typical form in the proportion of the breadth and length. There are also certainly four, if not more, umbonations on the face of each half-cell. These are nowhere distinctly spoken of as existing, and Mr. Archer states there are none visible in the front view of *E. ansatum*. They are, however, represented in the side view of the original figure, and are said to be very noticeable by Mr. Archer himself, when the desmid is so looked at. In the Saco Lake specimens they are always seen in the front view with great difficulty, and in some cases I failed entirely to demonstrate them, so that they do not afford a good character for the indication of a new species.

Fig. 1, pl. 13, represents a front view of a Saco specimen, magnified 450 diameters.

C. Lobi polares non-evidenter discreti.

C. End lobes not evidently distinct.

E. elegans, (BRÉB.) Ktz.

E. minus, oblongum, diametro duplo longius, utroque polo bifidum, segmentis introrsum rotundatis; semicellulis sursum modice attenuatis, utroque margine laterali bi-vel tri-sinuatis, sinu superiori vel intermedio profundiori, sub polo utrinque dente acuto prominente; cytodermate subtiliter punctato, punctis irregulariter sparsis; zygosporis globosis aculeatis, aculeis elongato-subulatis. (R.)

Long. 0.0012".—0.002". Lat. max. circiter 0.0011". (R.)

Syn.—*E. elegans*, (BRÉBISSEAU,) KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 185.

Hab.—South Carolina; Georgia; Florida; Rhode Island; Bailey. White Mountains, (F. W. Lewis).

Frond minute, scarcely twice as long as broad, oblong; segments with their basal portion emarginate at the sides, connected by a broad neck with the terminal portion; ends protuberant, rounded, acutely emarginate at the centre, pouting; s. v. with an inflation at the base of the segments, sides concave, ends rounded. Sporangium orbicular, spinous. (A.)

Remarks.—According to Prof. Rabenhorst *E. rostratum*, Ralfs, which is noted as an American species by Bailey, is a variety of *E. elegans*. Its peculiarities, according to Rabenhorst, are as follows: "Forma magis evoluta, profundior sinuata, segmentis polaribus latioribus, angulis acutis, dente paulo longiore."

Fig. 14, pl. 21, represents the outline of the frond as viewed laterally; fig. 2, pl. 13, a front view of the frond, magnified 750 diameters.

E. binale, (TURPIN) RALES.

E. minimum, diametro paulo vel subduplo longius, in sect. transversa oblongo-cylindricum, medio tumidum, utroque polo rotundatum; semicellulis indistincte trilobis, lobis basalibus

latis, rotundatis vel sinuato-bi-triarenatis; lobo polari abbreviato late truncato, leviter emarginato, angulis acutis lateraliter plus minus porrectis; cytiodermate subtilissime punctato.

Syn.—*E. binale*, (TURPIN) RALFS, British Desmidiæ, p. 90.

Hab.—Florida; Bailey. Rhode Island, (S. T. Olney,) Thwaites. Pennsylvania; Wood.

Fronde very minute, scarcely twice as long as broad, oblong; segments with their basal portion either entire or bicrenate at the sides, slightly contracted beneath the ends; ends dilated, not protuberant beyond the angles, its central notch acute, broad; tr. v. with two lateral inflations, ends truncate, angles rounded. (A.)

Remark.—Fig. 3, pl. 13, represents the front view of a frond, magnified 750 diameters.

Genus MICRASTERIAS, AG. (1827).

Cellulæ compressæ, profunde constrictæ, a fronte orbiculares vel late ellipticæ, a vertice fusiformes cum utroque polo acuto, semicellulæ tri- vel quinque-lobæ; lobi basales aut integri aut pluripliciter inciso-lobulati; lobus polaris aut integer aut sinuatus aut emarginatus, et interdum angulis productus et bifidus. Cytio-plasma chlorophyllacea in cellulæ lumen subæqualiter distributa, granula amylacea sparsa involvens. Cytioderma plerumque læve, nonnunquam punctatum, granulatum vel mucronatum.

Zygosporæ globosæ, ætate provecta aculeis simplicibus, apice bi-multi-fidis, nonnunquam repetito-multifidis armatæ.

Cells compressed, profoundly constricted, viewed from the front orbicular or broadly elliptic, from the vertex fusiform with acute ends. Semicells 3- or 5-lobed; basal lobes either entire or many times incisedly-lobulate; end lobe either entire or sinuate or emarginate, and sometimes with its angles produced and bifid. Chlorophyllous cytoplasm distributed nearly uniformly in the cavity of the cell, surrounding scattered starch granules. Cytioderm mostly smooth; sometimes punctate; granulate or mucronate.

Zygosporæ globose, at maturity armed with simple spines, whose ends bifid or multifid, and sometimes repeatedly multifid.

A. Semicellulæ trilobæ. Lobi basales horizontales; lobus polaris valde dilatatus, dorso plane convexus, truncatus vel leviter retusus, a lobis basalibus sinu amplissimo discretus.

A. Semicells trilobate. Basal lobes horizontal; end lobe strongly dilated, with the back convex, truncate, or slightly retuse.

M. arcuata, BAILEY.

M. mediocris, quadrangularis, paulo latior quam longa, profunde pinnatifida; lobis basalibus angustis elongatis, arcuatis, in apicem acutum attenuatis, divergentibus; lobis polaribus angustissimis, utrinque graciliter productis, in apicem acutum attenuatis, in medio dorso modice retusis. (R.)

Syn.—*M. arcuata*, BAILEY, Microscopical Observations: Smithsonian Contributions, vol. ii.

Hab.—In stagnis. Florida; Bailey.

“Quadrangular, segments three-lobed, the basal lobes long and arcuate, subtended by the transverse projections from the ends of the slightly notched terminal lobes.” (Bailey.)

M. expansa, BAILEY.

M. mediocris, tam longa quam lata, lobis stellatim expansis; lobis basalibus angustis in apicem acutum attenuatis, divergentibus, rectis; lobis polaribus e basi angusta sensim dilatatis, in medio dorso late sinuatis, angulis acutis (sed muticis). (R.)

Syn.—*M. expansa*, BAILEY, Microscopical Observations: Smithsonian Contributions, vol. ii.

Hab.—In stagnis, Florida; Bailey.

Segments three-lobed, basal lobes long, subconical, acute; terminal lobes slender, forked at the end, with the divisions much shorter than the basal lobes. (Bailey.)

M. quadrata, BAILEY.

M. arcuatæ similis, sed duplo major, semicellularum lobi basales minus arcuati, basi inflati, apice bidentati et cytoderma irregulariter granulatum. (R.)

Diam.—0.0043"—0.0049".

Syn.—*M. quadrata*, BAILEY, *Microscopical Observations: Smithsonian Contributions*, vol. ii.

Large quadrangular, three-lobed, basal lobes elongated, slightly curved, bidentate; terminal lobes with two slender transverse bidentate projections. Bailey.

M. disputata, WOOD.

M. magna, fere tam longa quam lata, subpinnatisecta, sinu acuto, lobis æqualibus; semicellulis profunde trilobis, lobis basalibus in apicem acute bidentatum valde attenuatis; lobo polari valde dilatato, dorso rotundato, angulis lateralibus acutissimis.

Long. $\frac{37}{500}$ " = .0074". *Lat.* $\frac{39}{500}$ " = .0078".

Syn.—*Micrasterias incisa*, KTZ. BAILEY, *Microscopical Observations: Smithsonian Contributions*, 1850.

Haud *Micrasteria incisa*, KÜTZING, *Spec. Algarum*, p. 171.

Tetrachastrum Americanum, ARCHER, PRITCHARD'S *Infusoria*, 1860, p. 725.

Hab.—South Carolina; Georgia; Florida; Rhode Island; Bailey. Pennsylvania; Wood.

M. large, about as long as broad, subpinnatisected; sinuses acute; semicells profoundly trilobate; basal lobes strongly attenuate into the acutely bidentate apex; distal lobes strongly dilated, rounded, with their lateral angles bidentate; end lobe broadly dilated, lateral angles very acute.

Remarks.—This desmid was first figured by the late Prof. Bailey in his *Microscopical Observations* (Smithsonian Contributions), as *M. incisa* of Ktz., and Rabenhorst, in his *Flora Europæa Algarum*, confirms this identification. He has probably, however, never seen the plant itself, but merely accepts the opinion of Professor Bailey. Mr. Archer (Pritchard's *Infusoria*), thinks the American plant is certainly distinct from the European, and this seems to me correct. The points of difference are—the American form is nearly twice the size of the European, the sinuses are much more widened outwardly, and the lobes are reduced rapidly in breadth to a mere point at the end, the dorsum of the distal lobes is also, I believe, more rounded. In his description of *T. Americanum*, as he calls it, Mr. Archer states the end lobe has its angles bidentate. In the only specimen I have seen, the angles end in a very sharp, almost spine-like point. Dr. Leidy found the species abundantly at Newport, Rhode Island, and his figure agrees with mine in this respect. In regard to the name, as there is already an *M. Americanum*, the specific name of Archer cannot be adopted, and for a similar reason it would not do to call it *M. Baileyi*. I have then been forced to give it a new title.

Fig. 4, pl. 13, was drawn by myself from the single specimen I have seen; fig. 4 a was drawn by Dr. Leidy from a Newport specimen.

M. oscitans, RALFS.

M. magna, pæne tam longa quam lata, subpinnatisecta, a vertice elliptico fusiformis, utroque polo bifida; lobis basalibus horizontalibus conico-productis, apice bifidis; lobo polari a lobis basalibus sinu amplo ac rotundato discreto, plus minus convexo, haud raro truncato, rarius leviter retuso, utrinque producto acuminato, plerumque bidentato. (R.)

Diam.—0.0047". *Long.* 0.0039". (R.)

Syn.—*M. oscitans*, RALFS, British Desmidiæ, p. 76. RABENHORST, Flora Europ. Algarum, Sect. III. p. 119.

M. pinnatifida Ktz. RABENHORST, Flora Europ. Algarum, Sect. III. p. 119.

Hab.—Florida; Rhode Island; Bailey.

Fronde about as broad as long, pinnatifid; lateral lobes separated from the terminal by a rounded sinus, horizontal, conical, their extremities bidentate; end lobe short, broad, its lateral projections short, conical, usually bidentate, narrower and shorter than the lateral lobes; ends convex at the centre; tr. v. fusiform, e. f. punctate. (A.)

Remarks.—According to Prof. Rabenhorst *M. pinnatifida*, Ktz., is a variety of *M. oscitans*, different from the typical form only in being smaller, and in having the lobes narrower.

B. Semicellulæ 3-vel 5-lobæ, plerumque radiatim inciso-lobulatæ. Lobi basales assurgentes aut non aut minus a lobo polari remoti.

B. Semicells 3, or 5-lobate, mostly radiately incisely lobulate. Basal lobes assurgent, either close to, or but slightly remote from the end lobes.

* *Semicellulæ trilobæ.*

* *Semicells trilobate.*

M. Americana, (EHRB.) Ktz.

M. magna, oblonga, subpinnatisecta, lobis polaribus paulum remotis, pæne duplo longior quam lata; cytodermate spinuloso unde laborum margines dentato-serrati conspiciuntur; cellula e latere conspecta oblonga, in medio leviter constricta, utroque polo bicornuta; semicellulæ basi tumore plus minus distincto instructæ, fere quinquelobæ, lobis basales latissimi iisdemque profunde bilobati, lobulis late excisis, segmentis dentato-serratis; lobis polaribus plus minus productis, in medio late excisis, segmentis profunde bifidis. (R.)

Diam.—0.0041". *Long.* circa 0.0051". (R.)

Syn.—*M. Americana*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 189.

Hab.—In stagnis, South Carolina; Florida; Bailey.

Fronde angular elliptic, more or less punctate; segments 3-lobed; lateral lobes broad, cuneate, their margins concave, inciso-serrate; and lobe broad, cuneate, end exserted, bipartite at the angles, the subdivisions narrow, and minutely dentate at the extremities; end concave. (A.)

Remark.—Fig. 17, pl. 12, represents a plate of this species.

M. Baileyi, RALFS.

M. parva, oblonga, granulata; semicellulis trilobis, lobis basalibus a lobo polari sinu amplo discretis, excisura acute triangulari in duas lacinas partitis, laciniis e basi latiori in apicem truncatum bidentatum attenuatis; lobo polari e basi angusta longe porrecto, sursum valde dilatato, in vertice leviter et late sinuato, angulis truncato, bidentato. (R.)

Syn.—*M. Baileyi*, RALFS, British Desmidiæ, p. 211.

Hab.—New York; Rhode Island; South Carolina; Florida; Bailey.

Fronde granulated; segments three-lobed; lobes bipartite, end one much exserted. (Ralfs.)

M. ringens, BAILEY.

M. mediocris, oblonga, margine granulata; semicellulis trilobis; lobis lateralibus bipartitis, laciniis divaricatis, apice obtusis, truncatis vel bidentatis; lobo polari e basi angusta sursum valde dilatato, exserto, in vertice leniter sinuato, angulis truncato. (R.)

Syn.—*M. ringens*, BAILEY, Microscopical Observations, pl. 1, fig. 11: Smithsonian Contributions, vol. ii.

Hab.—Florida; Bailey.

Oblong, segments three-lobed, coarsely granulated near the edge; basal lobes subdivided by a deep notch into two rather broad and obtuse or slightly bidentate projections; terminal lobes exserted, emarginate; extremities bidentate or obtuse.

* * *Semicellulæ quinque-lobatæ.*

* * *Semicells 5-lobed.*

M. truncata, (CORDA) BREB.

M. magna, orbicularis, aut lævis aut subtiliter punctata; semicellulis quinquelobis, lobis inter se sinu obtusangulo subangusto discretis, basalibus et intermediis inciso-lobulatis, segmentis acute bidentatis; lobo polari late cuneato, in dorso truncato, modo leviter convexo, modo leviter retuso, angulis aut bidentatis aut integris. (R.)

Diam.—0.003" *Long.* .0036".

Syn.—*M. truncata*, (CORDA,) BREBISSEON. RABENHORST, Flora Europ. Algarum, Sect. III. p. 191.

Hab.—Georgia; Florida; Rhode Island; Bailey. Pennsylvania; Wood. Rhode Island (S. T. Olney); Thwaites.

Frond orbicular, smooth; segments 5-lobed; basal and middle lobes obscurely bipartite, extremities bidentate; end lobe very broadly cuneate, bidentate at the angles, and with a slightly central concavity. (A.)

Remarks.—The dimensions given above were taken from the largest specimens I have seen, but do not at all equal those given by Prof. Rabenhorst, his breadth is .0041". According to the same authority, *M. crenata*, Bréb., is merely a variety of this species.

Fig. 15, pl. 21, represents the outline of a frond of this plant.

M. furcata, Ag.

M. permagna paulo longior quam lata, levis; semicellulis quinque lobis (pæne 7-lobis); lobis omnibus rectis; lobis basalibus angustioribus, bilobulatis, lobulis bifidis, sinu obtusangulo vel acutangulo, segmentis linearibus bidentatis (denticulis sæpe inæquilongis); lobis intermediis duplo latioribus, inciso-bilobis, lobulis iisdem ac loborum basalium; lobo polari nonnihil anguste cuneato, prominulo, in apice plus minus profunde sinuato-vel undulato inciso, angulis bidentatis.

Diam.— $\frac{60}{7500}$ " = .008".

Syn.—*M. rotata*, RALES, British Desmidiæ, p. 71.

M. furcata, AGARDH. RABENHORST, Flora Europ. Algarum, Sect. III. p. 191.

Hab.—South Carolina; Georgia; Florida; Rhode Island; Bailey. New Jersey; Wood.

M. very large, a little longer than broad, smooth; semicells 5-lobed (scarcely 7-lobed); lobes all straight; basal lobe narrower than the intermediate, bilobulate, lobules bifid, their sinuses acute or obtuse, segments linear, bidentate; teeth often long and unequal; intermediate lobes twice as wide as the basal, bilobate, their lobules of the same form as the basal lobe; end lobes narrowly cuneate, prominent, more or less profoundly sinuately or undulately cut at the apex, angles bidentate.

Remarks.—According to Rabenhorst and others, there is a European form of this species in which the marginal teeth are wanting. This may exist in this

country, but I have never met with it. All the specimens which have come under my notice were obtained in "Shepherd's Dam," near Greenwich, Cumberland County, New Jersey. None of them were as large as the maximum of the European measurements of which Rabenhorst gives 0.0109" as the diameter.

Fig. 5, pl. 13, represents a frond of this species, magnified 260 diameters.

M. denticulata, BRÉB. ?

M. permagna, paulo longior quam lata, lævis; semicellulis quinquelobis (pæne 9 lobis); lobis intermediis et basalibus simillimis, bilobatis, lobulis item in lobulis bifidis duobus divis; lobo polare angusto, cuneato, in apice plus minus inciso; margine minute denticulato.

Diam.—Lat. .0092". Long. .011."

Syn.—*M. denticulata*, BRÉBISSEON. RALFS, British Desmidiæ, p. 70, et ARCHER, PRITCHARD'S Infusoria.

M. denticulata, BRÉBISSEON. ? RABENHORST, Flora Europ. Algarum, Sect. III. p. 192.

Hab.—Pennsylvania; Wood. Florida; Bailey.

Very large, a little longer than broad, smooth; semicells with five lobes (scarcely 9); basal and intermediate lobes alike bilobate, lobules also divided into two bifid lobules; end lobe narrow, wedge-shaped, more or less incised at its apex; margin minutely denticulate.

Remarks.—Prof. Rabenhorst gives *M. denticulata*, BRÉB. as merely a variety of *M. furcata*, AG., stating that it only differs from the latter in the marginal incisions and teeth. Not having access to the original description of Brébisson I cannot express an opinion as to whether Prof. R. is correct or not, but the specimen from which the above description was drawn up (and which is figured on plate 13) certainly differs from *M. furcata* very essentially in the arrangement of its lobes, and is, I feel confident, *M. denticulata*, BRÉB. of RALFS and ARCHER.

Fig. 6, pl. 13, is a drawing of this plant, as seen by myself, magnified 260 diameters.

M. radiosa, AG.

M. maxima, orbicularis, lævis, antecedenti simillima, differt in primis segmentis ultimis tumidis in apicem bi-tri-fidum attenuatis, lobo polari vix prominulo, apice sinuato, ad utrumque angulum bi-tri-dentato. (R.) *Species mihi ignota.*

Diam.—0.0076". (R.)

Syn.—*M. radiosa*, AGARDH. RABENHORST, Flora Europ. Algarum, Sect. III. p. 192.

Hab.—Florida; Bailey.

Frond orbicular, smooth; segments 5-lobed; basal lobes twice, middle lobes generally thrice dichotomous, ultimate subdivisions inflated, attenuate towards the end, bidentate; end lobes emarginate, its angles dentate. (A.)

M. fimbriata, RALFS.

M. magna, orbicularis, lævis (nonnunquam superficie aculeis singulis sparsis); semicellulis quinquelobis, lobis omnibus confertis, basalibus angustioribus, repetito bilobulatis, lobis intermediis duplo latioribus, repetito-bilobulatis, lacinulis extremis leviter emarginatis, in angulis spinis elongatis armatis; lobo polari prominulo, in apice obtuse sinuato-vel-undulato-emarginato, angulis lateralibus rotundatis, ad marginem superiorem spinis singulis vel geminis obsito (rarius nudo). (R.) *Species mihi ignota.*

19 August, 1872.

Diam.—0051"—0078". (R.)

Syn.—*M. fimbriata*, RALFS, British Desmidiæ, p. 71, et RABENHORST, Flora Europ. Algarum, Sect. III. p. 193.

Hab.—South Carolina; Florida; Bailey.

Fronde orbiculari, smooth; segments 5-lobed, basal lobes twice, middle lobes thrice dichotomous; ultimate subdivisiones acutely bidentate; end lobe very slightly exserted, its angles very slightly produced, bidentate, ends emarginate. In transverse view is seen an inflated protuberance just over the central isthmus, which may possibly exist in other species of *Micrastrias*. (A.)

M. papillifera, BRÉB.

M. orbicularis, superficie laevis, margine extremo dentato papillifera; semicellulis quinquelobis; lobis basalibus et intermediis æquilatis, bilobatis; lobulis bifidis, laciniis linearibus bidentatis, dentibus papilliferis; lobo polari vix prominulo, in apice sinuato, angulis et margine dentato-mucronatis. (R.) *Species mihi ignota*.

Diam.—0.0045". (R.)

Syn.—*M. papillifera*, BRÉBISSEON. RALFS, British Desmidiæ, p. 72, et RABENHORST, Flora Europ. Algarum, Sect. III. p. 194.

Hab.—Florida; Rhode Island; Bailey.

Fronde orbiculari, having the principal sinuses bordered by a row of minute granules, otherwise smooth; segments 5-lobed; basal and middle lobes twice dichotomous, their ultimate shallow subdivisions terminated by two, sometimes three, gland-like teeth; end lobe emarginate, its angles dentate. Sporangium as in *M. denticulata*, but considerably smaller. (A.)

M. granulata, WOOD (sp. nov.)

M. magna, suborbicularis, arete granulata; semicellulis quinquelobis, lobis inter se sinu angusto discretis, basalibus et intermediis plerumque integris, lobo polari supra valde dilatato, in dorso medio leviter retuso; marginibus valde crenatis.

Diam.—Long. $\frac{1}{2}\frac{1}{40}$ " = .0043". Lat. $\frac{43}{12000}$ " = .0036".

Hab.—South Carolina, (Ravenel)

Large, suborbicular, closely granulate; semicells 5-lobed, lobes separated by narrow sinuses; basal and intermediate lobes mostly entire; end lobe distally broadly dilated, broadly and very shallowly emarginate; margin of frond strongly crenate.

Remarks.—The only specimens of this species that I have seen were collected by Prof. Ravenel in a shallow ditch near Aiken, South Carolina, where they formed a greenish, gelatinous mass, with numerous desmids and diatoms. It is most closely allied to *M. truncata*, from which it is separated by its entire lateral lobes, by its granulated surface, and its crenated margins. It also does not apparently attain as large a size as that species. The granules are very small in the central portion of the frond, but become larger as they approach the margin.

Fig. 16, pl. 21, represents an empty frond of this species, magnified 460 diameters.

M. Jenneri, RALFS.

M. magna, oblonga, plerumque subtiliter granulata; semicellulis quinquelobis; lobis basalibus et intermediis æquilatis, confertis, euneatis, bilobulatis; lobo polari late truncato vel late rotundato, in medio interdum leviter et obtuse emarginato, interdum nonnihil profunde emarginato.

Diam.—Lat. $\frac{45}{12000}$ "— $\frac{75}{12000}$ " = .006"—.0062". Long. $\frac{65}{75000}$ "— $\frac{75}{120000}$ " = .0062"—.0087".

Syn.—*M. Jenneri*, RALFS, British Desmidiæ, p. 76.

Hab.—Prope Philadelphia; Wood. South Carolina; (Ravenel)

Large, oblong, for the most part finely granulate; semicells 5-lobed; lobes wedge-shaped; basal and intermediate, about equally broad; end lobe broadly truncate or broadly rounded, in the middle sometimes slightly and obtusely emarginate, sometimes rather deeply emarginate.

Remarks.—I have found this species near Philadelphia, and also received it from Prof. Ravenel, by whom it was collected in South Carolina. The American plant differs from the typical form in not having the ultimate lobules emarginate, they being merely a little hollowed out in the centre, and sometimes scarcely this. The angles in some specimens are also more acute. Mr. Archer, however, speaks of a variety occurring in England, in which these lobules are not emarginate, and I do not think characters can be found separating the American from the European forms. The median suture is in all the specimens very narrow and deep, a mere line, as it were, extending nearly to the centre.

Fig. 7, pl. 13, represents a frond of this species.

M. Torreyi, BAILEY.

M. permagna, oblongo-orbicularis, lævis, profundissime lobata; semicellulis quinquelobis, lobis basalibus profunde bifidis, laciniis inferioribus apice bidentatis, superioribus integris, lobis intermediis profunde trifidis, laciniis superioribus bidentatis, inferioribus integris, lac. omnibus lanceolatis acuminatis, inferioribus paulum incurvis, superioribus recurvis; lobo polari non prominente, e basi angusta seusim dilatato, in vertice acute sinuato, angulis integris acuminatis. (R.) *Species mihi ignota.*

Syn.—*M. Torreyi*, BAILEY. RALFS, Brit. Desmidiæ, p. 210.

Hab.—Prope Princetown, New Jersey; Bailey.

Frond smooth; segments 5-lobed; basal lobes bifid, middle lobes trifid, the subdivisions nearest the opposite segments and those nearest the terminal lobe bidentate at the apex; the intermediate three terminating in acute points; all somewhat inflated and tapering; terminal lobe narrow, not exerted, spreading at the angles into divergent tapering points, ends slightly emarginate. (A.)

M. foliacea, BAILEY.

M. parva, subquadrata, lævis; semicellulis trilobis, lobis lateralibus profunde bifidis (unde rectior semicell. quinquelobæ), lobulis inæqualiter inciso-dentatis, lobulis inferioribus rectis, superioribus recurvis; lobo polari plus minus prominente, anguste cuneato, in vertice plus minusve emarginato, angulis aut acutis integris aut productis, bidentatis. R. *Species mihi ignota.*

Syn.—*M. foliacea*, BAILEY. RALFS, British Desmidiæ, p. 210.

Hab.—"Worden's Pond, Rhode Island; Bailey."

Frond subquadrate, smooth; segments 3-lobed; lateral lobes deeply bipartite, inciso-dentate, their margins concave, inciso-serrate; end lobe broad, cuneate, and exerted, bipartite at the angles, the subdivisions narrow, and minutely dentate at the extremities; end concave. (A.)

Genus STAUSTRUM, MEYEN.

Cellulæ libere natantes, in medio plus minus profunde constrictæ; semicellulæ a vertice 3-6 angulares vel radiatæ. Cytioderma aut læve aut punctatum aut verrucosum aut aculeatum, nonnunquam ciliis vel pilis obsessum.

Cells swimming free, more or less profoundly constricted in the middle; semicells when seen from the vertex 3 to 6 angular or radiate. Cytoderm either smooth or punctate, or verrucose or aculeate, sometimes covered with hairs or cilia.

A. CYTIODERMA LÆVE VEL RARISSIME SUBTILITER PUNCTATUM.

CYTIODERM SMOOTH OR VERY RARELY VERY FINELY PUNCTATE.

1. *Semicellularum anguli rotundati*.*Angles of the semicells rounded.***St. muticum**, BRÉB.

St. a fronte orbiculare, læve, profunde constrictum, nudum, vel mucro plus minusve firmo involutum; semicellulis ellipticis, a vertice conspectis 3-4 angularibus (rarius quinquangularibus) angulis rotundatis, lateribus leviter sinuato-retusis; zygosporis aculeatis, aculeis elongatis, subulatis, furcatim fissis. (R.) *Species mihi ignota.*

Diam.—0.0013"—0.00147". (R.)

Syn.—*S. muticum*, BRÉBISSEON. RABENHORST, Flora Europ. Algar., Sect. III. p. 200.

Hab.—South Carolina; Rhode Island; Bailey.

Segments in f. v. elliptic, smooth, without spines; e. v. with three or four broadly rounded angles, sides concave. Sporangium beset with numerous elongate somewhat stout spines, forked at the apex. (A.)

St. orbiculare, (EHRB.) RALFS.

St. suborbiculare, læve, sæpius mucro matricali involutum; semicellulis divergentibus, semiorbicularibus, dorso nonnunquam elevatis, angulis plus minus late rotundatis, lateribus plus minus sinuato-retusis; zygosporarum aculeis elongatis, subulatis, integris. (R.)

Diam.—.002".

Syn.—*St. orbiculare*, (EHRB.) RALFS, British Desmidiæ, p. 125. RABENHORST, Flora Europ. Algarum, Sect. III. p. 200.

Hab.—Rhode Island; Bailey. Pennsylvania; Wood. Rhode Island; (S. T. Olney) Thwaites.

Segments in f. v. semiorbicular, smooth, without spines; e. v. with three broadly rounded angles, sides slightly concave. (A.)

Remarks.—Fig. 17, pl. 21, represents the outline of the end view of a frond of this species. Fig. 8, pl. 13, is a drawing of the front view of a living frond.

2. *Semicellularum anguli mucronati vel aristati*.*Angles of the semicells mucronate or bristly.***St. longispinum**, (BAILEY) ARCHER.

St. magnum triangulare, læve, angulis in aculeos geminos validos subulatos longe productum, lateribus subplanum. (R.) *Species mihi ignota.*

Syn.—*Didymocladon longispinum*, BAILEY, Microscopical Observations.

Hab.—Florida; Bailey.

"Large, smooth, triangular, with two long spines at each angle." Bailey.

St. dejectum, BRÉBISSEON.

St. læve, parvum, sinu amplo, obtusangulo (vel acutangulo); semicellulis ellipticis (vel subtriangularibus), dorso nonnihil convexo, utroque fine in aculeum achroum rectum vel varie curvatis productis; a vertice triangularibus (vel quadrangularibus), angulis sæpe rotundatis aculeo interdum obsolete imposito.

Diam.—Lat. $\frac{10}{12000}$ "— $\frac{12}{12000}$ " = 0008"—.001". Long. $\frac{10}{12000}$ "— $\frac{12}{12000}$ " = .0008"—.0001".

Syn.—*Staurastrum dejectum*, BRÉBISSEON. RABENHORST, Flora Europ. Algarum, Sect. III. p. 203.

Hab.—South Carolina; (Ravenel) Wood.

Smooth, small; sinus ample, obtuse angled (sometimes acute angled?); semicells elliptic (or subtriangular?), with the dorsum slightly convex, at the angles with a straight or curved transparent spine; from the vertex triangular (or quadrangular?), angles often rounded, with a sometimes obsolete spine superimposed.

Remarks.—This species was collected near Aiken, South Carolina, by Prof. Ravenel, who found it forming with various diatoms and desmids a slimy mass in a feebly running ditch. It agrees very well with the European form, except that it is not so large (at least the largest I ever measured did not come up to the size of their transatlantic brethren), neither does it appear to vary quite so much. In the description, I have placed in brackets those characters in which the European form varies, and the specimens I have seen do not.

Fig. 18, pl. 21, represents outline of end of a semicell, magnified 750 diameters. Fig. 9, pl. 13, a front view, and 9 *a* the end view, of the living frond, magnified diameters.

St. aristiferum, RALFS.

St. læve, *St. cuspidatum* quodammodo simile, et eadem magnitudine sed isthmo destitutum; semicellulis tumidis, in media parte subrotundatis, lateraliter in lobum, basi constrictum, apice aristatum productis, lobis divergentibus, a vertice tri-quadrilobo-radiatis, radiis strictis æquidistantibus cruciatim dispositis, interstitiis profunde excisis. (R.) *Species mihi ignota.*

Diam.—Incl. arist. 0.0014". (R)

Syn.—*St. aristiferum*, RALFS, British Desmidiæ, p. 123. RABENHORST, Flora Europ. Algarum, Sect. III. p. 204.

Hab.—Georgia; Rhode Island; Bailey.

Segments smooth, in f. v. prolonged at each lateral extremity into a mamillate projection, which is terminated by a subulate, acute straight awn, the awns divergent, e. v. with three or four angles; angles inflated mamillate, terminated by an awn, sides deeply concave in the centre. (A.)

St. Lewisii, WOOD.

St. læve; sinu amplissimo, spinulo parvo armato et cum angulo obtuso; isthmo nullo; semicellulis a fronte late triangularibus, a vertice triangularibus et cum angulis nonnihil tumidis, et rotundatis; angulis spino maximo, robusto, acuto armatis.

Diam.—Long. cum. spin. $\frac{1}{4} \frac{1}{50}'' = .0025''$; lat. cum. spin. $\frac{27}{12000}'' = .00225''$. Sine spin.: long. $\frac{1}{8} \frac{1}{50}'' = .001666''$; lat. $\frac{3}{12000}'' = .001666''$. Spin.: long. $\frac{1}{5} \frac{1}{50}'' = .000666''$

Syn.—*St. Lewisii*, WOOD, Proc. Acad. N. S. 1870.

Hab.—In lacu Saco; (Lewis) Wood.

Smooth, with a very ample sinus, which is armed with a small spine and has a very obtuse angle; isthmus absent; semicells from the front broadly triangular, from the vertex triangular, with the angles somewhat tumid and rounded; angles armed with a very large acute robust spine.

Remarks.—This desmid is most closely allied to *St. aristiferum*, Ralfs, but differs from it in outline as seen from the front, there being no mamellation of the ends. The spines in the sinuses are always wanting in the European species.

Fig. 19, pl. 21, represents the outline of the end of a semicell, magnified 750 diameters. Fig. 11, pl. 13, represents the perfectly formed frond, magnified 750 diameters.

B. CYTIODERMA GRANULATUM VEL VERRUCOSUM.

CYTIODERM GRANULATE OR WARTY.

1. *Semicellulæ a vertice 3-7 angulares; anguli plus minus radiatim elongati.**Semicells seen from the vertex 3-7 angled; angles more or less radiately produced.***St. margaritaceum**, EHRB.

St. medioere, granulatum; semicellulis convergentibus, subfusiformibus, in medio tumidis, utrinque productis, truncatis, a vertice orbicularibus, 5-7 radiatis, radiis obtuse truncatis aërois, hyalinis, granulato-margaritaceis. (R.) *Species mihi ignota.*

Diam.—0.00135"—0.0017". (R.)

Syn.—*St. margaritaceum*, (EHRB.) MENEGHENI. RABENHORST, Flora Europ. Algarum, Sect. III. p. 206.

Hab.—South Carolina; Georgia; Florida; Rhode Island; Bailey.

Segments in f. v. gradually widening upwards, rough with pearly granules, outer margin convex, produced at each side into a colorless, more or less attenuate, short process, having the granules in transverse lines, blunt and entire at the apex, e. v. circular, bordered by from five to seven short, narrow, obtuse, colorless, granulate marginal rays. (Archer.)

St. dilatatum, EHRB.

St. parvum, granulatum; semicellulis rectis, cylindrico-fusiformibus, non tumidis, utroque fine obtusis vel subtruncatis, a vertice 3-4-5 radiatis, radiis latioribus, truncatis vel rotundatis, aërois, hyalinis, granulato-margaritaceis. (R.) *Species mihi ignota.*

Diam.—0.0008"—0.0011". (R.)

Var. **alternans**.

Semicellulis ellipticis rectis, utroque fine rotundatis, a vertice triradiatis, radiis obtusis, alternantibus cum semicellulæ inferioris. (R.)

Var. **tricornis**.

Semicellulis fusiformibus, nonnunquam in medio subtumidis, haud raro isthmo distincto conjunctis, a vertice 3-4 angularibus, angulis truncatis vel obtusis, plus minus radiatim productis. (R.)

Hab.—Georgia; Florida; Rhode Island; Bailey.

Syn.—*S. alternans*, BRÉBISON. *Var. alternans et tricornis*. RABENHORST, Flora Europ. Algarum, Sect. III. p. 207.

Remarks.—Prof. Rabenhorst considers *St. alternans* and *tricornis*, as simple varieties of *St. dilatatum*, whilst both Archer and Ralfs describe them as distinct. I have not seen either of them, and am not therefore entitled to offer an opinion. Mr. Archer describes the two species as follows:—

St. alternans, BRÉB.

Segments in front view elliptic or oblong, two or three times as broad as long, separated by a wide sinus, twisted, unequal; rough with very minute pearly granules; e. v. with three obtuse and rounded angles, forming short, not colorless rays, alternating with those of the other segments, sides concave. L. $10\frac{1}{37}$ ". Br. $11\frac{1}{6}$ ".

St. tricornis, BRÉB.

Segments in f. v. somewhat fusiform, often twisted, rough with minute puncta-like granules, tapering at each side into a short, usually colorless process, blunt or divided at the apex;

e. v. tri- or quadriradiate, processes short, usually colorless, sides somewhat concave. Sporangium orbicular, beset with spines ultimately branched at the apex. L. $1\frac{1}{278}''$ — $5\frac{1}{2}''$. B. $5\frac{1}{48}''$.

2. *Semicellulæ triangulares; anguli non producti, obtusi vel rotundati.*

Semicells triangular; the angles not produced, obtuse or rounded.

St. punctulatum, BRÉB.

St. parvum, punctulato-granulosum; semicellulis enormiter ellipticis, dorso late rotundatis, a vertice triangularibus; angulis non productis, obtuse rotundatis; lateribus modice retusis.

Diam.—Lat. $7\frac{1}{500}'' = .0012''$.

Syn.—*S. punctulatum*, BRÉBISSON. RALFS, British Desmidiæ. RABENHORST, Flora Europ. Algarum, Sect. III. p. 208.

Hab.—Pennsylvania; Wood.

Small, punctulate-granulate; semicells irregularly elliptic, with the dorsum broadly rounded from the vertex triangular; angles not produced, obtusely rounded; sides somewhat retuse.

Remarks.—This desmid is exceedingly common around Philadelphia, growing in the greatest abundance upon the face of wet dripping rocks. It is represented, fig. 10, pl. 13.

St. crenatum, BAILEY.

St. duplo circiter longius quam latum, in medio utrinque exsecutione profunda rotundata; semicellulis e basi cuneata flabelliformibus, margine superiore crenatis, a vertice triangularibus, angulis rotundato-truncatis, crenatis, lateribus sinuatis glabris. (R.) *Species mihi ignota.*

Syn.—*St. crenatum*, BAILEY. RALFS, British Desmidiæ, p. 214. RABENHORST, Flora Europ. Algarum, Sect. III. p. 220.

“Segments cuneate; outer margins crenate; end view with three truncate and crenate angles.”

3. *Semicellulæ vertice 3-7 radiata; radii in apice plerumque bi- tri- fidi vel bi- tri- spini.*

Semicells 3-7 radiate at the vertex; radii bi- or tri-fid, or bi- or tri-spinous at the apex.

St. polymorphum, BRÉB.

St. semicellulis ellipticis, subtiliter granulatis vel tenuissime spinulosis, in medio magis minusve inflatis, haud raro ventricosis, rectis, nonnunquam incurvis, utrinque processu plus minus elongato, lineari, in apice 3-4 fido vel spinulis 3-4 tenuissimis instructis, a vertice 3-4-5-6-7 radiatis, radiis achrois, aut trifidis aut rotundatis, trispinis. (R.) *Species mihi ignota.*

Syn.—*St. polymorphum*, BRÉBISSON. RALFS, British Desmidiæ, p. 135. RABENHORST, Flora Europ. Algarum, Sect. III. p. 209.

Diam.—Long. $0.001''$. Lat. $0.00087''$. (R.)

Hab.—Florida; Bailey.

Segments in f. v. broadly elliptic or almost circular, rough with minute granules (sometimes with a few minute scattered spines), processes short, stout, tipped by three or four divergent spines; e. v. with three, four, five, or six angles each produced into a short, stout process. Sporangium orbicular, beset with elongate spines, forked or branched at the apex. Archer.

Var. cyrtocerum. (*St. cyrtocerum*, BRÉB.)

Majus, ad $\frac{1}{9}''$, longum, semicellulis introrsum ventricosis, dorso late rotundatis, utrinque processu elongato, plerumque incurvo apice bi- vel tri-cuspidato instructis, a vertice trifidiatis, radiis rectis vel leniter curvatis, in apice aut bi- aut tri-cuspidatis. (R.)

Syn.—*Var. St. cystoceram*, BRÉBISSEON. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 210.

Hab.—Rhode Island; (S. T. Olney) Thwaites.

Segments in f. v. subeuneate, gradually widening upwards, truncate at the end margin, rough with minute granules, the lateral processes incurved, divided at the apex; e. v. triradiate, processes short, curved, sides slightly concave. L. $\frac{1}{800}$ ". B. $\frac{1}{500}$ ". (Archer.)

St. paradoxum, MEYEN.

St. semicellulis inflatis, dorso rotundatis vel rectilinearibus, angulis superioribus in radium elongatum achroum hispidum, apice trifurcatim productis, sæpius radio æquali interposito a vertice tri- vel quadriradiatis, radiis strictis, trifurcatis, longitudine corporis diam. æquantibus vel superantibus. (R.)

Diam.—Cum rad. .0015".

Syn.—*St. paradoxum*, MEYEN. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 210.

Hab.—In lacu Saco, New Hampshire; (Lewis) Wood.

Semicells inflated, dorsum rounded or rectilinear, with superior angles produced into elongate, transparent, hispid radii with trifurcate apices, often furnished also with intermediate equal radii; from the vertex three or four radiate, radii straight, trifurcate, equalling or longer than the diameter of the body.

Remarks.—I am indebted to Dr. Lewis for specimens of this species, which he collected at Saco Lake.

Fig. 20, pl. 21, represents the end view of an empty frond.

St. arachne, RALFS.

St. parvum, gracile, granulato-asperum; semicellulis introrsum ventricosoglobosis, angulis superioribus in cornu gracile, incurvum, apice obtusum, elongatis, a vertice pentagonis, quinque-radiatis, radiis elongatis linearibus achrois, obtusis, rectis vel leniter curvatis asperis. (R.)

Diam.—Sine rad. .0005", cum rad. .00167".

Syn.—*St. arachne*, RALFS, *British Desmidiæ*, p. 136. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 210.

Hab.—In lacu Saco, New Hampshire, (Lewis) Wood.

Segments in f. v. suborbicular, rough with minute granules, lower margin turgid, outer convex, tapering at each side into an elongate, slender, incurved process having the granules thereon in transverse lines, entire at the apex; e. v. circular, bordered by five slender, linear, colorless marginal rays.

Remark.—Fig. 21, pl. 21, represents an outline of the end view of the semicell.

St. gracile, RALFS.

St. medioere, granulato asperum, granulis in series transversas ordinatis; semicellulis ventre valde inflatis, dorso truncatis, angulis in cornu rectum achroum gracile apice trifidum productis, a vertice triradiatis, lateribus sinuatis. (R.) *Species mihi ignota.*

Diam.—0.0022". (R.)

Syn.—*St. gracile*, RALFS, *British Desmidiæ*, p. 136. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 211.

Hab.—South Carolina; Florida; Georgia; Rhode Island; Bailey. Rhode Island; (Olney) Thwaites.

Segments in f. v. triangular, ends truncate, rough with minute granules, tapering at each side into elongate, straight, slender, horizontal processes, terminated by three or four minute spines; e. v. triradiate, processes straight, sides concave. (A.)

C. CYTIODERMA PILOSUM, SPINULOSUM VEL ACULEATUM.

CYTIODERM PILOSE, SPINULOSE OR THORNY.

St. polytrichum, PERTY.

St. medioere, tam longum quam latum, profunde constrictum, sinu acutangulo ampliato, superficie undique setosum; semicellulis ellipticis vel subellipticis, divergentibus, dorso subplanis, ventre tumidis, margine setoso-ciliatis, a vertice triangularibus, angulis obtusis, lateribus subrectis. (R.)

Diam.— $\frac{1}{7} \frac{3}{100}$ " = .0017".

Syn.—*St. polytrichum*, PERTY. RABENHORST, Flora Europ. Algarum, Sect. III. p. 214.

Hab.—Prope Philadelphia; Wood.

Moderately large, about as long as broad, profoundly constricted, with the acute angled sinus widened, surface everywhere furnished with setæ; semicells elliptical or subelliptical, divergent, the dorsum nearly plane, their belly swollen, the margin setose-ciliate, from the vertex triangular, the angles obtuse.

Remarks.—This desmid appears to be rare in this country, as it probably is also in Europe. I have seen but a single specimen, which I found amongst other algæ near Cheltenham Hills, north of the city. It agreed in all respects with the description of Rabenhorst, as given above.

Fig. 12, pl. 13, is a drawing of this plant, also fig. 23, pl. 21.

St. Ravenelii, WOOD. (sp. nov.)

St. medioere, paulo longius quam latum; semicellulis a fronte ellipticis, a vertice triangularibus cum lateribus convexis vel leniter retusis et angulis rotundatis; isthmo connexivo subnullo, lato; sinu acutangulo; cytiodermate spinis acutis, robustis numerosis armato.

Diam.—Long. $\frac{7}{12000}$ " = 0.0014". Lat. $\frac{1}{1000}$ " = 0.001".

Hab.—South Carolina; (Ravenel) Wood.

Medioere, a little longer than broad; semicells from the front elliptical, from the vertex triangular, with the sides convex or slightly retuse, and the angles rounded; connecting isthmus obsolete, broad sinus acute-angled; cytioderm armed with numerous acute robust spines.

Remark.—Fig. 22, pl. 21, represents the front view of an empty frond of this plant; fig. 22 a, the side view, and fig. 22 b, the end, all magnified 750 diameters.

St. hirsutum, (EHRB.) BRÉB.

St. magnum, tertiam partem circiter quam longius quam latum, plus minus dense spinulosum, sinu plus minus lineari, acutangulo; semicellulis late ellipticis vel subsemiorbicularibus, spinis tenuibus strictis hirsutis, a vertice triangularibus, angulis obtuse rotundatis, lateribus rectis vel leniter convexis. (R.) *Species mihi ignota.*

Diam.—Sine spinis 0.0015". Zygospor. 0.0022". (R.)

Syn.—*St. hirsutum*, (EHRENBERG) BRÉBISSEON. RABENHORST, Flora Europ. Algarum, Sect. III. p. 211.

Hab.—Florida; Rhode Island; Bailey. Rhode Island; (S. T. Olney) Thwaites.

Segments in f. v. semiorbicular, separated by a linear constriction, covered with very minute, very numerous close set hair-like spines; e. v. with three broadly rounded angles, the spines evenly and numerous scattered; sides slightly convex. Sporangium orbicular, beset with short spines, branched at the apex. (A.)

St. Hystrix, RALFS.

St. parvum, tertiam partem longius quam latum, angulis aculeatum (cæterum læve), sinu acutangulo; semicellulis subquadratis, angulis late rotundatis, dorso planis, a vertice 3-4 angularibus, angulis late rotundatis, plus minus dense aculeatis. (R.) *Species mihi ignota.*

Diam.—0.001"—0.00089". (R.)

Syn.—*St. Hystrix*, RALFS, British Desmidiæ, p. 128. RABENHORST, Flora Europ. Algarum, Sect. III. p. 213.

Hab.—Rhode Island; (S. T. Olney) Thwaites.

Segments in f. v. subquadrata, extremities somewhat rounded, end margin nearly straight, furnished with a few scattered, subulate, acute spines, chiefly confined to the lateral extremities; e. v. with three or four broadly rounded angles, the spines scattered, chiefly confined to the extremities, sides concave. L. $1\frac{1}{10}7\frac{1}{2}$ "— $1\frac{1}{10}2\frac{1}{6}$ ". Br. $1\frac{1}{10}5$ "— $3\frac{1}{5}4$ ".

St. Cerberus, (BAILEY) ARCHER.

St. parvum, tam longum quam latum, sinu rotundato, superficie lævi; semicellulis oblongis utroque fine sinuato-truncatis, angulis in aculeum cuspidatum productis, in medio sursum et deorsum prominentiis geminis in aculeum elongatis instructis, a vertice triangularibus, angulis in apice truncato-vel sinuato-bi-cuspidatis, sub apice aculeis geminis brevibus præditis. (R.) *Species mihi ignota.*

Diam.—Cum acul. 0.00114"—0.0013". (R.)

Syn.—*Didymocladon Cerberus*, BAILEY, Microscopical Observations.

St. Cerberus, (BAILEY) ARCHER. RABENHORST, Flora Europ. Algar., Sect. III. p. 215.

Hab.—Florida; Bailey.

Small, deeply constricted, segments three-lobed, lobes with four teeth, two of which project upwards and two downwards at each truncated angle. (A.)

D. CYTIODERMA PROCESSIBUS NUMEROSIS, APICE PLERUMQUE TRUNCATIS ET DENTATO-FISSIS MUNITUM.

CYTIODERM WITH NUMEROUS PROCESSES, WHOSE APICES ARE MOSTLY TRUNCATE AND DENTATELY TORN.

St. furcigerum, BRÉB.

St. validum, submagnum, circiter tam longum quam latum, læve vel subtiliter granulatam, plerumque profundissime constrictum, sinu angusto lineari; semicellulis oblongo-ellipticis, plus minus tumidis, angulis in processus bifurcum aut rectum aut divergentem longe productis, dorso processibus similibus 2, 3, 4, instructis, omnibus processibus achrois granulato-dentatis, granulis in series transversas ordinatis, a vertice 3-, 4-, 6-, 7-, 8-, 9-angularibus vel radiatis, angulis plus minus tumidis, in processus crassum achroum asperum in apice fissum productis. (R.) *Species mihi ignota.*

Long. Sine process, 0.0018"—0.0019"; e. pr. 0.003"—0.0032". Lat. sine proc. 0.00185"; e. pr. 0.0027". (R.)

Syn.—*Staurastrum furcigerum*, BRÉBISSEON. RABENHORST, Flora Europ. Algar., Sect. III. p. 219.

Didymocladon furcigerus, RALFS, British Desmidiæ.

Hab.—South Carolina; Florida; Rhode Island; Bailey.

St. munitum, WOOD.

St. submagnum, fere $\frac{1}{2}$ plo longius quam latum, medio leviter constrictum, semicellulis a fronte

enormiter hexagonis, angulis in processus rectos et divergentes productis, dorso processibus similibus 4-5 instructo; semicellulis a vertice polygonis vel suborbicularibus margine processibus numerosis, plerumque 9 instructo; dorso processibus 5-8 instructis; processibus omnibus similibus, granulato-dentatis, apice achroo simplicibus, bifurcatis vel fissis.

Diam.—A vertice eum processibus, $\frac{5.1}{2000}'' = .00475''$. Sine process. $\frac{2.5}{2000}'' = .002''$.

Syn.—*St. munitum*, Wood, Proceed. Ac. Nat. Sc., 1869.

Hab.—In lacu Saco, New Hampshire; (Lewis) Wood.

S. rather large, about one-half longer than broad, slightly constricted in the middle; semicells from the front irregularly hexagonal, the angles prolonged in straight divergent processes, and the surface furnished with four or five similar ones; semicells from the vertex polygonal or suborbicular, the margin furnished with numerous processes, mostly about nine, and also with 5-8 on the dorsum; processes all similar, granulate-dentate, their transparent apices simple, bifurcate or torn.

Remarks.—This species is most closely allied to *St. furcigerum*, Bréb., from which it is at once distinguished by the orbicular vertex. The constriction between the semicells is also very different. In *St. munitum* it is a gradual, not very deep, hour-glass contraction; in *St. furcigerum* it is very narrow and linear.

Fig. 13 *a*, pl. 13, is a front view of this plant magnified 260 diameters; fig. 13 *b*, the end view of the same.

St. eustephanum, (EHRB.) RALFS.

St. laterum integrorum angulis productis apice spinulosis, spinularum furcatarum corona media dorsali. (R.) *Species mihi ignota*.

Syn.—*Desmidiium eustephanum*, EHRENBERG, Verbreitung und Einfluss der Mikrosk. Lebens in Süd- und Nord-Amerika, t. 4, f. 23.

Staurastrum eustephanum, (EHRB.) RALFS, British Desmidiæ, p. 215.

Stephanoxanthium eustephanum, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 221.

Staurastrum eustephanum, RALFS. RABENHORST (*loc. cit.*)

Hab.—West Point, New York; Bailey.

End view triangular with six emarginate spines on the upper surface; each angle terminated by a short ray tipped with spines. (Ralfs)

St. senarium, (EHRB.) RALFS.

Antecedenti simile sed laterum parietibus spinulis furcatis binis (sex), corona dorsali senaria. (R.) *Species mihi ignota*.

Syn.—*Desmidiium senarium*, EHRENBERG, Verbreitung. T. IV.

Stephanoxanthium senarium, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 220.

Staurastrum senarium, (EHRB.) RALFS, British Desmidiæ. RABENHORST, (*loc. cit.*)

Segments smooth in end view with three angles, each terminating in a short process tipped by minute spines, without lateral processes, but with six others confluent at their bases on the upper surface, divergent and forked. (Archer.)

Genus XANTHIDIUM, EHRB.

Cellulæ singulæ vel geminæ concatenatæ, inflato-rotundatæ, profunde constrictæ; semicellulæ compressæ, oblongæ, hemisphæricæ vel subquadrangulares, centro in tuberculum rotundatum vel truncatum et denticulatum protuberantes, ex transverso oblongo-rotundatæ. Cytoderma firmum setis, aculeis vel spinis simplicibus aut bi- tri-furcato-divisis armatum. Massa chlorophyllacea radiatim expansa. Zygosporæ armatæ. (R.)

Cells single or geminately concatenate, inflated, profoundly constricted; semicells compressed, oblong, hemispherical or subquadrangular, protruding in the centre as a rounded truncate or denticulate tubercle. Cytoderm firm, armed with setæ, or simple, or bi- tri-furcately divided spines. Chlorophyl radiately expanded. Zygosporæ armed.

Remark.—It has so happened that I have identified but a single species of this genus.

X. aculeatum, EHRB.

X. parvum, singulum, sparsum, diametro ipse subæquale, ex obliquo ellipsoideum, diametro duplo longius, constrictione obtusa lineari, semicellulis oblongis subreniformibus, basi subplanis, dorso late rotundatis, tuberculo centrali minus elevato, truncato, margine autem crenato-dentato; cytodermate undique aculeis subulatis obsito. (R.) *Species mihi ignota.*

Diam.—(Sine aculeis) 0.0025"—0.0029". (R.)

Syn.—*Xanthidium aculeatum*, EHRENBURG. RABENHORST, Flora Europ. Algarum, Sect. III. p. 222.

Hab.—Prope Savannah, Georgia; Bailey.

Frond in f. v. broader than long; constriction deep, linear; segments somewhat reniform; spines subulate, short, scattered, chiefly marginal; central protuberance cylindrical, truncate, border minutely dentate. (A.)

X. Arctiscon, EHRB.

X. semicellulis globosis, binis, aculeatis, aculeis numerosis undique sparsis crassis asperis apice trilobis. (R.) *Species mihi ignota.*

Syn.—*X. Arctiscon*, EHRENBURG. RABENHORST, Flora Europ. Algarum, Sect. III. p. 224.

Hab.—America borealis; Ehrenberg.

Frond in f. v. about as long as broad; constriction forming a wide notch; segments narrowed at the base, with broadly rounded ends; spines numerous, restricted to the outer margin, scattered, elongate, stout, terminated by three or four diverging points. (Archer.)

X. armatum, (BRÉB.) RALFS.

X. maximum, validum, solitarium vel binatim conjunctum, diametro plerumque duplo longius; semicellulis subcordatis vel angulari-rotundatis tuberculo centrali subelevato, truncato, margine granulato-dentato præditis; cytodermate verruculoso et processibus sæpius geminatis truncatis apice inciso-furcatis instructo. (R.)

Syn.—*Xanthidium armatum*, (BRÉBISSE) RALFS, British Desmidiæ et RABENHORST, Flora Europ. Algarum, Sect. III. p. 222.

Hab.—South Carolina; Florida; Bailey. Saco Lake; (Lewis) Wood.

Frond large, in f. v. twice as long as broad; constriction deep, linear; segments broadest at the base; ends rounded or somewhat truncate; spines in pairs, principally marginal, short, stout, terminated by three or four divergent points; central projections cylindrical truncate, the border dentate; e. f. punctate. Sporangium large, orbicular, with depressed tubercles, perhaps immature. L. $\frac{1}{8}$ ". B. $\frac{1}{7}$ ". (A.)

Remark.—Fig. 17, pl. 13, is a front view of a frond, magnified 260 diameters.

X. bisenarium, EHRB.

X. semicellulis globosis subangulosis, binis, aculeatis; aculeis fasciculatis, fasciculis in quovis globulo senis. *Species mihi ignota.*

Syn.—*X. bisenarium*, EHRENBURG. RABENHORST, Flora Europ. Algarum, Sect. III. p. 224.

X. Brébissonii, RALFS. ARCHER, PRITCHARD'S Infusoria, p. 736.

Hab.—America; Ehrenberg.

FronD in front view broader than long; constriction deep, acute not linear; segments subelliptic, sometimes irregular spines subulate, geminate, marginal, central protuberance cylindrical, truncate border minutely dentate. L. (not including spines) $\frac{1}{4}\frac{1}{8}$ ". B. $\frac{1}{4}\frac{1}{8}$ " to $\frac{1}{3}\frac{1}{8}$ ".

X. cristatum, BRÉB.

X. parvum, læve; semicellulis subhæmispherico-reniformibus, utroque polo aculeo unico incurvo, ambitu aculeis octo geminatis, a dorso ovato-ellipticis, utroque polo aculeis ternis, in medio plerumque aculeo abbreviato. (R.) *Species mihi ignota.*

Diam.—0.00196". R.

Syn.—*X. cristatum*, BRÉBISSEON. RALFS, British Desmidiæ, et RABENHORST, Flora Europ. Algarum, Sect. III. p. 224.

Hab.—South Carolina; Georgia; Florida; Bailey.

FronD rather longer than broad; constriction deep, linear; segments subreniform or truncate at ends; spines straight or curved, subulate, marginal, one at each side, at the base of the segment, solitary, the others geminate, in four pairs; central protuberance short, conical. (A.)

X. coronatum, EHRB.

X. semicellulis subglobosis binis, aculeatis, ubique asperis, aculeis crassis apice truncatis tridentato-coronatis quatuor utrinque dorsalibus, uno utrinque latere medio. (R.) *Species mihi ignota.*

Syn.—*X. coronatum*, EHRENBURG, Verbreitung, p. 138, et RABENHORST, Flora Europ. Algarum, Sect. III. p. 224.

Asteroxanthium coronatum, KÜTZING. RABENHORST, (*loc. cit.*)

Hab.—America; Ehrenberg.

Remark.—Mr. Archer appears to think that this species is simply a form of *Staurastrum furcigerum*. (BRÉB.); see Pritchard's Infusoria, p. 743.

X. fasciculatum, EHRB.

X. parvum, singulum, constrictione profunde lineari; cytodermate lævi vel sublævi; semicellulis oblongo-reniformibus vel hexagonis, diametro duplo longioribus, ambitu aculeis gracilibus geminatis 4-6, a dorso ellipticis, utroque polo aculeis quatuor instructis. (R.) *Species mihi ignota.*

Diam.—0.00223"—0.00256". (R.)

Syn.—*X. fasciculatum*, EHRENBURG. RABENHORST, Flora Europ. Algarum, Sect. III. p. 223.

Hab.—South Carolina; Georgia; Florida; Rhode Island; Bailey.

FronD about as long as broad; constriction deep, linear; segments somewhat reniform or subhexagonal, twice as broad as long, spines slender, subulate geminate, marginal, in four or six pairs; central protuberance short, conical, somewhat truncate. (A.)

Genus ARTHRODESMUS, EHRB.

Cellulæ profunde constrictæ; semicellulæ compressæ aut oblongæ, utroque polo aculeo subulato firmo instructæ, aut quadrangulares, angulis in aculeum rectum vel curvum productis, a dorso vel ellipticæ vel fusiformes. Massa chlorophyllacea in fascias quatuor radiantes disposita. (R.)

Cells profoundly constricted; semicells compressed or oblong, furnished at each end with a subulate spine, or else quadrangular with the angles produced into straight or curved spines, the dorsal aspect, elliptic or fusiform. Chlorophyl masses disposed in four radiating fascia.

Remarks.—I have found only a single undescribed species of this genus, but the following European forms have been detected in this country by Prof. Bailey. The genus appears to be, as Prof. Rabenhorst says, scarcely distinguishable from *Xanthidium* or *Staurastrum*.

A. octocornis, EHRB.

A. parvus, *lavis*, constrictione lata excavata; semicellulis trapezoideis, inciso-quadriradiatis, radiis in aculeum acutissimum strictum porrectis, a latere elongato-ellipticis, diametro fere triplo longioribus, utroque polo aculeum singulum gerentibus. (R.)

Diam.—0.00065". (R.)

Syn.—*Xanthidium octocorne*, RALFS. BAILEY, Microscopical Observations, p. 29.

Arthrodesmus octocornis, EHRENBURG. RABENHORST, Flora Europ. Algarum, Sect. III. p. 223.

Hab.—Florida; Rhode Island; Bailey.

Fronde smooth, minute, about as long as broad; constriction a wide notch; segments much compressed, trapezoid, each angle terminated by one or two straight, subulate, acute spines, the intervals between the angles concave. (A.)

a. Spine solitary at each angle. L. $\frac{1}{351}$ ". B. $\frac{1}{538}$ ". (A.)

b. Larger spines geminate at each angle. L. $\frac{1}{1020}$ ". B. $\frac{1}{906}$ ". (A.)

A. quadridens, WOOD.

A. late ovalis, vel suborbicularis, paulum longior quam latus, cum margine crenato-undulato; semicellulis nonnihil reniformibus, utroque fine aculeo subulato, modice robusto, acuto, recurvo, armatis; cytiodermate cum verruculis paucibus modice minutis in seriebus paucibus dispositis instructo; semicellulis a vertice acute ellipticis, et cum margine crenato et superficie sparse verruculosa.

Diam.—Lat. $\frac{3}{4000}$ " = .00075"; long. $\frac{5}{4000}$ " = .00125".

Syn.—*A. quadridens*, WOOD, Proc. A. N. S. 1869.

Hab.—In lacu Saco, (Lewis) Wood.

Broadly oval or suborbicular, a little longer than broad, with the margin crenately undulate; semicells somewhat reniform, at each end armed with a subulate, moderately robust, acute, recurved large spine; cytioderm with a few smallish tubercles arranged in three or four rows; semicells from the vertex acutely elliptical, with the margin crenate and the surface sparsely warty.

Remarks.—This species approximates *A. divergens*, from which it differs in the arrangement of its granules, its attaining not one-half the size, and, I believe, in the larger and more robust spines.

Fig. 2, pl. 20, represents an empty frond of this species.

A. Incus, (BRÉB.) HASSAL.

A. parvus tam longus quam latus, constrictione lineari obtusa vel late excisa; semicellulis oblongo-quadrangularibus, angulis externis aculeatis, internis rotundatis inermibus, aculeis longis singulis divergentibus. (R.)

Diam.—Max. 0.00098". Long. 0.00091". Spor. (sine acul.) 0.00085".

Syn.—*A. Incus*, (BRÉBISSEON) HASSAL, Fresh-Water Algæ, p. 357, et RABENHORST, Flora Europ. Algarum, Sect. III. p. 226.

Hab.—Georgia; Florida; South Carolina; Rhode Island; Bailey.

Fronde minute, smooth, as long, or longer than broad, constrictions a deep notch or sinus; segments with inner margin turgid, outer truncate; spines subulate, acute; sporangium orbicular, spinous; spines subulate. (Archer)

A. convergens, EHRB.

A. lævis mediocris, profunde et anguste constrictus, aculeis convergentibus armatus; semicellulis ellipticis vel ovato-oblongis, nunquam reniformibus, utroque fine aculeo longo firmo incurvo instructis. (R.)

Diam.—0.00185"—0.0016". (R.)

Syn.—*A. convergens*, EHRENBURG. RABENHORST, *Flora Europ.*, Algarum, Sect. III. p. 227.

Hab.—South Carolina; Georgia; Florida; Rhode Island; Bailey.

Fronde smooth, broader than long; constriction deep, acute; segments elliptic, each having its lateral spines curved towards those of the other; ends convex. L. $1\frac{1}{539}$ "— $1\frac{1}{598}$ " B. $1\frac{1}{477}$ "— $\frac{1}{584}$ ". (Archer)

FAMILY ZYGNEMACEÆ.

Cellulæ cylindricæ, æquipolares, similes, in familias filamentosas arcte conjunctæ, et cytioblasto centrali plasmate plerumque radiante involuto, et plasmate chlorophylloso aut effuso, aut effigurato, aut (plerumque) in fascias spirales ordinato, et granulis amylaceis instructæ. Filum simplex. Propagatio fit zygosporis conjugatione cellularum binarum ortis. Conjugatio triplici modo, aut lateralis aut scalariformis vel genuflexa. Vegetatio fit divisione transversali repetita.

Cells cylindrical, the same at both ends, closely conjoined into filamentous families, furnished with a central cytioblast wrapped up in generally radiating protoplasm, and with chlorophyllous protoplasm effused in shapeless masses or arranged in spiral filaments, and also with scattered starch-granules. Filament simple. Propagation takes place by means of zygosporis, arising from the conjugation of two cells. Conjugation occurring in three ways, lateral, scalariform, and genuflexuous. Growth taking place by means of transverse division of the cells.

Remarks.—The family under consideration is among the commonest and most widely diffused of all the fresh-water algæ. In almost every ditch or spring, or dripping moss-covered rock representatives of it are to be found, so that wherever quiet water is they may be confidently looked for. The single filaments are so minute that frequently the unaided eye cannot distinguish them, but multiplication with them is such a rapid process, that wherever found they are in great masses. These masses, when growth is active, are of a beautiful intense green, glistening and shining with the gelatinous matter which coats the threads and makes the mass so slippery. They may be found in greater or less abundance at all seasons, but as the specific characters are largely of sexual origin, non-conjugating specimens are of little value. For this reason, *Zygnemas* are only worth gathering when in fruit. The spores appear to be formed only in the spring and early summer, at least these are the only times in which I have found fertile filaments. In this neighborhood I have collected them in excellent condition as early as the beginning of April and as late as the latter part of June. Further south, conjugation of course commences earlier, and fine fruiting specimens received by myself from Mr. Canby were collected in Florida by him in February.

When conjugating freely the mass of *Zygnema* or *Spirogyra* loses its beautiful bright green color and become dingy and even brownish, often very dirty looking. The collector soon learns to pass by the beautiful vivid mass, as comparatively worthless, and fasten upon the pale, wan, sickly, apparently dying specimens as prizes worthy of a place in his cabinet.

In the *Zygnemaceæ* the individual plant, as ordinarily considered, is a filament

composed of a varying number of cells placed, end to end, all alike, and each of them apparently independent of its associates. Each cell in one sense is, therefore, a perfect, complete individual, capable of living dissociated from its companions. How far the life of one of these cells is influenced by that of its neighbors is uncertain, probably to a slight extent, possibly not at all. At any rate, they are so far independent that the filament is rather a composite body than a unit of life. These cells are cylindrical, with the ordinary cellulose wall, which can commonly be stained blue by iodine and sulphuric acid, and is often distinctly composed of layers, but never has any "secondary markings," each layer being precisely like that superimposed upon it. Outside of the wall is a jelly-like sheath, which is mostly not discernible from its thinness and transparency, although it no doubt exists, as is proven by the slipperiness of the general mass. The primordial utricle is always present. The chlorophyl is variously arranged, most generally in bands, either straight or spiral, sometimes in definite irregular masses, sometimes diffused through the cell. Imbedded in it are, at certain seasons, numerous minute, generally shining, granules, which are either minute specks of starch, or little drops of oil. Besides these there are contained in it, especially in the bands of chlorophyl, more or less numerous comparatively large, oval or roundish bodies, with a distinct outline and a deeper color than the surrounding portions. These masses are protoplasm, dyed with chlorophyl-green, and are believed to be especially active in the formation of starch. At times, iodine turns them simply brown; at others it colors their inner portions blue and their outer brown, showing them to contain starch. The general cavity of the cell is occupied by fluid, in which is placed the nucleus. This is mostly single, but rarely, according to Nægeli and other authorities, double, and De Bary states that he has seen three nuclei in a single cell. I have never seen more than one, and think that even this is not rarely absent, having certainly repeatedly failed to demonstrate its presence. It is colorless, often with a nucleolus, transparently bright, irregular in form, placed in the centre of the cell with numerous arms radiating out from it, some of them ending within the cell, others connecting it with the primordial utricle. De Bary states that this nucleus occasionally is tinged green with chlorophyl, I do not remember ever to have seen it so.

I have not infrequently seen numerous minute dark granules, similar to those seen in *Closterium*, scattered through the inside of the cell, in active motion. Sometimes they are to be found collected in vast numbers near the ends of the cells, dancing and swarming about one another, and passing off in small streams from one end to the other, coasting along close to the primordial utricle, in a word, exhibiting precisely the same motions as are so common among the desmids.

The *Zygnema* filament grows in length by a process of cell multiplication by division of the simplest kind. It seems to be somewhat uncertain whether the nucleus always divides into two as a part of the process or not. These plants multiply both by the separation of cells and their subsequent growth, and, by means of resting spores, the so-called *Zygospores*.

The first appearance of separation of the cells is an evident disposition to the rounding off of the ends of the cells. The corners are first rounded and separated

and this continues until only the centres of the ends are in apposition, and in a little while even these separate. This certainly, at least, is the process in certain species; but I have thought, that in other cases cells were separated by a simple splitting of the end wall, each cell retaining its half of the partition.

The *zygospores* are produced by a process of union of two cells, to which the name of *conjugation* has been given. Very rarely, if ever, is there any difference between the cells before conjugation, and it has not existed in any species which has come under my notice; but, after conjugation, the receiving cell is frequently enlarged, the other remaining cylindrical. De Bary, however, states that he has found a small but constant difference between the fertile and sterile cells of *Spirogyra Heeriana*.

The first perceptible change in a cell about to produce a resting spore, appears to be a loosening of the primordial utricle from the outer wall, and a contraction of it upon the cell contents, which thus are crowded together and more or less deformed. Simultaneously with this, or a little after or before it, the side wall of the cell is ruptured and a little pullulation or process is pushed out, which directly coats itself with cellulose and rapidly enlarges to a considerable diameter, at the same time growing in length until it meets a similar process pushing out from an opposing cell, or has attained as great a length as its laws of development will allow. When two processes meet they become fused together, the end walls are ruptured, and the contents of one cell passing over are received within those of the other, or else the contents of both cells meet within the connecting tube and there fuse together. This is the more common mode of conjugation, in which two cells of distinct filaments become joined together by a connecting tube. It is evident, that, if the filaments are fertile to their fullest extent, there will be as many of these connecting tubes as there are pairs of cells in the filaments, and a ladder-like body will be formed, the original filaments corresponding to the side pieces, the connecting tubes to the rounds. Hence this method of conjugation has received the name of *scalariform*.

In the so-called "*lateral conjugation*," instead of cells of different filaments joining, adjacent cells of one filament unite together to complete the process. The union of the two cells appears to take place in several ways. In accordance with one plan (fig. 1 *a*, pl. 14), connecting tubes, pushed out from near the ends of the cells, grow for a short distance nearly at right angles to the long axis of the filaments, and then bend at a right angle to themselves so as to run parallel to the filament-cells. The ends of these processes are, of course, opposed to one another, and coming in contact fuse together so as to form a continuous tube for the passage of the endochrome. Another method by which neighboring cells are sometimes connected is by the formation of coadjacent pouch-like enlargements of the opposing ends, and a subsequent fusion of these newly formed enlargements by the absorption of the end wall between them. (See fig. 2, pl. 14.)

Sometimes I think the union of two neighboring cells is facilitated by a curved neck forming to one or both of them, so that they are bent at an angle to one another, and can readily be united by means of a straight tube.

There is still another method of conjugation, the so-called *genyflexuous*, in which,

instead of a connecting tube being formed as the medium of union, two cells of opposing filaments become sharply bent backwards, so that their central portions are strongly thrust forward as obtuse points, which, coming in contact, adhere and allow of a passage-way between the cells being made by the absorption of their cohering walls.

A curious modification of, or departure from, the ordinary method of conjugation is sometimes seen, in the union of three instead of two cells. This is, I think, very rare, but has been seen by Meyen in the genus *Zygnema*, as well as by Schleiden and De Bary in *Spirogyra*. I myself have observed it once or twice in the latter genus. One of the cells plays the part of the female, receiving the contents of the other two, so that in it the primordial utricles of the three, with their contracted protoplasm, are fused into a zygospore.

The zygospore, however formed, varies in shape, but is mostly oval or globular, sometimes cylindrical, and when ripe is in most if not all species of a dark brownish color. It is described both by Pringsheim and De Bary as having three coats, but I have frequently found it impossible to demonstrate the presence of all of these, and I believe that not rarely one of them is absent. The outer coat is developed first and is the thickest and firmest. Occasionally it is double, *i. e.* composed of two distinct layers or parts, as in *Sp. protecta*, in which species the outer of these layers is the thickest, firmest, and most evident, whilst the inner layer is translucent and much less apparent. The second coat contains the coloring matter, which is sometimes brown, sometimes decidedly yellowish. The inner coat is not readily seen. It is elastic, thin, and is the last of the three to be formed.

The principal contents of the ripe spore are protein compounds (protoplasm), oil-drops, starch granules, and pigment. The oil is generally much more abundant than the starch, and not rarely the minute, bright drops entirely replace the little granules. According to Prof. De Bary, the pigment frequently, but not always, reacts with sulphuric acid, as does that of the fungal family, *Uredineæ*, striking with it a deep blue.

The germination of the spore, both in the genus *Spirogyra* and *Zygnema*, is very simple. The first step is an elongation and growth of the protoplasmic central mass, together with the inner transparent cellulose coat, and a consequent rupturing of the outer two coats, through which the newly forming plant protrudes and finally escapes. In this way in the genus *Spirogyra* an elongated club-shaped cell arises, one end of which is much larger than the other and contains all the chlorophyl. Sometimes a nucleus is perceptible in this cell, sometimes it is not. The larger end now becomes cut off by a partition wall from the smaller; if no nucleus has been previously apparent it now becomes so, and the first stage of development is completed. The filament after this grows by a simple repetition of the process of division in the larger end and the cells formed out of it. The smaller end undergoes little or no change. In the genus *Zygnema*, the cell that first emerges from the germinating spore is a perfect one, similar in all respects to those seen in the fully formed filament, which is developed out of it, by a simple process of cell division.

Besides the true *Zygospores*, Hassall many years since described bodies (Fresh-water Algæ, vol. i. pp. 132, 156, 170), which he found in filaments of this family, and which resemble in all respects ordinary *Zygospores*, but are produced each in a single cell without any aid from a second cell. He affirmed that he had observed this phenomenon especially in two species, *Spirogyra mirabilis* and *Zygnema notabilis*. These observations were doubted by some, whilst others, as Alexander Braun, supposed that there was a division of the cell protoplasm into two distinct portions, and then a conjugation of these within the original cell, and that Mr. Hassall had overlooked these changes. Prof. De Bary, however, states that he has seen a great many instances of this production of spores without conjugation (all in one species), and that there can be no doubt that Hassall's observations are substantially correct, and that no division of the primordial utricle such as was imagined by Prof. Braun takes place. Spores formed in this manner, as yet have not been seen to develop. There is, therefore, no certainty that they are capable of doing so. It is possible that they are merely the results of abortive attempts at reproduction, wanting the power of development because not fertilized.

Pringsheim and others have drawn from these bodies strong argument against the idea, that conjugation is to be looked upon at all as a sexual process.

The arguments both for and against regarding conjugation as the simplest expression of sexual life are ably elaborated by De Bary, *Untersuchungen über die Familie der Conjugatem*, p. 57, to which I must refer those desirous of following the subject further, contenting myself with expressing an agreement with the conclusions there arrived at, namely, that in conjugation the first dawns of sexuality are to be found. Looking at it in this light Prof. De Bary states his conviction that the spores formed in the manner last described, bear the same relation to the true *Zygospore* that the bud of a *Phanerogam* does to its seed, or the *Zoospore* of an *Ædognium* does to its resting spore.

Quite a number of bodies have been described by the older authorities as being found within the cells of plants of this family, which more recent observers have proven to be parasitic. Such are the "Spermatic spheres," transparent spheres motile by virtue of vibratile cilia, various monads, &c. &c., bodies for which it has been claimed, from time to time, that they were sexual elements, spermatozoids.

Genus SPIROGYRA, LINK.

Cellulæ vegetativæ cylindricæ, fasciis chlorophyllosis spiralibus instructæ. Conjugatio aut lateralis aut scalariformis aut et lateralis et scalariformis.

Syn.—*Spirogyra* et *Rhynchonema*, KÜTZING, RABENHORST, et auctores.

Salmacis, BORY.

Zygnema (partim), HASSALL.

Vegetative cells cylindrical, furnished with spiral chlorophyl bands. Conjugation either lateral or scalariform or both lateral and scalariform.

Remarks.—The genus *Spirogyra*, as defined above, has been divided by Kützing, Rabenhorst, and others into two genera, the characters being drawn from the method of union of the conjugating cells; in the one case the neighboring cells of a single filament (*Rhynchonema*), in the other cells of distinct filaments (*Spirogyra*),

uniting to form the spore. This at first sight appears to be a good ground for separation, but there are certain species in which, undoubtedly, both the former and the latter method of conjugation take place indifferently. Such species make a third group so precisely between the two others as, to my mind, to fuse them together and necessitate either the acknowledgment of three genera or the denial of more than one. The latter seems to me the more philosophical course.

A. CONJUGATIO LATERALIS (RHYNCHONEMA).

A. CONJUGATION LATERAL.

Sp. elongata, WOOD.

Sp. articulis vegetativis diametro 7-20 plo longioribus; articulis sporiferis multo brevioribus, valde tumidis; cytodermate utroque fine protenso et replicato; fascia unica, laxissime spirali; anfractibus plerumque 7; sporis ellipticis, diametro 1-2½ plo longioribus.

Diam.—Spor. $\frac{8}{500}$ " = .00106". Artic. vegetat. $\frac{4}{500}$ " = .0005".

Syn.—*Rhynchonema elongatum*, WOOD, Prodr. Amer. Philos. Soc. 1869, p. 137.

Hab.—In aquis limpidis, prope Philadelphia.

Sterile joints 7-20 times longer than broad; fertile joints much shorter, greatly swollen; cell wall at each end produced or folded in; chlorophyl filament 1, spiral lax; turns mostly 7; spores elliptical, 2-2½ times longer than broad.

Remarks.—I found this species about the middle of March, fruiting in a little pool near Cheltenham Hills, six or eight miles north of this city. It did not form a distinct stratum by itself, but was floating, intermingled with great numbers of other filamentous algæ, such as *fragillariæ*, *zygnemæ*, &c. It seems to be most closely allied to the European *R. minimum*; it however not only attains a somewhat larger size but also differs from that plant in the proportionate length of the sterile cells, in the number of the turns of the chlorophyl spiral in the cell, and in the proportionate length and breadth of the spore.

Fig. 1, pl. 14, represents portions of sterile filaments magnified 450 diameters; 1 a, a part of a fertile filament, magnified 450 diameters.

Sp. pulchella, WOOD.

Sp. articulis sterilibus diametro 2-3 plo longioribus; sporiferis nonnihil tumidis; fascia unica, anfractibus 3-4; sporis ellipticis, diametro fere duplo longioribus; cytodermate utroque fine protenso et replicato.

Diam.—Artic. Steril. $\frac{4}{500}$ "— $\frac{9}{700}$ " = .00033"—.0013". Spor. $\frac{9}{500}$ "— $\frac{10}{750}$ " = .0012"—.00133".

Syn.—*Rhynchonema pulchellum*, WOOD, Prodr. Amer. Philos. Soc. 1869, p. 138.

Hab.—In stagnis, prope Philadelphia.

Sterile joints 2-3 times longer than broad; fertile joints somewhat swollen; chlorophyl band one; turns of spiral 3-4; spores elliptical, almost twice as long as broad; cell wall at each end produced or folded in.

Remarks.—This species was found by myself fruiting in April, 1869, in stagnant ditches below the city, and in similar localities near Camden, New Jersey. It did not occur in masses but singly, intermixed with great numbers of other fruiting spirogyras. Most of the filaments seen were about .0010" in diameter; in but a single instance did they come much short of this. This species differs from *R. elongatum*, among other points, in the shortness of the tubes connecting the fertile

cells. I have never been able to identify an entirely sterile filament of this species; the measurements and description of the sterile cells were taken from infertile cells in filaments, which in other places had produced spores.

Fig. 2, pl. 14, represents a fertile filament, magnified 260 diameters.

B. CONJUGATIO SCALARIFORMIS (SPIROGYRA VERA).

a. Cytiodermate utroque fine protensum et replicatum.

a. Cytioderm folded in at the ends.

* *Fascia spiralis unica.*

* *Spiral filament single.*

Sp. Weberi, Kütz. ?

Sp. saturate viridis, lubrica; articulis vegetativis diametro 3-20 plo longioribus; fructiferis nonnihil inflatis; fascia dentata, plerumque unica sed fasciis duabus in quavis cellula; spirae anfractibus 3-8; cytiodermate plerumque utroque fine protenso et replicato; zygosporis ellipticis.

Diam.—Artic. steril. $\frac{7}{1000}'' - \frac{9}{1000}'' = .0008'' - .0012''$.

Syn.—*S. Weberi*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 233.

Hab.—In stagnis, prope Philadelphia.

Deep green, slippery; sterile joints 3-20 times longer than broad; fertile joints not swollen; chlorophyl filaments mostly single, but sometimes two in certain cells, dentate; turns of the spiral 3-8; cytioderma protruded or infolded at the ends; zygosporis elliptical.

Remarks.—This species, which is abundant around Philadelphia in stagnant ditches, I have found fruiting in the month of April. The number of spirals frequently varies even in the same filament. The infolding of the walls at the end of the cells is very often wanting in the fertile cells and occasionally is absent from one end of an ordinary vegetative cell. The American form agrees pretty well with the European, but is, however, larger, and also attains in its cells a greater proportionate length and has more turns of its chlorophyl spirals. The lower limits of the American form are, however, so overlapped by the upper limits of the European, that it seems to me they must be considered identical.

Fig. 19, pl. 12, represents a pair of fertile filaments of this species, magnified 260 diameters; 19 *a*, part of a sterile filament, magnified 260 diameters; 19 *b*, outline of a couple of fertile cells, magnified 260 diameters.

Sp. protecta, Wood.

Sp. saturate viridis, lubrica; articulis sterilibus diametro 6 plo longioribus; sporiferis vix tumidis; cytiodermate utroque fine protenso et replicato; fascia unica; anfractibus 6; sporis oblongis vel ellipticis; membrano crassissimo.

Diam.—Art. steril. $\frac{11}{1000}'' = .00146''$; spor. lat. $\frac{1}{1000}'' - \frac{2}{1000}'' = .00133'' - .0016''$ long. $\frac{25}{1000}'' = .0033''$.

Syn.—*Sp. protecta*, Wood, Prodromus, Proc. Am. Philos. Soc. 1869, p. 131.

Sp. deep green, slippery; sterile joints 6 times longer than broad; fertile cells scarcely swollen; cell wall folded in at the ends; chlorophyl band single; turns 6; spores oblong or elliptical, spore wall very thick.

Remarks.—I found this species in the latter part of April fruiting in a ditch in a meadow a little south of the mouth of Wissahicon Creek, near this city, and as late as the 25th of May in the "neck" below the city. It is remarkable

for the very great thickness of the walls of the spore. There are two very apparent coats separated by a thin not very evident one. The outer is the thickest; it is very thick, firm, and nearly colorless. The inner coat is of a decided orange-brown. The parent-cells which give origin to these spores are slightly enlarged in diameter. Sometimes the spores, instead of being elliptical, are irregular in shape.

Fig. 3 *a*, pl. 14, represents a sterile filament, magnified 250 diameters; fig. 3, a mature spore, magnified 450 diameters.

Sp. insignis, (HASSALL) KtZ.

Sp. articulis sterilibus diametro 5-14 plo longioribus; fasciis 2 (rarius 1-3), laxe spiralibus, angustis, crenatis; articulis fructiferis nonnihil tumidis; cytiodermate utroque fine replicato vel protenso; zygosporis rubido-brunneis, ovato-ellipticis.

Diam.—0.0015".

Syn.—*Zygnema insigne*, HASSALL, Fresh-Water Algæ, p. 440.

Spirogyra insignis, (HASSALL) KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 235.

Hab.—In stagnis, prope Philadelphia.

Sterile joints 5-14 times longer than broad; chlorophyl filaments mostly 2 (rarely 1-3), laxly spiral, narrow, crenate; fertile joints somewhat enlarged; cytioderm at each end folded in or produced; zygosporis reddish-brown, ovate elliptical.

Remark.—Fig. 6, pl. 16, represents this species.

b. Cytioderma cellulæ fine nec protensum nec replicatum.

Cytioderm not infolded in the end of the cell.

* *Fasciæ spirali unicæ (raro duæ).*

Chlorophyl band single (rarely two).

Sp. longata, (VAUCH.) KtZ.

Sp. dense cæspitosa, late luteolo-viridis, valde lubrica; articulis sterilibus diametro 2-6 plo longioribus, fertilibus saepe tumidis abbreviatis; fasciâ spirali lata, dentata; anfractibus sub-laxis 2-5; zygosporis ellipticis.

Diam.—0.001".

Syn.—*Conjugata longata*, VAUCHER, Histoire des Conferves d'Eau douce, p. 71.

Sp. longata, (VAUCH.) KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 238.

Hab.—In stagnis, prope Philadelphia; Wood. Rhode Island; (S. T. Olney) Thwaites.

Densely cæspitose, bright yellowish-green, very slippery; sterile joints 2-6 times longer than broad; fertile articles swollen, often abbreviate; chlorophyl filaments broad, dentate; turns of the spiral somewhat loose, 2-5; zygosporis elliptical.

Remarks.—According to Prof. Rabenhorst, this species attains in Europe a diameter of .0011" and the cells a length of 8 times their breadth. The same authority also describes the fertile cell as being either not swollen, or moderately so ("aut non aut modice tumidis"). In all the specimens of our American forms which I have seen, the sporangial cells are very decidedly swollen.

Fig. 4, pl. 14, represents portions of sterile filaments, magnified 250 diameters, and fig. 4 *a*, a part of a fertile pair of filaments containing immature spores enlarged 260 diameters.

Sp. quinina, (AG.) KÜTZ.

- Sp. saturate viridis, valde lubricata; articulis sterilibus diametro 1-6 plo longioribus; articulis fertilibus vel haud tumidis vel nonnihil tumidis; fascia unica; spiræ anfractibus modo densioribus, modo laxioribus, nonnunquam laxissimis, plerumque 3, interdum $1\frac{1}{2}$ -4; cytodermate cellulæ utroque fine nec protenso nec replicato; zygosporis aut globosis aut ovalibus aut cylindricis.

Diam.—Artic. steril. $\frac{7}{5000}''$ — $\frac{13}{7000}''$ = .0013''—0.0017''; sporis $\frac{11}{5000}''$ = .0014''.

Syn.—*Sp. quinina*, (AGARDH)KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 240.

Hab.—In stagnis, prope Philadelphia.

Deep green, very slippery, sterile articles 1-6 times longer than broad; fertile joints scarcely or not at all tumid; chlorophyl filament single; turns of the spiral sometimes denser, sometimes laxer, sometimes very lax, mostly 3 in number, sometimes $1\frac{1}{2}$ -4; cytoderm neither infolded nor protruded at the end; zygosporis polymorphous, globose, elliptical or cylindrical.

Remarks.—This species is very abundant in the ditches around Philadelphia, especially in the "neck" below the city. I have found it fruiting profusely in the month of April. The spores vary very much in form, some of them being globose, others elliptic, and still others cylindrical, with obtusely rounded ends. All these forms may occur in a single filament. The spore cell also varies in the amount of its enlargement. In many cases it preserves its cylindrical shape completely; in other instances it is markedly swollen.

Figs. 4 e, 4 c, pl. 19, represent portions of sterile filaments of this species; figs. 4 a, 4 b, and 4 d, portions of fertile filaments.

†† *Fasciæ spiralæ duæ vel plures.*

†† *Chlorophyl filaments two or many.*

Sp. decimina, (MULLER) KÜTZ.

Sp. sordide viridis, lubrica; articulis sterilibus diametro (0.00135''—0.00159'') plerumque duplo-, quadruplo fere longioribus, nonnunquam subæqualibus, fertilibus aut non aut modice tumidis; fasciis spiralibus plerumque 2, latis, decussatis, rarius 1 vel 3, anfractibus laxis 1- $1\frac{1}{2}$; zygosporis aut ovalibus aut late ellipticis vel subglobosis. (R.)

Syn.—*Sp. decimina*, (MÜLLER) KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 242.

Hab.—Prope Philadelphia.

Dirty green, slippery; sterile joints mostly from 2-4 times as long as broad (0.00135''—0.00159''), sometimes about as long as broad; fertile joints either moderately or not at all swollen; spiral filaments mostly 2, broad, decussating, rarely 1-3, turns loose 1- $1\frac{1}{2}$; zygosporis either oval, broadly elliptic, or subglobose.

Remarks.—I find this species marked in one of my note-books as having been found by myself near this city. I have no distinct recollection of seeing it, and, having preserved neither figure, specimen, nor description, am forced to content myself with copying the description of Prof. Rabenhorst.

Sp. dubia, KÜTZ.

Sp. viridis in fructe dilute viridis; articulis sterilibus cylindricis diametro $1\frac{1}{2}$ - $2\frac{1}{2}$ plo longioribus; fasciis spiralibus 2-3, angustissimis, nodosis, anfractibus laxis 1-2 (= 3-6); cytodermate utroque fine nec protenso nec replicato, nonnihil crasso; zygosporis polymorphis, aut sub-

globosis aut ovalibus, aut subcylindricis, diametro æqualibus aut $\frac{3}{4}$ plo longioribus; articulis fertilibus cylindricis, haud tumidis.

Diam.—Art. steril. $7\frac{15}{100}'' = .002$; spor $7\frac{15}{100}'' = .002''$.

Syn.—*Sp. dubia*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 243.

Hab.—In stagnis, prope Philadelphia.

Green, in fruit light green; sterile joints cylindrical, $1\frac{1}{2}$ – $2\frac{1}{2}$ times longer than broad; spiral filaments 2–3, very narrow, nodose, lax, turns 1–2; cytioderm neither infolded nor protruded at the end, rather thick; zygospores polymorphous, either subglobose, oval, or subcylindrical, as broad as long to $\frac{3}{4}$ times longer; fertile articles cylindrical, not enlarged.

Remarks.—I have found this species growing in the ditches below the city, fruiting abundantly in May. When in this condition it forms masses of a dirty, lightish, yellowish-green. The spores mostly fill pretty well the fertile cells. My specimens do not agree completely with the descriptions given of the European. The two forms, however, completely overlap one another, except in one character, namely, the shape of the sporangial cell. I have never seen it swollen or at all tumid in American specimens, whilst in the European it is said to be “modice tumidis.” This difference alone does not, however, seem to me sufficient to characterize a new species. I have seen specimens of this plant collected by Dr. Lewis at Cobble Mountain. They agree well with the Philadelphia specimens, except in attaining a little larger size, .0021", and in the sterile filaments having their walls very thick. The character of non-inflation of sporangial cells is perfectly preserved.

Fig. 4, pl. 17, represents this species.

Sp. rivularis, (HASSALL) RABENH. (*non* KÜTZ.)

Sp. saturate viridis, lubrica; articulis sterilibus diametro 7–11 plo longioribus; fertilibus cylindricis aut vix tumidis; cytiodermato tenuissimo, utroque fine nec protenso nec replicato, fasciis 4, laxè spiralibus, modice angustis, nodulosis et serratis, anfractibus $2\frac{1}{2}$; zygosporis ellipticis, diametro 2– $2\frac{1}{2}$ longioribus.

Diam.—Art. ster. $7\frac{9}{100}''$ — $7\frac{11}{100}'' = .0012''$ — $.00146''$; spor. $7\frac{10}{100}''$ — $7\frac{12}{100}''$.

Syn.—*Zygnema rivularis*, HASSALL, Fresh-Water Algæ, vol. i. p. 144.

Spirogyra rivularis, (HASSALL) (*non* KÜTZING) RABENHORST, Flora Europ. Algarum, Sect. III. p. 243.

Hab.—In rivulis, Florida; (CANBY) WOOD.

Deep green, slippery; sterile articles 7–11 times longer than broad, fertile cylindrical or slightly tumid; cytioderm very thin, neither infolded nor protruded at the end; chlorophyl filaments 4, laxly spiral, moderately narrow, nodose and serrulate, turns $2\frac{1}{2}$; zygospores elliptical, 2– $2\frac{1}{2}$ times longer than broad.

Remarks.—This species was collected by Mr. Wm. Canby in Pine Barren Run, near Hibernia, Florida. It is rather smaller than the European forms, but does not appear to be distinct from them. Rabenhorst, indeed, states that there are only two or three chlorophyl spiral bands in a cell, but Hassall in the description of the type states distinctly that in some instances there are four bands, and also figures the plant so.

Fig. 5 *a* and *b*, pl. 17, represents sterile cells of this species, magnified 260

diameters. Fig. 5 *c* is an outline of a pair of fertile cells enlarged to the same extent.

Sp. parvispora, Wood.

Sp. articulis sterilibus diametro 2-4 plo longioribus; fructiferis haud tumidis, diametro 1-2½ plo longioribus; fasciis spiralibus 4, angustis, nodosis, anfractibus pluribus; zygosporis parvisimis, ellipticis, diametro 1¼-2 plo longioribus; cytodermate utroque fine nec protenso nec replicato.

Diam.—Art. steril $\frac{7}{5000}$ " = .003"; spor. diam. transv. $\frac{7}{5000}$ " — $\frac{17}{6000}$ " = .002" — .0023", long. $\frac{7}{5000}$ " — $\frac{30}{6000}$ ".

Syn.—*S. parvispora*, Wood, Prodrum, Proc. Am. Philos. Soc. 1869, p. 139.

Hab.—In stagnis, Hibernia, Florida. (WM. CANBY)

Sterile joints 2-4 times longer than broad; fertile not tumid, 1-2½ times longer than broad; chlorophyl bands 4, narrow, nodose; turns many; zygosporis very small, elliptical, 1¼-2 times longer than broad; cell wall not infolded at the end.

Remarks.—I am indebted to Mr. Wm. Canby for specimens of this species, which he collected in a pond in the Pine Barrens near Hibernia, St. John's River, Florida. It is remarkable for the comparatively small size of the spores, which do not nearly fill the perfectly cylindrical mother-cells; indeed they are only about as long as the latter are wide. This species closely resembles *S. majuscula*, but is larger, does not, that I have ever seen, vary like it in the number of spores, and is especially separated from it by the very small size of the latter.

Fig. 7, pl. 15, represents a fertile pair of filaments of this species magnified 125 diameters.

Sp. majuscula, Ktz.

Sp. pallide et sordide viridis, fructus tempore fuscescens; articulis sterilibus diametro (0.0022" — 0.0025") 2½-4-10 plo longioribus; cytodermate tenui homogeneo; fasciis 3-4-5 (rarius 7), modo subrectis longitudinalibus, modo laxissime spiralibus, nodosis; zygosporis globosis vel ovalibus. (R.)

Syn.—*S. majuscula*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 244.

Hab.—Prope Philadelphia. ?

Pale and sordid green, fuscescens at the time of fruiting; sterile joints 2½-4-10 times longer than broad (.0022" — 0.0025"); cytoderm thin, homogeneous; spiral filaments 3-4-5 (rarely 7) partly straightish and longitudinal, partly laxly spiral, nodose; zygosporis globose or oval.

Remarks.—Shortly after I commenced to study the fresh-water algæ, I found below the city a fruiting *Spirogyra*, of which I preserved only a drawing, which I have since identified as apparently specifically one with the European *S. majuscula*, it differing only in not being quite so large; my measurement was $\frac{1}{5000}$ " = 0.002". Not having any specimens at hand, I have copied the description from the work of Prof. Rabenhorst.

Fig. 1, pl. 15, was copied from the drawing alluded to.

Sp. nitida, (DILLW.) LINK.

Sp. cæspitibus, lubricis, saturate viridibus; articulis sterilibus post divisionem diametro subæqualibus, ante divisionem 2-3 plo longioribus; articulis fertilis aliis similibus, haud tumidis; fasciis spiralibus 4 (3-4 R.), modice latis, anfractibus 1-2; zygosporis ellipticis.

22 August, 1872.

Diam.—0.0025".

Syn.—*S. nitida*, (DILLWYN) LINK. RABENHORST, Flora Europ. Algarum, Sect. III. p. 245.

Hab.—Prope Philadelphia.

Occurring in lubricous turfy masses, of a deep green color; sterile joints after division about as long as broad, before division 2-3 times longer; fertile joints similar to the others, not tumid; spiral filaments 4, moderately broad, turns 1-2; zygospores elliptic.

Remarks.—This species appears to be somewhat rare, at least I have found it but once, and then only in small quantity. Rabenhorst states that there are occasionally only three spirals, and his maximum diameter is 0.0031"; he also speaks of the fertile joints as "vix tumidis."

Sp. diluta, Wood.

Sp. articulata sterilibus diametro subæqualibus ad duplo longioribus, fructiferis haud tumidis; fasciis spiralibus 5, angustissimis, laxis, valde nodosis; anfractibus plerumque $\frac{1}{2}$, interdum 1; zygosporis sparsis, late ellipticis vel ovatis aut globosis; cytiodermate modice tenue, in utroque fine nec protenso nec replicato.

Diam.—Artic. steril. $\frac{2}{5000}$ " = .003".

Syn.—*S. diluta*, Wood, Prodromus, Proc. Am. Philos. Soc. 1869, p. 139.

Hab.—In stagnis, prope Philadelphia.

Sterile joints about as long as broad to twice longer, fertile cells not swollen; chlorophyl bands 5, exceedingly narrow, lax, strongly nodose; turns mostly $\frac{1}{2}$, sometimes 1; zygospores few, broadly elliptical, ovate or globose; cell wall moderately thin, not infolded at the ends.

Remarks.—I have found this species several successive seasons growing in the ditches in the Neck, below the city, especially in the neighborhood of the large stone barn, built by the great millionaire, and still known as "Girard's Barn." The spirals are very narrow and slender, and are moderately close to one another. They are chiefly made up of a number of chlorophyl nodules, the connecting thread between which is often very faint. In all the fruiting specimens, as I have seen them, the spores have been very few in number, most of the cells of the fertile filaments appearing to have aborted, so that they are simply empty. In most cases only about every third or fourth cell contained a spore.

Fig. 2, ul. 15, represents this species.

Sp. setiformis, (ROTH) KtZ.

Sp. saturata viridis, lubrica; articulis sterilibus diametro paullum brevioribus ad $1\frac{1}{2}$ plo longioribus; articulis fructiferis haud inflatis; fasciis 3-8, latis, dentatis, interdum nonnihil remotis, sed sæpe arete et dense conjunctis, nodosis; zygosporis globosis vel late ovalibus.

Diam.—.0035".

Syn.—*S. setiformis*, (ROTH) KUTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 246.

Hab.—In stagnis, prope Philadelphia.

Deep green, slippery; sterile joints a little shorter to one and a-half times longer than broad; fertile joints not inflated; spiral filaments 3-8, broad, dentate, sometimes somewhat remote, sometimes closely and densely conjoined, nodose; zygospores globose or broadly oval.

Remarks.—None of the descriptions which I have seen of this species state the number of the spiral filaments, but the other characters of the American form so agree with those of the European plant that it is probable that this one does also. The plant is not uncommon in the Neck, fruiting in the spring.

Fig. 3 *a*, pl. 15, represents part of a sterile filament of this species; 3 *b*, portion of a pair of fertile filaments, both magnified 125 diameters.

Sp. crassa, Ktz.

Sp. læte viridis, denique sordide viridis; articulis sterilibus diametro subæqualibus, post divisionem interdum fere $\frac{1}{2}$ plo brevioribus, ante divisionem sæpe fere 2 plo longioribus; cytodermate tenui, homogeneo, utroque fine nec protenso nec replicato; fasciis spiralibus 4, dentatis vel tuberculatis, sæpe arctis, subtransversis, tenuibus; anfractibus $1\frac{1}{2}$ -4; cellulis fructiferis aliis simillimis, haud inflatis; zygosporis globosis vel ellipticis.

Diam.—Max. .0065."

Syn.—*Sp. crassa*, Ktz. RABENHORST, Flora Europ. Algarum, Sect. III. p. 246.

Hab.—In stagnis, prope Philadelphia.

Bright green, but finally a dirty green; sterile articles about as long as broad, sometimes after division only half as long, sometimes before division twice as long; cytoderm thin, homogeneous, not infolded or produced at the ends; spiral filaments 4, dentate or tuberculate, often close, subtransverse, thin; turns from $1\frac{1}{2}$ to 4; fertile cells very like the others, not inflated; zygosporis globose or elliptical.

Remarks.—This species is very common in the neighborhood of this city, occurring in springs, &c., but especially in the ditches in the Neck. It forms long, lubricous masses, of a bright green color, readily distinguishable by the size of the filaments, which are separated with ease by the unaided eye. I have gathered it repeatedly, in fruit, from the middle of April to the middle of June. In this state the mass has lost its bright green color, and when the filaments are closely examined, even without a glass, minute dark points mark the positions of the spores.

Fig. 4 *a*, pl. 15, represents part of a filament commencing reproduction; 4 *b*, filaments which have matured the spores; 4 *c*, a pair of conjugating filaments.

Genus ZYGNEMA.

Cellulæ vegetativæ cylindricæ. Massa chlorophyllacea initio effusa et subhomogenea, postea distincte granulosa aut per cellulæ lumen distributa, granula amylacea duo centralia involvens, aut in corporibus duobus (in quaque cellula) plus minusve distincte stellatim radiantibus juxta nucleum centralium granum amylaceum unicum involventibus collocata. Conjugatio scalariformis vel lateralis.

Vegetative cells cylindrical. Chlorophyl masses in the beginning effused and subhomogeneous, afterwards distinctly granular, either distributed throughout the cavity of the cell, involving two central starch granules, or gathered together into two masses (in each cell), with more or less distinctly stellate radii and a central starch granule placed near the nucleus, one on each side of it.

Z. insigne, (HASSALL) Ktz.

Z. cæspitibus et plerumque natantibus vel in aqua diffusis, saturate viridibus vel sæpe sordide flavo-viridibus; articulis sterilibus diametro circiter æqualibus vel duplo longioribus; conjugatione scalariforme (et sæpe simul laterali, R.); zygosporis globosis; sporodermate lævi.

Diam.—Cell. $\frac{1.9}{15000}$ " = .00126"; spor. $\frac{1.4}{15000}$ "— $\frac{1.6}{15000}$ " = 0.00093"—0.00016".

Syn.—*Tyndaridea insignis*, HASSALL, Fresh-Water Algæ, vol. i. p. 163.

Zygnema insigne, (HASSALL) KÜTZING. RABENHORST, Flora Europ. Algæ, Sect. III. p. 249.

Hab.—In stagnis, prope Philadelphia; Wood. Rhode Island; (S. T. Olney) Thwaites.

Cæs pitose and mostly floating or diffused in the water, deep green, or a dirty yellowish-green; sterile joints about as long as broad, or twice as long; conjugation scalariform (according to Rabenhorst sometimes at the same time lateral); zygospores globose; spore coat smooth.

Remarks.—This species is very common around Philadelphia, forming great masses in the ditches of the "Neck," growing in the semistagnant water along the railroads, and forming with other algæ slimy coatings on the dripping rocks of the Wissahicon and various railroad cuttings. At certain times the cells are found crowded with endochrome, at other times they are almost empty. At certain seasons this plant multiplies with great rapidity after a somewhat peculiar fashion. Constrictions first appear in the filament at the junctions of the cells, which thus look as though their ends were rounding off. This goes on until the ends of the cells are greatly rounded, and are attached simply by their central parts, which soon separate. In this way (fig. 8b, pl. xv.) the filament is resolved into its component cells, or more generally into as many pairs of cells as compose it, which when once set free in the water rapidly grow into filaments by the ordinary process of cell multiplication by division. In most cases the zygospores are placed in one of the parent-cells, but I have seen instances in which some of them were formed in the connecting tubes.

Fig. 8, pl. 15, represents this species.

Z. cruciatum, (VAUCH.) AG.

Z. pallide viride, siccatum fuscescens vel fusco-nigrescens; articulis sterilibus brevicylindricis diametro (0.0016"—0.00195") æqualibus vel dimidio longioribus, rarius duplo longioribus, post divisionem factam haud raro dimidio brevioribus, fructiferis non tumidis; zygosporis plerumque globosis, maturis obscure fuscis, sporodermate subtiliter punctatis. (R.) *Species mihi ignota.*

Syn.—*Zygnema cruciatum*, (VAUCHER) AGARDH. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 251.

Tyndaridea cruciata, HASSALL, *Fresh-Water Algæ*, vol. i. p. 160.—HARVEY. BAILEY, *Microscopical Observations*, p. 21.

Hab.—Northern States; Virginia; Florida; Bailey.

Pale green, when dried subfuscous or blackish fuscous. Sterile joints shortly cylindrical, equal or a little longer, or more rarely twice as long as broad (diam. 0.0016"—0.00195"), after division sometimes shorter than broad; fruiting cells not tumid; zygospores mostly globose; when mature, obscure fuscous, their coat minutely punctate.

Genus SIROGONIUM, KtZ.

"Cellulæ vegetativæ cylindricæ, sporiferæ subinflatæ orculiformæ. Fasciæ chlorophyllosæ longitudinales, parietales, leviter flexuosæ, nodosæ (plerumque 2-3, rarius 4 in quaque cellula), granula amylacea 7-8 involutæ. Copulatio genuflexa, sine tubo connexivo." R. In specie Americana fasciæ chlorophyllosæ spirales et Spirogyræ illis similes.

Vegetative cells cylindrical, spore bearing cells somewhat inflated, or orculiform. Chlorophyl filament longitudinal, parietal, somewhat flexuous, nodose (mostly 2-3 rarely 4 in each cell), containing 7-8 starch granules; conjugation genuflexuous, without any connecting tubes. (Rabenhorst). In American species the chlorophyl filament spiral and like to that of Spirogyra.

Remarks.—This genus was originally made by Kützing to contain a single species, which possesses the characters given in the diagnosis of Prof. Rabenhorst

I have met with an American plant, which has some of these characters, and at the same time others which have been supposed to belong to the genus *Spirogyra*. It unites the method of reproduction of *Sirogonium* and the arrangement of the chlorophyl band of *Spirogyra*, standing as it were midway between them. It is not midway, however, but much nearer *Sirogonium*, for the passage from a very loose spiral to a longitudinal flexuous filament is a brief one, and although in some cells of *S. retroversum* the spiral makes a number of turns; in other long cells it scarcely gets around once, in other words the chlorophyl band is nearly straight. On the other hand, the reproduction is strictly that of *S. strictum*, at least in all cases which have come under my notice. There is, therefore, but one of two things to be done, either to unite *Sirogonium* with *Spirogyra*, or else to give up the arrangement of the chlorophyl as an essential character of the former genus. The great variance, in the latter respect, in our American species, greatly weakens the value of any such character, and I have, therefore, preferred the latter of the two courses.

S. retroversum, WOOD.

S. articulis sterilibus diametro 7-15 plo longioribus; fasciis spiralibus 1, rare 2, latis, granulatis; anfractibus 1-9; articulis fertilibus valde tumidis, retroversis; conjugatione genuflexa et sine tubo connexivo; cytodermate nonnihil crasso, utroque fine protenso vel replicato; sporis ellipticis.

Diam.—Art. steril. $\frac{7\frac{11}{500}}{500}$ " = .00146"; spor. lat. $\frac{7\frac{9}{500}}{500}$ "— $\frac{7\frac{13}{500}}{500}$ " = .00133"—.0016"; long. $\frac{7\frac{25}{500}}{500}$ " = .0033".

Syn.—*S. retroversum*, Wood, Prodr. Am. Phil. Soc. 1869, p. 139

Hab.—In stagnis, prope Philadelphia.

Sterile joints 7-15 times longer than broad; chlorophyl band 1, rarely 2, broad, granulate; turns 1-9; fertile article very tumid, retroverted; fertile cells scarcely swollen; cell wall folded in at the ends; chlorophyl band single; turns 6; spores oblong or elliptical, spore wall very thick.

Remarks.—I have found this species growing in stagnant ditches in the Neck below the city. In fruit the cells are almost always very markedly bent backwards, and have a broad pouch-like dilatation in front. The spores are elliptical, and, as I have seen them, greenish and with a thin coat, but may not have been completely matured.

Fig. 1, pl. 16, represents this species.

Genus MESOCARPUS, HASSALL.

Cellulæ massa chlorophyllosa initio diffusa, postea in fasciam longitudinalem, haud raro flexuosam contracta; nucleum centralem et granum amylaceum unicum vel duo involvens. Zygospora globosa vel ovata, in tubo connexivo inter cellulas binas plus minus genuflexas formata.

Chlorophyl mass in the beginning diffused in the cell, afterwards contracted into an often flexuous fascia, and involving a central nucleus and one or more starch granules. Zygosporæ globose or ovate, formed in the connecting tube between two more or less bent cells.

M. scalaris, HASSALL.

M. cellulis sterilibus diametro 3-6 plo longioribus, fertilibus valde curvatis; zygosporis ovalibus.

Diam.—Max. $\frac{7\frac{5}{500}}{500}$ " = .0011".

Syn.—*M. scalaris*, HASSALL, Fresh-Water Algæ, vol. i. p. 166, et RABENHORST, Flora Europ. Algarum, Sect. III. p. 257.

Hab.—In fossis, prope Philadelphia.

Sterile cells 3-6 times longer than their diameter, fertile strongly curved; zygospores oval.

Remarks.—This species is abundant in the stagnant ditches near Camden. It agrees well with the descriptions of the European form. I have, however, never seen it in the state in which it has "fuscous spores." They have always been greenish, but very possibly were not fully matured.

Fig. 5, pl. 15, represents a pair of cells of this species just commencing to conjugate.

M. parvulus, HASSALL.

M. cellulis diametro (0.00031"—0.00041") 5-12 plo longioribus; zygosporis globosis, plerumque 0.00062" latis, sporodermate fusco laevi. (R.) Species mihi ignota.

Syn.—*M. parvulus*, HASSALL, Fresh-Water Algæ, vol. i. p. 169, et RABENHORST, Flora Europ. Algarum, Sect. III. p. 257.

Hab.—Rhode Island; (S. T. Olney) Thwaites.

Cells 5-12 times longer than their diameter (0.00031"—0.00041"); zygospores globose, mostly 0.00062" broad, spore coat fuscous smooth.

Genus **PLEUROCARPUS**, A. BRAUN (1855).

Cellulæ eadem quæ in *Mesocarpus*; copulatio lateralis et sporifera, nonnunquam geniflexa et plerumque sterilis. (R.)

Cells like those in *Mesocarpus*; conjugation lateral and sporiferous, somewhat geniflexuous and mostly sterile.

P. mirabilis, BRAUN.

P. cellulis diametro (0.0011"—0.0013") 2-5 plo longioribus; zygosporis subglobosis, fuscis, levibus. (R.) Species mihi ignota.

Syn.—*Mougeotia geniflexa*, AGARDH. BAILEY, Silliman's Journal. New Series, vol. iii.
Pleurocarpus mirabilis, A. BRAUN. RABENHORST, Flora Europ. Algarum, Sect. III. p. 258.

Hab.—West Point, New York; Providence, Rhode Island; Detroit, Michigan; Fort Winnebago, Wisconsin; Bailey.

Cells 2-5 times longer than their diameter (0.0011"—0.0013"); zygospores subglobose, fuscous, smooth.

ORDER **Siphophyceæ.**

Algæ unicellulares. Cellula utriculiformis, plerumque ramulosa; ramuli vegetatione terminali præditi, sæpe demum septo discreti, et alteri in oosporangia, alteri in antheridia transmutantur. Cytoplasma viride, granulosum, mucilaginosum, vesiculis chlorophyllosis et granulis amylaceis repletum. Propogatio fit aut cytiogenesi libera, aut zoogonidiis aut oosporis.

Unicellular algæ. Cells utriculiform, mostly branched; branches with a terminal vegetation, often finally cut off by a partition wall and transformed into antheridia or oosporangia. Cytoplasm green, granular, mucilaginous, filled with chlorophyl vesicles and starch granules. Propagation either by forming minute spores by free cell formation, or by zoospores, or by oospores.

FAMILY HYDROGASTREÆ.

Plantulæ minimæ, terrestres, gregariæ. Cellula initio globosa, postea clavato-vel pyriformi-intumescens, basi attenuata elongata et in ramulos subtilissimos hyalinos partita. Cytoplasm mucilaginosum, ætate provecta gonidia divisione simultanea transformatum. Cytioderma lamellosum ætate provecta dilabens et contabescens et gonidia liberans.

Plants very small, terrestrial, gregarious. Cells in the beginning globose, afterwards clavate or pyriform, with an elongated, attenuated base, divided into very fine, hyaline branches. Cytoplasm mucilaginous, at maturity transformed by a simultaneous division into gonidia. Cytioderm lamellate, at maturity wasting, withering away and setting free the gonidia.

Remarks.—The *Hydrogastreæ* are curious little unicellular plants, which grow upon wet earth. The matured frond is swollen up at one end to form a subglobular or pyriform head, whilst at the other end it is produced into a long, much-branched, very fine root-like portion which enters the earth and maintains the little plant in its upright position. The green endochrome is contained almost entirely in the head, and forms generally a coat or layer in the outer portion of its cavity, the inner part of which appears to be occupied by a watery fluid.

The only specimens which I have seen of this family were found growing in the mud left by the receding water of a recently drawn mill pond, by Dr. Billings, U. S. A. When I got them they were thoroughly dried up, and consequently no opportunity of studying their development was afforded. According to Kützing and Braun, the species is propagated ordinarily by the breaking up of the chlorophyllous layer of protoplasm lining the wall of the cell into a larger number of very small globular spores. These, although not endued with the power of motion, seem from their method of formation and history to be homologous with zoospores. In most cases they are set free by the membrane of the parent-cell becoming gelatinously softened, swelling up, collapsing, and finally dissolving away. The little protococcoid cells then enlarging, develop at one end a hyaline prolongation which penetrates into the ground. Growth and development continuing the upper end of the cell swells up into the ovate or globular head, whilst the lower becomes the hyaline, branching, root-like portion of the new frond. No indication of this method of reproduction was discoverable in the plants which Dr. Billings sent me. The evident affinities of the family with the *Vaucheriaceæ* render it exceedingly probable that there is in it some method of sexual reproduction, as yet undiscovered, allied to that which occurs in the latter. In some of the specimens sent me, there were what appeared to be resting-spores (pl. XVI., fig. 2 a), occupying the whole of the cavity of the cell, from which they appeared to be finally discharged by a decay and rupture of the outer coat or wall. How these bodies were formed, and whether they really have power to reproduce the species I cannot tell.

Genus HYDROGASTRUM, DESV.

Character idem ac familiæ.

Characters that of the family.

H. granulatum, (LINN.) DESV.

H. plerumque gregarium, sæpe aggregatum, haud raro confluens; cellula e globoso-pyriformi, magnitudinæ seminis papaveris vel sinapios et ultra, prasino-viridi superficie pulverulenta. (R.)

Syn.—*Botrydium argillaceum*, WALLROTH. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 265.

Hydrogastrum granulatum, (LINNÆUS) DESV. RABENHORST, *loc. cit.*

Hab.—Delaware; (Dr. Billings) Wood; West Point, New York; Providence and Newport, Rhode Island; Bailey.

Mostly gregarious, often aggregate, not rarely confluent; cells pyriform, of the size of a poppy or mustard seed and larger; pea-green; surface pulverulent.

Remarks.—The above description is taken from Rabenhorst's work, and applies to the specimens collected by Dr. Billings in the State of Delaware, excepting that I did not discover any of them to be confluent, nor was their surface distinctly pulverulent. Prof. Kützing gives as a comparative character between this and *H. Wallrothii*, the smaller size of the spores; but Prof. R. says nothing about this. There were no spores in any of the American specimens, and I think it somewhat uncertain whether or not the plant is or is not either of the European species. It is very probable that it will be discovered that the only true specific characters are sexual, and consequently have not as yet been made out in any of the forms. Certainly the descriptions of the species as at present given seem to me not to contain any reliable characters.

Fig. 2 *a*, pl. 16, represents a very young state of our American plant; fig. 2 is the perfected frond, both magnified ninety diameters; fig. 2 *a* shows what is supposable a perfected resting spore magnified 160 diameters.

FAMILY VAUCHERiaceÆ.

Algæ monoicæ, cæspitosæ, unicellulares. Cellula vegetiva (thallus) vegetatione terminali, utriculi formi-elongata et ampliata, prominentiis plus minus elongatis ramosa.

Propagatio aut sexualis, fit oosporis ope spermatozoidiorum fecundatis, aut non sexualis zoogonidiis. Fructificatio triplex (melius organa fructificationis tria):—

1. *Sporangium* terminale, ex thalli apice plerumque globoso-clavato-tumido formatum, septo discretum, cytoplasmate obscure viridi, demum in zoogonium (zoosporam, Thur.) unicum permagnum, ciliis vibratoriiis dense obsitum abeunte faretum.

2. *Oogonium* (oosporangium) laterale, sessile vel prominentia, plus minus elongata vel simpliciter vel partita pedicellatum, cytoplasmate ætate provecta in oosporam singulam transmutato fetum.

3. *Antheridium* laterale, sessile vel e ramuli lateralis parte suprema septo discreta formatum, in quo *spermatozoidea* (antherozoidea, Thur.) numerosissima nascuntur, denique erumpunt. Spermatozoidea oblonga, ciliis duobus inæquilongis, subpolo antico ortis instructa. (R.)

Monœcious algæ, cæspitose, unicellular. Vegetative cells (thallus) growing at the ends, elongate, utriculiform, and ampliate, more or less profusely branched.

Propagation either sexual, with oospores which are fecundated by spermatozoids, or non-sexual, by means of zoospores. Organs of fructification of three kinds:—

1. *Sporangia*, which are terminal and mostly formed from the separation of clavately swollen, globose apex of the thallus (often of a branch) by means of a partition; in the sporangium arises a single, very large zoospore, which is densely clothed with cilia.

2. *Oogonia* (oosporangia), lateral, sessile or pedicellate simple bodies, whose cytoplasm is finally converted into an oospore.

3. *Antheridium* lateral, sessile, or formed out of the end of a branch; the spermatozoids formed in them oblong, furnished with two unequal cilia, arising near the front end.

Remarks.—The *Vaucheriaceæ* are amongst our most common fresh-water algæ. They occur generally in the form of vast numbers of individuals interwoven into broad mats, which have often both a felty look and feel. When growth is going on rapidly, these mats are of a beautiful vivid green; but when the process of sexual reproduction has checked the life of the individual they become dingy and dirty looking. The thallus is composed of a single cell and is almost always branched. The branches never have, at least in any of our species, a definite arrangement, save only in that they always arise from the side and not from the point of the thallus. In the European species, *V. tuberosa*, however, the branches are said to arise both from the point and sides of the frond.

The frond cell is generally nearly uniform in diameter and has a thick outer wall, which is composed of cellulose, as is proven by the action upon it of iodine and sulphuric acid and of the iodo-chloride of zinc solution. Within the cell are chlorophyllous protoplasm, starch granules, watery fluid, and a few scattered raphides or inorganic crystals. There is never any nucleus. The protoplasm is often very granular, and is mostly collected in a thick green layer upon the inner surface of the cell wall, leaving the centre of the cell free for the more watery contents.

Growth, except in the very young fronds, consists exclusively in an increase in length, and takes place only at the ends of the thallus or in the portions near it. The branches are almost always simple, but are said in some species to give origin to secondary branchlets, and even, at times, to tertiary ones. They grow in the same manner as the main thallus, *i. e.* by additions to their ends.

When the thallus of a *Vaucheria* is ruptured by external injury, or, at times, when it is dying from some hidden cause, a number of bright green globes of various sizes are formed out of the endochrome. These appear to have the power of independent existence for some time, but whether or not they ever actually grow into new thalli I am unable to state.

M. Walz asserts, that he has observed in certain species the formation of a quiet spore without the intervention of sexual organs, and that the process is as follows. The end of a long or short twig swells up, and the chlorophyl and protoplasm from the neighboring parts accumulate in the enlarged portion. A partition wall then forms at the base of the latter, which is thus changed into a closed chamber, a sporangium. The green contents then slowly gather themselves together into a denser and denser ball, becoming more and more separated, in so doing, from the wall of the sporangium, and finally secreting around themselves a distinct membrane. After the formation of a spore in this way, the sporangium opens at the apex and allows it to escape. The spore, after remaining quiet for some time in the water, at last germinates into a new frond, in a similar manner to an ordinary zoospore. In my earlier studies of fresh-water algæ, I noticed something very similar to this in one of our species, but convinced myself that the little body was nothing but a zoospore, whose normal development had been perverted by untoward influences, and therefore paid no more attention to the matter. It is probable that the life-history of the bodies observed by M. Walz is capable of the same explanation.

Although I have very frequently cultivated *Vaucherias*, I have never been so fortunate as to see them form their zoospores, nor indeed to see a zoospore in its motile state. The life-history of these bodies has, however, been fully and repeatedly worked out by other observers. It is described by such as occurring in the following manner. One end of a branch first enlarges into a bulbous, often conical, point, into which the neighboring endochrome crowds itself. This point is next divided off by a partition wall from the remainder of the thallus and constitutes the *zoosporangium*, the contents of which rapidly condense into one or two masses, generally oval in shape, each of which eventually forms a zoospore. When the latter are matured, the apex of the *zoosporangium* opens, and the little bodies within slowly and gradually emerge, without any apparent cause for their motion. Sometimes, according to Cohn, instead of this steady outward passage, there are repeated forward and backward movements of the zoospores within the case. The zoospore after its perfection is generally oval, and very large. Within it there are one or more vacuoles, and surrounding it is a layer of colorless protoplasm. It is remarkable for having its whole surface densely covered with short cilia. Its period of motile life appears to be very brief; according to Walz, that of the zoospore of *V. sericea*, Lyngb., lasts only from one-half to one and a half minute, after which time the cilia are lost and a cellulose wall secreted around the mass. Germination takes place by the growth of the cylindrical thread out from each end of the zoospore.

True sexual reproduction takes place in this family by means of *antheridia* and *oogonia*, male and female organs. All known species are mostly if not absolutely monœcious, both organs being contained in the one individual and always placed in proximity. All of the species in which the development and structure of the sexual organs have been studied, agree in the essential points.

The first appearance of the *antheridium* is as a little pouch projecting out from the side of the thallus. This increases in size and soon assumes the peculiar shape of the species. At the same time there is a diminution, according to M. Walz, of the chlorophyl in the antheridium, so that, when the partition wall forms and shuts off the cavity of the latter from that of the thallus, there are only a very few scattered green granules remaining. The antheridium at the time of separation contains, therefore, only transparent protoplasm, which soon becomes granular, and shortly afterwards exhibits the moving spermatozoids, which appear to be formed out of the thick layer of protoplasm that lines the inner surface of the cell wall. The point of the antheridium opens so soon as the spermatozoids are perfected, and allows them to escape.

The formation of the *oogonia* takes place very similarly to that of the antheridia. There is the same little protrusion from the side of the thallus in the commencement of the process, the same after-growth and increase of this pouch, and the same formation of a separating wall between it and the main body of the frond. A very marked difference, however, is to be found in the contents of the two, the oogonium from the very commencement being crowded with chlorophyl and oil globules. When the oosporangium is completed, the end of it opens, and, at the same time, the contents gather themselves into a dense protoplasmic ball, which lies in the

centre. The spermatozoids, which are at this time already free in the water, are very minute, longish, ellipsoidal or ovate masses, provided with two unequal cilia. These commonly both arise together from one end of the body, and are directed in opposite directions—one backwards, the other forwards. According to M. Walz, however, in *V. sericea* the cilia arise from the opposite ends. According to De Bary, the spermatozoids of *V. aversa*, Hassall, contain reddish pigment-granules. M. Walz states that he has twice seen the process of impregnation in *V. sericea*, Lyngh., and describes it essentially as follows: After the bursting of the antheridium and the formation of the opening in the oogonium, the spermatozoid clustered around the little orifice in the latter, but were apparently debarred entrance by the presence of a glutinous jelly. After a time, however, one, and then another, forced a passage through this obstacle until finally a number gained access to the protoplasmic ball within. Over this they swarmed, pushing it and retiring and butting against it until some of them actually forced their way into it and were absorbed by it. Impregnation being now completed, the oospore acquired a very sharp definite outline, and secreted in a very short time a membrane around itself. The changes which followed during its maturing consisted of the acquiring of a thick coat and the replacing of the chlorophyl within by a reddish-brown coloring matter. The ripened resting spore of almost all the *Vaucheria* is provided with three coats, of which the middle is the thickest. The contents consist of protoplasm, reddish-brown pigment, and numerous oil globules.

Genus VAUCHERIA.

Genus unicum, character idem ac familiæ.

The only genus of the family, having the same characters.

V. sessilis, (VAUCH.) DE CANDOLLE.

V. laxe intricata, pallide et subsordide viridis; thallo capillari, parce ramoso; oogoniis 2-3 approximatis, rarius singulis, ovatis vel ovali-oblongis, plus minusve obliquis, rostratis; antheridio intermedio, ramuli modo brevi hamato, modo recto subulato, subclavato, modo elongato et incurvato, haud raro circinato sustentato; oosporis maturis fusco-punctatis, membrana triplici involutis. (R.)

Syn.—*V. sessilis*, (VAUCH.) DE CANDOLLE. RABENHORST, Flora Europ. Algar., Sect. III. p. 267.

V. cæspitosa, (VAUCH.) AGARDH. RABENHORST, *loc. cit.*

Hab.—Salem, North Carolina; Schweinitz. Common at West Point, New York; Waterville, Maine; Culpepper Co., Va.; Bailey.

Laxly intricate, pale and subsordid green; thallus capillary, sparsely branched; oogonia 2-3, approximate, rarely single, ovate or oval-oblong, more or less oblique, rostrate; antheridia intermediate, sustained upon branches partly shortly hamate, partly straight subulate, subclavate, partly elongate and incurved, and not rarely circinnate; oospores at maturity, fuscous-punctate, surrounded by a three-fold membrane.

Remark.—I think I found this species near Philadelphia in my earliest researches, but cannot speak certainly, having preserved neither notes nor specimens.

V. velutina, Ag.

V. thallo repente, ramulis erectis, numerosis, fastigiatis, in cœspitem velutinum læte viridem intricatis; oogoniis lateralibus singulis, globosis, sessilibus, antheridio paulo longiore unico subulato leviter incurvato consociatis (R.) *Species mihi ignota.*

Diam.—Oogonii 0.0023"—0.0027". (R.)

Syn.—*V. velutina*, AGARDH. RABENHORST, Flora Europ. Algarum, Sect. III. p. 274.

Hab.—Salem, North Carolina; Schweinitz. Common at West Point, New York; Waterville, Maine; Culpepper Co., Va.; Bailey.

"Filaments exceedingly tough, interwoven into a dense, velvety, green stratum, pellucid below and creeping over the mud; branches near the extremity erect, fastigate, and more or less crooked; vesicles solitary, globular, on short lateral peduncles." Carmichael.

V. geminata, (VAUCH.) DE CANDOLLE.

V. obscure vel sordide viridis, in cœspites dense intricata; thallo capillari, tenaci, dichotomo; oogoniis duobus (rarius I vel 3), ovatis vel obovatis, oppositis, distincte pedunculatis, antheridio intermedio subulato, plus minus recurvo; oosporis maturis fusco-maculatis, sporodermate achroo e stratis tribus composito involutis; sporangiis in eodem vel proprio thallo, cyathiformi-ampliatis truncatis et angulato-cornutis. (R.)

Syn.—*V. geminata*, (VAUCH.) DE CANDOLLE. RABENHORST, Flora Europ. Algarum, Sect. III. p. 269.

Hab.—In stagnis, prope Philadelphia; Wood.

Obscure or sordid green, densely interwoven into a turfy mass; thallus capillary, tenacious, dichotomous; oogonia two (rarely 1-3), ovate or obovate, opposite distinctly pedunculate, antheridia intermediate, subulate, more or less recurved; oospores at maturity spotted with fuscous, their coat transparent and composed of three strata; sporangia in the same or a separate thallus swollen cup-shaped, truncate and horned at the angles.

Remarks.—I have found this species in fruit but once, then it grew in a ditch below the city. Not having mounted any of it, nor having written a description of it at the time, I have been forced to simply copy that of Prof. Rabenhorst.

V. polymorpha, WOOD.

V. in cœspites dense intricata; thallo capillari, tenui; antheridiis corniculatis ex ramuli lateralis apice formatis; ramulis fertilibus interdum et oogoniis et antheridiis instructis, interdum antheridiis solum; oogoniis plerumque geminis, interdum singulis, globosis vel ovatis, sæpe breve rostratis, plerumque distincte pedunculatis sed rarius sessilibus; oosporis enormiter subglobosis vel ovatis; sporodermate achroo, e stratis duobus composito.

Syn.—*V. polymorpha*. WOOD, Prodromus, Proceedings Amer. Philos. Society, 1869, p. 140.

Hab.—In aquis, prope "Buffalo Bayou," Texas; (Ravenel.)

Cæspitose; thallus hair-like, thin; antheridia corniculate, formed of the apex of lateral branches; fertile branches sometimes furnished both with oogonia and antheridia, sometimes with antheridia alone; oogonia sometimes single but mostly in pairs, occasionally shortly rostrate, generally distinctly pedunculate but sometimes sessile; oospores irregularly subglobose or ovate, surrounded by a transparent double spore coat.

Remarks.—This species was collected by Prof. Ravenel near the city of Houston, Texas. As I received the mass, it was labelled as being obtained from "a shallow slimy pool formed by drippings from the side of a ravine near Buffalo Bayou." The species probably grows in the water, evidently forming turfy mats. It is

remarkable from the fact that, whilst in many cases the little branches which produce the antheridia give origin to the spores also, in others they do not; so that there are numerous antheridia, which are unconnected with any female organs. When a branch does produce both of the reproductive organs it usually forks into three short branchlets, thus giving origin to a pair of sporangia and a single curved, hooked antheridia. Sometimes, however, there is but a single female branchlet, and I have even seen a sporangium, immediately sessile upon a branch, which at its apex gave origin to a male organ. In the coat of the perfected spore, I have not been able to find more than two distinct strata.

Figs. 3 and 3 *a*, pl. 20, represent sporangia and antheridia of this species; 3 *b*, a simple, young and only partly formed antheridia, magnified 160 diameters; 3 *c*, a perfected spore magnified 260 diameters.

V. sericea, LYNGBYE.

V. aquatica vel terrestris, cæspitosa, vel sordide vel læte vel luteolo-viridis; thallis tenuibus, dense intricatis, laxè et vage ramosis, ramisque sæpe adscendentibus vel erectis; oogoniis sessilibus vel brevissime pedicellatis, 1-6 seriatis, unilateralibus, oblique et enormiter ovalibus, ore laterali producto rostellatis; antheridiis in thallo ipso juxta oogoniis sessilibus, cylindraceo-subclavatis, deflexis; spermatozoidis oblongis, puncto rubro notatis (teste de Bary), in utroque polo cilio unico præditis.

Syn.—*V. aversa*, HASSALL, Fresh-Water Algæ, p. 54.

V. sericea, LYNGBYE. RABENHORST, Flora Europ. Algarum, Sect. III. p. 271.

Hab.—Prope Philadelphia; Wood.

Aquatic or terrestrial, occurring in turfy mats of a yellowish, dirty, or bright green color; fronds thin, densely intricate, laxly and vaguely branched, often together with the branches ascending or erect; oogonia sessile or very shortly pedicellate, 1-6 seriate, unilateral, obliquely irregularly oval, their lateral mouths produced into a rostellum or beak; antheridia sessile upon the thallus itself near the oogonium, somewhat cylindrical, subclavate, deflexed especially in age; spermatozoids (according to De Bary) oblong, marked with a red point and furnished with a single cilia at each end.

Remarks.—I can perceive no constant differences between *V. sericea*, Lyng. and *V. aversa*, Hass. The extreme forms differ somewhat, but both are very common about Philadelphia, and everywhere grade into one another. Prof. Rabenhorst thinks that the two forms are scarcely distinct, and states that the most characteristic differences are, that in *V. aversa*, the thallus is much thicker, and the oogonia larger and more erect, whilst the oospores are smaller and consequently do not fill the cavity of their case. These differences are, except the last, simply differences in size, and seem to me to depend simply upon circumstances of growth. The relatively smaller size of the spore is a very frail hook indeed to hang a species upon.

The plant grows in springs and actively running water abundantly in this neighborhood; also on very wet ground, especially on that which is habitually overflowed, such as the face of dams, neighborhood of springs, &c. In the water, it is frequently on the ground, but also often clothes such objects as stones, largish sticks, &c.

ORDER **Nematophycæ.**

Algæ multiceulares, chlorophyllosæ, membranacæ vel filamentosæ, ramificatione aut instructæ aut destitutæ. Propagatio fit aut oosporis aut zoogonidiis, sed nunquam conjugatione.

Multicellular, chlorophyllous algæ, membranaceous or filamentous, furnished with or destitute of branches. Propagated by oospores or zoospores, never by conjugation.

FAMILY ULVACEÆ.

Thallus membranaceus vel foliaceus, vel filiformis (Schizomeris?) rarius crustaceus, e cellularum strato unico formatus, aut expansus aut tubuloso- vel vesiculoso-concretus.

Propagatio fit zoogonidiis, cytoplasmatis divisione repetita ortis. Zoogonidia oblonga, polo antico ciliis vel binis vel ternis vel quarternis instructa.

Thallus membranous or foliaceous, rarely crustaceous, composed of a single stratum of cells, either expanded or tubularly or vesicularly concreted.

Propagation by means of zoogonidia, formed by the repeated division of the cytoplasm. Zoogonidia oblong, furnished with two, three, or four cilia at the anterior end.

Genus PROTODERMA, Ktzt.

Thallus crustaceus, indeterminatus, substrato arcte adhærens, e cellulis anguloso-rotundatis, irregulariter ordinatis, arcte connexis compositus.

Propagatio ignota.

Thallus crustaceus, indeterminate, closely adherent to the substratum, composed of closely conjoined irregularly arranged angularly rounded cells.

Propagation unknown.

P. viride, Ktzt.

P. viride, lubricum.

Syn.—*P. viride*, KUTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 307.

Hab.—In aquario; Wood.

Green; slippery.

Remark.—I have seen a plant, which I take to be this species, growing on the glass and on pebbles in the aquarium of my friend, Dr. Frické.

Genus ULVA, LINN.

Thallus membranaceus, plane expansus, angustus vel latus, nonnunquam latissimus, magis minusve undulato-crispatus, sæpe laciniatus, haud raro perforatus, e cellularum strato unico formatus, callo disciformi parvo affixus, ætate provecta sæpe libere natans. Cellulæ anguloso-rotundatæ, cœloplasmaticæ, parenchymaticæ connexæ.

Vegetatio cellularum divisione in duas directiones repetita. Propagatio fit zoogonidiis, in cellulis quibusdam cytoplasmatis divisione 4, 8-16 ortis, ciliis vibratoriiis quaternis longitudine corporis longitudinem vix superantibus instructis.

Thallus membranous, expanded, narrow or broad, sometimes very broad, more or less undulately curled or crisped, often lacinate, not rarely perforate, formed of a single stratum of cells, fastened by a small discoid thickened portion, in advanced age often swimming free. Cells angularly globose, joined into a sort of parenchyma.

Growth occurring by the repeated division of the cells in two directions. Propagation by zoospores, 4-8-16 of which are formed at once by a division of the endochrome of certain cells, and are furnished with four vibratile cilia scarcely longer than the body.

U. merismopedioides, Wood.

U. ampla, membranacea, late expansa, dilute viridis, tenuis, radiatim et enormiter plicata, ambitu sæpe subrotundata; margine undulato, interdum suberenato; cellulis enormiter ovalibus vel angularibus, nucleo destitutis, quarternariis et in familias *Merismopediarum* modo obscure associatis.

Diam.—Cell. max. $\frac{5}{12000}$ " = .00041', plerumque $\frac{2}{12000}$ " — $\frac{3}{12000}$ " = .00016' — .00025.

Syn.—*U. merismopedioides*, Wood, Botanical Report of the United States Geological Exploration of the Fortieth Parallel, p. 415.

Hab.—In torrentibus, Diamond Range (alt. 6000 ft.), Rocky Mountains; (Sereno Watson) Wood.

Thallus ample, broadly expanded, membranaceous, dilute green, thin, radiately and irregularly plicate with its outline often somewhat rounded; its margin undulate or at times almost crenate; the cells irregularly oval or angular, destitute of nucleus, quarternary and obscurely arranged in families after the manner of a merismopedia.

Remarks.—The largest fronds of this species that have come under my notice are about three inches long by two broad, thin, easily torn, and not all gelatinous. The portion by which they have been attached is very evident, near one of the margins, and from it broad undulations or folds radiate. Sometimes the frond is split up into palmate, lobe-like parts.

The cells are not closely approximate, but are placed in a homogeneous translucent membrane, in such a way as to remind one of a *Merismopedia*.

I do not feel certain that this plant is distinct from *U. orbiculata* of Rabenhorst, though for the present I have preferred so to consider it. His description is very brief and incomplete, as is also the original one of Thuret, which I have consulted. Prof. R., however, gives *U. latissima* of authors as a synonym of *U. orbiculata*, and certainly this plant is distinct from *U. latissima*, Harvey, of our coast. Again it seems impossible that a plant growing near the summit of the Rocky Mountains should be identical with one found on the coast of France. Prof. Sereno Watson found this plant growing on rocks in a mountain stream of the Diamond Range, at an altitude of 6000 feet.

Genus ENTEROMORPHA, LINK.

Thallus membranaceus, tubulosus vel utriculiformis, basi affixus (saltem initio, postea sæpe libere natans), e cellularum strato unico compositus, sæpe ramosus, haud raro ramosissimus. Propogatio fit zoogonidiis. Hæc zoogonidia proceantur in cellulis quibusdam 8-16 cytoplasmatis divisione repetita, in polo anteo rostriformi ciliis duobus corpus duplo superantibus prædita. (R.)

Thallus membranaceous, tubular or bladder-shaped, affixed by the base (at least in the beginning, often afterwards floating freely), composed of a single stratum of cells, often branched, not rarely very much branched. Propagation by means of zoospores, 8-16 of which are formed by the repeated division of the protoplasm of a cell. Their anterior beak-like portion provided with two cilia whose length is not less than twice that of the body.

***E. intestinalis*, (LINN.) LINK.**

E. teres, forma et magnitudine admodum varia, sæpe pedalis etiam supra, leptoderma, saturate vel pallide viridis, filiformis vel intestiniformis, plana vel bullosa; cellulis 3-5-6 angularibus. (R.) *Species mihi ignota.*

Diam.—0.00048" — 0.0008". (R.)

Syn.—*E. intestinalis*, (LINNÆUS) BAILEY, Silliman's Journal, N. S., Vol. III., et RABENHORST, Flora Europ. Algarum, Sect. III. p. 312.

Hab.—Hudson River, from Newburgh to New York City; Narragansett Bay, Rhode Island; Bailey.

Terete, very various in size and shape, often a foot or more in length, smooth, deep or pale green, filiform or intestiniform, plain or bullose; cells 3-5-6 angular; their diameter 0.00048" — 0.0008".

Genus SCHIZOMERIS, Ktz. ?

Thallus filiformis, cylindricus, hic illic valde contractus, basi attenuata affixus. Vegetatio fit cellularum divisione initio in duas postea in tres (?) directionem. Propogatio fit zoogonidiis. Zoogonidia in thalli juvenis cellulis orta, ovata, polo antico ciliis tribus instructa.

Thallus filiform, cylindrical, here or there strongly contracted, adnate by the strongly contracted base. Growth in the beginning by the division of the cells in two directions, afterwards in three directions. Zoogonidia formed in the cells of the young thallus, ovate, their anterior end furnished with three cilia.

Remarks.—The plant from which the above generic description has been drawn up grows abundantly in our ditches below the city. Whether it really belongs to the genus *Schizomeris* or is the representative of a new group is somewhat uncertain. I have never seen the European plant, but, if I understand the descriptions of it, the cells in it are all arranged in a single plane. This certainly is not the case in the old plants of our North American form, for in them the cells are so placed as to make a thick opaque filament, the outside of which everywhere presents the outer walls of cells. The life history of the European species has not been at all worked out, and I have refrained from actually indicating a new genus, in the absence of absolute knowledge upon the subject, because the specific characters of the two plants are so much alike.

I have had some opportunities for studying the life history of our American plant. The zoospore (Fig. 1 c. pl. XVII.) is of the ordinary conical or ovate form, with a very decided transparent anterior end, from which arise three cilia. As the number three is a rare one for cilia to exhibit, I have examined several zoospores with care, and am very certain that they had no more or less. It is, therefore, probable that the number is fixed for the species, although just possible that my finding several individuals in agreement was accidental. The zoospore after a period of free life, during which its motion is very active, becomes quiescent, and, its cilia withering away, attaches itself by its smaller end to some twig, stone, or other support. At the same time it appears to change its shape somewhat, growing longer and narrower, and the smaller end spreading out to form a little foot. Simultaneously with these changes the young plant acquires a cellulose coat, and so becomes a perfect cell, in which I have never been able to detect any nucleus. After a while the cell thus formed divides transversely into two, which, of course, lay end to end. Each of these cells then grows until it attains a certain size, and then the transverse division is repeated. In this way the process goes on until finally a long filament is produced, which is composed of but a single series of cells. These cells are much broader than long, and are placed end to end, so that the cylindrical frond is made up as it were of disks laid one upon the other. When the filament has in this way reached a certain stage of development, one of two things occurs, either the cells begin to divide at right angles to the plane of their previous division, or else the production of zoospores takes place. In the first instance each cell divides into two, four, or more cells. This division, I believe occurs in three if not all directions, so that each original cell is represented by a number of cells, and a sort of compound filament arises, out of which the matured

large trichoma is formed by a continuation of growth, and, perhaps, by a repetition of the division. I have never been able to discover that any reproductive process whatever takes place in this compound filament, and am very confident it never produces zoospores. It is very possible, however, that it may in some way give origin to resting spores, although, as above stated, no indication of this has ever come under my notice. The zoospores are formed in the young fronds as follows: The endochrome in the cell concerned gradually separates in the ordinary manner into several distinct masses, which soon assume a more or less irregularly globular or pyriform shape. Whether the number of these masses is fixed for the single cell or not I am unable to state. These changes occur almost simultaneously in a number of consecutive cells, commencing with the most distal and rapidly spreading towards the base of the filament. When they are pretty well advanced, the walls of the cells undergo some alteration, probably a gummy degeneration, whereby they become soluble in the water. As the division of the endochrome occurs first in the most distal cells of the filament, so does also this change in the cellulose coat. When the endochrome masses are well shapen and distinct, they begin to exhibit motion, becoming uneasy, restless, changing their position, rolling on themselves, and pushing against one another. At the same time solution of the cell walls commences, the partitions between the cells disappearing, and the outer walls spreading. These changes go rapidly forward, and in a little while the zoospores stream out from the fading end of the frond, jostling and crowding as though eager to enter upon their new life.

Fig. 1 *a*, pl. 17, represents the basal portion of an old filament which has failed to form zoospores, magnified 125 diameters. Fig 1 *b* was drawn from a young filament during the process of forming zoospores; owing to their rapid motion, the cilia of the latter could not be seen. This figure is enlarged 250 diameters. Fig. 1 *c* represents a zoospore which has just become quiescent, and still retains its cilia, although they have lost their motile power. Fig. 1 *d*, *e*, *e*, represent the very young plant in different stages of growth. They are all magnified 450 diameters.

***S. Leibleinii*, Ktz. ?**

S. læte viridis vel saturate nigro-viridis.

Diam.—Max. $\frac{1}{125}$ " = .08".

Syn.—*S. Leibleinii*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 311.

Hab.—In fossis, prope Philadelphia.

Bright green to deep blackish-green; largest diameter of the frond $\frac{1}{125}$ ".

Remarks.—Owing to the profusion of zoospores produced by a single filament at one time, it is very usual to find large numbers of the younger plants attached so closely to some central body as to form dense masses of a beautiful green color. The support of these small masses is often entirely concealed, and I have frequently seen them moving freely about the jar, without any apparent cause, until the mystery was solved by finding that some unfortunate snail carried the forest on his back.

The oldest filaments are perfectly opaque, showing, under the microscope, by transmitted light, no trace of their structure.

The species is exceedingly common in the later summer and early fall months in the ditches and sluggish streams around the city, especially in the Neck.

FAMILY CONFERVACEÆ.

Fila articulata aut simplicia aut ramosa, vegetatione terminali non limitata instructa. Articuli plerumque plus minusve elongati, sed nonnunquam diametro breviores, cylindrici, rarius tumidi. Cytioderma plerumque manifesto lamellosum. Massa chlorophyllosa granulata, vesiculas amylaceas involvens, parietalis vel in ætate provecta sæpe in cellulæ centro contracta.

Vegetatio fit utriculi primordialis divisione semper in unam eandemque (transversam) directionem repetitia. Propagatio fit zoogonidiis.

Filaments articulate, simple or branched, growth terminal, unlimited. Joints mostly more or less elongated, but sometimes shorter than long, cylindrical rarely tumid. Cytioderm mostly plainly lamellate, chlorophyll masses granular, surrounding fine starch granules, parietal or often in the centre of the cell.

Growth taking place by division of the primordial utricle always in one direction, namely transversely. Propagation by means of zoospores.

Genus CONFERVA, (LINN.) LINK.

"Fila articulata simplicia. Articuli cylindrici. Massa chlorophyllosa homogenea vel granulata, vesiculas amylaceas involvens. Propagatio ignota." (R.)

Threads articulate simple. Articles cylindrical. Chlorophyll mass homogeneous or granulate, including amylaceous vesicles.

Remarks.—A large number of forms of the genus *Conferva* have been described as distinct species by Kützing and other authors. The characters assigned to these species, however, do not seem to me in any way distinctive. I cannot believe it possible at present to recognize, define, and describe species in this genus, and believe that further studies must be made in their life-history, and other characters discovered before the different forms can be separated. Probably, as was the case with the *Ædlogoniaceæ*, when their sexual life is made out, in it will be found the vital differences. No doubt there are many species common to Europe and America, but I have been entirely unable to determine them. Among the very earliest of my observations upon the fresh-water algæ, before experience had taught how and what to observe, was one made upon what I suppose was a species of this genus. I have never met with the plant since, but as the observation has direct bearing upon the method of propagation, I mention it here, imperfect as it unfortunately is. The plant was found growing on the mud along the Schuylkill River, near Gray's Ferry Bridge, below the city. The filaments were simple, of great length, and uniform in diameter; fig. 7 *a*, pl. 18, represents a portion of one magnified 500 diameters. The cells varied from about as long as broad to three times as long. The amount of endochrome in the cells also varied very much. In most of them, it was not nearly sufficient to fill the cavity, and was arranged as a central superficial band. Many of the cells were seen engaged in the production of zoospores. (Fig. 7 *b*, pl. 18.) Such were well filled with endochrome, which gradually condensed itself into a globular or pyriform mass in the centre of the cell. This, after a short time, began to exhibit activity, rolling upon itself and finally pushing about as much as its confined quarters would allow, until at last it

escaped into the water, through the cell wall. Each cell in this way gave origin to a single zoospore. The walls did not melt away in the water, and, as a number of consecutive cells underwent these changes at the same time, the filament or a portion of it was left as an empty shell. The zoospores were of the usual shape, with a bright anterior spot or beak. The number of cilia was not noted. After a time they settled down generally in clusters, attaching themselves to some foreign particle, dropping their cilia and acquiring a cellulose wall. (Fig. 7 *e*, pl. 18.) They then elongated, underwent the ordinary cell division in a transverse direction, and, by the repetition of this, gradually grew into filaments similar to that from which they sprang.

Fig. 7 *d*, pl 18, represents a young filament just formed in this manner, magnified 500 diameters.

Genus CLADOPHORA, Ktz. (1843.)

Fila cellularum serie simplici formata, varie ramosa. Rami filo centrali similes. Cytiderma plerumque crassum, lamellosum. Cytioplasma parietale.

Filaments composed of a simple series of cells and variously branched. Cytiderm mostly thick and lamellate. Cytioplasm parietal.

Remarks.—The *Cladophora* are branched plants of rather rigid habits, which grow both in salt and fresh water. They are readily recognizable by their comparatively stiff appearance, the absence of gelatinous matter about them, and by the want of regularity in their branching. A large number of species have been described, most of which are marine. They are exceedingly difficult to define, and it is very possible that their hitherto undiscovered sexual reproduction may be finally found to afford the only true characters. I have identified two European forms as growing near this city, and a third has been recognized by Prof. Harvey, as found in our northern States.

I have never seen the production of zoospores in this family, but they are said to be formed by the simultaneous division of the layer of chlorophyllous protoplasm, which fills the outer part of the cell cavity. They exhibit the power of very active motion even before their exit from the cell, which occurs through a papilloid orifice, mostly at the end of the cell, sometimes in its side. Their cilia are sometimes two, sometimes four in number, and their life-history appears to be precisely similar to that of other zoospores.

Cl. glomerata, (LINN.)

Ramuli fili primarii in parte superiore atque ramorum ordinis secundi et tertii plerumque fasciculato- vel penicilliformi-aggregati. Cellulæ maximæ vegetæ cytioplasmate cellularum parieti retiformi- vel subspiraliter applicato. Cellulæ fructiferæ semper terminales, inferiores semper steriles videntur. (R.)

Syn.—*Cl. glomerata*, (KÜTZING) RABENHORST, Flora Europ. Algarum, Sect. III. p. 337.

Hab.—Lake Ontario; Pickering. Falls of Niagara; Lakes Erie, Huron, and Michigan; Fourth Lake, near Madison, Wisconsin; Bailey.

“Filaments tufted, bushy, somewhat rigid, much branched, bright grass-green; branches crowded, irregular, erecto-patent, repeatedly divided; ultimate ramuli secund, subfasciculate; articulations 4–8 times as long as broad.”

Remarks.—Prof. Harvey says (Smithsonian Contributions): “I have received North American specimens from Milton, Saratoga County, N. Y., and from Lake Erie; also from the Mexican Boundary Surveying Expedition.”

Cl. fracta, DILLW.

Clad. prima juventute affixa sed postea libere natans et cæspites formans; ramis ramulisque sparsis, divaricatis, nonnunquam refractis; ramulorum cytioplasmate non spiralter ordinato; cytiodermate sæpe crassissimo; cellulis fertilibus haud terminalibus, plerumque in ramulorum medio, aut eorum basi.

Syn.—*Cl. fracta*, (DILLW.) RABENHORST, Flora Europ. Algarum, Sect. III. p. 334.

Hab.—In flumine Schuylkill, prope Philadelphia; Wood. West Point, New York; Providence, Rhode Island; Bailey.

In the young state fixed, but afterwards floating free and forming matted masses; branches and branchlets scattered, divaricate, somewhat refracted; cytioplasm of the branches not spirally arranged; cytioderm often very thick; fertile cells not terminal, mostly in the middle of the branches, sometimes in their base.

Cl. brachystelecha, RABENHORST.

C. per totam vitam innata, obscure viridis, sicca pallida, pygmæa, 2–4, rarius 6 linea longa, ramosissima, intricata, plerumque culmigena; ramis primariis $\frac{1}{30}''$ — $\frac{1}{40}'' = 0.00295''$ — $0.0022''$ crassis, ramulis ultimis $\frac{1}{60}''$ — $\frac{1}{70}'' = 0.00147''$ — $0.00128''$ crassis; articulis diametro 4–12 plo longioribus; cytiodermate suberasso, hyalino, subtiliter plicato-striato; cytioplasmate imprimis cellularum superiorum laxè spiralter ordinato. (R.)

Syn.—*Cl. brachystelecha*, RABENHORST, Flora Europ. Algarum, Sect. III. p. 343.

Hab.—Prope Philadelphia; Wood.

Fixed through the whole life, obscure green, pale when dried, dwarfish, 2–4, rarely 6 lines long, very much branched, intricate, mostly attached to culms; primary branches $0.00295''$ — $0.0022''$ thick, ultimate ramuli $0.00147''$ — $0.00128''$ thick; articles 4–12 times longer than thick; cytioderm thickish, hyaline, subtilely plicately striate; cytioplasm, especially of the upper cells, laxly spirally arranged.

Remarks.—I have notes of having identified this species at some time, but, having kept neither specimens nor detailed memoranda, have simply copied the description of Prof. Rabenhorst.

FAMILY OEDOGONIACEÆ.

Algæ monoicæ vel dioicæ. Fila articulata aut simplicia aut ramosa, cellula basali obovato-clavata, basi plerumque lobato-partita vel scutata innata. Propagatio fit tum zoogonidiis tum oosporis fecundatione sexuali ortis. Zoogonidia formantur singula in quavis cellula, forma late ovali vel globosa, polo anteo achroo corona ciliorum vibratoriorum prædita.

Oogonia singula vel plura (2–5) continua, plus minusve tumida, in quoque oospora singula, matura rubro- aut flavo-fusco-colorata, ante germinationem in zoosporas plerumque quatuor dilabens se format.

Antheridia brevi-filiformia, 1–2–3–10–articulata, plerumque singula aut oogonio aut filo vegeto insidentia aut in individuis variis sæpe cellula obovato-clavata subtentata.

Monæcious or diæcious algæ. Filaments articulate, either simple or branched, fixed by the basal cell which is obovate-clavate, mostly with its base lobately parted or shield shaped.

Propagation sometimes by zoospores, sometimes by resting spores, the result of sexual impregnation. Zoospores formed simply in certain cells, broadly oval or globose, their anterior end transparent, and furnished with a crown of vibratile cilia. Resting spores single or in series of from

two to five, more or less tumid, single in each sporangium, at maturity reddish or yellowish fuscous, before germination dividing themselves into (mostly four) zoospores.

Antheridia shortly filiform, 1-2-3-10 articulate, mostly single, either upon the sporangium or vegetation cell.

Remarks.—The *Ædogoniaceæ* have been by previous writers simply divided into two genera, *Ædogonium* and *Bulbochæte*. The plants represented by these two divisions have certainly many characters in common, as in the production of their zoospores and spermatozoids as well as in their peculiar method of cell division. Yet they are so very diverse in some particulars in regard to the latter, as well as in their habit of growth and in the formation of their sporangia, that it has seemed to me that the differences between them were more than sufficient to characterize merely genera, and that to each of these groups should be awarded the rank of a sub-family.

Again, in the old genus of *Ædogonium*, we have very distinct groups, separated by differences in the most important of all the characteristic portions of the plant—the sexual apparatus. These groups are the so-called *Monœcious*, *Gynandrous*, and *Diœcious* *Ædogonia*; the *monœcious* division comprising those plants in which one individual gives origin both to the female and male germs; the *gynandrous*, those species in which the plant that produces the female germ gives origin also to a peculiar zoospore, the so-called *androspore*, which, after a period of motile life, settles down and develops a dwarf plant, the *andrœcium*, in which the *spermatozoids* are developed; and the *diœcious* group containing species in which the male and female plants are distinct individuals. Dr. Pringsheim states (*Morphologie der Ædogon.*, p. 43) that these groups pass into one another, but in my opinion, by his own showing, they are sharply distinct. The nearest approach to such passage is between the first and second groups, and consists simply in the fact that in certain species the androspore when it settles down develops into a one-celled instead of a two or three-celled antheridium. This to me does not seem to indicate a union of the groups, for the essential difference is not in the form or complexity of the antheridium, but in the circumstance that in the one case the female filament develops a spermatozoid capable of fertilizing the germ, whilst in the other it gives rise to a body which does not possess that power at all, but does have the capability of giving origin to a second plant, in which the spermatozoid is developed. The groups, therefore, appear to be sharply and distinctly definable.

In the *Bulbochætiæ* but a single genus has as yet been discovered, and this is distinctly gynandrous, but it seems probable that hereafter other plants of this subfamily will be found which are monœcious or diœcious, so that we will have in the two subfamilies two parallel groups of genera.

For the reasons above indicated I have ventured to divide the family into two subfamilies, the one comprising three, the other a single genus. The peculiarities of growth, production of zoospores, and sexual development will be found described under the particular subfamilies.

SUBFAMILY CÆDOGONIEÆ.

Filamenta stricta, haud ramosa, sine setis veris, sed sæpe apice setiforma, elongata, hyalina.

Filaments simple, not ramose, without true seta, but often with their apex seta-like, elongate, hyaline.

Remarks.—The *Cædogoniaceæ* are small filamentous plants, whose size is sufficient to render them visible to the unaided eye, and yet not sufficient to make each individual distinctly apparent. They grow mostly in quiet water, attached to almost any and every thing that can afford a foothold, fringing with apparent indifference stones, twigs, sticks, dead leaves, bits of glass, boards, etc. I have seen such masses of them crowding the whole surface of a *phrysa* as to entirely conceal the animal and its shell, and present the curious spectacle of a perambulating, waving forest of bright green. The individual filament is composed of cylindrical cells, which are always without a nucleus, and have their chlorophyl diffused instead of being collected into bands or stripes. The walls are mostly quite thick and marked near the distal end with circular striæ, whose numbers bear relation to the edge of the cell, for these striæ are the results of the peculiar method of cell multiplication by division, each one marking one such division. When an *cædogonium* cell has attained sufficient maturity and is about to divide, the first perceptible change is the appearance of a little circular line or streak near its distal end. About the same time and in the same place a fine partition is formed by an outgrowth from the primordial utricle, a probably double delicate wall of condensed protoplasm separating the upper end of the parent cell from the lower or main portion. The upper end now begins to develop into a new cell. This development takes place by the formation of an entirely new layer of cellulose inside the little cell, *i. e.* between the new primordial utricle and the old cell wall, and afterwards by the lengthening of this layer by interstitial deposit in the usual way; the thick wall of the parent cell in no way directly participates in the growth (fig. 2 *b*, pl. 17). It is evident that as the new wall grows the old cell wall must be as it were raised up upon it, borne away as a little capping from the basal portion of the parent cell. Consequently when a young cell is watched during this process the little line-like incisure of the parent cell is seen to widen until it becomes an evident trench, and this trench grows wider and wider, until at last it is so broad as to be no longer a trench, and the little end of the parent cell simply caps its offspring. When the latter has fulfilled its allotted period of growth, the process is repeated, the line of separation appearing this time just below the edge of the first cap. It is plain that the second new cell when formed must have a double cap crowning its extremity. At each repetition a new layer is added to the thickening cap, until at last it may be composed of six distinct layers, each projecting just beyond the next older one. Under the microscope the increased thickening of the distal end of a cell bearing such a crown-piece is not sufficiently evident to at first attract attention, whilst each edge of a layer appears as a stria. It is plain that the number of these striæ represents the number of

times division has occurred; if there be four striæ, four times; six striæ, six times, &c.

Besides this method of development, in many species new cells are formed by a sort of pullulation, occurring in the end cell of the filament. The primordial utricle appears to rupture the wall of the distal extremity of the latter and grow out into a little pullulation, or teat, which very soon becomes separated from the parent cell, by the reformation, as it were, of the end wall of the latter. The new little cell thus formed coats itself with cellulose, and rapidly grows, especially in length, always, however, or at least for a length of time, remaining of a smaller diameter than the cell from which it sprang. By a repetition of this process a succession of cells is formed, each one of which, like the successive joints of the field telescope, is a little smaller than its proximal neighbor and contains less chlorophyl, until finally the cells are reduced to exceedingly fine, perfectly transparent, colorless cylinders, which together form a seta or hair.

Reproduction takes place among the *Ædogoniaceæ*, both by means of zoospores and sexual organs. The former of these are quite peculiar, and, therefore, require especial notice.

Only a single one is ever produced in a cell, and there is consequently no division of the chlorophyllous protoplasm preceding their formation. The first change noticeable is a sort of confusion of the cell contents, the protoplasmic portion of which loosens itself, as it were, from the walls, and collects in a mass at the distal end of the cell. This mass after a short time assumes a more or less irregularly globose shape, and simultaneously the parent cell begins to separate from its distal neighbor. This separation appears to take place commonly by a solution of an exceedingly fine ring of the wall of the parent-cell, just at the origin of the transverse partition separating the two cells, and it is therefore brought about not by a splitting of the end partition wall, but by a circumcision of the side walls of the cell, and consequently the cavity of the latter is thrown open, the end wall remaining with and closing the distal cell, whose contents have not undergone change. On the other hand, observation leads me to think that sometimes there is a splitting of the end wall. According to my observation, sometimes the filament is completely broken in two, but very commonly the two cells remain attached by one corner, opening from one another as it were on a hinge-joint (fig. 2 *f*, pl. 17).

The gathering of the protoplasm, already spoken of, into a ball, is a slow process, and the escape of this ball, through the opening formed in the manner described, takes place even more slowly. The motion is not at all perceptible, with a power of a thousand or twelve hundred diameters. During the passage the ball becomes more or less twisted and deformed, but as it emerges the uncompressed portion shortens and swells out, and when the mass of protoplasm is at last free in the water, it soon assumes a globular or regularly ovate shape. The mother-cell, thus bereft of its contents, is left dead and void. The primordial utricle indeed still remains within, but it has lost all its wonderful powers, and is nothing but a shrunken, twisted, or folded dead membrane. What is the cause of the motion of the zoospore within the cell it is very difficult to determine. It certainly is not vibrating cilia. When the zoospore first escapes, it is, as already stated, an irregular lump

of strongly chlorophyllous protoplasm, homogeneous or with one or more roundish masses of darker green within it. As it assumes its shape, however, a very distinct transparent spot appears at its smaller end. Whether this is an absolute vacuole or not, I have never been able to satisfy myself, but I am rather inclined to believe that it contains highly refractive transparent protoplasm. As this spot is perfected the cilia make their appearance. Whether they are actually first formed there, or whether, as is more probable, they are formed inside the cell, and are so folded against the general mass as to be invisible, I have never determined. Dr. Pringsheim, however, figures them within the cell. I have seen them in their early development long before motion commenced in them, but they were always perfectly formed as soon as apparent. They are present in great numbers, making a crown or ring around the edge of the transparent beak-like end. When they commence to vibrate, their action is at first very slow, and the waves of motion run through them deliberately from one cilium to the other, but soon, however, the motile impulses succeed one another more and more rapidly, until the general mass of the zoospore begins to tremble, then to rock, and finally darting off the little body hastens hither and thither through the water. The zoospore of an *Ædogonium* is always readily distinguished from most other similar bodies by its large size and peculiar motion, which is a forward movement combined with a distinct rolling on its long axis. After a time the zoospore, coming in contact with some speck of matter to which it can attach itself, ceases its movements, the cilia rapidly wither away, and the end to which they have been attached swells out or elongates into a broad, or narrow, simple, bifid, or trifid process, placed at an angle to the main axis of the cell, so as to form the so-called foot, the holdfast that anchors and fixes the new plant. Whilst this is taking place, the general form of the zoospore alters into that of a cylinder, a cellulose wall is secreted all about it, and the first cell of the new plant is complete. As soon as this cell is sufficiently matured, it begins to undergo division in the manner already described, and to develop into the new filament.

In regard to the time when these zoospores are given off most abundantly, and the circumstances that influence the process, I can only state that it occurs when there is least tendency to the production of resting spores, probably in youngish plants, and I have thought was favored by a full supply of light, with a moderate temperature.

Sexual reproduction occurs among the *Ædogoniaceæ* in accordance with three distinct types, to which the name of *monœcious*, *diœcious*, and *gynandrous* has been severally applied. The characteristic differences are to be looked for in the production of the antheridiæ or male plant, the female germ being always prepared in essentially the same way. In most instances two cells are requisite for the production of the latter. At first there is nothing by which cells set apart for the formation of the female germ can be distinguished from ordinary cells. The proximal one of the pair finally, however, undergoes changes similar to those seen when a zoospore is to be formed, namely, a sort of confusion of the endochrome, and finally a gathering of it into a mass at the distal end of the cell. Instead of there being a solution of the side wall of the cell, however, the end wall

undergoes absorption, so that the cavities of the two cells are more or less completely thrown into one. All or nearly all of the contents of the proximal cell now slowly pass into the distal one, which thus becomes crowded with chlorophyllous protoplasm. At or before this period, the distal receiving cell undergoes a change in form, widening out greatly, and sometimes appearing actually to shorten, so that it is in most instances resolved into a more or less regular globose or oval cell. As the sporangium or spore-case thus formed perfects itself the endochromes of the two cells become completely fused into one mass, which gradually condenses and assumes a regular shape, until, in the form of the perfected female or receptive germ, it is a dark, opaque ball more or less completely filling the sporangial cell. At the same time, in order to afford passage for the male germ, an opening is formed through the walls of the sporangium. This happens in two ways. The simplest of these is by the formation of one or more circular openings or pores in the wall. This pore is sometimes below, sometimes above the equatorial line. Its position, numbers, and form afford good specific characters. The second method is by the development of a little trap-door entrance at the distal end of the spore-case. This method is unknown in our American flora, and, never having seen it, I must refer to the papers of Pringsheim for details.

The above-described mode of origin of the sporangium is the common one. In *O. mirabile*, WOOD, however, but one cell is concerned. This cell grows to an enormous size, far beyond that of its fellows, and its endochrome collects into the upper half of it, to be at last shut off from the lower half of the cell by the formation of a new cellulose partition or end wall; or, in other words, the parent cell divides by a modified process of cell division, different from that common in the family. The distal daughter-cell contains all the endochrome. After the changes are completed, the appearance is the same as ordinarily presented, namely, an empty cell surmounted by the sporangium. Sometimes, even in plants in which the ordinary process occurs elsewhere, a single cell appears at times to have sufficient vitality to develop into a sporangium without aid from its neighbor, so that the latter will preserve its integrity, and the resting spore finally lie in proximity to a cell full of endochrome.

In the *monœcious Edogoniaceæ*, a single filament produces both the male and female germs. Certain cells appear to be set apart to develop into sporangia, whilst others give origin to the spermatozoids. No such plants have as yet been detected in North America, and I, therefore, pass on without speaking more in detail.

The second method in which the spermatozoids are produced is the most common in our flora; it is the so-called *gynandrous* plan. In this the single filament produces the female germs directly and the male germs indirectly. The former arise in the way previously described, whilst the latter are the resultant of a complex series of life actions, as follows: One of the main cells of the originating filament, differing in no perceptible way from its fellows, instead of like them developing new cells, divides up by a simple process of cell division into two or more cells, each one of which contains very largely of chlorophyllous protoplasm. The protoplasm within each of these secondary or daughter-cells soon condenses into an irregularly ovate or conical mass, which often, even within the cell, may be seen to

have the transparent beak of the zoospore (pl. 18, fig. 2 *d*). Inside of the cell the *andros pore*, as it is called, shows no cilia, but when it is set free by a more or less complete solution of the cell wall, it assumes the form of the ordinary *Ædlogonium* zoospore, with a crown of cilia, whose vibrations soon cause it to dart through the water. These androspores are of course much smaller than an ordinary zoospore, and after a period of active motion, they attach themselves to the parent filament, generally either on or near the sporangial cell. Their first life-actions, after settling, are precisely like those of the zoospore, namely, dropping of the cilia, enlargement of the smaller end into the so-called "foot," an elongation of the general mass, and the secretion of an outer coating of cellulose. In this way a peculiar-shaped, somewhat ovate cell is formed, which contains a great quantity of rich protoplasm with mostly a small amount of chlorophyl. From such cells are developed the mostly two- or three-celled, perfect antheridia, which in *gynandrous* *Ædlogonia* are generally to be seen, during the period of fructification, in numbers attached to the filament, mostly in the neighborhood of the sporangium. Their protoplasmic contents are remarkable for the activity of their movements, and I have seldom seen more beautiful and rapid cyclosis than they display—currents setting in all directions—particles actually brushing against one another (pl. 17, fig. 2 *h*). The spermatozoids are formed in the distal cell, sometimes one, sometimes more. In the species *O. mirabile*, WOOD, (pl. 18, fig. 2 *g*, 2 *b*) in which I have most carefully studied their origin, two are produced in the single cell. This cell is in the commencement of the process, although comparatively poor in chlorophyl, crowded with a rich solid protoplasm, which divides into two distinct masses, somewhat in the manner seen in the commencement of ordinary cell division. As there is no distinct nucleus, of course there are no precedent nuclear changes. The masses thus formed gradually assume a more or less perfectly globular shape inside the cell, although I have never been able to see that they there develop cilia, and finally are set free by the lifting up of the end of the mother-cell, like a little trap-door. Their mode of escape through the exit thus offered is similar to that of the ordinary zoospore, which they resemble, except that they are much smaller, are much less rich in chlorophyl, and have the anterior clear space less defined. They are said to be furnished with a crown of cilia similar to that of the zoospore. I myself have never seen these, but do not doubt their existence.

In the *diocious* *Ædlogonia* there are distinct filaments, male and female, one of which produces the oosporangium with its contained germ, whilst the other gives rise directly to the spermatozoids.

The resting spore which develops after impregnation is variously shaped, but in most instances is round or oval. It is often, if not always, furnished with two coats, the outer of which is thick, firm, and frequently provided with surface appendages, such as tubercles, ridges, spines, etc. Besides these there is also, probably, a very delicate inner coat. The spore appears to be set free from its case by the decay of the latter, there being never, at least that I have seen, any regular dehiscence. Although I have made several attempts, it has never been my good fortune to observe anything like germination of these resting spores. Prof. Chr. Vaupell, however, has published an account of the manner as observed by himself. Some water containing fruitful

Edogonia was allowed to dry in a glass, towards the close of September, and the greenish residue was placed in water in the following January. By March the resting spores were everywhere in active germination. The first change was a rupture of the two outer coats and the escape, through the slit, of the contents, still surrounded by a very delicate hyaline membrane. By this time the protoplasm had divided into usually four (sometimes only two or three) greenish masses, each of which was oval in shape and had its own extremely thin, hyaline coat, and was therefore a perfect cell. The old outer shell of the spore laid discarded in the water and soon decayed, and in a little while the hyaline sac surrounding the four daughter-cells itself disappeared, leaving them exposed and naked. After awhile each of these cells opened at one end by means of an annular split, cutting off the apex of the wall and allowing it to lift off like a little lid. Through the circular opening thus made, the contents now emerged. The point of the inner mass was colorless and directed towards the orifice, and the whole moved vigorously backwards and forwards until it finally escaped, as a perfected zoospore. This little body simulated very closely the ordinary zoospore, both in appearance and life-history, growing, after a brief period of activity, into an ordinary filament, in precisely the same manner as the zoospores.

Genus *ÆDOGONIUM*.

Antheridia et oogonia in individuo unico.

Antheridia and oogonia in the same individual.

Remark.—No species of the genus *Edogonium*, as here defined, has as yet been discovered in this country.

Genus *PRINGSHEIMIA*.

Dioica. Antheridia et oogonia in individuis distinctis orta.

Diœcious Antheridia and oogonia arising in distinct individuals.

***P. inequalis*, WOOD.**

P dioica; cellula basali biloba; plantis femineis quam plantis masculis permulto majoribus; oogoniis enormiter globosis vel subovoideis, poro laterale supra medium posito instructis; oosporis forma eadem, sed paulo minoribus.

Syn.—*Edogonium inequale*, WOOD, Proc. Amer. Philos. Soc., 1869, p. 141

Hab.—In stagnis, prope Philadelphia.

O. diœcious, basal cell bilobate; female plant very much larger than the male plant; oosporangium irregularly globose or subovoidal, opening by a lateral pore above the middle; resting spores of the same form as the sporangium, but a little smaller.

Remarks.—This plant seems to be more closely allied to *O. gemelliparum*, Pringsheim, than to any other species. It agrees with it in the inequality of the male and female plants, in the shape of the sporangium, and the position of the lateral pore. The diameter of the female plant is often nearly four times that of the male, and the difference in length is much more apparent. The mother-plant is composed of from 3-6 cells in the most distal of which the spermatozoids are formed. I am not able to state how many of these bodies are formed in a single cell, having

only seen the latter when more or less completely emptied, but, judging from the relative sizes, there must be several. In a cell containing a single spermatozoid, that body moved about freely, and at last escaped, apparently through an orifice in the end wall of the cell. It made two attempts before getting out, and during its passage was distinctly constricted in the middle. It resembled in appearance an ordinary zoospore, but was of course much smaller, and was nearly devoid of color, having but a slight greenish tint. I found this species growing abundantly in the stagnant ditches of the Neck, below the city.

Fig. 1 *a*, pl. 18, represents a young female plant; 1 *b*, a fertile plant with immature spores. 1 *c* was taken from the supposed male plant alluded to in the text. The latter figure is magnified 450 diameters, the others 250.

Genus ANDROGYNIA.

Gynandra. Androspore in plantis femineis ortæ; postea hanc affixæ et in antheridiis se formant.

Gynandrous. Androspores arising in the female plant; after affixing themselves to this and developing into antheridia.

A. multispora, WOOD.

A. oogoniis singulis, vel binis vel ternis continuis, globosis instructa; poro laterale distale; oosporis globosis, oogonii lumen replentibus; antheridiis plerumque pluribus, planta feminea insidentibus, cellula inferiore multo majoribus.

Syn.—*Oedogonium multispora*, WOOD, Proc. Amer. Philos. Soc., 1869, p. 141.

Hab.—In stagnis, prope Philadelphia.

Oosporangia single or bi- or triseriate, globose with a distal lateral pore; oospore globose, about the same size as the sporangial cavity; antheridia bi- or tricellular, curved, with the lower cell much the largest, generally adhering in considerable numbers to all parts of the female plant.

Remarks.—This species differs from its nearest European congeners, *Ædogon. Rothii* and *Æ. depressum*, very markedly in the bicellular antheridia. I have never seen the spermatozoids actually emerging from their mother-cell, but have seen in the terminal antheridial cell a pair of oval bodies, which I took to be those bodies.

Fig. 3, pl. 17, was taken from a filament of this species magnified 500 diameters. It shows spores in different stages of maturity, with an empty basal cell in one case, and in the others without. Also male plants, one of them containing partially formed spermatozoids. The small arrows indicate the direction of cyclotic currents.

A. mirabilis, WOOD.

A. rare setigera; articulis diametro 2–8 plo longioribus; oogoniis plerumque singulis, rare geminis, nonnihil ovatis, infra latis sed supra contractis et medio tumidis; poris lateralibus duobus supra medium positis; oosporis aut late ovalibus aut subglobosis; sporodermate haud signato; antheridiis plerumque bicellularibus, interdum tricellularibus, plerumque in filo vegetativo infra oogonium aut in oogonio insidentibus; spermatozoideis singulis et geminis.

Diam.—Artic. veget $\frac{7}{1000}$ "— $\frac{13}{1000}$ "=.0004"—.0017". Spor. $\frac{18}{1000}$ "— $\frac{20}{1000}$ ".0024"—.0027".

Syn.—*Oedogonium mirabile*, WOOD, Proc. Amer. Philos. Soc., 1869, p. 142.

Hab.—In rivulis quietis, prope Philadelphia.

A. rarely setigerous; articles 2-8 times longer than broad; oosporangia mostly single, rarely geminate, subovate, in the lower portion broad, in the middle swollen, in the upper part contracted; the 2 lateral pores situated above the middle; oospore subglobose or broadly ovate, its coats without markings; antheridia generally bicellular, sometimes tricellular, numerous, placed generally upon the female filament either upon or below the oosporangia.

Remarks.—This species was found growing in a rather stagnant brook in the meadow by “Robinson’s Knoll,” at the junction of the Schuylkill River and Wissahickon Creek, near Philadelphia. The filaments, which vary very greatly in size, are in their early history attached to dead leaves and sticks, but finally, I think, float free in the water. The larger, fruit-bearing filaments are remarkable for their crookedness. None of the threads that I have seen ended in a seta-like portion.

The fruit is produced in abundance, but very rarely is there more than a single spore in any one place. The method of the formation of the sporangia differs from that of all the other *Edogonia* which have come under my notice. Instead of two cells being concerned but one cell is employed. The cell (fig. 2 *a*, pl. 18) that is to be used for such a purpose grows much beyond the ordinary size, until it is nearly or quite twice as large as its neighbors. All the time it is well filled with chlorophyllous protoplasm. This now contracts and finally is all packed into the upper half of the cell. At or even before this time the lateral openings become apparent. There are two of them, situated just in the angle where the cell at its upper end commences to contract to the size of its fellow. At this time I think fertilization takes place, although I have never actually seen the spermatozoids enter the orifices. The cell (fig. 2 *b*, pl. 17) now divides into two by forming a wall separating the lower empty half from the upper full one, which is to be the sporangium. The contents of the latter now condense into a ball, and it itself becomes more tumid in the middle. Finally a reddish-brown broadly globular spore (fig. 2 *c*, pl. 18) is formed. I have not been able to make out more than one distinct thick coat. The surface of the spore is smooth. The androspores are formed in a cell (fig. 2 *d*, pl. 18) which has grown beyond the normal size and then divided into four or five short cells, each of which gives origin, I believe, to a single androspore in its interior. The antheridia are numerous, from 2 to 6 being commonly attached to the lower portion of the sporangium, or to the cells just beneath it. They (fig. 2 *e*, pl. 18) have a rather large foot, and are generally curved at the base. The distal of the two cells composing them is crowned with a little cap, and produces one or sometimes two spermatozoids. These (figs. 2 *b* and 2 *g*, pl. 18) during their escape are always very much squeezed out of shape, but when free become globular or slightly pear-shaped. They are highly transparent and contain a few green granules. Their motion is at first slow, but soon becomes very active. The mode of egress from the cell is obtained by the cutting off of the upper end of it, the little cap opening like a trap-door. After this cell has been emptied, sometimes a second similar one is formed, which bears it aloft. I have never seen spermatozoids produced by this second cell.

A. *Huntii*, Wood.

Filum plerumque in setam longam, terminalem coloris expertam productum; oogoniis plerumque singulis, globosis, interdum nonnihil hexagoniis, medio nonnihil tumidis, poro laterale

infra medium posito; oosporis globosis, oogonii lumen haud replentibus, superficie lineis elevatis spiralibus quatuor instructa; antheridiis bicellularibus (interdum tricellularibus?).

Diam.—Spor. $\frac{1}{600}$ " = .002".

Syn.—*Edogonium Huntii*, Wood, American Naturalist, 1868.

Hab.—In aquario meo.

Filaments mostly produced into a long apical seta; **oogonia** mostly single, globose, sometimes somewhat hexagonal, somewhat tumid in the middle, the lateral pore placed below the middle; oospore globose, not filling the cavity of the spore case, its surface with four spiral elevated lines or ridges; antheridia bicellular (sometimes tricellular?).

Remarks.—This little plant appeared in my aquarium some years since, forming a delicate fringe upon the various aquatic plants growing therein. Its color is a bright yellowish green, deepening to a very dark green in cells which are crowded with granular protoplasm. The filaments vary very greatly in size, the largest I have seen were $\frac{1}{600}$ of an inch in diameter. They are provided with long, terminal seta, which are much more universally present than in any of the other species I have met with. The first step in the formation of a spore is the emptying of a cell into its distal neighbor, so that each spore case is placed at the end of an empty cell. These sporangia may be single or they may be in series of two or more, separated only from one another by the eruptive cells just spoken of. The color of the mature spore is a very dark reddish-brown. The antheridia is bicellular, slightly curved, somewhat stipate, with a distinct foot. Its most common position is on the vacated cell just below the spore case. The zoospores, as I have seen them, are always globose.

I have named this species after my friend, Dr. J. Gibbons Hunt, a well-known microscopist of this city, to whom I am greatly indebted for aid in my earlier microscopic studies.

Fig. 2, pl. 17, represents different forms and parts of this plant. 2 *a* shows the end of a filament and the long seta-like lip. 2 *b* was taken from two cells, one of which had just undergone division, and shows very plainly the method of procedure; lying as it were between the cells, and bearing the end of the lower one upon it, is the new little cell. Fig. 2 *c* represents a fertile filament with two mature spores and one not fully grown. Fig. 2 *d* was drawn from a filament just forming a spore, and shows the male plant *in situ*. Fig. 2 *e* represents a male plant (magnified some 1300 diameters) with the outer terminal cell scarcely more than a primordial utricle. The contents of the lower cell were in a state of intense motion; and the arrows are meant to indicate the directions of the currents. Fig. 2 *f* represents a portion of a filament with a zoospore just escaped and still quiescent.

A. echinata, Wood (sp. nov.)

A. valde elongata; articulis diametro 6–14 plo longioribus; oogoniis globosis, plerumque depresso, ad .0014" crassis; oosporis oogonii forma et ejus lumen replentibus, valde aculeatis poro laterale supra medium posito; antheridiis bicellularibus?

Diam.—Spor. $\frac{1}{12000}$ " = .001". Cell. $\frac{3}{2000}$ " — $\frac{1}{2000}$ " = .00033" — .0005".

Hab.—In stagnis, Alleghany Mountains.

O. gynandrous, very elongate; joints 6-14 times longer than broad; sporangia globose, mostly depressed, about .0014" in diameter; oospores of the same form as sporangia, whose cavity they almost fill; covered with sharp spines; the lateral pore placed above the middle; antheridia bicellular?

Remarks.—I found this distinct species in a little stagnant pool in the wilderness, known as Bear Meadows, in Centre County, of this State. The filaments are very long, and were matted together into a sort of fibrous mass. The male plants were few in number, and were attached to the female plant in the neighborhood of the sporangia. I have not seen any composed of more than two cells. They are furnished with a well-marked foot, above which there is a short neck. As I have seen them they are nearly straight.

I have not been able to make out more than one coat to the spores. This coat is very thick, and is furnished with numerous thorn-like spines. These are very sharp at the points, but at their bases are mostly very robust.

Fig. 3, pl. 18, represents a spore of this plant magnified 750 diameters.

SUBFAMILY BULBOCHÆTEÆ.

Filuma ramosum, setis strictis hyalinis achrois e basi bulbosa et plus minus elongatis instructum.

Filaments branching, furnished with straight, hyaline, more or less elongated seta, arising from a bulbous base.

Remarks.—The *Bulbochæteæ* are at once separated from their allies the *Ædognoniæ* by their bushy, branched habit of growth. The shape of the individual cell is also entirely different, for instead of being regularly cylindrical they are almost always markedly dilated at their distal end, so as to be somewhat clavate, nor is the filament or its branches ever ended by a long seta-like series of narrow colorless cells. Many or all of the cells are, however, furnished with a single very long unicellular unbranched hair. These hairs are colorless, hyaline, and provided with a markedly and abruptly bulbous base. The *Bulbochæteæ* grow in similar positions to their allies, but are not nearly so common, nor when present do they grow in such abundance, very rarely, if ever, forming the dense forest-like fringes or the matted masses that some species of the *Ædognoniæ* do. They are reproduced both by zoospores and resting spores.

The manner of the development of and growth of the plant from the zoospore is very peculiar. I have never myself studied it, but Prof. Pringsheim gives the following account: When the zoospore first settles down it produces a cell closely resembling that of an *Ædognonium*. The first change which occurs in this cell is the formation of a small, conical, transparent, colorless space at the apex, which space in a little while becomes separated from the mother-cell by a distinct partition-wall, and at the same time the apex itself is ruptured, and the point of the little growing cone pushed through the opening. This rupture does not take place irregularly, but by a sort of circumscribed dertiscence, similar to that of the *Ædognonium*, the top of the mother-cell being lifted up like a little trap-door, and finally pushed aside as the new conical cell grows elongate and becomes converted into a hair. After the formation of this apical hair, the mother-cell undergoes division in a manner similar to that of an *Ædognonium*. Near its distal end a

circular slit appears, and at the same time a partition forms, so that from the mother-cell are developed a small apical and a large basal daughter-cell. The history of the former of these is simply one of growth as regards the main axis. It increases in size but does not give origin to new cells. All such cells are formed out of the basal daughter-cell, which, as already described, divides into a new apical and basal cell—the apical only to grow in the main filament—the basal to divide anew. It is always the basal cell that undergoes division, throughout the whole life-history of the plant, one cell alone contributing to the growth of the main filament. The filament thus formed bears upon its distal end the hair which grew upon the original spore-cell, and this hair is, save only the basal cell, the oldest part of the filament. The cell upon which it rests is the next oldest, the next to it in position, the next in age, and so on (from older to younger) down to the basal cell, the oldest of all, lying next to the latest born.

Although the cells of the main filaments do not contribute to its development, yet it is from them that the lateral branches are formed. The production of a branch begins by the appearance of a clear space near the apex of the cell, but this clear space is placed, not exactly at the apex, but a little to one side. It soon becomes distinctly conical, enlarges, bursts through the old cell-wall, is cut off by a cellulose partition from its parent, and develops into a hair similar to that first formed, but placed at an angle to the long axis. It is remarkable that the opening for the exit of the growing hair occurs, not by a circular transverse slit, but by a longitudinal one, the two halves of the old cell-walls separating as the little cone pushes its way between them and persisting as a sort of sheath to its base. When the hair is perfected the cell from which it grew undergoes division in the usual way, save only that the cutting off of the old wall is done obliquely instead of transversely, so that the partition is oblique instead of horizontal, and the new cell grows at an angle to the old, instead of in the line of its axis. The new cell, consequently, is the starting point to a branch at an angle to the main filament. This branch, like the main filament, grows only by the repeated divisions of its primal basal cell, and bears aloft its seta. Secondary branches may arise from it precisely in the way that it arose from the parent stem, and thus at last is formed the bushy plant of the *Bulbochateæ*.

The zoospores closely resemble those of the *Ædlogoniæ*, and are oval or globose masses of chlorophyllous protoplasm, with a transparent space at the smaller end, surmounted by a crown of cilia. Their mode of formation and whole life-history are also similar to that of the *Ædlogoniæ* zoospores, up to the time when in their germination they begin to produce new cells.

Sexual reproduction amongst all the known *Bulbochateæ* is similar in its general aspect to that seen among the gynandrous *Ædlogoniæ*, but differs considerably in detail. The oogonia are mostly formed in lateral branches. Their position in these branches varies in the various species.

Since any cell from the next to basal to the most distal of all crowned with the terminal seta may be converted into a oogonium, according to Pringsheim, the cell which is to form the oogonium arises in the usual way, by the division of a cell into two daughter-cells. The new daughter-cell, which is to develop into the

sexual part, does not, however, rupture the old wall of the mother-cell, but grows out beyond it, and there dilates. The new cell is therefore divisible into two parts, a proximal cylindrical portion, contained within the walls of the mother-cell, and a distal more or less globular piece beyond the latter. The chlorophyllous protoplasm now collects in this dilated portion, leaving the basal cylindrical part bare and empty. The oogonium is not, however, formed directly from this upper portion (the primitive oogonium, as it may be called), but a new wall forms within the latter and then it undergoes division much as did the primary cell. In this way it is that the upper and lower portions of the old wall, *i. e.* that of the primitive oogonium, remain as a sort of basal sheath and cap to the fully-formed sporangium. The little hole by which the spermatozoids find entrance to the contents of the oogonium is always formed in the upper half of the wall of the latter.

As stated, all the species of *Bulbochaeteæ* as yet known are gynandrous. The antheridia resemble those of similar *Ædogoniææ*, and their life-history is very similar. The development of the resting spores is said to take place as follows: The first change is in the color of the spore, the bright red becoming green, especially near the margins of the cavity. The outer wall is then ruptured and the spore grows into a long oval body, whose contents are chiefly green with a sprinkling of the original red. The protoplasm of this oval body gradually divides into four masses, which become more and more distinct, until they are at last well formed zoospores, similar to those produced in the more ordinary method, except, perhaps, that they are redder. They are finally set free in the water by a solution of the cell wall surrounding them, and enter upon a brief free existence, to settle down after a little and grow into a fully-formed plant.

Genus BULBOCHÆTE.

Androspore in planta femineâ ortæ, postea hanc affixæ et in antheridiis se formantes.

Androspore arising in the female plant, afterwards affixed to it and developing into the antheridia.

B. ignota, WOOD.

B. sparse ramosa, elongata; articulis diametro max. ($\frac{1}{13000}'' = .00077''$) $1\frac{1}{2}$ -3 plo longioribus; oogoniis long. $\frac{1}{4000}'' = .0025''$, lat. $\frac{1}{8000}'' = .0018''$, interdum lateralibus et sessilibus, interdum inter ramulorum cellulas vegetativas positis, dissepimento nullo; oosporis ovalibus, longitudinaliter nonnihil oblique et distante costatis, in ætate provecta aurantiaco-brunneis, sporodermate crasso; antheridiis 3-4 cellularibus, stipitatis.

Syn.—*B. ignota*, Wood, Prodrômus, Proc. Amer. Philos. Soc., 1869.

Hab.—In aquis quietis, prope Philadelphia.

B. sparsely branched, elongate with the joints $1\frac{1}{2}$ -3 times longer than broad ($\frac{1}{13000}'' = .00077''$); oosporangia .0025" long by .0018" broad, sometimes lateral and sessile, sometimes placed upon the apex of a branch, sometimes situated in the length of the branches between their cells; the empty cell which supports the sporangium without dissepiment; oospores, oval, filling rather closely the cavity of the spore-case, longitudinally somewhat obliquely and distantly costate, when mature orange brown; spore-coat rather thick; antheridia 3-4 celled, scarcely stipitate.

Remarks.—When I described and figured this species I had never seen the mature fruit, but very recently Mr. Quimby has communicated specimens to me.

The color of the spore is orange brown, and the thick coat is slightly tinged with yellowish. The mature oosporangium is somewhat flattened at the sides, not so elliptical as the young spore, which I have figured.

Fig. 5 *a*, pl. 18, represents a fragment of a filament showing young sporangial cells magnified 260 diameters; 5 *b*, represents a branch with a youngish spore in it, magnified 160 diameters; fig. 5 *c*, was taken from a male plant.

B. dumosa, Wood.

B. articulata diametro $1\frac{1}{2}$ –2 plo longioribus; oogoniis plerumque in ramorum brevissimorum apicibus positis sed interdum lateralibus, plerumque setam terminalem gerentibus; oosporis enormiter ovalibus aut ovatis, nonnihil indistincte longitudinaliter oblique subarete striatis; antheridiis bicellularibus, stipite instructis, cellula basale medio tumida, supra sæpe contracta.

Syn.—*B. dumosa*, Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 142.

Hab.—In aquario meo.

Joints $1\frac{1}{2}$ –2 times longer than broad; oosporangia generally placed upon the ends of short branches but sometimes lateral, mostly carrying a terminal seta; resting spores irregularly oval or ovate, somewhat indistinctly obliquely longitudinally and rather closely striate; antheridia bicellular, furnished with a little stipe, their basal cell tumid in the middle, frequently contracted above.

Remarks.—This species appeared spontaneously during the latter part of the winter upon some large fresh-water algæ which I was cultivating. It branches irregularly and sometimes somewhat profusely, so as to have quite a bushy habit. The antheridia appear to produce a single spermatozoon in the terminal cell; at least as far as my observation has gone this is true. I think I have always found the distal cells of fertile plants emptied of their contents, as though they had furnished the androspores which had grown into the antheridia. This species is closely allied to *B. gracilis*, of Pringsheim, from which it differs in the position of the oogonia, in the relative breadth and length of the cell, and the number of cells composing the antheridia.

Fig. 6 *a*, pl. 18, represents a filament of this species magnified 260 diameters; 6 *b*, a male plant magnified 750 diameters.

B. Canbyii, Wood.

B. permagna ad .035" longa, sparse ramosa; articulis sterilibus diametro 2–8 plo longioribus; oogoniis lateralibus vel in ramulorum apicem positis, transverse enormiter ovalibus; oosporis, transverse enormiter ovalibus, plerumque nonnihil triangularibus, oogonii lumen replentibus; sporodermate crasso, haud costato, enormiter punctato; antheridiis bicellularibus.

Diam.—Cell. steril. $\frac{75}{500}$ " — $\frac{8}{500}$ " = 00066–001. Spor. transv. $\frac{717}{500}$ " = .00226.

Syn.—*B. Canbyii*, Wood, Proc. Amer. Philos. Society, 1869, p. 142.

Hab.—In aquis quietis, prope Hibernia, Florida; (William Canby).

B. very large, attaining a length of more than one-third an inch, sparsely branched; sterile joints 2 to 8 times longer than broad; oosporangia lateral or placed upon the ends of branches, irregularly transversely oval; oospores of a similar shape, often a little triangular, filling the cavity of the sporangium; spore coat thick, not costate but irregularly punctate.

Remarks.—It affords me great pleasure to dedicate this very handsome species to Mr. William Canby, by whom it was collected in Florida, as an acknowledgment of favors received, and as a testimony of respect and high regard for him

personally, and as being among the foremost students of American phanerogamic botany.

This species is more nearly allied to *S. minor* than to any other of the European forms, but differs from it very essentially in size and habit. It is always, as I have seen it, except in very young plants, sparsely and mostly dichotomously branched, and attains a very great length, at times probably exceeding the third of an inch. The spore is mostly sessile upon the distal ends of the cells of the filament; in all such cases I have noticed that the cell upon which it was borne was divided in its middle by a partition into two cells. Not unfrequently the spore is raised upon a short branch. The male plants are attached to the female filaments generally in the neighborhood of the sporangium, to which they sometimes fasten themselves immediately. They are shortly stipitate, and composed of two cells. The mature spore is transversely oval, now and then slightly triangular, and is nearly of the color of burnt sienna. Its coat is thick, often slightly yellowish, and has on its outer surface irregular punctations, looking like corrosions. These are not detachable, except when the ruptured spore is more or less completely emptied of its contents. The sporangium closely invests the spore, and when the latter is matured undergoes a circular division, so that the top falls off and allows the spore to escape.

Fig. 6 *c*, pl. 16, represents a portion of a filament, magnified 260 diameters, with a young sporangium and young male plants attached; 6 *b*, represents a very young plant, magnified 260 diameters. Fig. 6 *a*, was taken from a mature plant, and shows the mature spore. Fig. 6 *e*, shows in outline a sporangium and male plants attached; whilst 6 *d*, was drawn from a sporangium which had perfected its spore and undergone the natural dehiscence.

FAMILY CHROOLEPIDÆ.

Algæ aeræ, aureo-, aurantiaco- vel rubro-fusco-coloratæ, siccatae sæpe canæ. Fila varie ramosa, cytiodermate crasso vel subcrasso, firmo, subcartilagineo prædita, in pulvinulos minulos vel in stratum tenue aut incrassato-tomentosum densissime aggregata vel implicata. Cytioplasma oleosum vel granulosum, aut rubellum, aureum, aut flavo-fuscum, interdum viride tinctum, post mortem plerumque expallescens. Propagatio fit zoogonidiis.

Ærial algæ. Golden orange, or reddish fuscous, often grayish when dried. Filaments variously branched, furnished with a thick, or thickish, subcartilaginous cytioderm, densely aggregated into minute cushions, or a thin or tomentosely thickened stratum. Cytioplasm granular or containing oily particles, reddish-golden, or yellowish-fuscous, sometimes tinged with green; after death often colorless or nearly so. Propagation by zoospores.

Remarks.—The plants of this family are so different from the others of the order, that it is a matter of considerable doubt whether or not they should be classified with them. They rarely possess distinct, well-pronounced chlorophyl, and form mats or strata of some shade of reddish, grayish, or brownish, so that they are very different in appearance from the other *Confervaceæ*.

I do not think their position can be certainly fixed until their life-history has been more fully developed. In assigning them this place I have simply followed Prof. Rabenhorst.

The only specimens that have come to my notice are in a dried condition, and consequently no possible opportunity has been afforded of studying the manner of reproduction. No one has as yet, at least to my knowledge, discovered any sexual reproduction in the family, but the method in which the zoospores are produced has been carefully studied, especially by Drs. Caspary (*Regensburg Flora*, 1858) and Hildebrand (*Botanische Zeitung*). The little motile bodies are not produced in the cells indiscriminately, but in certain ones set apart for the purpose, to which the name of zoosporangia is very applicable. These are large, globular, thick walled cells, which are generally provided with a protuberance at the top and marked by transverse wrinkles. They are most frequently situated upon the end of the filament or one of its branches, but are rarely placed in the middle of the thread, and still more rarely the cell next below the zoosporangium elongates itself sideways and upwards into a thread, so that the reproductive cell is left as a lateral one-celled branch or process. When the zoosporangium is sufficiently matured the endochrome breaks up into a number of minute masses, the future zoospores. Finally the crowning papilla of the mother-cell ruptures and allows the contents to escape as a well-formed vesicle, containing the perfected zoospores. It is said, however, that sometimes the vesicle is wanting, and the zoospores are discharged into the water. In the ordinary course, after a little while the vesicle lying in the water bursts and sets its motile contents free. The zoospores themselves are very small, according to Hildebrand, $\frac{5}{500}$ — $\frac{6}{500}$ mm. in length, by $\frac{1}{500}$ — $\frac{2}{500}$ mm. in breadth. In accordance with the same authority they are, when first discharged, cylindrical, but in a little while become flattened, and shaped like a flaxseed. They are biciliate and contain a large number of small, orange-colored particles. From thirty-two to sixty-four of them are formed in one zoosporangium, and neither light nor time of day appear to have any influence upon their birth. Hildebrand states that their motile life lasts from eighteen to thirty-six hours, but according to Caspary, after continuing in motion for about an hour, they grow sluggish, sink, become globular, then elongate themselves and shortly undergoing transverse division, actively commence to form the new filament.

Genus CHROOLEPUS, Ag.

Fila distincte articulata, intricata, enormiter ramosa.

Filaments distinctly articulate, intricate, irregularly branched.

C. aureum, (LINNÉ.) KtZ.

C. filis ramossimis, in stratum aureo-brunneum, ad duas tres lineas crassum, cæspitosum et molle intricatis vel in cæspitulos aggregatis; articulis enormibus, diametro sesqui-, duplo triplove longioribus.

Diam.—Max = .001".

Syn.—*C. aureum*, (LINNÉ.) KÜTZING. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 371.

Hab.—Little Falls, New York; Godwinsville, New Jersey; (Austin). Texas; (Ravenel).

Filaments very much branched, interwoven to form a yellowish-brown softish mat, two or three lines in thickness; joints irregular, $1\frac{1}{2}$ –3 times longer than broad.

Remarks.—I am indebted to Mr. Austin for specimens which are labelled "Forms dense yellow-brown cushions on rocks, at Little Falls, New York and

Godwinsville, New Jersey." As dried, the plant is in extended, gray, felt-like masses. The walls of the articles as seen with the microscope are thick and irregular, and the joints themselves are also very irregular, the end ones being often swollen and rounded so as to give the branches a sort of bulbous termination.

Among the Algæ collected in Texas by Prof. Ravenel, is a dried specimen (No. 100), labelled "On Bark, Houston, Texas," which I cannot separate from this species. It occurs in small tufts, which, as dried, are of a very decided orange, and, no doubt, were still brighter during life. The articles are not so irregular as in Mr. Austin's specimens, but excepting in this and color when dried they agree very well. Besides these I have several specimens from the same source, which are in extended mats and agree in all respects with their northern brethren.

Our American form appears to attain a greater diameter in its individual filaments than does the European variety, but I know of no other character separating it from the latter; and consequently must consider them identical. The measurement given is an extreme one, .009" being commonly the limit.

Genus BULBOTRICHIA, KUTZ.

Fila indistincte articulata, achroa, firma, ramosa; rami in apice intumescens, sporangia constituentes.

Filaments indistinctly articulate, translucent, firm, branched; the ends of the branches swollen so as to form sporangia.

B. albida, Wood (sp. nov.).

B. strato albido, coriaceo vel crustaceo; filis arcte intertextis, enormiter ramossissimis, coloris expertibus; sporangiis viridibus.

Hab.—In muscis, Northern New Jersey; (Austin.)

Forming a white leathery or crustaceous stratum; thread closely interwoven, irregularly and plentifully branched, colorless; sporangia greenish.

Remarks.—This curious little plant, which was sent me by Prof. Austin, occurs in minute white patches growing on mosses at the base of stumps in woods. Sometimes these are encrusted abundantly with the carbonate of lime, when they are hard and crustaceous. The sporangia appear to vary greatly in size; sometimes they resemble very closely a single spore (probably their commencing stage). The bases of the branches are rarely, if ever, furnished with the bulbous swelling, given by Rabenhorst as a generic distinction, but such enlargements do occasionally occur in the course of the filaments and branches. The filaments are composed of a series of cells, which are in places long, and have their end walls thin and not readily seen.

Fig. 5, pl. 16, represents a part of a plant magnified 460 diameters.

FAMILY CHÆTOPHORACEÆ.

Algæ aquaticæ vel palustres, rarius terrestres, plerumque monoicæ vel dioicæ. Fila varia, sæpe dichotome ramosa, hæud raro fasciculatim ramulosa, plerumque in cæspites vel pulvinulos cumulata, in mucro gelatinoso subliquido vel firmo nidulantia. Propagatio fit tum oosporis, tum zoogonidiis. Zoogonidia oriuntur aut singula aut geminis aut cytoplasmatis divisione 8-16 in quoque sporangio.

Aquatic, paludal, or rarely terrestrial algæ, mostly monœcious or diœcious. Filaments various, often dichotomously, but not rarely fasciculately branched, mostly aggregated into turfy masses or little cushions, and generally surrounded by a firm or subliquid gelatinous mucus. Propagation both by zoospores and resting spores. Zoospores arising either singly or by the division of the cytoplasm into 8-16 in each sporangium.

Genus STIGEOCLONIUM.

Fila articulata, simpliciter ramosa; rami ramulique sparsi, rarius fasciculatim approximati, in apicem acutum, sæpe piliferum achroum attenuati et plerumque longe protensi, sæpius ramellis brevibus subulatis instructi. (R.)

Filaments articulate, simply branched; branches and branchlets sparse, rarely fasciculately approximated, with their ends acute and frequently prolonged into an attenuate transparent seta or hair, and very often furnished with short subulate branches.

Remarks.—Plants which are certainly referable to this genus are abundant in every place in which I have ever looked for fresh-water algæ. I confess, however, that although very much time has been given to their study, I have not been able to make out any distinct specific characters, nor any identifications from the diagnoses of M. Rabenhorst. In a certain spring northeast of the city, there grows one of these forms, which I have closely watched for several seasons. In the earlier state it appears at times to possess the characters of a young *Chatophora* (pl. 19, fig. 1), forming a small gelatinous base out of which the threads soon escape as they lengthen. It constituted a sort of mucoid layer adhering to the boards lining the stones with waving masses of projecting filaments six or even eight inches in length. The filaments were mostly about $\frac{1}{2250}$ " in diameter and much interlaced.

The cells varied greatly in length, some being scarcely as long as broad, whilst others were eight or ten times longer. The short cells were generally densely filled with endochrome, whilst the long ones were nearly empty. The branches often ended abruptly, but were more frequently tipped with a long seta-like point. The method of branching is as varied as can be imagined, as is shown by fig. 4, pl. 16, and fig. 1, pl. 20, all taken from different plants of this species. I have frequently seen the production of zoospores, but no other method of reproduction. In all cases a single motile body (fig. 4, pl. 16) was formed in each cell. These minute bodies are globular or pyriform, and within the cell exhibit no motion whatever. Their escape takes place very slowly through a lateral slit in the wall. No cause of the motion is visible, and during the passage the zoospore is often very much squeezed out of shape. According to Braun (*Verjüngung*), these zoospores possess a red eye-spot. I had not read his description at the time my observations were made, but did not notice any. The zoospores germinated in the usual way, elongating and growing into a cell with a transparent seta-like end, and finally undergoing repeated divisions to form the plant.

M. Braun states that he has observed another process, in which the contents of a single cell undergoes a perpendicular division, so as to form four small zoospores, which escape from the cell in the same way as the larger one, and further says that he has never known these microgonidia to germinate.

Genus DRAPARNALDIA, Ag.

Fila articulata ramosa, e cellulis magnis, maxime hyalinis, fascia chlorophyllosa latiuscula ornatis, semper sterilibus formata, fasciculis penicillato-ramulosissimis, e cellulis minoribus fertilibus compositis, plus minus dense obsessa. Articuli terminales omnium ramulorum inanes achooli steriles, in pilum hyalinum plus minus elongati.

Filaments articulated, branched, formed of large cells which are chiefly hyaline, but furnished with a transverse chlorophyllous fascia, more or less densely clothed with penicillately ramulose fasciculi, formed of smaller fertile cells. Terminal articles of all the joints empty, transparent, sterile, and elongate, in a more or less hyaline hair.

D. glomerata, (VAUCH.) AG.

D. filis ramisque primariis achrois vel subachrois, ad 0.00147" crassis, articulis inferioribus diametro æqualibus vel paulo brevioribus, geniculis manifesto constrictis, fasciis chlorophyllous angustis dilute viridibus; ramis primariis subrectangulo-patentibus, sæpe oppositis; ramulorum fasciculis confertis, patentibus, alternantibus vel oppositis, dense ramellosis, subovalibus, obtusis. (R.)

Syn.—*D. glomerata*, (VAUCHER) AGARDH. RABENHORST, Flora Europ. Algarum, Sect. III. p. 38.

Hab.—Rhode Island; (S. T. Olney) Thwaites.

Filament and primary branches colorless or subcolorless, and reaching 0.00147" in diameter, lower articles about as long or a little shorter than broad, manifestly constricted at the joints, chlorophyl fascia narrow, light green; primary branches subrectangularly patent, often opposite; fasciculi of branches crowded, patent, alternating, or opposite, densely ramellose, suboval, obtuse.

Remarks.—According to M. Thwaites the true *Dr. glomerata* grows in Rhode Island, as he so identified specimens sent to him by Mr. Olney. These specimens were, however, in all probability dried, and if this was so, I confess not to attaching much weight to the identification. The *Draparnaldia*, common near Philadelphia, is at once so like and yet so different from the description of *D. glomerata*, that I am unable to fully satisfy myself whether it be a variety of the European species or distinct from it. It differs very greatly in the thickness of the stem and primary branches. I have given above Prof. Rabenhorst's description of the European variety, and now append one of the plant growing in this neighborhood.

Var. maxima.

Dr. filis achrois, ad 0.004" crassis, articulis plerumque diametro duplo longioribus, in medio sæpe valde tumidis; ramis primariis achrois vel subachrois, oppositis vel alternantibus vel ternatis, elongatis, dense ramellosis, cum ramulis lanceolatis; ramulorum extremorum fasciculis dense ramellosis, ovatis vel late lanceolatis, plerumque confertis; ramulorum articulis inferioribus plerumque diametro (ad $\frac{1}{875}$ ") subæqualibus, articulis superioribus diametro duplo aut triplo longioribus, plerumque piliferis.

Hab.—Prope Philadelphia; Wood.

Filament transparent, attaining a diameter of 0.004", its articles mostly twice as long as broad, strongly swollen in the middle; primary branches colorless or subcolorless, opposite, alternate or ternate, elongate, densely ramellose with the ramuli lanceolate; fasciculi of extreme branches densely ramellose, ovate, or broadly lanceolate, mostly crowded, inferior articles of the branches mostly about as long as broad ($\frac{1}{875}$ "), superior articles two to three times as long, mostly piliferous.

Remarks.—In this form there are almost always numerous little clusters of branchlets, growing immediately from the main stem or large branches; such clusters are more rigid, more open, more broadly ovate, and less markedly piliferous than the others.

D. plumosa, (VAUCHER) AGARDH.

D. filis ramisque primariis hyalinis, plerumque $\frac{1}{50}''' = 0.00179''$ crassis; articulis diametro æqualibus vel dimidio brevioribus, rarius paulo longioribus, geniculis vix aut modice constrictis, fasciis chlorophyllosis angustis læte viridibus; articulis inferioribus ramulorum diametro ($\frac{1}{15}''' - \frac{1}{25}'''$) æqualibus vel subduplo longioribus, pæne torulosis, superioribus cylindricis ad $\frac{1}{4}'''$ attenuatis, diametro duplo triplo-quintuplo longioribus, plerumque non piliferis; ramulorum *fasciculis dense ramellosis, elongatis, acute lanceolatis, crecto-subappressis.* (R.)

Syn.—*Dr. plumosa*, (VAUCHER) AGARDH. RABENHORST, Flora Europ. Algarum, Sect. III. p. 382.

Hab.—In rivulis et aquis quietis.

Filament and primary branches hyaline, mostly $\frac{1}{50}''' = 0.00179''$ in diameter; articles as long as broad or one-half shorter, rarely a little longer, scarcely or slightly constricted at the joints, chlorophyl fascia bright green, narrow; lower articles of the branches about as long as broad ($\frac{1}{15}''' - \frac{1}{25}'''$) or nearly twice as long, somewhat torulose, the upper ones cylindrical, as small as $\frac{1}{4}'''$, two to five times longer than broad, mostly not piliferous; *fascicles of branches densely branched, elongate, acutely lanceolate, actually subappressed.*

Remarks.—I have found a *Draparnallia* frequently, which I believe to represent the European *D. plumosa*. As I have preserved, however, no specimens or descriptions, I have simply copied the description of Prof. Rabenhorst.

D. Billingsii, WOOD.

D. valde gelatinosa; filis et ramis primariis achrois ad $\frac{3}{100}''$ crassis, sparsissime ramosis, articulis diametro 2-6 plo longioribus, sæpe medio valde tumidis; fasciis chlorophyllis dilute viridibus, sæpe nullis aut subnullis; ramulorum fasciculis distantibus, late ovalibus vel late triangularibus, alternantibus vel oppositis vel triplice verticellatis, sparse ramosis, patentissimis; ramulis pilis longissimis robustis terminalibus instructis; oosporis globosis, moniliforme conjunctis; sporodermate crasso.

Syn.—*D. Billingsii*, WOOD, Proc. Am. Philos. Soc., 1869, p. 143.

Hab.—In aquis quietis, prope Philadelphia.

Fronde very gelatinous, filament and primary branches attaining a diameter of $\frac{1}{20}''$, very sparsely branched, their articles 2-6 times longer than broad, often very much swollen in the middle; chlorophyl band light green, frequently almost or entirely wanting; fascicles of branches distant, broadly oval or triangular, alternate, opposite, or in whorls of three, very open; ultimate branchlets terminating in a long, robust, hyaline hair; resting spores globose, with thick walls, arranged in long moniliform sometimes branched filaments.

Remarks.—I found this plant about the middle of March, 1869, floating on the surface of a little pool in the woods near Cheltenham Hills, a few miles north of Philadelphia. To the naked eye it appears as a gelatinous mass, resembling a *Tetraspora*, but when closely examined this translucent jelly is seen to be filled with rather distant greenish points, which are the little clusters of branches. The largest specimens I have seen had attained a length of nearly two inches. The filaments are very transparent and have the branches placed at long intervals.

The ultimate branch groups are ovate or oval, and are remarkable for their openness, the branchlets being few in number and widely separated. Most of the ultimate branchlets are prolonged into a remarkably strong long hair.

The cells of the main filaments are beautifully transparent, and are sometimes cylindrical but more generally are barrel-shaped. Both secondary and primary branches are often arranged singly, sometimes in pairs, not unfrequently in threes. When placed between two plates of glass and examined closely by the unaided eye, this species is readily distinguishable from our other *Draparnaldia*, by its fasciculi of branches being so widely separated as to be not at all confused with one another.

I have a single specimen which I believe to be in fruit. The resting spores (fig. 6, pl. 14) are in long branched chains. They are more or less globose, with a very thick outer transparent wall, and an inner green endochrome, which very probably becomes brownish at maturity. Except when they are branched, these series of spores remind one very strongly of the filaments of some nostocs.

I dedicate this very beautiful species to Dr. J. S. Billings, U. S. A., to whom I am under the greatest obligations for aid in the prosecution of this research, and whom I have ever found to unite the greatest scientific liberality with a strong enthusiasm for and able prosecution of the study of these lower vegetable forms.

Since describing this species I have received the *Microscopical Journal* for 1869, containing Dr. Hicks's paper upon *D. cruciata*. The original description in the *Linnean Transactions* had escaped my notice. *D. cruciata* and *D. Billingsii* are exceedingly closely related, yet if Dr. Hicks's description and figures be accurate they are probably distinct. Thus in the last species the ramuli are not placed at right angles to the main filament, nor are they ever in fours, both of which are given as characters of *D. cruciata*. They are, on the contrary, in *D. Billingsii* at various angles, and commonly arise singly, but not unfrequently in pairs, and very rarely in threes. It is worthy of remark, on the other hand, that the figures of Dr. H. do not entirely agree with his description, as in no case are there more than two and frequently but a single branch at one place. The cells of the main filament are also more barrel-shaped in our species than one would infer to be the case with *D. cruciata*.

After all, however, I think it very possible that both forms belong to the one species.

Fig. 6, pl. 14, represents a small portion of the frond with fertile branches magnified 460 diameters.

Genus CHÆTOPHORA, SCHRANK.

Fila articulata ramique primarii radiatim dispositi, e cellulis vegetativis elongatis, fascia chlorophyllosa in morem *Draparnaldia*e et *Stigeoclonii* ornatis compositi, sursum in ramulos numerosissimos, brevius articulatos, articulis extremis attenuatis sæpe inanibus non aut vix piliferis instructos, fasciculatos plus minus dense congestos divisi, massa gelatinosa firma, coriacea vel dura involuti, thallum globosum vel subglobosum aut plane expansum varie lobatum et fissum constituent. (R.)

Filaments articulated, with the primary branches radiately disposed, composed of elongated vegetative cells, ornamented with a chlorophyllous fascia like a *Draparnaldia* or *Stigeoclonium*, distally

resolved into very numerous fasciculate, more or less densely congested branches, with shorter joints, their end joints alternate, often empty, either not or scarcely piliferous; surrounded by a firm coriaceous or hard jelly, so as to form a globose, subglobose, or expanded thallus.

Remarks.—I have never seen the production of the zoospores in this genus, but they are said to arise one in a cell, and to escape by a sort of lateral splitting of the wall.

C. elegans, (ROTH) AGARDH.

Ch. thallo globoso vel subgloboso, pisi vel cerasi magnitudine, dilute vel saturate viridi, nitido, superficie levi vel quasi tuberculata, elastice molli, nonnunquam indurato; fasciculorum ramulis laxis vel confertis, articulis extremis brevi-cuspidatis, sæpe piliferis.

Syn.—*C. elegans*, (ROTH) AGARDH. RABENHORST, Flora Europ. Algarum, Sect. III. p. 384.

Hab.—United States.

Thallus globose or subglobose, of the size of a pea or cherry, light green, with the surface smooth or quasituberculate, elastic but soft, sometimes indurated; branches of the fasciculi lax or crowded; end articles shortly cuspidate, often piliferous.

Remarks.—One of the commonest of our fresh-water algæ is a plant belonging to this genus, which I think is probably the *C. elegans* of Roth. I am, however, unable to discover any characters separating *C. pisiformis*, *C. elegans*, and perhaps *C. tuberculosa*, and hardly know by which of the three names our American form should be known. Our plant grows generally in shaded pools, springs, and ditches in great abundance, adhering as little translucent balls to grasses, leaves, twigs, or anything that may be in the water. The size of the frond varies from the young one, not so large as a pin's head, to the old matured one, which may be nearly an inch in diameter. The color also varies greatly. It is always some shade of a pure green. The surface is mostly smooth, but sometimes it is so puckered up as to be a mass of large flat tubercles. It is these forms that I suppose to represent *C. tuberculosa*. The thallus is generally elastic, but at the same time soft, so that although readily compressed and pushed out of shape, it is entirely mashed with some difficulty, especially as, owing to its slipperiness, it constantly escapes from the grasp.

In regard to the individual filaments, the method of their branching and the proportionate length and breadth of the cells vary very much in different individuals and probably at different ages of the same individual.

Fig. 5, pl. 6, represents rather indifferently well a young individual of this species.

C. endiviæfolia, (ROTH) Ag.

Ch. thallo lineari, subplano, semipollicari vel pollicari, nonnunquam valde elongato, læte vel obscure viridi, dichotomo-subreticulatum-laciniato (nonnunquam habitu *Ricciæ fluitantis*); filis ramisque primariis plerumque æbrois, passim viridi-zonatis, parallelis; ramulorum fasciculis lateralibus, plus minus densis, divaricato-patentibus; articulis plus minus tumidis, diametro æqualibus vel subæqualibus; geniculis constrictis; cytoplasmate granuloso effuso. (R.) *Species mihi ignota*.

Syn.—*C. endiviæfolia*, (ROTH) AGARDH. RABENHORST, Flora Europ. Algarum, Sect. III. p. 383.

Hab.—South Carolina; (Ravenel) Wood. Rhode Island; (S. T. Olney) Thwaites.

Thallus linear, flattish, of half to a whole thumb's breadth, sometimes greatly elongate, bright or obscure green, dichotomously subreticulately laciniate (sometimes with the habit of *Riccia fluitans*); filament and primary branches mostly colorless, sometimes zoned with green, parallel; lateral fasciculi of branches more or less dense, divaricately patent; joints more or less tumid, diameter equal or subequal; joints constricted; cytoplasm effused granulate.

Remarks.—I have never seen a living or well-preserved specimen of this species, and have, therefore, here simply copied the description of Prof. Rabenhorst. Prof. Ravenel has sent to me dried algæ labelled, and I think correctly, as belonging to this species, but their condition did not allow any scientific study of them.

Genus PILINIA, Ktz.

Fila articulata, erecta, simplicia vel dichotome ramosa, basi affixa, in stratum crustaceum subspongiosum, fragile aggregata. Propagatio adhuc ignota.

Filaments articulate, erect, dichotomously branched, fixed by the base, aggregated into a somewhat spongy fragile crustaceous stratum. Method of propagation unknown.

P. diluta, WOOD, (sp. nov.)

P. rupicola, in strato cano-viridi disposita; filis ramisque fasciculatis, apice obtusis; articulis diametro $1\frac{1}{2}$ plo— $3\frac{1}{2}$ plo longioribus.

Diam.—Max. 0.0004".

Hab.—In fontibus maximis, prope Bellefonte, Centre County, Pennsylvania; Wood.

Growing on stones and rocks, forming a grayish-green stratum; filaments and branches fasciculate, with the apices obtuse; joints $1\frac{1}{2}$ — $3\frac{1}{2}$ times longer than broad.

Remarks.—Near Bellefonte, Centre County, Pennsylvania, there issues from the limestone rocks the largest spring I have ever seen, giving rise to a creek-like torrent, which supplies the city with water, and passes on scarcely diminished in volume. In this spring grows the curious algæ under consideration, forming a somewhat lubricous crustaceous and stony stratum on the stones and rocks in the basin. This stratum is of a grayish-green color, and is quite friable, breaking in the direction of the filaments with the greatest possible readiness. When placed under the microscope it is seen to be composed of filaments whose course is a direct one from the under to the upper surface. They are apparently rigid, preserving their courses, and not being intermatted. They are composed of cylindrical, confervoid cells, and are dichotomously branched, and yet when viewed as a whole the filament and its branches form a sort of fasciculus. The basal cell or cells appear to be globular. When I collected this plant I was forced by circumstances to put the specimens in carbolic-acid water for future study, and, therefore, I have had no opportunity of studying their method of reproduction. I am not altogether satisfied in referring this plant to the *Pilinia*, and yet all the most important of the characters given by Rabenhorst are preserved by it. It certainly, however, differs very greatly from *P. rimosa*, Ktz.

Genus APHANOCHÆTE, BRAUN.

Fila distincte articulata, prostrata, repentia, interdum in stratum irregulare plus minusve concreta; ramulis repentibus vel adscendentibus; cellulis chlorophyllaceis, apice vel dorso setigeris. Propagatio zoogonidiis.

Threads distinctly articulate, prostrate, creeping, sometimes more or less concreted into an irregular stratum; branches creeping or ascending; chlorophyllous cells with the dorsum or apex setigerous. Propagation by zoospores.

Remarks.—Sexual reproduction has not as yet been discovered in this genus. According to Dr. Braun (Verjüng., Translation of the Ray Society, p. 184, &c.) two zoospores are generally formed in a cell by a division of its contents parallel to the septa, but occasionally this division not taking place, the cell contents are resolved into a single zoospore. The zoospores themselves are nearly globular, biciliate, and unprovided with any reddish eye-spot.

A. repens, BRAUN.

A. filis procumbentibus plerumque simplicibus; articulis cylindricis aut tumidis, diametro subæqualibus ad 1-2 plo longioribus; setis e cellularum dorso egressis, plerumque singulis sed interdum geminis, interdum nullis.

Diam.—Artic. $\frac{3}{12000}$ "— $\frac{5}{12000}$ " = .00025—.0004".

Syn.—*A. repens*, BRAUN. RABENHORST, Flora. Europ. Algarum, Sect. III p. 391.

Hab.—In *Ædogoniis*, prope Philadelphia; Wood.

Filaments procumbent, mostly simple; articles cylindrical or tumid, from as long as broad to twice as long; seta arising from the back of the cells, generally single, sometimes geminate, sometimes wanting.

Remarks.—The specimens from which the above description was drawn up, were found growing on the filaments of *Ædogonium mirabile*, WOOD. They were remarkable for the rarity with which they were branched, for in but two or three cases out of a great number, were any branches detected. The articles were frequently twice as long as broad. In both these particulars the plant differs from the typical European *A. repens*, but the descriptions of that form are so short and imperfect that I have preferred retaining the name for the American plant.

Fig. 5, pl. 14, represents an ordinarily formed specimen magnified 460 diameters. It had been kept for some time in weak carbolic-acid solution, and although the green of the chlorophyll was perfectly preserved, the stumps only of the setæ were visible. How long the perfect setæ are I cannot at present say, not having made any notes on the fresh specimens.

Genus COLEOCHÆTE, BRÉB. (1844).

Fila articulata ramosa aut in pulvinulum conjuncta aut in thallum planum subdisciformem parenchymaticum concreta; articuli oblongi, antice plus minus dilatati, angulo superiori vel dorso sæpe in setam basi vaginatam producti. Propagatio fit tum oosporis fecundatione sexuali ortis, tum zoogonidiis. Zoogonidia in quaque cellula fructifera unica, forma subglobosa vel late ovalia, polo antico ciliis vibratoriiis binis instructa. (R.)

Filaments articulated, branched, either conjoined into a little cumulated mass or parenchymatously concreted into a plain subdisciform thallus; articles oblong anteriorly, more or less dilated, often furnished with a long seta on their dorsum or superior angle. Propagation occurring by means of oospores, formed by sexual organs or by zoospores. Zoospores subglobose or broadly oval, formed singly in the fertile cell, furnished at their anterior pole with vibratile cilia.

Remarks.—I have seen a large number of specimens of, as I believe, two distinct species of this genus, but never having found any fruiting fronds, have not been

able to identify them. One of the forms grows in this immediate locality, and is very probably *C. scutata*, Bréb. The other was collected in Northern Michigan. It is characterized by its frond never being disciform, although composed of a single plane of cells parenchymatously united.

CLASS RHODOPHYCÆ.

Algæ multicellulares, vegetatione terminalis non limitata præditæ plerumque trioicæ.

Thallus e cellularum seriebus vel stratis singulis vel pluribus compositus, aut nudus aut e cellularum strato corticatus, forma quam maxime varius; membranaceus (*Porphyridium*), crustaceus (*Hildenbrandtia*), filamentosus et verticillatim ramosus (*Batrachospermum*, *Thorea*), fasciiformis (*Bangia*), foliaceus, etc.

Cytoplasmata plerumque rhodophyllo (Colm), rarius phyco-chromate coloratum, granula amyloidea vel amylacea et sæpe guttulas oleosas includens.

Propagationis organa triplicis indolis, sæpissime in plantas distinctas disposita.

1. *Organa mascula vel antherida* e fasciculis cellularum plerumque moniliformibus ramosis, denique in spermatozoidea vel spermata fœcundantia (*Sporidia* I. Ag.) oblonga vel ovalia, achrod, immobilia dissolutis formata.

2. *Organa feminea vel cystocarpia* Ktz. e soris nonnunquam moniliformibus formata, qui e placenta sæpissime corticali evolvuntur, nudi vel cuticula mucilaginosa vel involucro inclusi, denique sporas (*polysporas*) numerosas immobiles mox germinantes emittunt. Fœcundatur cystocarpium statu primordiali ope organi piliformis (*trichogyne* Thuret et Bornet) quorum spermata copulantur.

3. *Tetrasporangia* e cellula corticali unica valde intumescente formata, divisione utriculi primordialis cruciata quadrilocularia; in quoque loculo (*cellulis secundariis, sororiis*) spora unica (*tetraspora*) se format, quae sine fœcundatione germinat. (R.)

Multicellular algæ, mostly triœcious, furnished with unlimited not terminal vegetation.

Thallus composed of cells in rows or in a simple or multiple stratum, either bare or provided with cortical strata of cells, exceedingly various in form; membranaceous (*Porphyridium*), crustaceous (*Hildenbrandtia*), filamentous and verticillately branched (*Batrachospermum*, *Thorea*), fasciate (*Bangia*), foliaceous, &c.

Cytoplasm mostly rhodophyllous, rarely phycochromatously colored, including amyloid granules or starch and frequently oil drops.

Propagation by means of three immotile organs, generally placed upon distinct plants.

1. *Antheridia* composed of mostly moniliformly branched fascicles of

cells, which dissolve into oblong, oval, transparent immotile spermatozooids (*Sporidia* Ag.).

2. *Cystocarpia* Ktz., or *Pistillidia*, formed of somewhat moniliform sori, which are evolved from a generally cortical placenta, and are naked or surrounded by a mucilaginous cuticle or involucre, and finally emit numerous immotile spores (*polyspores*), which quickly germinate. The fecundation of the cystocarpia occurs in their primordial state by contact of the spermatia with a piliform organ known as *trichogonia*.

3. *Tetrasporangia* formed of single, greatly swollen cortical cells, becoming cruciately quadrilocular by division of the primordial utricle; in each loculus (*secondary or sister cells*) a single spore (*tetraspore*) forms, which germinates without fecundation.

FAMILY PORPHYRACEÆ.

Thallus mucoso-membranaceus, foliaceus vel filamentosus, e cellularum seriebus vel strato unico formatus, plerumque purpurascens, valde lubricus.

Vegetatio fit cellularum divisione in duas vel omnes directiones repetita.

Propagatio fit tetrasporis. Cystocarpia nondum observata.

Thallus mucous-membranous, foliaceous or filamentous, formed of cells in series or in a single stratum, mostly purplish, very slippery.

Growth taking place by repeated division of the cells in two or all directions.

Propagation by means of tetraspores. Cystocarps not yet observed.

Remarks.—The only species of this family as yet observed in North America can hardly be said to have a definite thallus. They are rather multitudes of cells heaped together and closely attached to one another into a shapeless expanded mass.

Genus PORPHYIDIUM, NAEG. (1849).

Thallus mucoso-membranaceus, suberustaceus, longe lateque expansus, e cellulis globosis vel polyedricis compositus. Propagatio adhuc ignota.

Thallus mucous-membranous, suberustaceous, long and widely expanded, composed of globose or polyhedral cells. Propagation unknown.

P. cruentum, (AG.) NAEG.

P. thallo saturate purpuro-sanguineo, lubrico; cellulis anguloso-rotundatis. (R.)

Diam.—0.00027"—0.00035". (R.)

Hab.—New York.

Syn.—*P. cruentum*, (AGARD.) NAEGEL. RABENHORST, Flora Europ. Algarum, Sect. III. p. 397.

Thallus deep crimson purple, slippery; cells angled and rounded.

Remarks.—The only specimen I have seen of this species was a little speck, adherent to a bone picked up on Governor's Island, in New York Harbor. It is very probable that it was a recent arrival, brought over, perchance, by some emigrant. For it I am indebted to Dr. Billings, U. S. A. The description and

measurements given above are copied from Prof. Rabenhorst's work. My specimen agrees well with it.

P. magnificum, WOOD.

P. cellulis globosis vel subglobosis, sæpe nonnihil polygonis et in massam indefinite expansam confluentibus; cytoplasmate purpureo, granulato; cytiodermate crasso, haud lamelloso.

Diam.—Cell cum. tegum. $\frac{8}{24000}$ — $\frac{14}{24000}$. Tegum. $\frac{1}{30000}$ — $\frac{1}{16000}$.

Syn.—*P. magnificum*, WOOD, Proc. Am. Philos. Soc., 1869, p. 144.

Hab.—In terra humida, Texas; Prof. Ravenel.

Cells globose or subglobose, often somewhat polygonal and conjoined into an indefinite mass; endochrome purple, granulate; cell wall thick, not laminate.

Remarks.—This species, which was collected in Texas by Prof. Ravenel, growing, I believe, on wet sand, is very distinct from the European plant, differing essentially in size and form. In some instances the cells have a greenish tint, but this is possibly owing to immaturity, as such cells seem smaller than others. The whole mass to the eye has a very rosy purple tint, and although under the microscope it appears much darker and more purple, yet it often retains some of the roseate hue. At the edges of the masses the dark-reddish color often gives way to a very decided greenish tint, presenting an appearance which is very well represented in the drawing of the preceding species, in M. MENGEHINI'S *Monographia Nostochinearum Italicarum, &c., Memoire della Reale Accademia delle Scienze di Torino*. The cells are often closely united by their thick coats into a very coherent mass. With the ordinary cells I have occasionally seen other larger ones, of an orange color, with very thick walls. Are these resting spores?

Fig. , pl. 19, represents single cells of this plant magnified 750 diameters.

FAMILY CHANTRANSIACEÆ.

Thallus filamentosus. Fila articulata, e cellularum serie unica formata, ramosa, stricta, nuda, raro passim corticata, rami superne fasciculatim ramellosi; articuli cylindrici. Cytioderma, homogenum, maxime hyalinum. Cytoplasma homogenum, plerumque purpurascens. Propagatio fit polysporis immobilibus, ovalibus, in ramellorum apice vel lateraliter formatis, corymboso aggregatis. Antheridia subglobosa, terminalia. Tetraspora raro observatæ.

Thallus filamentous. Threads articulate, formed of a single series of cells, branched, straight, bare, rarely here and there articulate; branches above fasciculately branched; joints cylindrical. Cytioderm homogeneous, mostly hyaline, cytoplasm homogeneous, mostly purplish. Propagation by immovable oval polyspores formed on the ends of the branches or laterally and corymbosely aggregate. Antheridia subglobose terminal. Tetraspores rarely observed.

Genus CHANTRANSIA, FRIES.

Familie genus unicum.

The only genus of the family.

C. expansa, WOOD.

C. caespitosa, in lapide stratum saturate violaceo-purpureum lubricum, indefinite expansum, formans; filis purpureis, modice ramosis, fere 2 lineas longis et ramis plerumque strictis et rectis, sæpe elongatis; ramulis fertilibus brevibus, ascendentibus; articulis diametro 3–8 plo

longioribus, extremis obtusis; polysporis in ramellis lateralibus racemosim et confertim cumulatis, ovalibus vel nonnihil obovatis.

Diam.—Fil. $\frac{1}{2500}$ " = .0004". Spor. transv. $\frac{1}{3650}$ " = .00027 long. $\frac{1}{2500}$ " = .0004".

Syn.—*C. expansa*, Wood, Prodomus, Proc. Amer. Philos. Soc., 1869.

Hab.—In rivulis, prope Philadelphia.

Cæspitose, forming a dark purple, slippery, indefinite stratum on stones; filaments purple, moderately branched, almost 2 lines long, together with the branches strict and straight, often elongate; infertile branches sometimes very few, sometimes very numerous; fertile branches short, ascending; joints 3–8 times as long as their diameter, the final articles obtusely rounded: polyspores racemose, crowded on the fertile branches, oval or somewhat ovate.

Remarks.—This species was found growing in a running stream, forming a felty slimy coating upon large stones, looking so much like a stratum of *Oscillatoria*, that when I gathered it I thought it probably was a representative of that genus. The stratum, however, when carefully examined, is seen to be made up of an indefinite number of minute, very closely approximate tufts. The color was a dark dull purple. The plant may possibly be the *Chantransia violacea*, of KÜTZING, which it resembles in many particulars, but it is nearly twice as long and the filaments are considerably thicker. Its habit of growth also seems to be essentially different from that of the European plant, so that I have finally decided to consider it a distinct species. The exact locality of its growth is in a thickly-shaded portion of the stream that runs along the North Pennsylvania Railroad, just this side of Cheltenham Hills.

Fig. 2, pl. 19, represents a filament magnified 125 diameters; fig. 2 a, a part of a fertile branch magnified 460 diameters.

C. macrospora, WOOD (sp. nov.).

C. cæspitosa, subpollicaris, olivaceo-grisea vel saturate violaceo-purpurea; filis ramosis et ramis plerumque strictis et rectis, et elongatis; articulis diametro 3–8 plo longioribus; ramulis fertilibus brevissimis; polysporis singulis vel geminis, sparsis, sæpe distantibus, globosis, interdum nonnihil ovalibus.

Diam.—Fil. plerumque .0008—max. .001. Polysp. .0009.

Hab.—South Carolina; (Ravenel).

Cæspitose, about an inch long, olive-gray to deep-violet purple; filaments a good deal branched, with the branches mostly straight and elongated; fertile branches very short; articles 3–8 times longer than broad; spores single or geminate, few, often distant, globose, or sometimes slightly oval.

Remarks.—I am indebted to Prof. Ravenel for specimens of this species preserved in carbolic-acid water. They are labelled, "Dull olive green, growing against wooden boards in spring, Nov. 5, 1869. Aiken, South Carolina." The most of the mass is of the color noted, or at least approaches it, but a portion is almost blackish purple. The species is a very distinct one, characterized by the larger diameter of its articles and spores, by the paucity and shape of the latter, as well as by its variance in coloration. In some old specimens the cell wall is distinctly lamellate. I have only seen fruit on the purple filaments. The

spores, apparently not mature, have a greenish-brownish tint. I have also received from Prof. Ravenel dried algæ, which, apparently, are the same species as those from which this description has been written, but which, not being in fruit, cannot be absolutely identified. They are, as dried, of a bright bluish-green, and attain the length of an inch and a half or more.

Fig. 3, pl. 19, represents a part of a branch of this plant magnified 460 diameters.

FAMILY BATRACHOSPERMACEÆ.

Algæ dioicæ. Thallus filamentosus, articulatus, ramosus, aut violaceus, violaceo-purpureus vel cæruleo-viridis, muco matricali involutus; filis primariis ramisque e cellularum serie unica centrali primaria et seriebus numerosis secundariis parallelis continuis vel interruptis externis compositis, aut ramulorum fasciculis verticillatis globoso vel subgloboso dense conglobatis æquali distantia obsitis, aut ramulis simplicibus vel dichotomis dense ubique vestitis. Vegetatio terminalis.

Diœcious algæ. Thallus filamentous, articulate, branched, violet or violet-purple or bluish-green, covered with mucous; primary filament and branches composed of a single central series of cells, and numerous external, parallel, continuous, or interrupted secondary series; either furnished with globosely or subglobosely densely conglobate, equally distant verticillate fasciculi of branches, or everywhere densely covered with simple or dichotomous branches. Vegetation terminal.

Genus BATRACHOSPERMUM, ROTH, 1800.

Thallus filamentosus, moniliformis, e cellularum serie unica medullari, accessoriis parallelis corticata compositis, ramulorum fasciculis subgloboso-conglobatis obsessus.

Thallus moniliform, composed of a simple series of medullary cells and cortical accessory parallel series, clothed with subglobosely conglobate fasciculi of branches.

Remarks.—The *Batrachosperms* are amongst the very largest of the fresh-water algæ, forming gelatinous branched masses from a few inches to even more than a foot in length. The fronds are very freely and very irregularly branched, and are evidently composed throughout, *i. e.*, both in regard to the main filaments and the branches, of two portions, a central axis and much more slender short transverse branchlets, which often end in a long hair, and are arranged more or less exclusively in groups, so as to form, to the naked eye, at regular intervals, little balls or knots, the whole plant thus presenting a sort of moniliform aspect. Sometimes, however, these glomeruli are placed so closely together, and grow so large that they become confluent, and the branch to which they are attached appears as a uniform thick and very gelatinous cylindrical cord.

The axis both of the stem and the branches of a *Batrachosperm* consist originally of but a single series of cells. The development of new cells takes place in two ways, the one of which results simply in an increase in the length of the axis, the other in the production of branches. The first of these is the ordinary process of cell multiplication by division, and occurs only in the end cells, so that no new cells are ever formed in the central portions of the axis, which increases in length solely by the addition of new cells at the end, and by longitudinal growth of the old ones. The first step towards the formation of a branch is the production of a little pouch-like protrusion near the upper end of a cell. This increases

in size and soon being cut off from the parent-cell by a partition, forms a complete cell, the starting point of a new branch. If this cell has been formed alone, without companions, it is the beginning of a main branch, and divides after a very brief period transversely, the new cell thus arising in a little while itself divides, and so the process goes on until the axis of a large branch, similar to the parent axis is developed, and which, like the parent axis, increases only by a division of the end cell and longitudinal growth of the central ones.

When a glomerulus is to be formed instead of a single pouch, a number appear around the upper end of a cell, and become cut off as new cells. Each of these is the starting point of a new row of cells, which not only grows, at least up to a certain point, by the division of the end cells, but which also gives rise to a large number of branches in a way precisely similar to that in which it itself was developed, *i. e.*, by the formation of little lateral protrusions, &c. These secondary branches have a life-history similar to that of the branch whose offspring they are. They continually give origin to new branchlets in the way just described, which branchlets themselves produce fresh offshoots, and so it goes on until at last the forest of branchlets making up the dense glomerulus is evolved. It has been just stated that the original axis of the main filament or any branch is composed of a single simple series of large cells; when an old *Batrachosperm* is placed under the microscope, however, it is at once evident that the axis is in reality formed of such a series lying in the centre and covered over and often hidden by numerous longitudinal series of smaller cells. These latter do not belong to the original axis, but are secondary additions to it, and arise in this way. Whilst a glomerulus is being developed certain of the basal cells of its constituent branches give origin in the usual manner to branchlets, which, instead of growing outward to form a part of the glomerulus, grow upwards or downwards, closely hugging and finally enveloping the original axis, and at last forming a distinct cortical layer to it.

Very frequently in well-advanced *Batrachosperms* there will be seen scattered among the glomerulus large, round, firm, dense balls composed of a great number of small closely-attached cells. These are the reproductive bodies. According to H. Graf zu Solms-Laubach (*Botanische Zeitung*, 1867, p. 161), they are the result of sexual reproduction, and are developed from *antheridia* and *trichogonia* (female organs) in the following manner:—

The antheridia are small roundish cells full of a colorless protoplasm, which is remarkable for the very numerous bright granules which it contains. They occur either scattered or in groups, and are placed upon the upper ends of peculiar ovate cells, also filled with a colorless protoplasm. Most frequently there is a single antheridium to the basal cell, sometimes two; the latter number appears never to be exceeded. When matured, the antheridia open and allow their contents to escape in the form of roundish or flattened bodies, which never, as far as known, acquire cilia, and have, therefore, no power of spontaneous motion. These bodies, which are believed to be spermatozoids, are unprovided with anything like an external membrane, and are composed of protoplasm identical with that in the antheridium.

Whilst these changes are occurring, certain cells in other localities are being transformed into female organs, to which our author applies the name of *Trichogonia*.

These are borne upon cells similar to those supporting the antheridia. At first they are not markedly different from the other cells, but soon undergo a very rapid growth. This is not, however, regular, and is not partaken of by a band of tissue about one-third way from the basal end, so that at last a long somewhat flask-shaped cell is produced, with a very marked contraction at the point indicated, separating it into two portions. The wall of this cell is thin but very distinct, and the cavity is filled with a homogeneous or very sparsely granular protoplasm, which is continuous through the narrow neck-like portion. After a time there appear one or more large irregular vacuoles, with actively moving corpuscles in them, and at the same time the neck appears to be stopped with a slimy substance. Careful examination with reagents shows that this is cellulose, and that it does not completely block the passage-way through the isthmus. At this time there appear lying upon the free end of the trichogonia globular or flattened bodies, without external membrane, corresponding in all respects with those already described as being produced in the antheridia. The end of the trichogonium generally enlarges at this period into a sort of roundish knob, and by and by the end wall between this and one of these globules becomes absorbed, so that there is a free communication between the two. Whilst this is going on the globule acquires a thin, delicate coat, and there appears in it a vacuole similar to those preexisting in the trichogonium.

The first result of this impregnation of the trichogonium is the deposit of new cellulose, and the complete blocking up of the passage-way through the isthmus or narrowed portion. Already before the fecundation, the upper cells of the branches supporting the trichogonia have produced numerous branchlets, which growing upwards more or less completely cover that organ. After impregnation the cells near to the trichogonium become much larger and broader, their vacuoles disappear, and are replaced by a dense granular dark greenish-brown protoplasm.

These cells now show a great activity in the production of numerous branches in the usual way, but it is the upper two alone which, with the *trichogonium* that they support, are concerned in the formation of the fruit glomerulus. These put out all over their surface an immense number of protrusions, which soon in the ordinary way become the parents of as many twigs or branchlets, which growing and branching, precisely as do the vegetative branches, soon become excessively crowded. The base of the trichogonium participates also in this production of branches, and at last a dense ball is formed of pseudoparenchymatous tissue by the forced adhesion of the crowded twigs. The central cells of the glomerulus thus formed are very large and bladder-like. The outer part of the ball is composed of innumerable radiating rows of small cells, the end cell of each branch being roundish so as to present a convex external face. At maturity these cells open and allow their contents to escape as round masses, which appear to have no membrane, but begin at once to grow and secrete cellulose. Their after-history has not been made out with absolute certainty, but they are believed to directly develop the new plant.

B. moniliforme, (ROTH.)

B. pollicare, bi- tripollicare, raro pedale, mucro gelatinoso plus minus firmo involutum, violaceum, fuscum, rufo-brunneum, purpureum vel cæruleo-viridiscens, vage ramossissimum; ramulorum articulis omnibus conformibus, oblongo-subclavatis, extremis nonnunquam setigeris; internodiis nudis vel ramulis accessoriis singulis sparsis instructis.

Diam.—Tetrasp. globulus $\frac{27}{4 \times 100} = .006$.

Syn.—*B. moniliforme*, ROTH. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 405.

Hab.—In aquis puris, Michigan; Gray. New York; Bailey. Virginia; Jackson, Alabama; Tuomey. South Carolina; (Ravenel) Pennsylvania; New Jersey; Wood.

One inch to a foot in length, clothed with a more or less firm gelatinous mucus, violet, fuscous, reddish-brown, purple, or bluish-green, vaguely and profusely branched; joints of the branches similar, oblong-subclavate, the outer ones sometimes setigerous; internodes naked or furnished with a few scattered accessory branchlets.

Remarks.—This species is very abundant in fresh, cool rivulets, in springs, in limestone waters, in pine-barren streams, and even occasionally in ditches, wherever I have botanized in Pennsylvania and New Jersey. It varies greatly in size, in color, and other particulars.

The branchlets, as I have observed them, are most generally not setigerous, but at times they are provided with seta of moderate length.

I have found numerous fruiting fronds, but in none of them was the fruit in great abundance, not nearly so much so as in the Rocky Mountain species.

B. vagum, (ROTH) AGARDH.

B. vage ramossissimum, uni- vel tripollicare, fuscum vel ærugineum; internodiis inferioribus ramellis numerosis obsessis, superioribus nudis vel subnudis; ramulorum articulis extremis setis longissimis instructis.

Diam.—Tetrasp. globulus $\frac{15}{4 \times 100} = .00333$.

Syn.—*B. vagum*, (ROTH) AGARDH. RABENHORST, *Flora Europ. Algarum*, Sect. III. p. 406.

Hab.—In aquis quietis, Uintah Mountains, Nevada; (S. Watson).

Vaguely branched, one to three inches long, brownish or æruginous; internodes—the inferior covered with a dense mass of branchlets—the superior naked, or nearly so; last articles of the branchlets provided with an extremely long seta.

Remarks.—I have received from Mr. Screno Watson some half a dozen dried algæ, which I have referred to *B. vagum*, with some doubt. They are labelled as having grown in shallow water, in a beaver pond, in Pack's Cañon, Unita, Uintah Mountains, Nevada, at an altitude of 7000 feet. All the descriptions of *B. vagum* which I have seen are singularly imperfect; in none is it stated how large the spore masses grow, and how plentifully the branchlets are provided with seta. As far as the descriptions go, however, my specimens agree with them, and I have, therefore, refrained from indicating a new species. The plants are remarkable for the profusion and extreme length of the seta, and for the quantity of fruit which they produce. The fruit masses are small but very compact, scarcely more than half the size of those of the preceding species. The verticles of branchlets are often completely joined, and as it were almost swallowed up by the mass of intervening scattered branchlets which arise directly from the main axis. In the distal

portions of the fronds, however, the glomeruli are more fasciculate and more distinct, for although sometimes so close as to be almost confluent at their spreading edges, at their bases they are distinct. This species very probably attains a much larger size than indicated by my specimens, and possibly varies as much in color as *B. moniliforme*.

Genus TUOMEYA, HARVEY.

"Frond cartilaginous, continuous, solid, at first transversely banded, afterwards annularly constricted; composed of a longitudinal axis, and two strata of peripheric cells. Axis columnar, consisting of several longitudinal cohering filaments, beset with closely placed whorls of moniliform ramelli, whose branches anastomose horizontally and vertically into a cellular peripheric membrane, which is coated externally with moniliform filaments, gradually developed. Fructification probably in the superficial filaments.

T. fluviatilis, HARVEY.

Hab.—On stones, in rivers and streams. River in Alabama; Prof. Tuomey. Near Fredericksburg, Virginia; Prof. Bailey.

Fronds tufted, an inch or two in height, scarcely as thick as a hog's bristle, much and irregularly branched, bushy; the branches alternate or secund, scattered or crowded, twice or thrice divided, and set with scattered patent ramuli which are slightly constricted at the interstices, and taper to an obtuse point. When young the branches and ramuli are perfectly cylindrical, and when examined under a low power of the microscope show a surface composed of minute, dotlike cells, placed close together, and marked at short intervals with dark-colored transverse bands. These bands disappear under a higher magnifying power. They are indications of the nodes of the axis of the frond seen through the peripheric stratum. In old, fully developed specimens the branches and ramuli are annularly constricted at short intervals, the nodes becoming swollen, whilst the internodes remain unchanged. When a young branch is bruised between two pieces of glass the axis may be readily extracted. It consists of several parallel longitudinal jointed threads combined together at closely-placed nodes, from which issue horizontal dichotomous filaments, composed of roundish or angular cells. These excurrent filaments spread both horizontally and vertically, and their branches anastomose into a cellular mass or fleshy membrane, which forms the inner peripheric stratum. In young plants a portion of the frond, between the axis and periphery, is hollow, but in older ones the cavity is quite filled up with cells. The external surface of the cellular periphery is clothed with a coat of moniliform filaments gradually developed, and forms what is above called the second peripheric stratum. These are found only in fully-grown specimens; they consist of much smaller cells than those of the inner stratum; they are more strongly colored, and I consider them to be connected with fructification. The color is a dark olive. The substance is brittle, rigid when dry, and the plant scarcely adheres to the paper. The generic name is in memory of the late Prof. Tuomey, of Tuscaloosa."

Remarks.—I have no knowledge of this plant, and have simply copied the description of Prof. Harvey; Smithsonian Contributions, 1846.

FAMILY LEMANEACEÆ.

Algæ rivulares vel fluviatiles. Thallus e præembryone confervacea enascens, setaceus, subsimplice vel fasciculatim ramosus, cavus, nodosus, e cellularum stratis internis et corticatis formatus. Noduli plerumque papillarum corona instructi. Polysporæ numerosæ, in seriebus ramosis moniliformibus fasciculatim aggregatæ, sine fecundatione germinantes.

Algae growing in streams and rivers. Thallus developing from a confervoid prothalloid filament, setaceous, almost simple or fasciculately branched, hollow, nodose, composed of internal and cortical strata of cells. Nodules generally provided with a corona or papilla. Polyspores numerous, fasciculately aggregated in branched moniliform series, germinating without fecundation.

Genus LEMANEA, BORY.

Genus unicum.

The only genus.

Remarks.—The plants belonging to the genus *Lemanea* are quite peculiar in aspect and habit. They grow exclusively in fresh water, especially frequenting streams whose current is rapid, and whose waters are chilled by the mountain air. Their frail, tubular, scarcely-branched fronds offer but little resistance to the water, whilst their lower end is swollen into a sort of discoid root, which adheres firmly to the stones. The frond is mostly blackish or brownish, and is formed of two distinct portions or layers, of which the outer or cortical is composed of small closely cohering, colored cells; the inner of much larger cells, which have thick colorless walls, and are placed so as to leave more or less numerous interspaces. In the immature frond there is also a longitudinal central column, besides some slender many-jointed filaments, passing obliquely through the cavity, but as final development takes place these seem to disappear. The mature frond is alternately contracted and expanded throughout its length. In the narrow portions the inner tissue often blocks up the tube entirely, whilst the dilated parts are loosely filled with the spores, which are produced within the frond. The spores themselves are oval, thickish-walled cells, whose endochrome changes from greenish to a very decided yellow during the process of maturing. They are joined together to form rows or series, which are not simple, but are very much branched, so that from a central basal row arises a complex bush-like mass (pl. 20, fig. 4). These spore-clusters are always distinct, a number of them existing in each sporangial node of the frond.

Dr. B. Wartmann described, nearly twenty years ago, very fully the way in which the spores germinate and develop into the frond. The first step, according to this authority, consists in the elongation of the spore and the projection of one end, which is soon cut off by the formation of a transverse partition, and constitutes a new cell. This multiplying in no strikingly peculiar way soon develops into a branched confervoid filament. A large number of these filaments are generally produced in one place at one time and form a very apparent greenish layer. Finally certain cells in branches of these filaments swell up and become very much broader than their fellows, undergoing, at the same time, division so rapidly that they become very short. By and by they divide also in the direction of their breadth, so that instead of a simple series of cells there arises a compound mass. This is the beginning of the new frond. At first it is dependent upon the parent filament, but soon acquires a root-like process at the base and develops rapidly into the complex cartilaginous plant.

L. torulosa, (ROTH) AG.

L. subsimplex, plerumque arcuata, cartilaginea et nonnihil rigida, 1-2 pollices longa; nodulis approximatis, papillis applanatis, plerumque 4-6 enormiter verticellatis, vel nonnihil sparsis, interdum nonnihil confluentibus; sporis ovalibus.

Diam.—Sporis. transv. max. $1\frac{8}{10}000''$ — $1\frac{4}{10}000''$.

Syn.—*L. torulosa* (ROTH) AG. RABENHORST, Flora Europ. Algarum, Sect. III. p. 411.

Hab.—In flumine, Kentucky; (Short) Harvey. Pennsylvania; Virginia; New York; New Jersey; Wood.

Subsimple, mostly arcuate, cartilaginous and somewhat rigid, 1-2 inches long; nodules approximate, with their papules applanate, mostly 4-6, irregularly verticillate or somewhat scattered, sometimes slightly confluent; spores oval.

Remarks.—This plant attains a length of about two inches, and grows in masses attached to rocks, often forming a sort of turfy covering to them, in rapidly running water. In mass it has a grayish or blackish appearance. The filament has a grayish groundwork, with a dark band at the position of the nodes, which are enlarged and inclose the spores. The transverse outline of the filament is a very irregular circle. I have found this species very abundant in the rapid water of the Schuylkill, just above Flat Rock Tunnel, on the Reading Railroad, eight or nine miles above Philadelphia. Prof. E. D. Cope has sent me specimens collected by himself in swift streams in Western Virginia, and Mr. Austin has obtained it in similar situations in Northern New Jersey. Mr. Austin has also sent me specimens collected in Canada West.

L. fluviatilis, AG.

L. simplex vel parce ramosa, quatuor uncias longa (interdum spithamea?), recta vel subrecta; nodulis subremotis, papillis verticillatis magnis obsseis; sporis globosis vel subellipticis.

Diam.—Spor. $7\frac{10}{10}000''$ — $1\frac{3}{10}000''$.

Syn.—*L. fluviatilis*, AGARDH. RABENHORST, Flora Europ. Algarum, Sect. III. p. 411.

Hab.—In rivulis, Alabama; T. M. Peters.

Simple or sparsely branched, 4 inches long (sometimes growing of a span length?), straight or nearly so; nodules rather distant, papillæ verticillate, large, prominent.

Remarks.—The only specimens I have seen of this species were sent me by Prof. Ravenel. This plant is larger and heavier than *L. torulosa*, from which it is also readily distinguished by its very large prominent papillæ. These are in slightly irregular whorls of three or more. The spores vary in shape from that of a globe to that of a somewhat four-sided ellipse; in the latter case being sometimes nearly twice as long as broad. Prof. Rabenhorst speaks of the plant attaining the length of a span. I have never seen it over four inches.

L. catenata, Kütz.

L. ad uncias 5 longa, regulariter constricta, simplex, compressa, arcuata, in massa obscure violacea; papillis nullis; sporis enormiter ovalibus vel subglobosis.

Diam.—Spor. transv. max $1\frac{2}{10}000'' = .001''$.

Syn.—*L. catenata*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sec. III. p. 412.

Hab.—In rivulis frigidis montanis Diamond Range, Rocky Mountains; (Sereno Watson).

About 5 inches long, regularly constricted, simple, compressed, arcuate, in mass obscure violet; papules wanting; spores irregularly oval or subglobose.

Remarks.—I have received specimens of the plant from which the above diagnosis was drawn, from Mr. Sereno Watson, labelled "Mountain stream, Diamond Range, altitude 6500 feet." In the dried state they are closely interwoven into a dark purple, rigid thin mass. When soaked out they preserve the same color in mass, but each individual stem has a general light yellowish, neutral ground tint, with dark-purplish or greenish-black bands at regular intervals. At the position of these bands the filament is nearly round and contracted, whilst between them it is compressed and enlarged. The spores are placed, not at the swelling, but at the constrictions, corresponding to the dark rings in position. They are quite irregular in shape, and of a faint yellow tint. The filaments between the little knots of spores appear to be hollow. Their walls are everywhere very thin when compared with *L. torulosa*, hence they are more flaccid. The species agrees in every respect with Prof. Rabenhorst's diagnosis of *L. catenata*, KtZ., a native of cold mountain streams of Germany and Switzerland. I regret, however, very greatly that I have had no opportunity of comparison with European specimens, or a fuller description.

S U P P L E M E N T.

THE following species, of which the author has not seen specimens, were inadvertently omitted from their proper places in the monograph. They are all contained in the *Nereis Boreali-Americana* of Prof. Harvey. The following descriptions and remarks are simply copied from the work mentioned.

Tetraspora lacunosa, CHAUV.

Fronde at first tubular, then flat, or irregularly lobed, membranaceous-gelatinous, pale-green, everywhere pierced with roundish holes of various sizes. *Chauv. Alg. Norm. Breb. Alg. Fal. p. 11, t. 1. Kütz. Sp. Alg. p. 227. T. Godeyi, De Breb. Kütz. Tab. Phyc. t. 30, f. 3. T. perforata, Bailey, M.S.*

Hab.—In fresh-water streams. Abundant near Westpoint, Prof. Bailey; Providence, Rhode Island, Mr. Olney. (v. s. in Herb. T.C.D.)

Fronde at first funnel-shaped, afterwards splitting open, and then flat, expanding upwards and irregularly lobed, everywhere pierced with roundish holes of various sizes, large and small intermixed. These holes increase in size and numbers with age, and thus at last the frond becomes an open network. The substance is very gelatinous, but rather firmer than in some other species of the genus. The color is a pale green; and the hyaline gelatinous membrane is filled with roundish granules set in fours.

Kützing's figure of *T. Godeyi* answers well to our plant. I have not seen any authentic specimens of *T. lacunosa*, which is referred by Kützing to his *T. lubrica*, var. β ., but the description given of it applies to the American plant. When carefully dried, it forms a very pretty object for the herbarium. (*Chlorospermeæ*, p. 61.) (*Harvey*, p. 61.)

Nostoc (Hormosiphon) arcticum, BERK.

Fronde foliaceous, variously plaited, green or brownish; filaments at length (their gelatinous envelope being dissolved) free. *Berk. in Proc. Lin. Soc. fide An. Nat. Hist. 2d Ser. vol. 10, p. 302.*

Hab.—On the naked soil, in boggy ground. Assistance Bay, lat. $75^{\circ} 40'$ N. Dr. Sutherland. (v. s.)

“**Fronde** foliaceous, variously plicate, sometimes contracted into a little ball. Gelatinous envelope at length effused; connecting cells at first solitary, then three together; threads, which are nearly twice as thick as in *N. commune*, breaking up at the connecting cells, so as to form new threads, each terminated with a single large cell, the central cell becoming free.”
Berk. l. c.

“It grows,” says Dr. Sutherland, “upon the soft and almost boggy slopes around Assistance Bay; and when these slopes become frozen at the close of the season,

the plant lying upon the surface in irregularly plicated masses becomes loosened, and if it is not at once covered with snow, which is not always the case, the wind carries it about in all directions. Sometimes it is blown out to sea, where one can pick it up on the surface of the ice, over a depth of probably one hundred fathoms. It has been found at a distance of two miles from the land, where the wind had carried it. At this distance from the land it was infested with *Poduræ*, and I accounted for this fact by presuming that the insects of the previous year had deposited their ova in the plant upon the land, where also the same species could be seen in myriads upon the little purling rivulets, at the side of which the *Nostoc* was very abundant." At p. 205 of his Journal, Dr. Sutherland further mentions having tried it as an article of food, and found it preferable to the *Tripe de Roche* of the arctic hunters. Its nutritive qualities are probably equal to those of the jelly derived from other *Algæ*. (*Chlorospermeæ*, p. 113.)

***Nostoc flagelliforme*, BERK. and CURT.**

Terrestrial; frond cartilaginous, linear, very narrow, compressed and often channelled, much branched, irregularly dichotomous; branches solid, densely filled with moniliform curved threads. *Berk. and Curt. No. 3809.*

Hab.—On naked aluminous soil, at San Pedro, Texas, Mr. Charles Wright. (v.s.)

Fronde several inches in length, half a line in diameter, lying prostrate on the surface of the soil, much branched in an irregularly dichotomous manner; branches exactly linear, compressed, often channelled on one or both sides, thinned in the middle and incrassated to the edge. *Substance* firm and elastic, cartilaginous, solid, densely filled with moniliform, curved or curled, interlaced threads, which are set longitudinally in the frond, and lie nearly parallel to each other. *Color* dark olive.

A very curious and most distinctly marked species, differing from others of this genus, much in the same manner that *Chatophora endiviaefolia* does from the ordinary globose forms of *Chatophora*. (*Chlorospermeæ*, p. 115.)

***Nostoc microscopicum*, CARM.**

Fronde densely aggregated, very minute, globose or oblong, immersed in a blackish crust; filaments few. *Carm. in Hook. Brit. Fl. 2, p. 399. Harv. Man. Ed. 1, p. 184. N. muscorum, Hass. Br. Fr. Wat. Alg. p. 292, t. 74, fig. 4.*

Hab.—"Stones in a small stream, Baffin's Bay," Dr. Sutherland, *vide* Prof. Dickie.

I have not seen American specimens. In Britain this species grows among mosses on exposed calcareous rocks, but not in water. The above specific character is taken from the British plant. The fronds are rarely more than the tenth of an inch in diameter, and contain two or three beaded filaments lying in a copious transparent jelly. (*Chlorospermeæ*, p. 115.)

Genus HYDRURUS, AG.

Frond fixed at base, cylindrical or compressed, elongated, branched, gelatinous. *Structure*: seriated, but separate, cellules, filled with bright-green endochrome, inclosed in gelatinous parallel tubes, ranged longitudinally in the frond, and surrounded by a common gelatinous envelope.

Of this genus several species have been described by authors, all having a close resemblance to each other, and all very variable in ramification. Indeed it is almost impossible to fix characters by which they can be permanently kept apart; and instead of adding another specific name to the already too numerous list, I prefer to consider the American specimens received as constituting a luxuriant variety of the best known of the established species. All previously recorded species or varieties of these plants are natives of rapid rivers and streams in various parts of Europe. (*Chlorospermeæ*, p. 118.)

Hydrurus penicillatus, var. **occidentalis**, HARV.

Fronde very long (1–2 feet or more), much branched; branches very irregular, scattered or crowded, wormlike, tapering to a fine point, naked or clothed with feathery villous ramuli; cells ellipsoidal or pear-shaped, twice as long as their diameter.

Hab.—On the rocky bottom of rivers and streams, in a strong current. Santa Fe, New Mexico, Mr. Fendler, February to April, 1847. (v.s. in Herb. T.C.D.)

Fronde attached at base, one or two feet long, from one to four lines in diameter, very much and irregularly branched; branches scattered or crowded, simple or divided, a foot or more in length, attenuated to a fine point, sometimes smooth and naked, but generally densely clothed with slender, villous ramenta, spreading to all sides. The gelatinous tubes or sheaths in which the cells are seriated are very obvious, and lie close together in longitudinal, parallel strata. The cells are of large size, bright-green color, and variable shape; some are twice as long as others.

This I had at first supposed to be a new species, but now regard it as a very gigantic state of *H. penicillatus*, Ag., which under various forms and of various sizes is common in alpine streams in Europe. I fear characters derived from the shape and size of the cellules are not more to be depended upon than are those taken from the ramification. (*Chlorospermeæ*, p. 118.)

Draparnaldia opposita, Ag.

Fronde vaguely much branched; joints of the main filament as long as broad, or shorter; pencils of ramuli mostly opposite, densely set, lanceolate-acuminate in outline, plumose, bi-tripinnate, the apices much attenuated. *Ag. Syst.* p. 59. *Kütz. Sp. Alg.* 357. *Lyngb. Hyd. Dan. tab.* 65, fig. A. *Batrachospermum Americanum*, Schweinitz.

Hab.—In clear streams. New York, Professor Bailey. New Jersey, Mr. Jackson. (v.s.)

Fronde 2–3 inches long, gelatinous, capillary, irregularly much branched; the branches patent, lateral, more or less divided, and set with lesser ramuli. Main filaments with short articulations, as long as their breadth, or shorter, transversely banded. At every two or three nodes and sometimes at every node a pair of opposite penicillato-multifid ramuli are thrown off. These are bright green, ovato-lanceolate in outline, much acuminate and twice or thrice pinnate, their pinnules somewhat constricted at the nodes, and tapering at the apex into long, needle-like, hyaline points. Their cells are commonly nucleated and filled with endochrome.

Whether this be permanently distinguishable from *D. glomerata* is doubtful. It has externally the aspect of that species, but its microscopic characters are nearer those of *D. plumosa*.

GEOGRAPHICAL LIST OF SPECIES.

CLASS PHYCOCHROMOPHYCEÆ.

ORDER CYSTIPHORÆ.

Family CHROOCOCCACEÆ.

Chroococcus

- refractus, *Wood.* *Hab.* near Philadelphia.
 multicoloratus, *Wood.* *Hab.* near Philadelphia.
 thermophilus, *Wood.* *Hab.* Benton Springs, Owen
 Co., California.

Gloeocapsa

- sparsa, *Wood.*

Cælosphærium

- dubium, *Grün.* *Hab.* near Philadelphia.

Merismopedia

- nova, *Wood.* *Hab.* near Philadelphia.
 convoluta, *Bréb.* *Hab.* Spring Mills, Montgomery
 Co., Pa.

ORDER NEMATOGENEÆ.

Family OSCILLARIACEÆ.

Oscillaria

- chlorina, *Ktz.* *Hab.* near Philadelphia.
 corium, *Ag.* *Hab.* New York.
 decorticans, *Gener.* *Hab.* Northern U. States.
 Fröhlichii, *Ktz.* *Hab.* Schuylkill River, near
 Philadelphia.
 imperator, *Wood.* *Hab.* near Philadelphia.
 limosa, *Ag.* *Hab.* near Camden, New Jersey.
 muscorum, *Ag.* *Hab.* West Point, New York.
 neglecta, *Wood.* *Hab.* near Philadelphia.
 nigra, *Vauch.* *Hab.* New York; Philadelphia.
 tenuis, *Ag.* *Hab.* Rhode Island; New York;
 Virginia.
 tenuissima, *Ag.* *Hab.* Warm Springs of Washita.

Chthonoblastus

- repens, *Ktz.* *Hab.* New York; Massachusetts;
 Rhode Island.

Lyngbya

- bicolor, *Wood.* *Hab.* Schuylkill River, near Phila-
 delphia.
 muralis, *Ag.* *Hab.* Whale Fish Islands, Davis
 Straits, British America.

Family NOSTOCHACEÆ.

Sub-Family NOSTOCÆÆ.

Nostoc

- Austinii, *Wood.* *Hab.* New Jersey.
 alpinum, *Ktz.* *Hab.* Alleghany Mountains; Clover
 Mts., Nevada; Baffin's Bay, British America.
 calcicola, *Ag.* *Hab.* Catoosa Springs, Georgia.
 calidarium, *Wood.* *Hab.* Benton Springs, Owen
 Co., California.
 cæruleum, *Lyn.* *Hab.* New Jersey.
 Cesatii, *Bals.* *Hab.* Kansas.
 comminutum, *Ktz.* *Hab.* near Philadelphia.
 commune, *Vauch.* *Hab.* New Jersey; Rio Bravo.
 depressum, *Wood.* *Hab.* New Jersey.
 punctatum, *Wood.* *Hab.* New Jersey.
 pruniforme, *Agh.* *Hab.* New Jersey.
 verrucosum, *Vauch.* *Hab.* Maine.
 sphericum, *Vauch.* *Hab.* Centre Co., Pennsylvania.

Sub-Family SPERMOSIREÆ.

Anabæna

- gelatinosa, *Wood.* *Hab.* near Philadelphia.
 gigantea, *Wood.* *Hab.* near Philadelphia.
 flos-aquæ, *Ktz.* *Hab.* Round Pond, West Point,
 New York.

Cylindrospermum

- comatum, *Wood.* *Hab.* Niagara, Canada.
 flexuosum, *Rab.* *Hab.* near Philadelphia.
 macrospermum, *Ktz.* *Hab.* South Carolina.
 minutum, *Wood.* *Hab.* near Philadelphia.

Dolichospermum

- polyspermum, *Ktz.* *Hab.* near Philadelphia.
 subrigidum, *Wood.* *Hab.* New Jersey.

Family RIVULARIACEÆ.

Nostochopsis

- lobatus, *Wood.* *Hab.* Schuylkill River, near Phila-
 delphia.

Gloetotrichia

- angulosa, *Roth.* *Hab.* Hudson River, near West
 Point.

Gloiotrichia

incrustata, *Wood.* *Hab.* Schuylkill River near Philadelphia.

Rivularia

cartilaginea, *Wood.* *Hab.* Northern Michigan.

Zonotrichia

minutula, *Wood.* *Hab.* Clear Pond, Adirondack Mountains.

mollis, *Wood.* *Hab.* Cave of the Winds, Niagara.

pareezonata, *Wood.* *Hab.* Cave of the Winds, Niagara.

Dasyactis

mollis, *Wood.* *Hab.* Cass River, Northern Michigan.

Mastigonema

elongatum, *Wood.* *Hab.* Philadelphia.

fertile, *Wood.* *Hab.* Alleghany Mountains, Centre Co., Pennsylvania.

halos, *Wood.* *Hab.* Stonington, Connecticut.

sejunctum, *Wood.* *Hab.* Cass River, Northern Michigan.

Mastigothrix

fibrosa, *Wood.* *Hab.* near Philadelphia.

Family SCYTONEMACEÆ.

Scytonema

Austinii, *Wood.* *Hab.* Little Falls, New Jersey.

calotrichoides, *Ktz.* *Hab.* South Carolina.

cataracta, *Wood.* *Hab.* Niagara River, Niagara.

cortex, *Wood.* *Hab.* South Carolina.

dubium, *Wood.* *Hab.* Cumberland Co., New Jersey.

immersum, *Wood.* *Hab.* Cumberland Co., New Jersey.

Myochrous, *Ag.* *Hab.* West of Crow's Neck, West Point.

Nægeli, *Ktz.* *Hab.* near Bellefonte, Centre Co., Pennsylvania.

Ravenellii, *Wood.* *Hab.* South Carolina.

simplice, *Wood.* *Hab.* Aiken, South Carolina.

thermale. *Hab.* South Carolina.

Tolypothrix

distorta, *Mul.* *Hab.* near Philadelphia; West Point, N. Y.; Rhode Island; Madison, Wisconsin.

Family SIROSIPHONACEÆ.

Sirosiphon

acervatus, *Wood.* *Hab.* South Carolina.

argillaceus, *Wood.* *Hab.* South Carolina.

compactus, *Ag.* *Hab.* Salem, Massachusetts; New Jersey.

Cramerii, *Br.* *Hab.* Mount Tahawus, Adirondack Mountains.

Sirosiphon

guttula, *Wood.* *Hab.* South Carolina.

lignicola, *Wood.* *Hab.* South Carolina.

neglectus, *Wood.* *Hab.* New Jersey.

pellucidulus, *Wood.* *Hab.* near Hibernia, Florida.

pulvinatus. *Hab.* Northern New Jersey.

scytenematoides, *Wood.* *Hab.* South Carolina.

Stigonema

Ravenellii, *Berkeley.* *Hab.* Lookout Mountains, Georgia.

CLASS CHLOROPHYLLACEÆ.

ORDER COCCOPHYCEÆ

Family PALMELLACEÆ.

Pleurococcus

pulvereus, *Wood.* *Hab.* Boiling Springs, near Bellefonte, Centre Co., Pennsylvania.

seriatus, *Wood.* *Hab.* New Jersey.

Palmella

dura, *Wood.* *Hab.* near Philadelphia.

hyalina, *Lyn.* *Hab.* From Rhode Island to Wisconsin.

Jesenii, *Wood.* *Hab.* near Philadelphia.

Pagerogalla

stellio, *Wood.* *Hab.* Bear Meadows, Alleghany Mountains, Centre Co., Pennsylvania.

Tetraspora

bullosa, *Roth.* *Hab.* Salem, North Carolina.

gelatinosa, *Roth.* *Hab.* Salem, North Carolina; Newburgh, New York.

lubrica, *Roth.* *Hab.* Northern Atlantic States.

Dictyosphaerium

pulchellum, *Wood.* *Hab.* near Philadelphia.

Rhaphidium

falcatum. *Hab.* near Philadelphia.

polymorphum, *Fr.* *Hab.* near Philadelphia.

Family PROTOCOCCACEÆ.

Polyedrium

enorme, *Ralfs.* *Hab.* Florida.

Scenedesmus

acutus, *Meyen.* *Hab.* Rhode Island; near Philadelphia.

obtusus, *Meyen.* *Hab.* Georgia; Rhode Island.

polymorphus, *Wood.* *Hab.* near Philadelphia.

quadricauda, *Turp.* *Hab.* Rhode Island; Pennsylvania.

rotundatus, *Wood.* *Hab.* near Philadelphia.

Hydrodictyon

- utriculatum, *Roth.* *Hab.* West Point and Weehawken, New York; Mexican Boundary; Pennsylvania; New Jersey.

Pediastrum

- Boryanum, *Tur.* *Hab.* Rhode Island; Pennsylvania; Georgia; Florida.
 constrictum, *Hassall.* *Hab.* South Carolina; Georgia; Rhode Island.
 duodenarius. *Hab.* South Carolina; Rhode Island.
 Ehrenbergii, *Corda.* *Hab.* Rhode Island; South Carolina; Georgia; Florida.
 pertusum, *Ktz.* *Hab.* Rhode Island.
 Selenæa, *Ktz.* *Hab.* Rhode Island.

Family VOLVOCINEÆ.

Chlamydococcus

- nivalis. *Hab.* Greenland; Rocky Mountains.

Volvox

- globator, *Linn.* *Hab.* United States.

ORDER ZYGOPHYCEÆ.

Family DESMIDIACEÆ.

Palmogloea

- clepsydra, *Wood.* *Hab.* near Philadelphia.

Penium

- Brébissonii, *Men.* *Hab.* South Carolina.
 closterioides, *Ralfs.* *Hab.* South Carolina.
 Digitus, *Ehrb.* *Hab.* Pennsylvania; New York; Georgia.
 interruptum, *Bréb.* *Hab.* near Grahamsville, South Carolina.
 Jenneri, *Ralfs.* *Hab.* Florida.
 lamellosum, *Bréb.* *Hab.* Rhode Island.
 margaritaceum, *Ehrb.* *Hab.* Rhode Island.
 minutum, *Clerc.* *Hab.* Rhode Island; South Carolina; Georgia.

Closterium

- acerosum, *Schr.* *Hab.* South Carolina; Georgia; Florida.
 Amblyonema, *Ehrb.* *Hab.* West Point, New York; Providence, Rhode Island.
 angustatum, *Ktz.* *Hab.* Rhode Island; New Hampshire; Pennsylvania.
 areolatum, *Wood.* *Hab.* Northumberland Co., Pennsylvania.
 Cucumis, *Ehrb.* *Hab.* New York.
 Dianæ, *Ehrb.* *Hab.* Georgia; Florida; Pennsylvania; Rhode Island.
 Ehrenbergii, *Men.* *Hab.* Philadelphia.
 Jenneri, *Ralfs.* *Hab.* Rhode Island.
 juncidum, *Ralfs.* *Hab.* Saco Lake, New Hampshire; South Carolina.

Closterium

- Leibleinii, *Ktz.* *Hab.* Georgia; South Carolina; Pennsylvania.
 lineatum, *Ehrb.* *Hab.* Pennsylvania.
 Lunula, *Müller.* *Hab.* South Carolina; Florida; Georgia; Pennsylvania.
 maximum, *var.* *Hab.* Pennsylvania.
 moniliferum, *Bory.* *Hab.* Georgia; Rhode Island.
 parvulum, *Næg.* *Hab.* near Philadelphia.
 rostratum, *Ehrb.* *Hab.* near Philadelphia.
 setaceum, *Ehrb.* *Hab.* Stonington, Connecticut; Providence, Rhode Island; Pennsylvania; Georgia; Florida.
 striolatum, *Ehrb.* *Hab.* Centre Co., Pennsylvania.
 Venus, *Ktz.* *Hab.* South Carolina.

Tetmemorus

- Brébissonii, *Men.* *Hab.* Atlantic States.
 giganteus, *Wood.* *Hab.* Centre Co., Pennsylvania.
 granulatus, *Bréb.* *Hab.* Rhode Island; Pennsylvania; South Carolina.
 levis, *Ktz.* *Hab.* near Philadelphia.

Pleurotænium

- Baculum, *Bréb.* *Hab.* Georgia.
 breve, *Wood.* *Hab.* District of Columbia.
 clavatum, *Ktz.* *Hab.* South Carolina; Georgia.
 constrictum, *Bailey.* *Hab.* Rhode Island.
 crenulatum, *Ehrb.* *Hab.* Rhode Island; New Jersey; Pennsylvania; South Carolina; Georgia; Florida.
 gracile, *Rab.* *Hab.* Florida.
 hirsutum, *Bailey.* *Hab.* United States.
 nodosum, *Bailey.* *Hab.* South Carolina; Georgia; Florida; Pennsylvania.
 Trabecula, *Ehrb.* *Hab.* Pennsylvania; New Jersey; South Carolina; Georgia; Florida.
 undulatum, *Bailey.* *Hab.* Florida.
 verrucosum, *Bailey.* *Hab.* Rhode Island.

Triploceras

- gracile, *Bailey.* *Hab.* Rhode Island; New Jersey; New Hampshire; Florida; Georgia.
 verticillatum, *Bailey.* *Hab.* with the last.

Spirotænia

- bryophila, *Bréb.* *Hab.* near Philadelphia.
 condensata, *Bréb.* *Hab.* Pennsylvania; Rhode Island; Florida.

Barbusina

- Brébissonii, *Ktz.* *Hab.* Florida; Georgia; South Carolina; Rhode Island.

Didymoprium

- Grevillii, *Ktz.* *Hab.* Pennsylvania; South Carolina; Georgia.

Sphærozozma

- excavatum, *Ralfs.* *Hab.* Rhode Island; South Carolina; Georgia; Florida.
 pulchrum, *Bailey.* *Hab.* New York; New Jersey.
 serratum, *Bailey.* *Hab.* South Carolina; Georgia; Florida.

Hyalotheca

- disilliens, *Smith.* *Hab.* Rhode Island; Pennsylvania; South Carolina; Florida.
 mucosa, *Mert.* *Hab.* Rhode Island.

Desmidium

- aptogonium, *Bréb.* *Hab.* South Carolina; Georgia.
 quadrangulatum, *Ktz.* *Hab.* South Carolina.
 Swartzii, *Ag.* *Hab.* Atlantic States.

Aptogonium

- Baileyi, *Ralfs.* *Hab.* Rhode Island; New Jersey.

Cosmarium

- amœnum, *Bréb.* *Hab.* Florida; Rhode Island.
 bioculatum, *Bréb.* *Hab.* Rhode Island.
 Botrytis, *Bory.* *Hab.* Pennsylvania.
 Brébissonii, *Men.* *Hab.* White Mountains, New Hampshire.
 Broomei, *Thu.* *Hab.* Pennsylvania; Georgia.
 cælatum, *Ralfs.* *Hab.* near Albany, New York; South Carolina.
 commissurale, *Bréb.* *Hab.* White Mountains, New Hampshire.
 connatum, *Bréb.* *Hab.* Florida.
 crenatum, *Ralfs.* *Hab.* Rhode Island.
 cucumis, *Corda.* *Hab.* New Hampshire; Pennsylvania; South Carolina; Georgia; Florida.
 depressum, *Bailey.* *Hab.* Florida.
 margaritifera, *Turp.* *Hab.* Pennsylvania; South Carolina; Florida; Mexico.
 Meneghenii, *Bréb.* *Hab.* Pennsylvania.
 ornatum, *Ralfs.* *Hab.* Rhode Island.
 ovale, *Ralfs.* *Hab.* Pennsylvania.
 pyramidatum, *Bréb.* *Hab.* Pennsylvania; Georgia; Florida.
 Quimbyii, *Wood.* *Hab.* near Philadelphia.
 sublobatum, *Bréb.* *Hab.* Rhode Island; Georgia; Florida.
 suborbiculare, *Wood.* *Hab.* Lake Saco, New Hampshire.
 tetropthalmum, *Ktz.* *Hab.* New Jersey.
 Thwatesii, *Ralfs.* *Hab.* Florida.
 undulatum, *Corda.* *Hab.* Rhode Island; South Carolina.

Euastrum

- affine, *Ralfs.* *Hab.* South Carolina; Georgia.
 ampullaceum, *Ralfs.* *Hab.* South Carolina; Florida.

Euastrum.

- binale, *Turp.* *Hab.* Florida; Pennsylvania; Rhode Island.
 circulare, *Hassal.* *Hab.* Rhode Island.
 crassum, *Bréb.* *Hab.* United States.
 Didelta, *Turp.* *Hab.* South Carolina; Georgia; Pennsylvania; Rhode Island.
 elegans, *Bréb.* *Hab.* United States.
 gemmatum, *Bréb.* *Hab.* Rhode Island.
 insigne, *Ralfs.* *Hab.* Florida; Rhode Island.
 multilobatum, *Wood.* *Hab.* Saco Lake, New Hampshire.
 oblongum, *Greville.* *Hab.* Rhode Island.
 ornatum, *Wood.* *Hab.* Saco Lake, New Hampshire.
 Ralfsii, *Rabenh.* *Hab.* South Carolina; New Hampshire; Rhode Island.
 verrucosum, *Ehr.* *Hab.* Rhode Island; South Carolina; Georgia; Florida.

Micrasterias

- Americana, *Ehrb.* *Hab.* Florida; South Carolina.
 arcuata, *Bailey.* *Hab.* Florida.
 Baileyi, *Ralfs.* *Hab.* New York; Rhode Island; South Carolina; Florida.
 denticulata, *Bréb.* *Hab.* Pennsylvania; Florida.
 disputata, *Wood.* *Hab.* Atlantic States.
 expansa, *Bailey.* *Hab.* Florida.
 fimbriata, *Ralfs.* *Hab.* South Carolina; Florida.
 foliacea, *Bailey.* *Hab.* Worden's Pond, Rhode Island.
 furcata, *Ag.* *Hab.* Atlantic States.
 granulata, *Wood.* *Hab.* South Carolina.
 Jenneri, *Ralfs.* *Hab.* near Philadelphia.
 oscitans, *Ralfs.* *Hab.* Florida; Rhode Island.
 papillifera, *Bréb.* *Hab.* Florida; Rhode Island.
 pinnatifida, *Ktz.* *Hab.*
 quadrata, *Bailey.* *Hab.* Florida.
 radiosa, *Ag.* *Hab.* Florida.
 ringens, *Bailey.* *Hab.* Florida.
 Torreyi, *Bailey.* *Hab.* near Princeton, New Jersey.
 truncata, *Corda.* *Hab.* Atlantic States.

Staurostrum

- alternans, *Bréb.* *Hab.* Georgia; Florida; Rhode Island.
 arachne, *Ralfs.* *Hab.* Saco Lake, New Hampshire.
 aristiferum, *Ralfs.* *Hab.* Georgia; Rhode Island.
 Cerberus, *Bailey.* *Hab.* Florida.
 crenatum, *Bailey.* *Hab.* Rhode Island.
 cyrtoceram, *var., Bréb.* *Hab.* Florida.
 dejectum, *Bréb.* *Hab.* New York; South Carolina.
 dilatatum, *Ehrb.* *Hab.* Southern Atlantic States.
 eustephanum, *Ralfs.* *Hab.* West Point, New York.
 fureigerum, *Bréb.* *Hab.* South Carolina; Florida; Rhode Island.

Staurastrum

- gracile*, *Ralfs.* *Hab.* South Carolina; Georgia; Florida; New York; Rhode Island.
- hirsutum*, *Ehrb.* *Hab.* Florida; Rhode Island.
- Hystrix*, *Ralfs.* *Hab.* Rhode Island.
- Lewisii*, *Wood.* *Hab.* Saco Lake, New Hampshire.
- longispinum*, *Arch.* *Hab.* Florida.
- margaritaceum*, *Ehrb.* *Hab.* Saco Lake, New Hampshire.
- munitum*, *Wood.* *Hab.* Saco Lake, New Hampshire.
- muticum*, *Bréb.* *Hab.* South Carolina; Rhode Island.
- orbiculare*, *Ehrb.* *Hab.* Rhode Island; Pennsylvania.
- paradoxum*, *Mey.* *Hab.* Saco Lake, New Hampshire.
- polymorphum.* *Hab.* Florida.
- polytrichum*, *Per.* *Hab.* near Philadelphia.
- punctulatum*, *Bréb.* *Hab.* Pennsylvania.
- Ravenellii*, *Wood.* *Hab.* South Carolina.
- senarium*, *Ehrb.* *Hab.* America.
- tricornis*, *Men.* *Hab.* Georgia; Florida; Rhode Island.

Xanthidium

- aculeatum*, *Ehrb.* *Hab.* near Savannah, Georgia.
- Aretiscum*, *Ehrb.* *Hab.* North America.
- armatum*, *Bréb.* *Hab.* South Carolina; Florida; New Hampshire.
- bisenarium*, *Ehrb.* *Hab.* America.
- cristatum*, *Bréb.* *Hab.* Southern Atlantic States.
- coronatum*, *Ehrb.* *Hab.* America.
- fasciculatum*, *Ehrb.* *Hab.* South Carolina; Georgia; Florida; Rhode Island.

Arthrodesmus

- convergens*, *Ehrb.* *Hab.* South Carolina; Georgia; Florida; Rhode Island.
- Incus*, *Bréb.* *Hab.* Georgia; Florida; South Carolina; Rhode Island.
- octocornis*, *Ehrb.* *Hab.* Florida; Rhode Island.
- quadridens*, *Wood.* *Hab.* Lake Saco, New Hampshire.

Family ZYGEMACEÆ.

Spirogyra

- crassa*, *Ktz.* *Hab.* near Philadelphia.
- decimina*, *Mül.* *Hab.* near Philadelphia.
- diluta*, *Wood.* *Hab.* near Philadelphia.
- dubia*, *Ktz.* *Hab.* near Philadelphia.
- elongata*, *Berk.* *Hab.* near Philadelphia.
- insignis*, *Hus.* *Hab.* near Philadelphia.
- longata*, *Vauch.* *Hab.* Rhode Island; near Philadelphia.

Spirogyra

- majuscula*, *Ktz.* *Hab.* near Philadelphia.
- nitida*, *Hüll.* *Hab.* near Philadelphia.
- protecta*, *Wood.* *Hab.* near Philadelphia.
- parvispora*, *Wood.* *Hab.* Hibernia, Florida.
- pulehella*, *Wood.* *Hab.* near Philadelphia.
- quinina*, *Ag.* *Hab.* near Philadelphia.
- rivularis*, *Hassall.* *Hab.* Florida.
- setiformis*, *Roth.* *Hab.* near Philadelphia.
- Weberi*, *Ktz.* *Hab.* near Philadelphia.

Zygnema

- insigne*, *Hassall.* *Hab.* Rhode Island; near Philadelphia.
- cruciatum*, *Vauch.* *Hab.* Virginia; Florida; Northern States.

Sirogonium

- retroversum*, *Wood.* *Hab.* near Philadelphia.

Mesocarpus

- scalaris*, *Hassall.* *Hab.* near Philadelphia.
- parvulus*, *Hassall.* *Hab.* Rhode Island.

Pleurocarpus

- mirabilis*, *Braun.* *Hab.* New York; Rhode Island; Michigan; Wisconsin.

ORDER SIPHOPHYCEÆ.

Family HYDROGASTRÆÆ.

Hydrogastrum

- granulatum*, *Linn.* *Hab.* Delaware.

Family VAUCHERIACEÆ.

Vaucheria

- aversa*, *Hassall.* *Hab.* near Philadelphia.
- geminata*, *Vauch.* *Hab.* near Philadelphia.
- polymorpha*, *Wood.* *Hab.* Texas.
- sessilis*, *Vauch.* *Hab.* New York; Maine; Virginia; North Carolina.
- velutina*, *Ag.* *Hab.* New York; Maine; Virginia; North Carolina.

ORDER NEMATOPHYCEÆ.

Family ULVACEÆ.

Protoderma

- viride*, *Ktz.* *Hab.* Philadelphia.

Ulva

- merismopedioides*, *Wood.* *Hab.* Diamond Range, Rocky Mountains.

Enteromorpha

- intestinalis*, *Linn.* *Hab.* Hudson River; Narragansett Bay.

Schizomeris

- Leibleinii*, *Ktz.* *Hab.* near Philadelphia.

Family CONFERVACEÆ.

Conferva.*Hab.* United States.**Cladophora**brachystelecha, *Hab.* near Philadelphia.fracta, *Dill.* *Hab.* Pennsylvania; New York; Rhode Island.glomerata, *Linn.* *Hab.* Lakes Ontario, Erie, Huron, and Michigan.

Family CÉDOGONIACEÆ.

Androgyniaechinata, *Wood.* *Hab.* Florida.Huntii, *Wood.* *Hab.* near Philadelphia.mirabilis, *Wood.* *Hab.* near Philadelphia.multispora, *Wood.* *Hab.* near Philadelphia.**Pringsheimia**inequalis, *Wood.* *Hab.* near Philadelphia.**Bulbochæte**Canbyii, *Wood.* *Hab.* Hibernia, Florida.dumosa, *Wood.* *Hab.* near Philadelphia.ignota, *Wood.* *Hab.* near Philadelphia.

Family CHROOLEPIDÆ.

Chroolepusaureum, *Ktz.* *Hab.* New York; New Jersey; Texas.**Bulbotrichia**albida, *Wood.* *Hab.* Northern New Jersey.

Family CHÆTOPHORACEÆ.

Stigeoclonium.*Hab.* Eastern United States.**Draparnaldia**Billingsii, *Wood.* *Hab.* near Philadelphia.glomerata, *Vauch.* *Hab.* Rhode Island.maxima, *var., Wood.* *Hab.* near Philadelphia.**Draparnaldia**plumosa, *Vauch.* *Hab.* near Philadelphia.**Chætophora**elegans, *Roth.* *Hab.* Eastern United States.endiviæfolia, *Roth.* *Hab.* Rhode Island; South Carolina.**Pilinia**diluta, *Wood.* *Hab.* Centre County, Pennsylvania.**Coleochæte.***Hab.* Eastern United States.**Aphanochæte**repens, *Eraun.* *Hab.* near Philadelphia.

CLASS RHODOPHYCEÆ.

Family PORPHYRACEÆ.

Porphyridiumcruentum, *Ag.* *Hab.* New York.magnificum, *Wood.* *Hab.* Texas.

Family CHANTRANSIACEÆ.

Chantransiaexpansa, *Wood.* *Hab.* near Philadelphia.macrospora, *Wood.* *Hab.* South Carolina.

Family BATRACHOSPERMACEÆ.

Batrachospermummoniliforme, *Roth.* *Hab.* Eastern United States.vagum, *Roth.* *Hab.* Uintah Mountains, Nevada.**Tuomeya**fluviatilis, *Harr.* *Hab.* Alabama; Virginia.

Family LEMANEACEÆ.

Lemaneacatenata, *Ktz.* *Hab.* Diamond Range, Rocky Mountains.fluviatilis, *Ag.* *Hab.* Alabama.torulosa, *Roth.* *Hab.* Virginia; Kentucky; Pennsylvania; New Jersey.

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CORRECTIONS OF THE PLATES.

Plate IV., for *Zonotricha* read *Zonotrichia*.

Plate VIII., for *um* in the terminations of the specific name of figures 2, 3, and 4, read *us*.

Plate IX., fig. 3, for *argillacea* read *argillaceus*.

Plate X., fig. 4, for *Botryococcus pulchellus* read *Dictyosphaerium pulchellum*.

Plate XI., fig. 5, 6, and 7, for *Cosmarium* read *Closterium*.

Plate XII., fig. 1 and 20, for *Cosmarium* read *Closterium*.

Plate XV., fig. 8, for *insignis* read *insigne*.

Plate XVI., fig. 4, for *Bulbotricha* read *Bulbotrichia*.

Plate XVIII., fig. 1, for *Pringsheimii* read *Pringsheimia*.

Plate XXI., fig. 7, for *tetraophthalmum* read *tetrophthalmum*.

EXPLANATION OF PLATES.

PLATE I.

- Fig. 1. A single filament of *Oscillatoria chlorina*, Kützing, magnified 750 diameters.
- Fig. 2. The end of a filament of an *Oscillatoria* supposed to be identical with *O. Fröhlichii* Kützing.
- Fig. 3. *O. nigra*, Vaucher.
- Fig. 3 a. Represents a portion of a mat or mass of *Oscillatoria nigra*, Vaucher; there is too much green in the color.
- Fig. 3 b. Represents several filaments separated from the edge of the mass and slightly magnified.
- Fig. 3 c. A portion of a filament.
- Fig. 3 d. A portion of another filament still more highly magnified. The color in 3 c is more natural than that of 3 d.
- Fig. 4. A portion of a filament of *O. limosa*, Agardb, magnified 1250 diameters. The articles in this filament are more distinctly separated than natural.
- Fig. 5. *O. neglecta*, Wood.
- Fig. 5 a. An outline view of a filament, magnified 450 diameters.
- Fig. 5 b. A full figure of the same, magnified 500 diameters.
- Fig. 6. *O. imperator*, Wood.
- Fig. 6 a. Represents the end of a filament, magnified 250 diameters. In the centre of the plate is a fragment (marked simply fig. b.), out of which the endochrome has been partially squeezed to show the markings of the sheath at the joints.
- Fig. 7. *Lyngbya bicolor*, Wood. Fig. 7 represents a moderately magnified portion of a filament. Fig. a (near to fig. 8) represents a portion of an ordinary filament very slightly magnified.
- Fig. 7 c. A portion of a filament containing a heterocyst, magnified 800 diameters.
- Fig. 7 d. A broken end of a filament showing the sheath extending beyond the endochrome, magnified 800 diameters.
- Fig. 8. A variety(?) of *Lyngbya bicolor*, Wood, from the Schuylkill River, magnified 200 diameters.
- Fig. 9. *Cosmarium Quimbyii*, Wood. The bands between the cells are too heavy and prominent.

PLATE II.

- Figs. 1 b and 1 c. Different stages of germination of the spore of a *Cylindrospermum* of unknown species, magnified respectively 800 and 1200 diameters.
- Fig. 1 a. A chain of spores, believed to belong to the same species; one of these spores has commenced to germinate.
- Fig. 2. A portion of the upper surface of a frond of *Nostoc calidarium*, Wood.

- Fig. 2 *b*. A "first form" filament of the same species.
 Fig. 2 *a*. A filament from an old frond of the same plant.
 Fig. 2 *c*. Fragments of tissue from the upper surface of a mature, actively growing plant of the same species.
- Fig. 3. A filament of *Nostoc comminutum*, Ktz., magnified 800 diameters.
 Fig. 4. A filament of *Anabæna gelatinosa*, Wood, magnified 750 diameters, showing the large globular body at the end, supposed to be a spore.
 Fig. 5. A filament of *Anabæna gigantea*, Wood, magnified 750 diameters.
 Fig. 6. A portion of a filament of *Cylindrospermum minutum*, Wood, magnified 800 diameters. The number has been omitted from this figure on the plate; the figure is immediately under *A. gigantea*, Wood; the hairs on the heterocyst are too coarse and rigid.
 Fig. 7. A spore and outline of heterocyst of *Cylindrospermum macrospermum*, Ktz., magnified 750 diameters.
 Fig. 8. The end of a filament of *Cylindrospermum comatum*, Wood, magnified 1375 diameters. The appendages to the heterocyst are coarser than natural.
 Fig. 9 *a*. A section of an immature frond of *Rivularia cartilaginea*, Wood.
 Fig. 9 *b*. The base of a fertile filament, showing the spore and basal cells, magnified 800 diameters.
 Fig. 10. An ordinary filament of *Nostoc sphaericum*, Poiret.
 Figs. 10 *a* and 10 *c*. Filaments enlarging preparatory to longitudinal division.
 Fig. 10 *b*. A filament already partially divided into two.

PLATE III.

- Fig. 1 *a*. *Cylindrospermum flevuosum* (Ag.), a fertile filament, magnified 450 diameters.
 Fig. 1 *b*. The end of a fertile filament, magnified 750 diameters.
 Fig. 2. *Dolichospermum (Sphaerozyga) subrigidum*, Wood, magnified 975 diameters.
 Fig. 3. Portion of a fertile filament of *Dolichospermum (Sphaerozyga) polysperma*, Ktz., magnified 750 diameters.
 Fig. 4 *a*. A section of a frond of *Gloiothrichia incrustata*, Wood, showing youngish filaments, masses of lime, and an organic body of unknown nature, all inclosed in a transparent jelly.
 Fig. 4 *c*. Single filaments with immature spores, magnified 260 diameters.
 Fig. 4 *b*. The base of a filament, showing the nearly matured spore, and empty cells situated beyond it.
 Fig. 5. *Chlorococcus* of undetermined species.
 Fig. 5 *a*. The motile state.
 Fig. 5 *b*. The condition of the plant after having lost its cilia and commenced its quiescent life.
 Figs. 5 *a* and 5 *c*. Different stages in this life after division.
 Fig. 5 *d*. The *Hæmatococcus* or resting condition, the form assumed by the plant during slow desiccation.
 Fig. 6. *Nostochopsis lobatus*, Wood.
 Fig. 6 *a*. Part of a section, from within outwards, of the frond, showing the tortuous branched filaments, without sheaths in the gelatinous matrix.
 Fig. 6 *b*. A portion of a fertile filament with the lateral spores.
 Fig. 6 *c*. A sterile filament.
 Fig. 7. *Protococcus* of undetermined species.
 Fig. 7 *a*. A cell supposed to belong to the resting or winter condition of the plant.

Fig. 7 *b*. The first breaking-up of the contents of the large cell into a brood of cells.
 Figs. 7 *d* and 7 *c*. Different stages in the life of the latter brood-cells and their progeny.

PLATE IV.

Fig. 1. *Mastigonema fertile*, Wood, a single plant (*not* magnified 750 diameters, as marked on the plate).

Fig. 1 *b*. A heterocyst magnified 750 diameters, also a spore cell and spore similarly amplified.

Fig. 2 *a*. A portion of the frond of *Mastigonema sejunctum*, Wood, amplified 250 diameters.

Fig. 2 *b*. A single filament magnified 800 diameters.

Fig. 3. A single filament of *Zonotrichia mollis*, Wood, enlarged 260 diameters.

Fig. 4. A section of the frond of *Zonotrichia parvezonata*, Wood, magnified a few diameters.

Fig. 5. The base of a filament of *Dasyactis mollis*, Wood.

Fig. 5 *a*. Section of the frond, magnified 450 diameters.

Figs. 5 *b* and 5 *c*. Young filaments; each magnified 450 diameters.

Fig. 6. Fronds of *Calosphærium dubium*, Wood.

PLATE V.

Fig. 1 *a*. A portion of the frond of *Mastigonema elongatum*, Wood, slightly magnified to show the filaments radiating from the fragment of matter to which they are attached.

Fig. 1 *b*. A single filament magnified 460 diameters.

Fig. 2 *c*. A cluster of youngish filaments of *Mastigonema halos*, Wood.

Fig. 2 *b*. A portion of an older filament to show the spore-like divisions of the endochrome, magnified 460 diameters.

Fig. 3. A pair of young connate filaments of *Mastigothrix fibrosa*, Wood, magnified 450 diameters.

Fig. 3 *a*. An old filament magnified 800 diameters.

Fig. 3 *b*. A young filament with heterocyst, enlarged 450 diameters.

Fig. 3 *d*. A filament with two basal cells magnified 450 diameters.

Fig. 4 *a*. A portion of filament of *Scytonema Ravenellii*, Wood, magnified 160 diameters.

Fig. 4 *b*. The end of a branch magnified 450 diameters.

Figs. 5 *a*, *b*, *c*, & *c*. Different forms of *Chroococcus refractus*, Wood. Fig. 5 *h* is not a good one. I was not able to express well the peculiar translucent shining tint, and the artist who copied my drawing failed even more decidedly in simulating it; the pink shade is altogether wrong, I never saw any such color in the plant.

Fig. 6. Different forms of *Chroococcus multicoloratus*, Wood.

Fig. 6 *c*. Represents what was thought to be possibly a hibernating form of the species.

PLATE VI.

Fig. 1 *a*. A portion of a frond of *Scytonema thermale*, Ktz., magnified 260 diameters.

Fig. 1 *b*. Outline sketch, showing the form of the heterocyst, magnified 750 diameters.

Fig. 2. Trichoma or frond of *Scytonema callitrichoides*, Ktz., amplified 250 diameters.

Fig. 3 *a*. Outline sketch showing the cells or chambers of *Scytonema dubium*, Wood, magnified 750 diameters.

Figs. 3 *b* and 3 *c*. Portions of the filaments or trichoma, magnified 460 diameters.

Figs. 4 and 4 b. *Scytonema cortex*, Wood. Portions of filaments, magnified 750 diameters.

Fig. 5. Piece of a small twig with a well-formed and also a very young frond of *Chætophora elegans*, Ag., growing upon it, magnified a few diameters.

PLATE VII

Fig. 1. A perfect trichoma of *Scytonema cataractum*, Wood.

Fig. 1 b. A terminal part of a filament, magnified 250 diameters.

Fig. 2 a. A portion of a filament of *Scytonema immersum*, Wood, magnified 750 diameters.

Fig. 2 b. Nearly a whole filament or trichoma, amplified 260 diameters.

Fig. 3. *Rhaphidium polymorphum*, Wood, different forms, magnified 750 diameters.

Fig. 4. *Protococcus* of undetermined species.

Fig. 4 a. The largest and most mature form, probably the hibernating or winter cell.

Fig. 4 b. The same, commencing its active life.

Fig. 4 d. Colony cells believed to have been developed out of the cell represented by fig. 4 b, magnified 750 diameters.

Figs. 4 and 4 c. The motile state of the species.

PLATE VIII.

Fig. 1 b. Portion of the frond of *Tolypothrix distorta* (Müller), magnified 500 diameters.

Fig. 1 a. Heterocysts magnified 800 diameters.

Fig. 2. Portion of a frond of *Sirosiphon pellucidulus*, Wood, magnified 260 diameters.

Fig. 2 a. End of the branch.

Fig. 3. Portion of a frond of *Sirosiphon compactus* (Ag.), magnified 260 diameters.

Fig. 3 a. End of a filament, magnified 160 diameters.

Fig. 3 c. Portion of a filament showing the heterocyst magnified 460 diameters.

Fig. 4. Portion of a frond of *Sirosiphon neglectus*, Wood.

Fig. 5. Frond of *Sirosiphon guttula*, Wood.

Fig. 5 b. End of a branch, magnified 460 diameters.

Fig. 6. Portion of a frond of *Scytonema Nägelii*, Ktz.

Fig. 7. Different forms of *Glaucapsa sparsa*, Wood, magnified 700 diameters.

Fig. 8. *Merismopedia nova*, Wood, magnified 400 diameters.

PLATE IX.

Fig. 1. Fragment of a frond of *Sirosiphon scytenematoïdes*, Wood.

Figs. 2 and 2 c. Portions of fronds of *Sirosiphon lignicola*, Wood, magnified 260 diameters.

Fig. 2. The end of a branch of the same, magnified 460 diameters.

Fig. 3 a. Portion of a very old frond of *Sirosiphon argillaceus*, Wood, magnified 460 diameters.

Fig. 3 b. A terminal branch of a growing frond of the same.

Fig. 4 a. A frond of *Stigonema Ravenelii*, Berkeley, magnified 125 diameters; also a fragment of the same plant, magnified 450 diameters.

PLATE X.

Fig. 1 *a*. A frond of *Sirosiphon pulvinatus*, Bréb., var. *parvus*, from a specimen collected by Dr. J. G. Hunt, near Philadelphia. The ground color of this figure is too yellow.

Fig. 1 *b*. A fragment of the same, magnified 460 diameters.

Fig. 2. A row of cells of *Pleurococcus scriatus*, Wood, magnified 460 diameters.

Fig. 3 *a*. A portion of the old external part of a mass of *Palmella Jessenii*, Wood, magnified 750 diameters.

Fig. 3 *b*. A fragment from the interior of such a mass of the same amplification.

Fig. 3 *c*. A portion of the soft jelly of a young actively growing mass, magnified 750 diameters.

Fig. 4. A frond of *Dictyosphaerium pulchellum*, Wood, magnified 460 diameters. I at first referred this plant to the genus *BOTRYOCOCCUS*, and distributed some specimens under that generic title, and so marked my original drawing.

Fig. 5. A slice of a youngish frond of *Palmella dura*, Wood, magnified 460 diameters.

Fig. 5 *b*. A fragment from an old frond, showing the spores in various stages of growth. The color of the large spores is not nearly dark enough, it should be much more brownish.

PLATE XI.

Fig. 1. Different forms of *Scenedesmus polymorphus*, Wood, magnified 450 diameters.

Fig. 2. *Scenedesmus quadricauda*, Bréb., magnified 750 diameters.

Fig. 3. *Scenedesmus rotundatus*, Wood, magnified 750 diameters.

Fig. 4. Ordinary vegetative cells of *Palmogloia clepsydra*, Wood, in different stages or conditions of life-history, magnified 750 diameters. Those cells which have the endochrome much broken up are believed to be preparing for conjugation.

Fig. 4 *a*. A pair of cells uniting in conjugation.

Fig. 4 *b*. Cells which have united so that the young spore is very apparent with the empty semi-cells of the parents attached to it.

Fig. 4 *c*. A more advanced spore and empty semi-cells.

Figs. 4 *d* and 4 *e*. Matured or nearly matured spores, as seen with different focussing; in the first the upper surface of the spore is especially brought out. All these figures, except 4 *b*, are magnified 750 diameters.

Figs. 5 and 5 *a*. Different forms of *Closterium acerosum* (Schr.), magnified 250 diameters.

Fig. 5 *b*. Empty conjugating cells with nearly matured spore.

Fig. 6. Outline of *Closterium areolatum*, Wood, magnified 160 diameters.

Fig. 6 *a*. End of a dead, empty frond, enlarged 1375 diameters.

Fig. 7. Outline of *Closterium Venus*, Ktz., magnified 450 diameters.

(These last three species are incorrectly labelled on the plate, *Cosmarium*.)

PLATE XII.

Fig. 1. *Closterium lineatum*, Ehrb. (Incorrectly labelled on the plate *Cosmarium*.) Magnified 160 diameters.

Fig. 2. *Closterium Ehrenbergii*, Menegh., magnified 160 diameters.

Fig. 3. *Closterium rostratum*, Ehrb., magnified 260 diameters.

Fig. 4. *Closterium Diana*, Ehrb., magnified 260 diameters.

Fig. 5. *Closterium parvulum*, Nag., magnified 450 diameters.

- Fig. 6. *Closterium Leibleinii*, Ktz., magnified 260 diameters.
- Fig. 7. *Tetmemorus giganteus*, Wood., magnified 260 diameters.
- Fig. 8. *Tetmemorus granulatus* (Bréb.), magnified 450 diameters.
- Fig. 9. *Pleurotanium Trabecula* (Ehrb.), magnified 160 diameters.
- Fig. 10. *Spirotania bryophila* (Bréb.).
- Fig. 11. *Spirotania condensata*, Bréb.
- Fig. 12. *Hyalotheca dissiliens*, Bréb.
- Fig. 13. *Didymoprium Grevillii*, Ktz.
- Fig. 13 a. End view.
- Fig. 14. *Cosmarium Botrytis* (Bory.), magnified 460 diameters.
- Fig. 15. *Cosmarium Cucumis*, Corda.
- Fig. 15 d. A frond in which the neck or isthmus has begun to elongate previous to division.
- Fig. 15 b. An abnormal frond which has attempted division, but in which the inner semicells of the new frond have failed to form perfectly and to separate.
- Fig. 16. *Euastrum multilobatum*, Wood; front view.
- Fig. 17. *Micrasterias Americana* (Ehrb.).
- Fig. 18. *Cosmarium Meneghenii*, Bréb., magnified 750 diameters. The sinus should be very narrow but distinct, instead of being absent as in the figure.
- Fig. 19. *Spirogyra Weberi*, Ktz., portions of conjugating filaments, magnified 260 diameters.
- Fig. 19 a. A portion of a sterile filament, magnified 160 diameters.
- Fig. 19 b. Conjugating cells with nearly mature spores, magnified 260 diameters.
- Fig. 20. *Closterium juncidum*, Ralfs., magnified 260 diameters.
- Fig. 21. *Cosmarium margaritifera* (Turp.), magnified 460 diameters.

PLATE XIII.

- Fig. 1. Front view of *Euastrum Ralfsii*, Rabenh., magnified 450 diameters.
- Fig. 2. Front of *Euastrum elegans*, Bréb., enlarged 750 diameters.
- Fig. 3. Front view of *Euastrum binale* (Turp.), magnified 750 diameters.
- Fig. 4. Front view of *Micrasteria disputata*, Wood, drawn from a Philadelphia specimen.
- Fig. 4 a. The same after a figure drawn by Dr. Jos. Leidy, from a Newport specimen.
- Fig. 5. *Micrasterias furcata*, Agardh., front view, magnified 260 diameters.
- Fig. 6. Front view of *Micrasterias denticulata*, Bréb., magnified 260 diameters.
- Fig. 7. *Micrasteria Jenneri*, Ralfs. Front view.
- Fig. 8. *Staurastrum orbiculare* (Ehrb.). Front view.
- Fig. 9. *Staurastrum dejectum*, Bréb. Front view, magnified 750 diameters.
- Fig. 10. *Staurastrum punctulatum*, Bréb. Front view.
- Fig. 10 a. View from the apex.
- Fig. 11. Front view of *Staurastrum Lewisii*, Wood, magnified 750 diameters.
- Fig. 12. Front view of *Staurastrum polytrichum*, Perty.
- Fig. 13 a. Front view of *Staurastrum munitum*, Wood.
- Fig. 13 b. End view of the same.
- Fig. 14. *Cosmarium pyramidatum*, Brébisson. Front view.

Fig. 15. *Cosmarium Broomei*, Thw. Front view, magnified 460 diameters.

Fig. 16. *Cosmarium commissurale*, Bréb. Front view, magnified 750 diameters.

Fig. 17. *Xanthidium armatum*, Bréb. Front view, magnified 260 diameters.

PLATE XIV.

Fig. 1. A sterile cell of *Rhynchosnema elongatum*, Wood, magnified 450 diameters.

Fig. 1 a. Portion of a filament containing a fertile cell, with the spore nearly matured, amplified 450 diameters.

Fig. 2. A filament of *Rhynchosnema pulchellum*, Wood, containing both fertile and sterile cells, magnified 260 diameters.

Fig. 3. The ripened spore of *Spirogyra protecta*, Wood, magnified 450 diameters.

Fig. 3 a. Outline of conjugating filaments, and figure of a sterile filament, enlarged 250 diameters.

Fig. 4. Sterile cells of *Spirogyra longata* (Vauch.), magnified 250 diameters.

Fig. 4 a. Fertile filaments, magnified 260 diameters.

Fig. 5. A filament of *Aphanochate repens*, Wood, which has lost its cilia, magnified 460 diameters.

Fig. 6. A fertile branch of *Draparnaldia Billingsii*, Wood, showing the chains of spores, magnified 460 diameters.

PLATE XV.

Fig. 1. Portion of a filament of *Spirogyra majuscula*, Ktz., containing cells with mature spores and others just commencing the process of conjugation.

Fig. 2. A portion of a sterile filament of *Spirogyra diluta*, Wood, magnified 125 diameters, also the outline of a pair of conjugating filaments of the same amplification.

Fig. 2 b. Conjugating filaments of *Spirogyra diluta*, Wood, magnified 125 diameters.

Fig. 3 a. Portion of a sterile filament of *Spirogyra setiformis* (Roth) Ktz., magnified 125 diameters.

Fig. 3 b. Conjugating filaments of the same species, similarly amplified.

Fig. 4 a. Cells of *Spirogyra crassa*, Ktz., preparing for conjugation.

Fig. 4 c. Conjugating cells of the same plant in the first stage of union.

Fig. 4 b. Conjugating cells containing nearly matured zygospores, enlarged 125 diameters.

Fig. 5. Filaments of *Mesocarpus scalaris*, Hassall, commencing the process of conjugation, magnified 125 diameters.

Fig. 6. Sterile cells of *Spirogyra insignis* (Hassall) Ktz.

Fig. 6 b. Conjugating filaments of the same species.

Fig. 7. Conjugating filaments of *Spirogyra parvispora*, Wood, containing nearly matured spores, magnified 125 diameters.

Fig. 8. Portion of an ordinary sterile filament of *Zygnema insigne* (Hassall) Ktz.

Fig. 8 a. Fertile filaments of the same, magnified 250 diameters.

Fig. 8 b. Sterile filament in which multiplication of the species is taking place by the separation of the cells, magnified 260 diameters.

PLATE XVI.

Fig. 1 a. Cells of *Sirogonium retroversum*, Wood, just commencing the process of conjugation.

Fig. 1 b. Sterile cells.

Figs. 1 d and 1 e. Outlines of fertile cells; all of these figures are magnified 260 diameters.

- Fig. 2. A matured frond of *Hydrogastrum granulatum* (Linn.), enlarged 90 diameters.
 Fig. 2 a. A resting spore of the same, enlarged 160 diameters; also a minute, very young frond, magnified 90 diameters.
 Fig. 4. A branch of a *Stigeoclonium*, emitting zoospores, enlarged 460 diameters.
 Fig. 5. *Bulbotrichia albida*, Wood, magnified 460 diameters.
 Fig. 6 a. A fertile branch of *Bulbochate Canbyii*, Wood, showing the matured spore, magnified 260 diameters.
 Fig. 6 b. A young plant.
 Fig. 6 c. A branch with young male plants, and a forming zoosporangium, magnified 260 diameters.
 Fig. 6 d. The empty cup left after the discharge of the oospore.
 Fig. 6 e. Outline of sporangium.
 Fig. 9. A young plant of *Stigeoclonium*.

The globular figures in the lower part of the plate are separate cells of *Porphrydium magnificum*, Wood, magnified 760 diameters. The numbering of the figures at the bottom of the plate are wrong, fig. 3 should read fig. 4, 4, 5, &c.

PLATE XVII.

- Fig. 1 a. The basal portion of an old frond of *Schizomeris Leibleinii*, Ktz., ? magnified 120 diameters.
 Fig. 1 b. A filament emitting zoospores, magnified 250 diameters.
 Fig. 1 c. A perfected zoospore.
 Figs. 1 d, 1 e, 1 e. Young plants formed by the germination of the zoospore, magnified 450 diameters.
 Fig. 2 a. The distal end of a filament of *Ædogonium Huntii*, Wood.
 Fig. 2 b. Cells showing the formation and growth of a new cell.
 Fig. 2 c. A portion of filament containing spores in different conditions of maturity.
 Fig. 2 d. A young female plant with attached dwarf plant.
 Fig. 2 f. Cells emitting a zoospore, magnified 250 diameters.
 Fig. 2 g. The perfected zoospore.
 Fig. 2 h. Outline sketch of a young male plant, magnified about 1200 diameters. The arrows are meant to represent cyclotic currents.
 Fig. 3. A fertile filament of *Ædogonium multispora*, Wood, showing spores in different states of maturity, and dwarf male plants.
 Fig. 4 c. Sterile cells of *Spirogyra dubia*, Ktz., enlarged 260 diameters.
 Figs. 4 and 4 d. Outline sketches of cells containing spores, magnified 260 and 160 respectively.
 Figs. 5 a and 5 b. Sterile cells of *Spirogyra rivularis* (Hassall), magnified 260 diameters.
 Fig. 5 c. Outline sketch of conjugating cells with spore similarly amplified

PLATE XVIII.

- Fig. 1 a. A young female plant of *Pringsheimia inæquale*, Wood, magnified 250 diameters.
 Fig. 1 b. A portion of an adult female plant, containing immature spores, and showing in outline in the upper sporangium the orifice through which the spermatozoa enter, magnified 250 diameters.
 Fig. 1 c. The supposed young male plant, magnified 450 diameters.
 Fig. 2. *Ædogonium mirabile*, Wood. A portion of a filament with a partially matured spore.

Fig. 2 *a*. A portion of a female plant, showing the beginning of the development of the female germ, *i. e.* the formation of a very large cell.

Fig. 2 *b*. A further stage of the process, showing the cell divided into an upper and lower portion, with the outline of the attached male plant.

Fig. 2 *c*. A fertile filament containing a matured spore. All of these figures are magnified 160 diameters.

Fig. 2 *d*. A couple of cells, one of which has divided into four daughter-cells, each of which contains a nearly perfected androspore, magnified 460 diameters.

Figs. 2 *b* and 2 *g*. Different views of dwarf male plants discharging spermatozoids, the first figure offering a profile view of the cap, the second a view from behind, magnified 400 diameters.

Fig. 2 *e*. A three-celled dwarf male plant, magnified 460 diameters.

Fig. 3. Matured spore of *Edogonium echinatum*, Wood, uncolored and magnified 750 diameters.

Fig. 4. Spores in sporangia of a Florida *Edogonium* of undetermined species.

Fig. 5. *Bulbochate ignota*, Wood.

Fig. 5 *a*. Branches of a frond, showing different stages in the early development of the female germs.

Fig. 5 *b*. Sporangium containing a nearly matured spore. All magnified 460 diameters.

Fig. 6 *a*. Part of a frond of *Bulbochate dumosa*, Wood, with female germs and dwarf male plants in different stages of development, magnified 260 diameters. The fine markings on the spores have not been reproduced in the chromo-lithograph from my drawing.

Fig. 6 *b*. Male plant discharging spermatozoid, magnified 750 diameters.

Fig. 7 *a*. Part of a sterile filament of a *Conferva* of unknown species.

Fig. 7 *b*. The same discharging zoospores.

Fig. 7 *c*. A cluster of germinating zoospores.

Fig. 7. A young plant. All these figures are magnified 500 diameters.

PLATE XIX.

Fig. 1. *Stigeoclonium*, showing chætophoroidal stage.

Fig. 2. Portion of a fertile filament of *Chantransia expansa*, Wood, magnified 125 diameters.

Fig. 2 *b*. A fragment of a fertile branch, magnified 260 diameters.

Fig. 3. A portion of a fertile filament of *Chantransia macrospora*, Wood, magnified 460 diameters.

Fig. 4 *a*. Outline of some fertile cells of *Spirogyra quinina*, Ag.

Fig. 4 *b*. Filaments in an advanced stage of conjugation.

Figs. 4 *c* and 4 *e*. Fragments of sterile filaments.

PLATE XX.

Fig. 1. *Stigeoclonium*, found near Philadelphia.

Fig. 2. *Arthrodesmus quadridens*, Wood, as viewed from the end, and magnified 250 diameters, also a front view of similar amplification.

Figs. 3 and 3 *a*. Different forms of fructification of *Vaucheria polymorpha*, Wood, showing the emptied antheridia and fertile sporangium.

Fig. 3 *b*. An immature antheridium.

Fig. 3 *c*. Spore of same species.

Fig. 4. Section through fertile node of *Lcmanca torulosa* (Roth).

Fig. 5. Outlines of *Euastrum multilobatum*, Wood. The lower figure represents a lateral view; the upper a two-thirds view.

Fig. 6. Outline of *Penium digitus*, Bréb.

PLATE XXI.

Fig. 1. *Pleurotaenium crenulatum* (Ehrb.), an outline view, magnified 160 diameters.

Fig. 2. *Pleurotaenium breve*, Wood, the empty dead frond, magnified 750 diameters.

Fig. 3. *Tetmemorus Brébissonii* (Mench.), an empty semicell.

Fig. 3 a. Outline of a whole frond.

Fig. 4 b. *Closterium lineatum* (Ehrb.), an empty semicell, magnified 260 diameters.

Fig. 5. *Cosmarium Botrytis* (Bory), an empty frond, magnified 750 diameters.

Figs. 5 a and 5 b. Outlines of semicells to show the variety of form.

Fig. 6. *Cosmarium Brébissonii*, Menegh., an empty frond, magnified 750 diameters and outline of apex view.

Fig. 7 a. *Cosmarium tetropthalmum* (Ktz.), outline of the empty frond, magnified 460 diameters.

Fig. 8. *Cosmarium margaritifera* (Turp.), view of an empty semicell, magnified 750 diameters; the outline of this should be more regular.

Fig. 9. *Cosmarium suborbiculare*, Wood, an empty frond, magnified 750 diameters.

Fig. 9 a. An outline of end view of similar amplification.

Fig. 10. *Cosmarium Broomei*, Thw., lateral outline of the frond.

Fig. 11. *Micrasterias Jemmerii*, Ralfs, an empty semicell.

Fig. 12. *Euastrum ornatum*, Wood, front view, magnified 450 diameters.

Fig. 12 a. Lateral view.

Fig. 13. *Euastrum Didelta*, Turpin, outline of the front view.

Fig. 14. *Euastrum elegans*, Bréb., outline of the lateral view.

Fig. 15. *Micrasterias truncata*, Corda, outline of front view, magnified 260 diameters.

Fig. 16. *Micrasterias granulata*, Wood, front view of an empty frond, magnified 460 diameters.

Fig. 17. *Staurastrum orbiculare*, Ehrb., outline of the end view.

Fig. 18. *Staurastrum dejectum*, Bréb., outline of the end view, magnified 750 diameters.

Fig. 19. *Staurastrum Lewisii*, Wood, outline of the end view, magnified 750 diameters.

Fig. 20. *Staurastrum paradoxicum*, Mey., outline of end view.

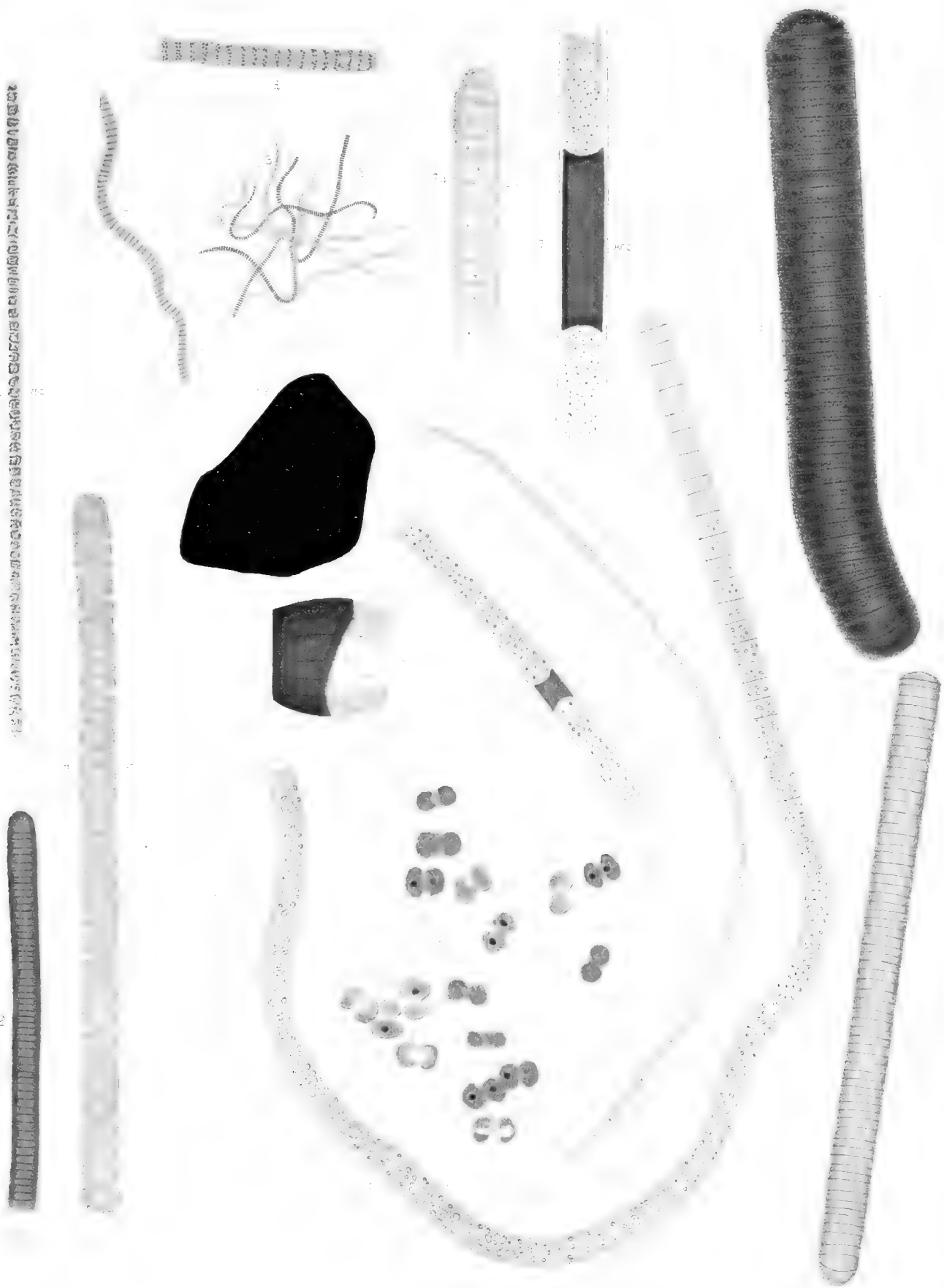
Fig. 21. The five radiate figure is an end view of *Staurastrum Arachne*, Ralfs, the triradiate of *Staurastrum paradoxum*, Meyen, magnified 750 diameters.

Fig. 22. *Staurastrum Ravenellii*, Wood, Front view of the empty frond, magnified 460 diameters

Fig. 22 a. The side view of an empty semicell, magnified 750 diameters.

Fig. 22 b. The end view with the same amplification.

Fig. 23. *Staurastrum Polytrichum*, Perty, outline of the frond as seen from the end.



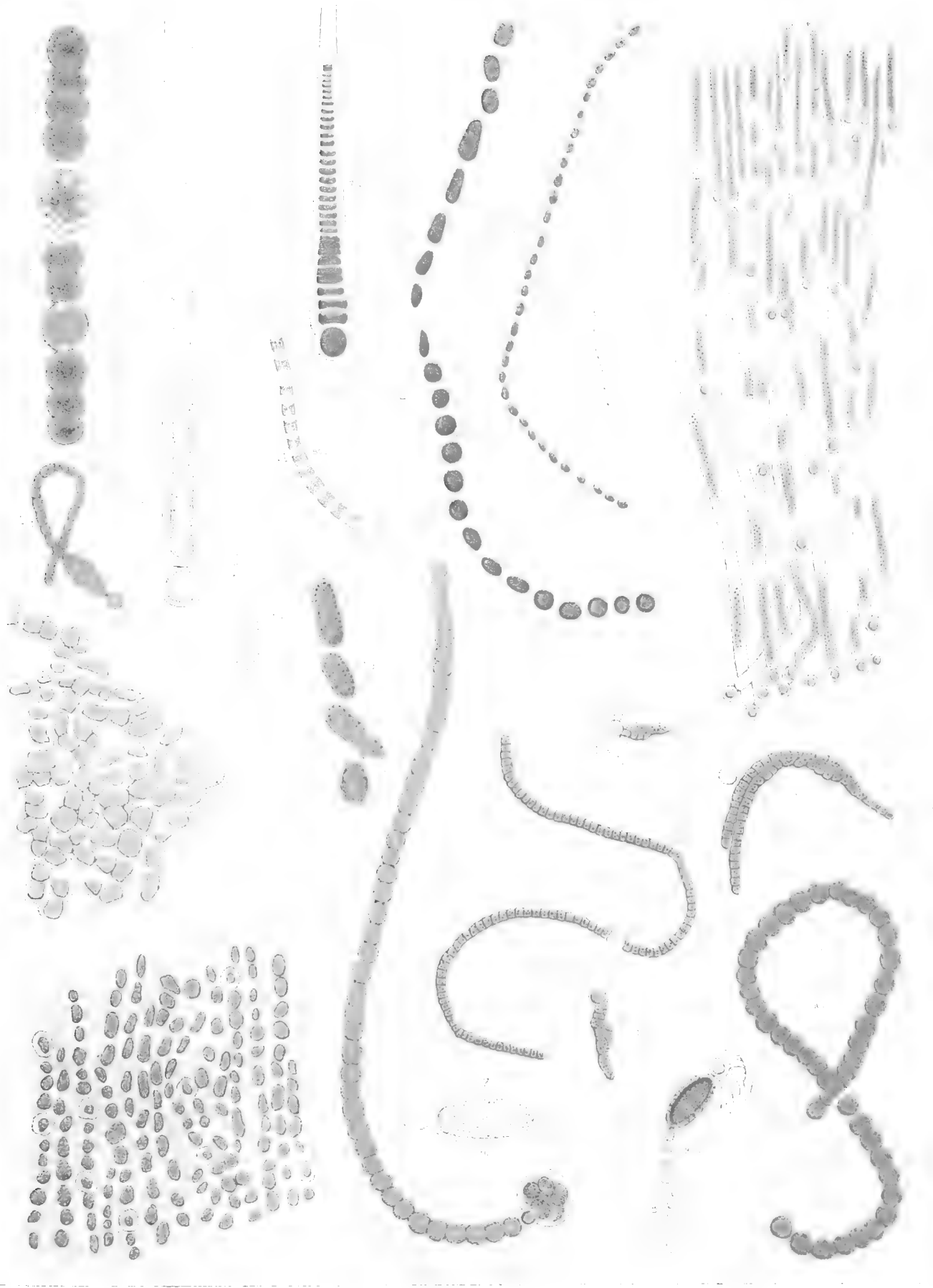
AFTER NATURE BY DR. H. C. WOOD

T. SINCLAIR & SON, PHILADELPHIA

Fig. 1. *O. CHLORINA*.
 " 2. *O. FRÖHLICHII*.
 " 3. *O. NIGRA*.

Fig. 4. *O. LIMOSA*.
 " 5. *O. NEGLECTA*.
 " 6. *O. IMPERATOR*.

Fig. 7. *LYNGBYA BICOLOR*.
 " 8. *LYNGBYA VARIETY*.
 " 9. *C. QUIMBYII*.



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FIG. 1. CYLINDROSPERMUM
 2. NOSTOC CALIDARIUM
 3. NOSTOC COMMINUTUM

FIG. 4. ANANENA GELATINOSA
 5. ANANENA GIGANTEA
 6. CYLINDROSPERMUM MINUTUM
 7. C. MACROSPERMUM

FIG. 8. RIVULARIA CARTILAGINEA
 9. NOSTOC SPHAERICUM



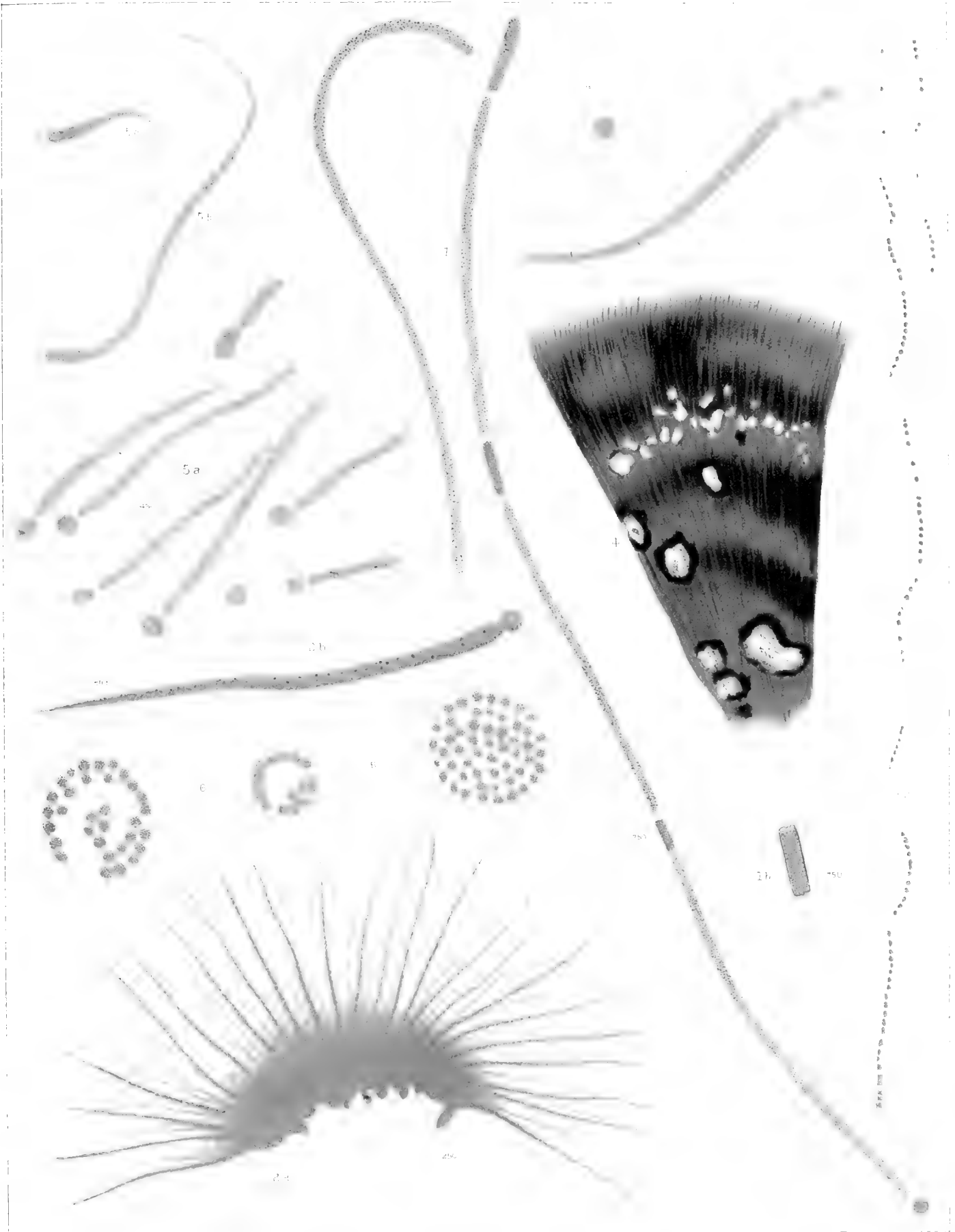
AFTER NATURE BY DR. H. C. WOOD

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Fig. 1. CYLINDROSPERMUM FLEXUOSUM
 " 2. SPHÆROZYGA SUBRIGIDA

Fig. 3. S. POLYSPERMA
 " 4. GLOIOTRICHA INCRUSTATA

Fig. 5. CHLOROCOCCUS
 " 6. NOSTOCOPSIS LOBATUS
 " 7. PROTOCOCCUS



ARTS AND CRAFTS BY R. H. L. ADRIJ

V. SINGH & SON, DELHI, INDIA

Fig. 1. MASTIGONEMA FERTILE.
 " 2. MASTIGONEMA SEJUNCTUM.

Fig. 3. ZONOTRICHA MOLLIS
 " 4. ZONOTRICHA PARCEZONATA

Fig. 5. DASYACTIS MOLLIS.
 " 6. COELOSPHAERIUM DUBIUM.

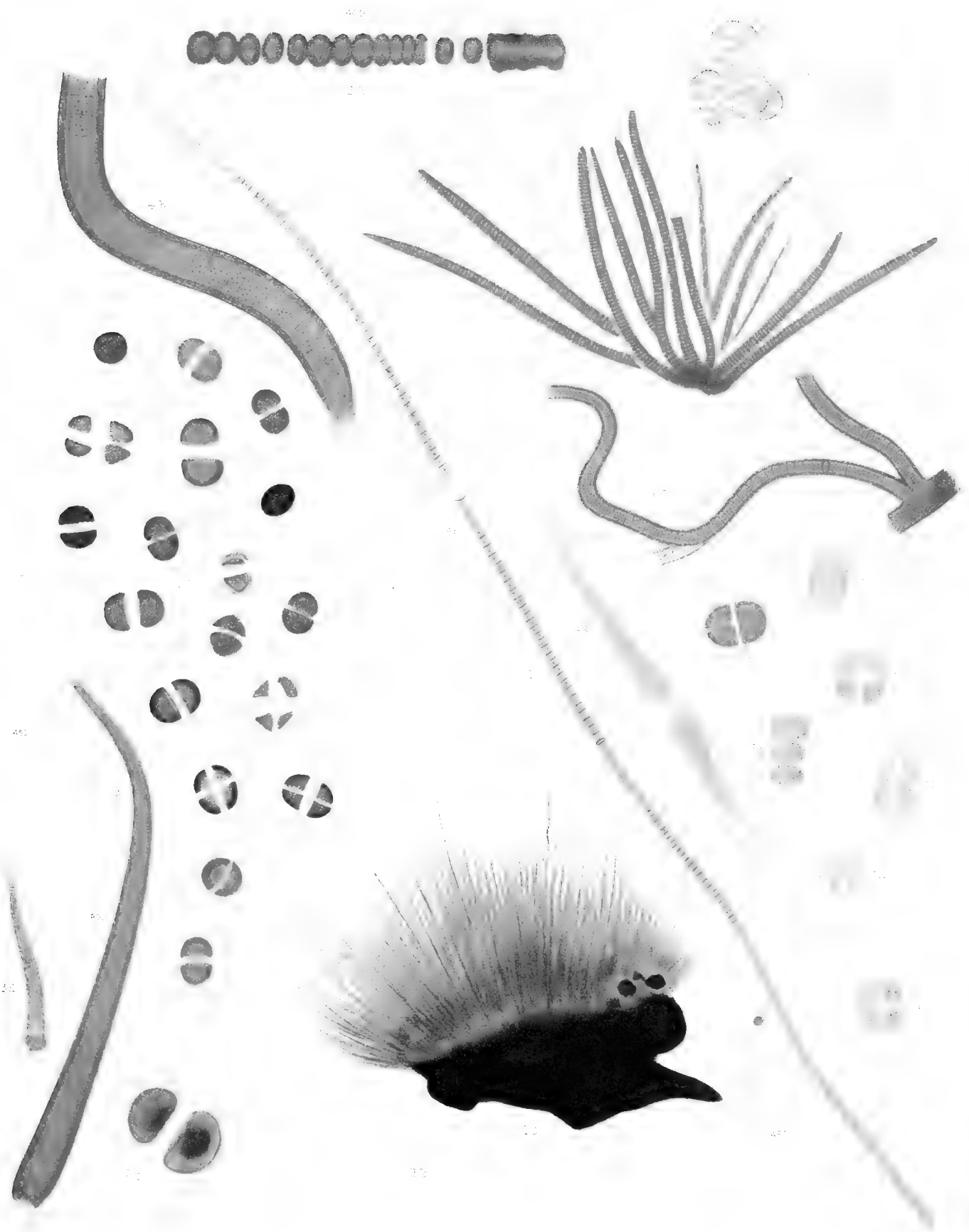
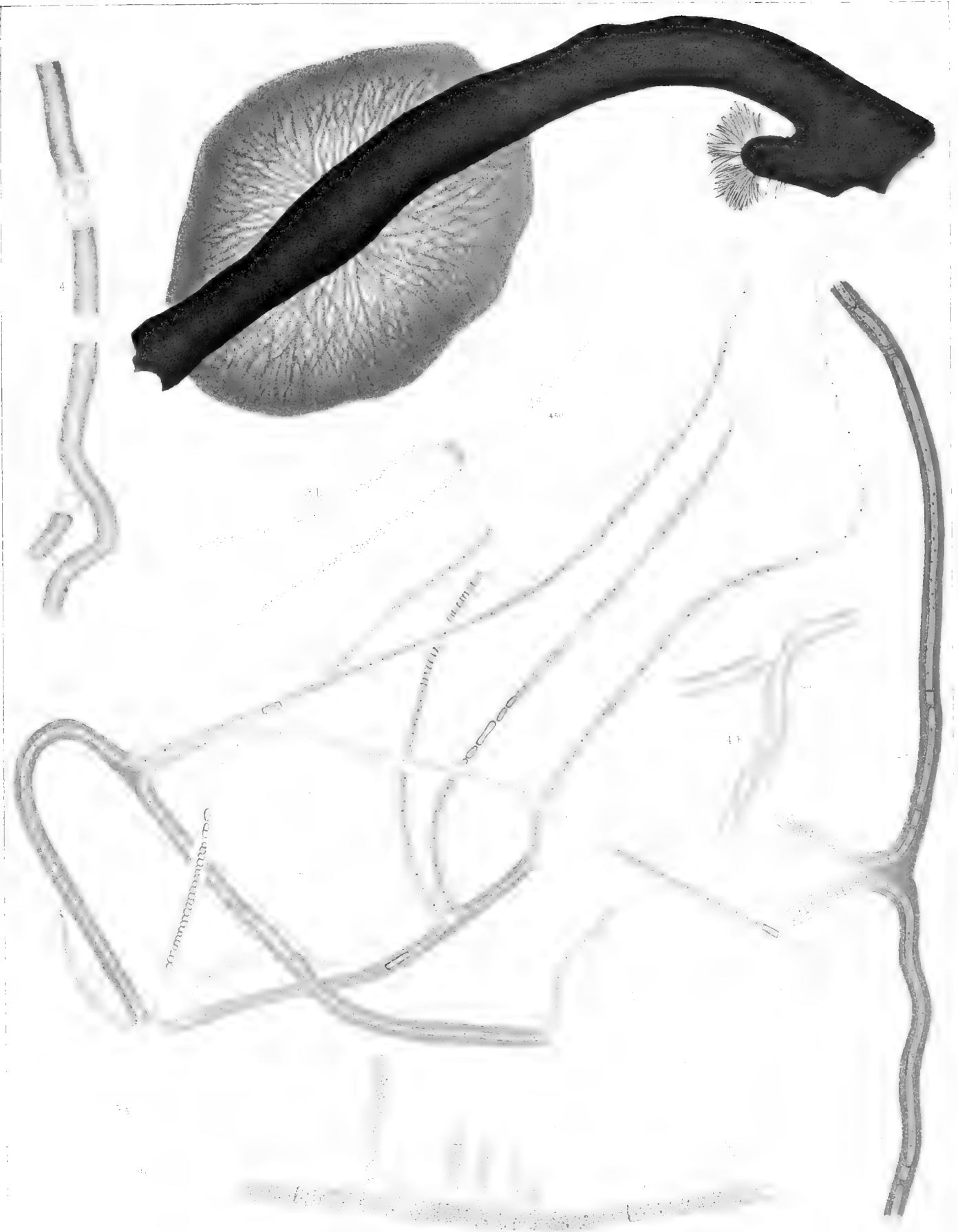


Fig. 1. MASTIGONEMA ELONGATUM
 " 2 MASTIGONEMA HALOS.

Fig. 3. MASTIGOTHRIX FIBROSA.
 " 4. SCYTONEMA RAVENELII.
 " 5. CHROOCOCCUS REFRACTUS.

Fig. 6. CHROOCOCCUS MULTICOLOR-
 ATUS.



AFTER NATURE BY DR H. C. WOOD

T. BINCLAIR & SON, PHILADELPHIA.

Fig. 1. SCYTONEMA THERMALE.

" 2. S. CALLITRICHOIDES.

" 3. S. EUBIUM.

Fig. 4. S. CORTEX.

" 5. CHÆTOPHORA ELEGANS.

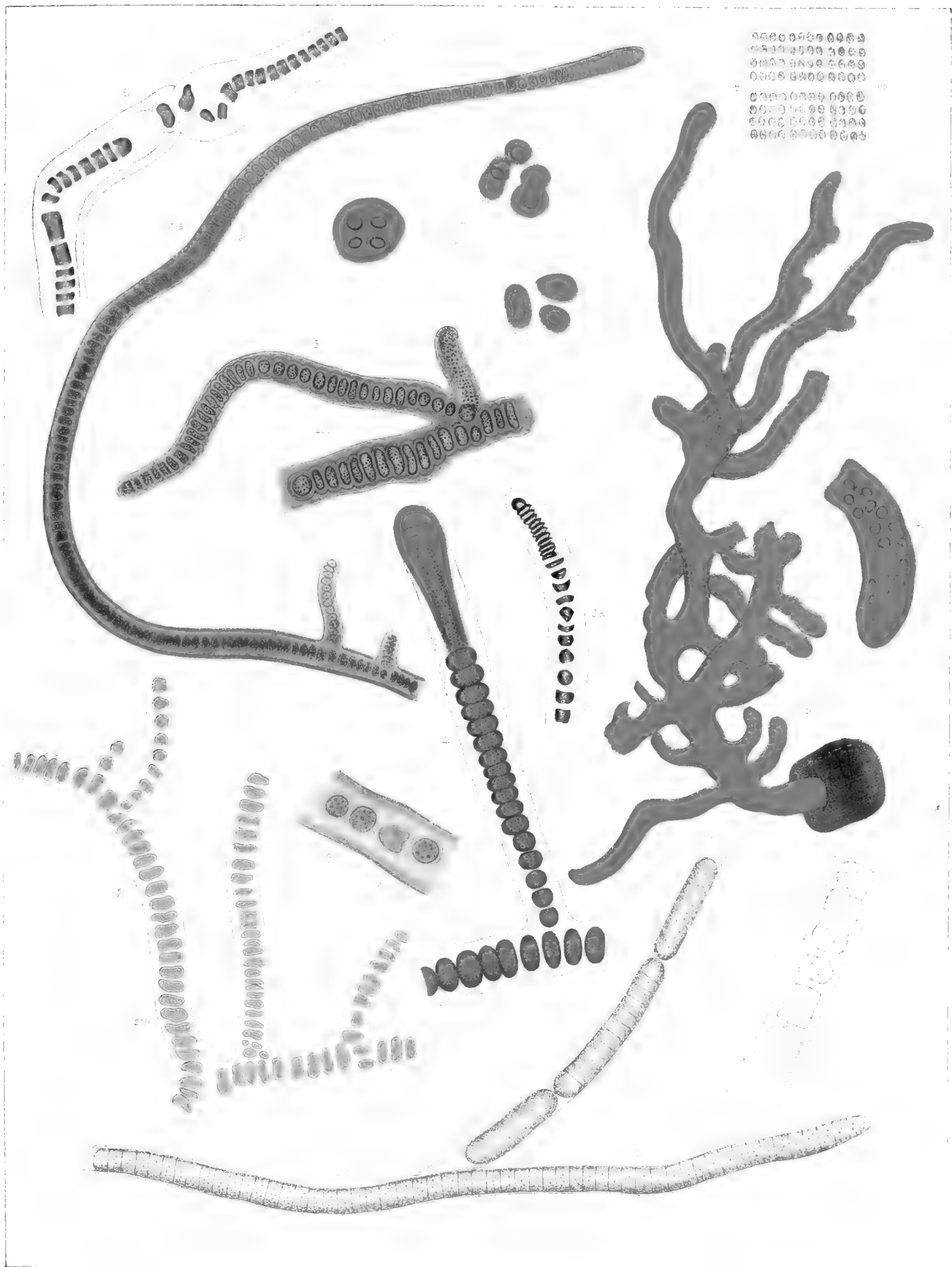


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T. SINCLAIR & SON, PHILADELPHIA

FIG. 1. SCYTONEMA CATARACTUM
2. SCYTONEMA IMMERSUM

FIG. 3. RHAPHIDIUM POLYMORPHUM,
4. PROTOCOCCUS



AFTER NATURE BY DR. H. C. WOOD.

SCHEPPE & SON, PHILADELPHIA

Fig. 1. TOLYPOTHRIX DISTORTA.

" 2. SIROSIPHON PELLUCIDULUM.

" 3. SIROSIPHON COMPACTUM.

Fig. 4. SIROSIPHON NEGLECTUM.

" 5. SIROSIPHON GUTTULA.

" 6. SCYTONEMA NÆGELLII.

Fig. 7. GLOEOCAPSA SPARSA.

" 8. MERISMOEDIA NOVA.



AFTER NATURE BY DR. H. C. WOODS

SCIENCE & ART PHOTODUPLICATION SERVICE

Fig. 1. SIROSIPHON SCYTENEMATOIDES.

Fig. 2. SIROSIPHON LIGNICOLA.

Fig. 3. S. ARGILLACEA.

Fig. 4. STIGNEMA RAVENELLI.



AFTER NATURE BY DR. H. C. WOOL.

T. SINCLAIR & SON, PHILADELPHIA.

Fig. 1. *S. ROSIPHON PULVINATUS*
Fig. 2. *PLEUROCOCCUS SERIATUS*

Fig. 3. *PALMELLA JESSENI*

Fig. 4. *BOTRYOCOCCUS PUJCHELLUS*
Fig. 5. *PALMELLA DURA*

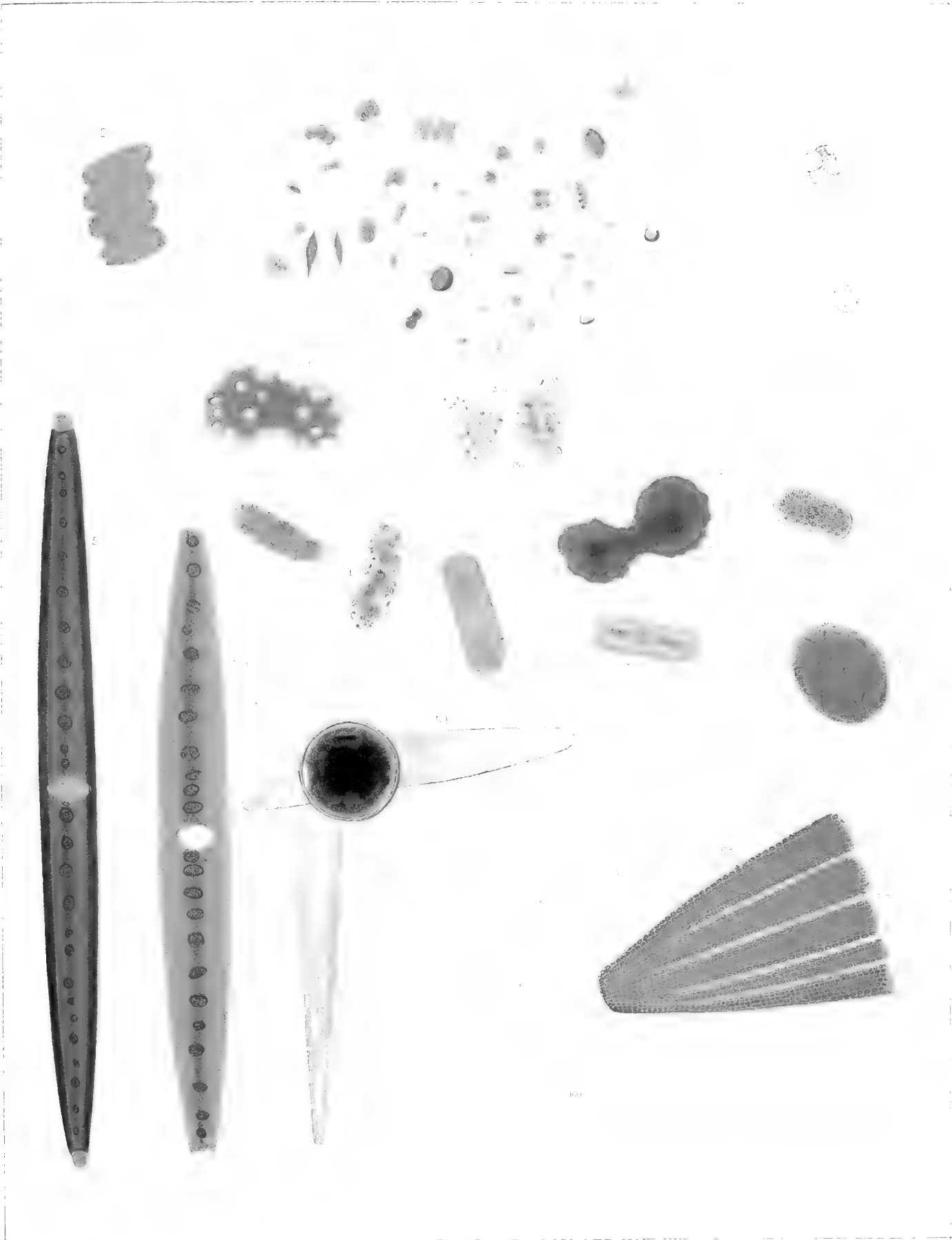


Fig. 1. SCENEDESMUS POLYMORPHUS.

2. SCENEDESMUS QUADRICAUDA

Fig. 3. SCENEDESMUS ROTUNDATUS

4. PALMOGLOEA CHEPSYDRA

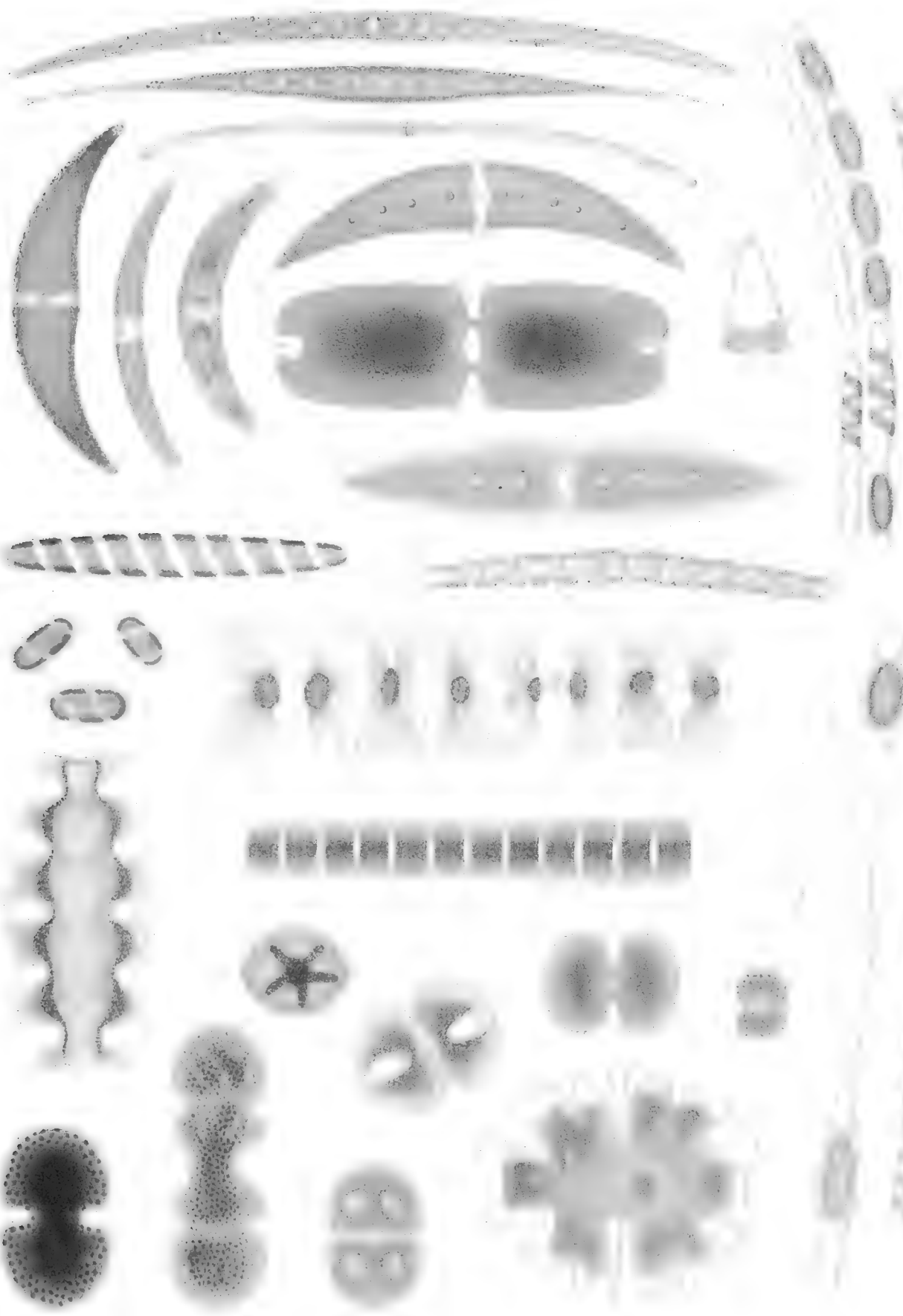
5. COSMARIUM ACEROSUM

Fig. 6. COSMARIUM AREOLATUM.

7. COSMARIUM VENUS.

AFTER CAPTURE BY CH. H. ALLEN

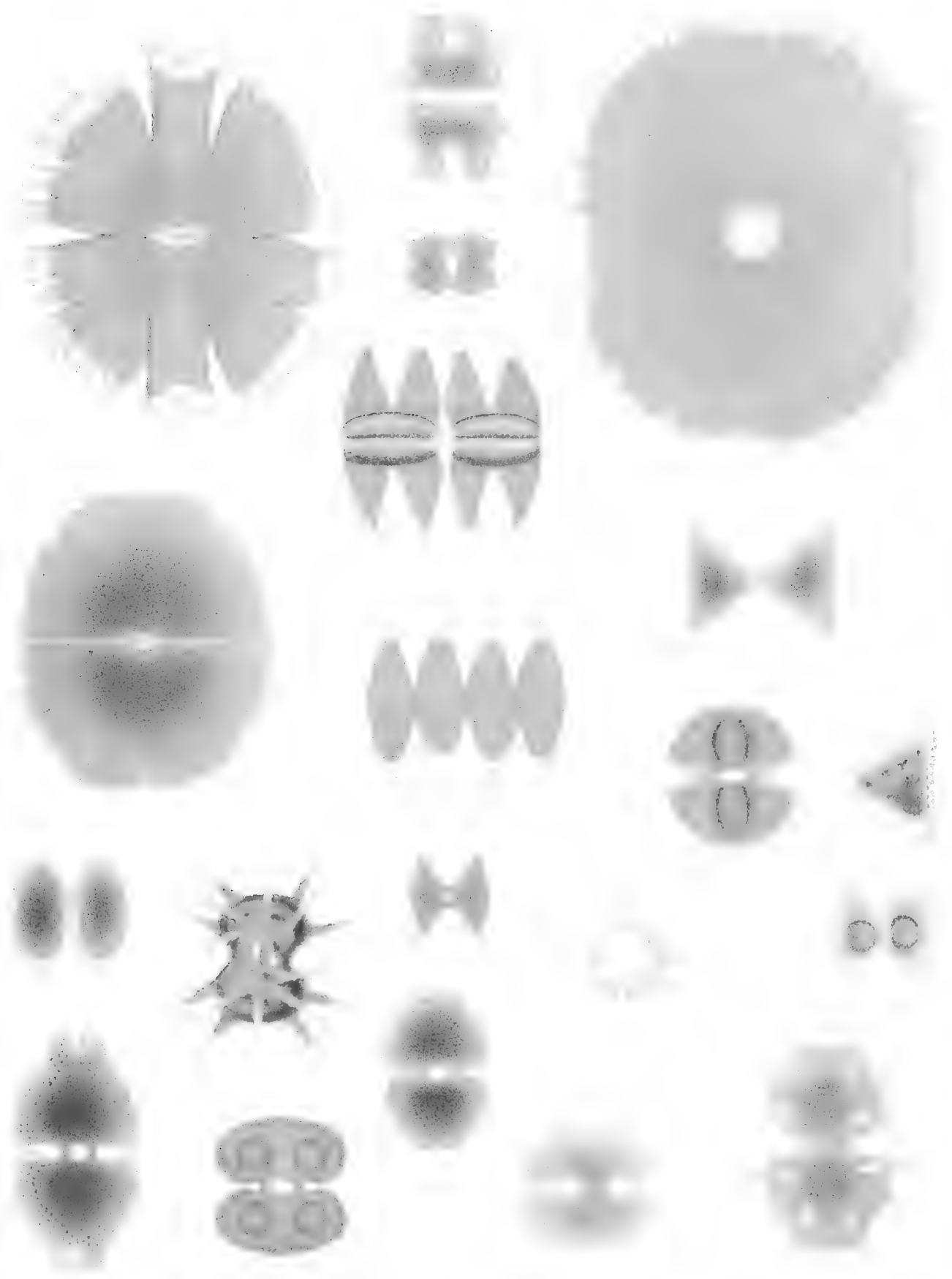
T. S. YOUNG & ASSOCIATES, INC., PHILA.



1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.

Fig. 1. Development of the embryo.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.



ART BY DR. H. J. WOOD

Fig. 1. *EUASTRUM BALFOI* (100x) Fig. 6. *LEPIDO-VARIETY GEMMIGUTSCHI* (100x) Fig. 11. *STAUSTRUM* sp. (100x) Fig. 16. *STAUSTRUM* sp. (100x)
 Fig. 2. *EUASTRUM ELEGANS* (100x) Fig. 7. *M. JENNERSII* (100x) Fig. 12. *STAUSTRUM* sp. (100x) Fig. 17. *STAUSTRUM* sp. (100x)
 Fig. 3. *EUASTRUM R. NALE* (100x) Fig. 8. *STAUSTRUM URBEQUARUM* (100x) Fig. 13. *STAUSTRUM* sp. (100x) Fig. 18. *STAUSTRUM* sp. (100x)
 Fig. 4. *MICRASPERMUM SPUTA* (100x) Fig. 9. *STAUSTRUM* sp. (100x) Fig. 14. *STAUSTRUM* sp. (100x) Fig. 19. *STAUSTRUM* sp. (100x)
 Fig. 5. *MICRASPERMUM* sp. (100x) Fig. 10. *STAUSTRUM* sp. (100x) Fig. 15. *STAUSTRUM* sp. (100x) Fig. 20. *STAUSTRUM* sp. (100x)

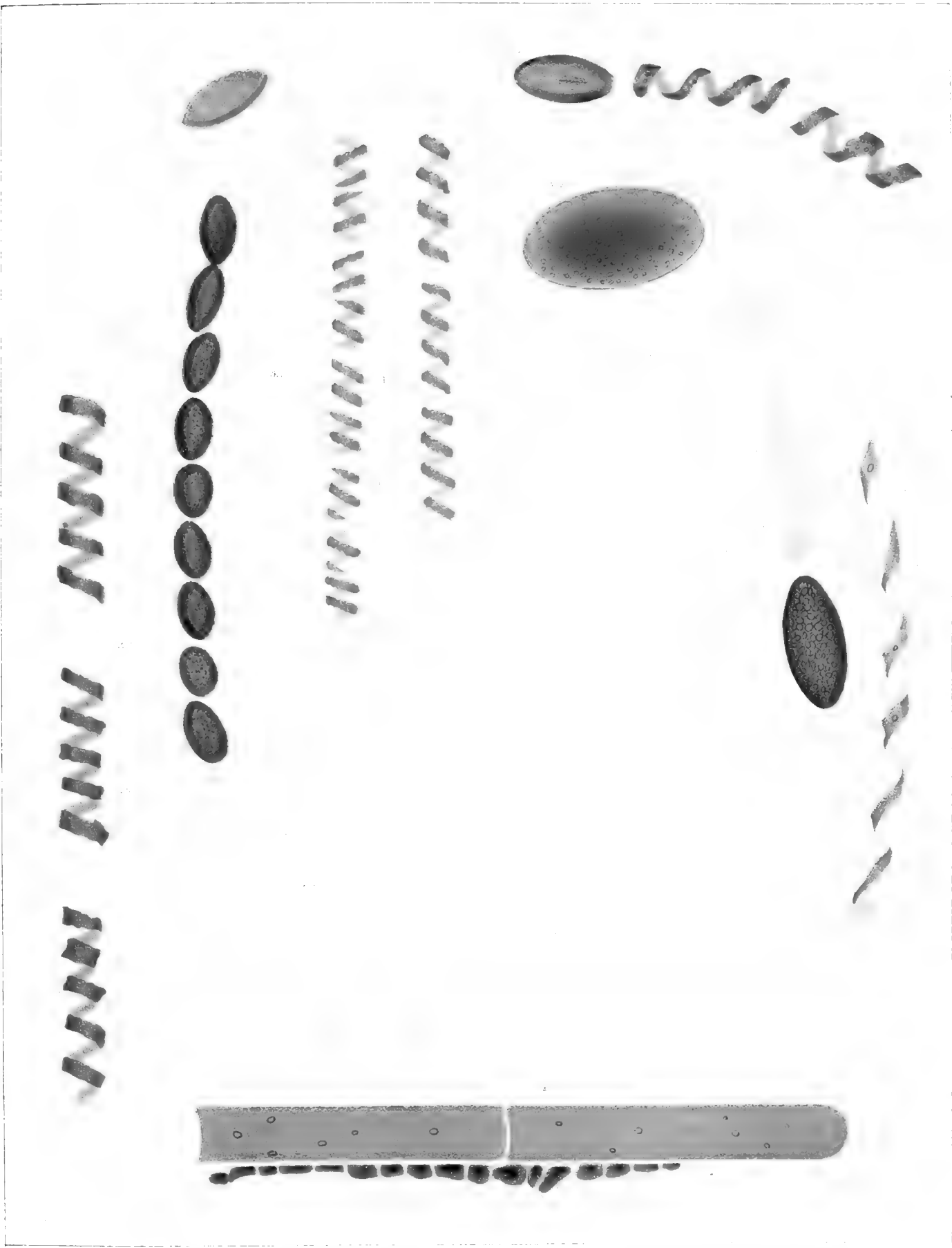


Fig. 1. RHYNCHONEMA ELONGATUM

" 2. RHYNCHONEMA PULCHELLUM

Fig. 2. SPIROGYRA PROTECTA.

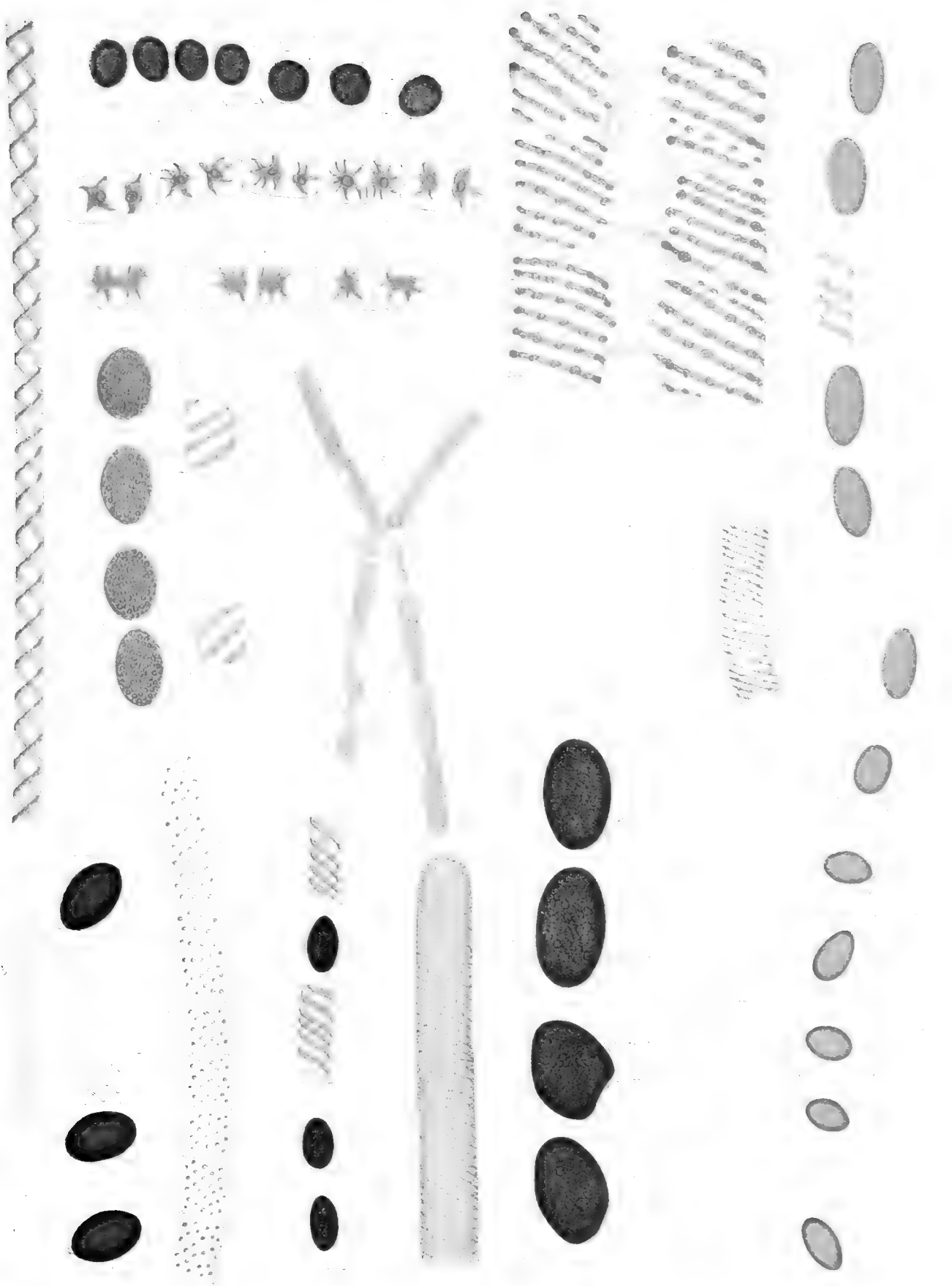
" 3. SPIROGYRA LONGATA.

Fig. 5. APHANOCHAETE REPENS.

" 6. DRAPARNALDIA BILLINGSII.

AFTER NATURE BY DR H. C. WOOD

UNIVERSITY OF CHICAGO



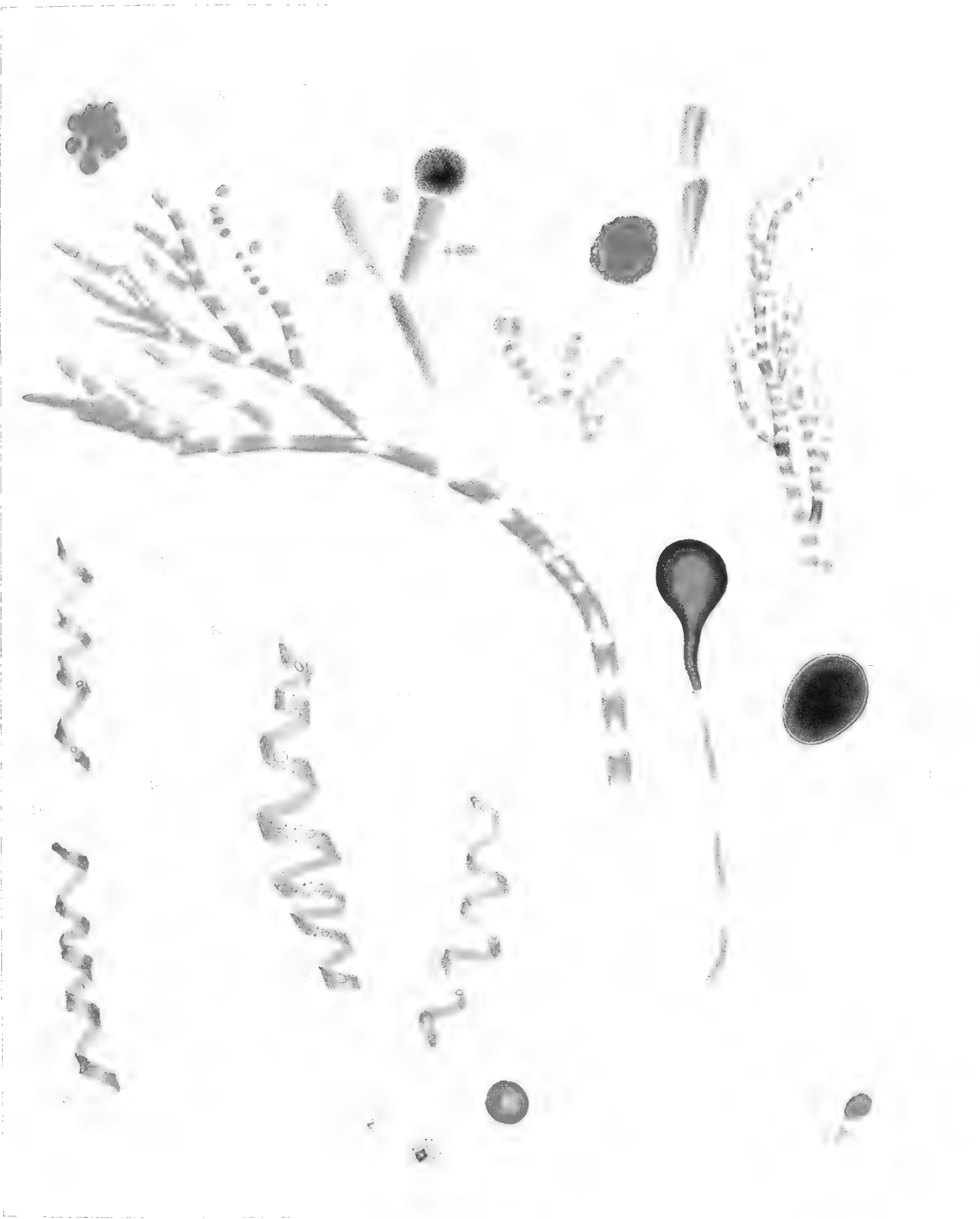
REPRODUCED BY DR. H. C. WOOD

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FIG. 1. SPIROGYRA MAJUSCULA
 " 2. SPIROGYRA DILUTA

FIG. 3. SPIROGYRA SETIFORMIS
 " 4. SPIROGYRA CRASSA
 " 5. MESOCARPUS SCALARIS

FIG. 6. SPIROGYRA INSIGNIS
 " 7. SPIROGYRA PARVISPORA
 " 8. ZYGNEMA INSIGNIS

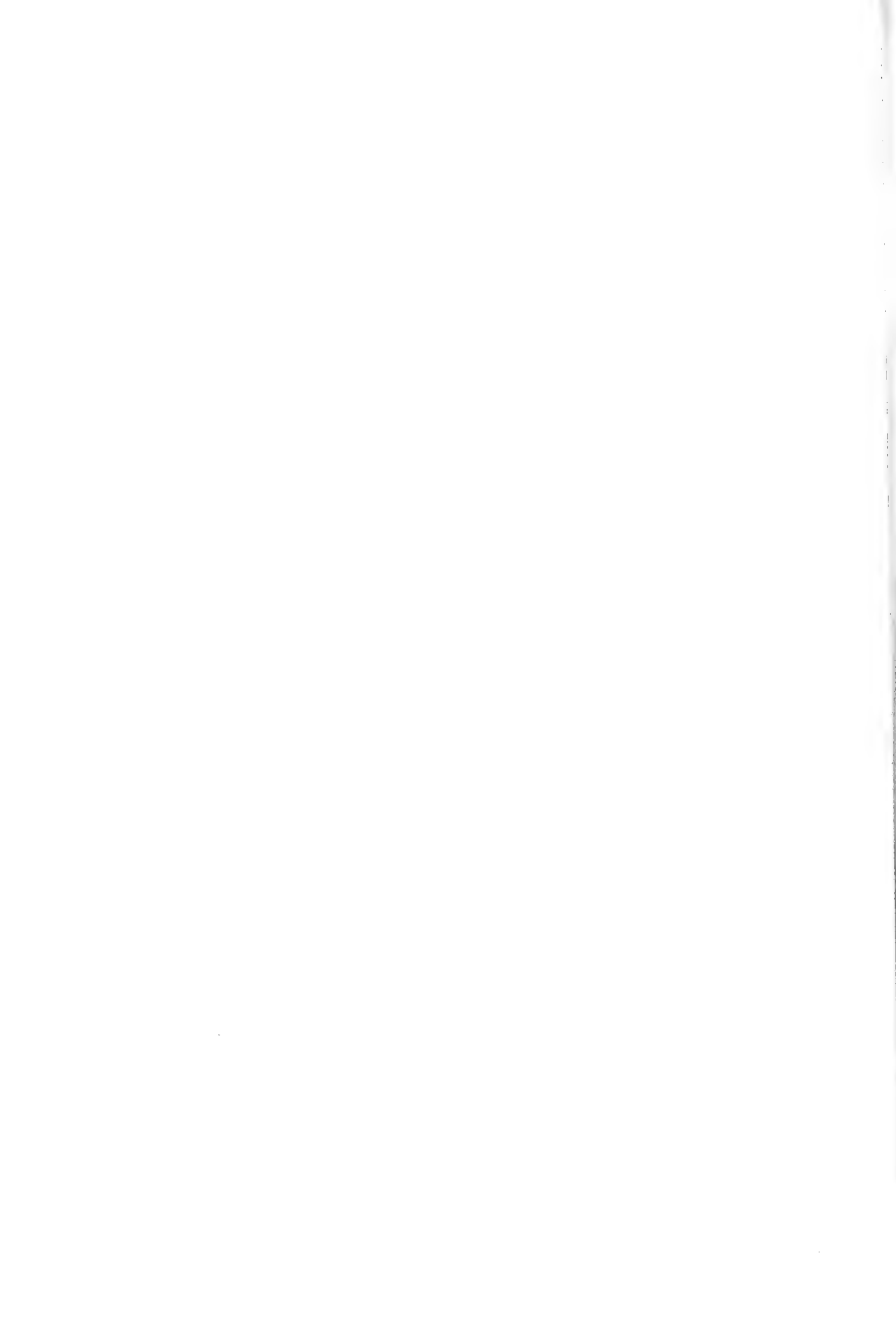


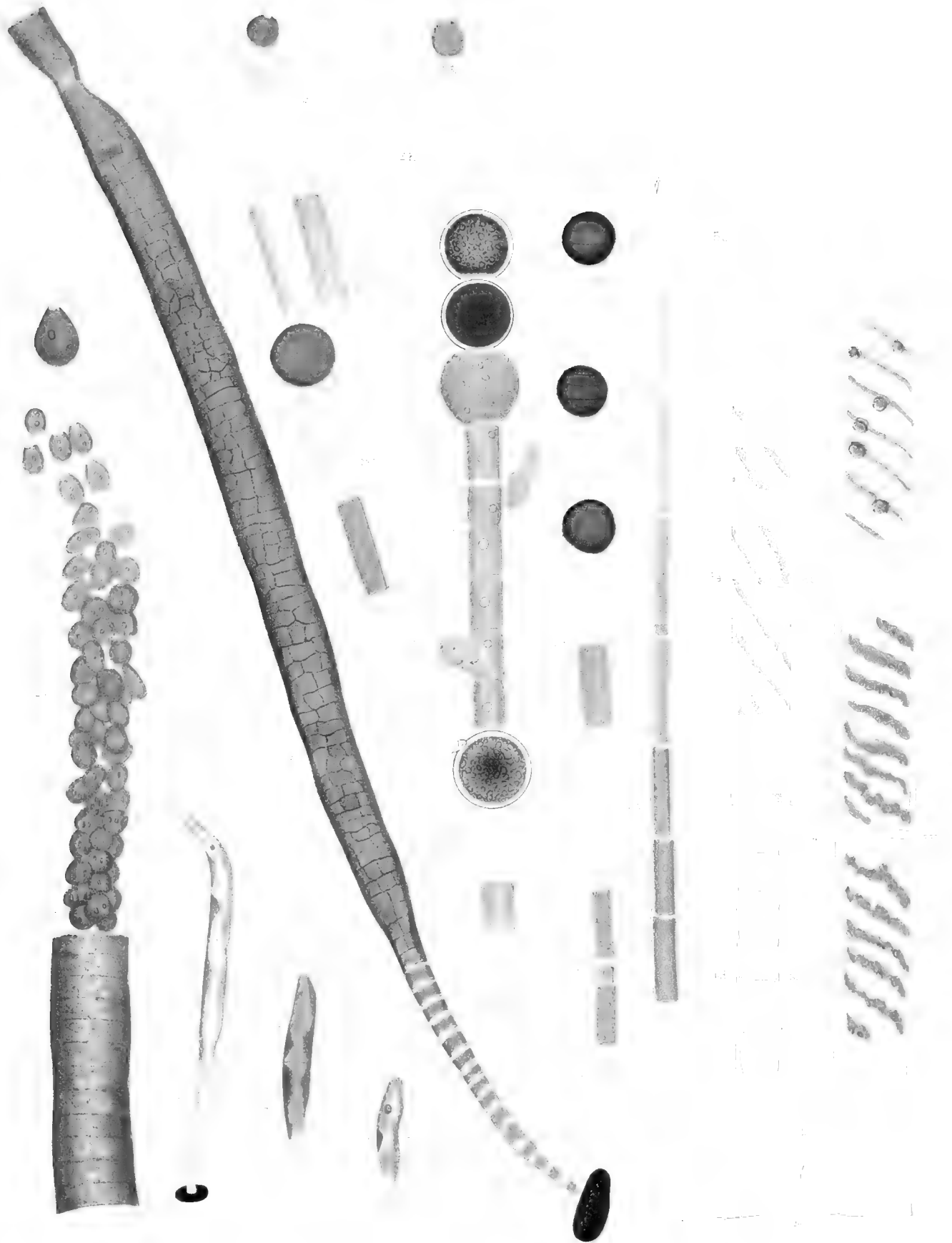
AFTER NATURE BY DR H. C. WOOD

T. SINCLAIR & SONS, PHILADELPHIA

Fig. 1. SIROGONIUM RETROVERSUM.
" 2. HYDROGASTRUM GRANULATUM
" 3. STIGEOCLONIUM.

Fig. 4. BULBOCTICHA ALBIDA.
" 5. BULBOCHETE CANBYI
" 6. STIGEOCLONIUM.





AFTER NATURE BY DR. H. F. WOOD

BY SINCLAIR & SON, LITHOGRAPHERS

FIG. 1. SCHIZOMERIS LEIBLEINII.

" 2. OEDOGONIUM HUNTII

" 3. O. MULTISPORA

FIG. 4. SPIROGYRA DUBIA.

" 5. S. RIVULAIS

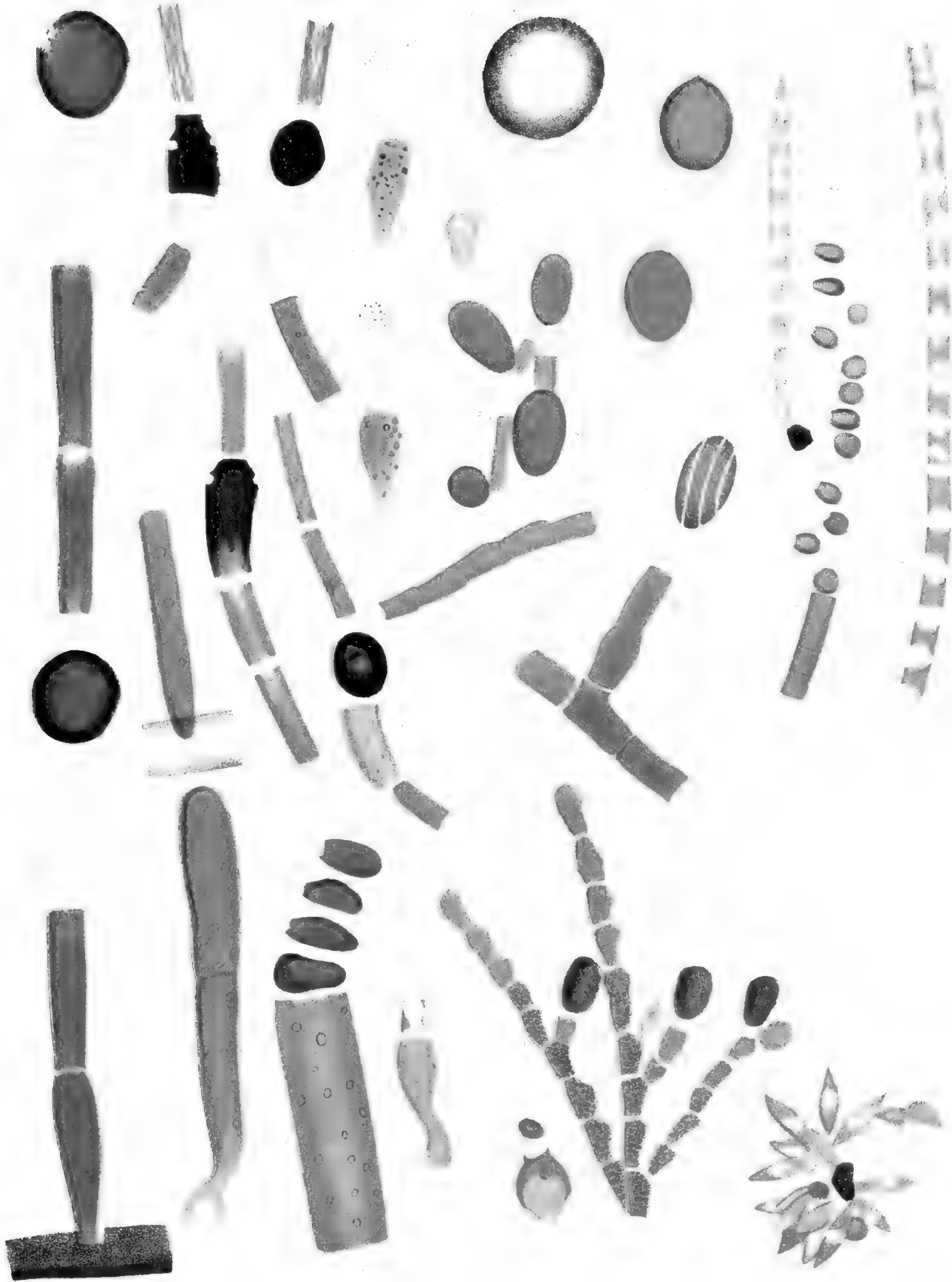


FIG. 1. PRINGSHEIMII INAEQUALE.
 " 2. OEDOGONIUM MIRABILE
 " 3. OEDOGONIUM ECHINATUM
 " 4. OEDOGONIUM.

FIG. 5. BULBOCHÆTE IGNOTA.
 " 6. BULBOCHÆTE DUMOSA
 " 7. CONFERVA.

AFTER NATURE BY DR. H. C. WOOD.

1. SINCLAIR & SON, PHILADELPHIA.



1877 NATURE BY L. R. H. J. ADAM.

1877 NATURE BY L. R. H. J. ADAM.

Fig. 1. STIGEOCLONIUM
 Fig. 2. CHANTRANSIA EXPANSA

Fig. 3. CHANTRANSIA MACROSPORA
 Fig. 4. SPIROGYRA QUININA



Fig. 1. *ARTHROSPERMUM QUARRIENS*

Fig. 2. *VAUGHANIA POLYMORPHA*
Fig. 3. *HEMANGIA TORULOSA*

Fig. 4. *EGASTYRUM MULTICORNUTUM*
Fig. 5. *PENIUM DIGITUS*



ART. DRAWN BY DR. H. C. WOOD

PLATE XXI BY DR. H. C. WOOD

| | | | |
|---------------------------------|-------------------------------|------------------------------|---------------------------------|
| Fig. 1. PLEUROTÆNIUM GRENULATUM | Fig. 6. COSMARIUM BRÉBISSONII | Fig. 11. COSMARIUM BRIGMÆ | Fig. 17. STAUROSTRUM ORBICULARE |
| " 2. PLEUROTÆNIUM BREVE | " 7. COSMARIUM TETRAPODUM | " 12. MICRASTERIAS FINNERII | " 18. ST. REJECTUM |
| " 3. TETRAMORUS BRÉBISSONII | " 8. COSMARIUM MARGARETÆ | " 13. E. DELTA | " 19. ST. HEWSONII |
| " 4. CLOSTERIUM LINEATUM | " 9. COSMARIUM SUBORBICULARE | " 14. E. ELEGANS | " 20. ST. PARADOXICUM |
| " 5. COSMARIUM BOTRYTIS | | " 15. MICRASTERIAS TRUNCATA | " 21. ST. RAFFINESII |
| | | " 16. MICRASTERIAS GRANULATA | " 22. ST. RAVENELII |
| | | | " 23. ST. POLYTRICHUM |

AN

INVESTIGATION

OF THE

ORBIT OF URANUS,

WITH GENERAL TABLES OF ITS MOTION.

BY

SIMON NEWCOMB,

PROFESSOR OF MATHEMATICS, UNITED STATES NAVY.

[ACCEPTED FOR PUBLICATION, FEBRUARY, 1873.]

ADVERTISEMENT.

IN the investigation of the Orbit of Uranus which forms the subject of the accompanying memoir, as well as in that of the Orbit of Neptune previously published in the Smithsonian Contributions, a large amount of arithmetical computation has been required, especially in the reduction and comparison of observations. The cost of this, in accordance with the spirit of the Institution in advancing science, has been defrayed from the income of the Smithson fund.

As required by the rules of the Institution, the accompanying memoir was referred to competent authority for examination, and the persons selected for this purpose were Professor J. H. C. Coffin, of the Nautical Almanac Office, and Professor Asaph Hall, of the Naval Observatory.

JOSEPH HENRY,
Secretary S. I.

WASHINGTON, 1873.

PHILADELPHIA:
COLLINS, PRINTER,
705 Jayne Street.

P R E F A C E.

THE present work was undertaken as far back as the year 1859. But the labor devoted to it at first amounted to little more than tentative efforts to obtain numerical data of sufficient accuracy, and to decide upon a satisfactory method of computing the general perturbations of the planet. The elements of Neptune employed in the earlier computations were found to deviate too widely from the truth to be used in computing the perturbations of Uranus with the first order of accuracy, and it became necessary to correct them. This was done during the years 1864 and 1865, and the investigation was printed by the Smithsonian Institution in the latter year. It was then found that the adopted elements of Uranus also differed too widely from the truth to serve as the basis of the work, and they were provisionally corrected by a series of heliocentric longitudes derived from observations extending from 1781 to 1861. Finally it was found that the adopted method of computing the perturbations, that of the "variation of elements," though not deserving of the disfavor into which it has fallen of late years, was practically inapplicable to the computation of the most difficult terms, namely, those of the second order with respect to the disturbing forces. Indeed, it appeared to the author that the only method of computing those terms which was at the same time general, practicable, and fully developed, was that of Hansen. But, were this method adopted, all that had previously been done would have been useless, even for the purpose of comparison and verification, owing to the expression of the coordinates in terms of a disturbed mean anomaly. It appeared to the author that, although this form of theory led to expressions having fewer terms than the other, it was not without its relative disadvantages. Other considerations being equal, he conceived that astronomers generally would greatly prefer to see the perturbations expressed directly in terms of the time, owing to the ease with which the results of different investigators could then be compared, and with which corrections to the theory may be introduced.

Under these circumstances the method described in the first chapter of the present paper was worked out. The question how much it contains that is essentially new is one that the author has never closely examined: it is, however, certain

that the mode of considering the subject is well known, being that employed by La Place, Herschel, De Pontécoulant, Encke, and perhaps others. The method of forming the required derivations of the perturbative function from the analytical development of that quantity, he has not seen elsewhere.

With these improved elements and methods the work was recommenced in 1868. The earlier investigations being merely provisional, it has not been deemed necessary to present them in the present work. Some of the results, corrected for errors of the older elements, are, however, given for the purpose of comparison.

Although this investigation has absorbed the greater part of the author's leisure for more than five years, it is only through the aid of the Smithsonian Institution and Nautical Almanac that he has been enabled to bring it to a conclusion within that time. At an early stage of the work Professor Henry responded favorably to a request for aid by the employment of computers; it was, however, not found practicable to use such aid until the perturbations had been completed, and the provisional theory concluded. Then, the comparison of theory and observation, and the construction of the tables, involved a large amount of mechanical computation, and on this part of the work a number of persons have been employed by the Institution at various times, among whom may be mentioned Professor F. W. Bardwell, of the University of Kansas, and Dr. C. L. F. Kampf, late of the Observatory of Leiden. Every part of the work has, however, been done under the author's immediate direction, and, as nearly as possible, in the same way as if he had done it himself, a result which, in one or two cases, has been attained only by the expenditure of an amount of labor approximating that saved by the employment of the computer.

In presenting the steps of the investigation, the end has been kept constantly in view to render as easy as possible the detection and correction of any error, or the introduction of any alteration in the elements or other data. It is, of course, impossible to present the steps of the computation with any approach to fulness without far transcending the limits of the printed work: The results given are, therefore, those which it was supposed would be most useful to the future investigator of the same subject. There is reason to believe that the original computations will ultimately become the property of the National Academy of Sciences, so that they may always be referred to for the clearing up of any difficulty in the printed text.

The author's acknowledgments are due to Professor J. H. C. Coffin, Superintendent of the Nautical Almanac, and Mr. E. J. Loomis, of the Nautical Almanac Office, for reading the proof sheets of the last twelve tables during the absence of the former abroad.

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ERRATA.

Pages 100 to 105. In computing the latitude from the provisional theory the values of the secular terms of $\delta\gamma$ and δk on page 97 have been interchanged. The provisional latitude, therefore, requires the correction

$$-0''.53 T \sin v + 0''.53 T \cos v$$

where

$$v = g + 12^\circ 45' + 2e \sin g.$$

This correction is not applied in the subsequent investigation. Its effect would have been to change the value of b deduced on p. 176 by probably $0''.2$ or $0''.3$. The effect on the other elements of latitude would have been much smaller, and therefore unimportant.

Page 122, line 15. Add: the corrections in the sixth column being omitted.

Page 151. Add foot-note: In forming these comparisons the corrections to the heliocentric longitude in the sixth column of the provisional ephemeris, pages 100 to 105, are not applied.

Page 159. Equation 7. In this equation the coefficients of $\delta\lambda$ and $\delta\rho$ have been multiplied by $\frac{1}{5}$, instead of $\frac{2}{5}$, the factor of δl . The effect of this error enters into all the subsequent results, but in the comparisons of theory and observation it is corrected.

Page 184. The element here represented by κ (kappa) is the same which, in the preceding chapters, has by mistake been represented by k , and which is defined on p. 24. The k of Chapter VIII is, therefore, not the same with that of preceding chapters.

ON THE ORBIT OF URANUS.

INTRODUCTION.

THE connection of the planet Uranus with the most brilliant astronomical achievement of the century lends a peculiar interest to its theory. The researches of Adams and Le Verrier showed that the observed motions of that planet were represented, at least approximately, by the action of a theoretical planet having the longitude of Neptune. Peirce showed that the action of Neptune itself accounted for these motions within the limits of possible error of the observations used by Le Verrier. It remains to be seen whether the agreement between theory and observation still subsists when the comparatively few observations used by those investigators are reduced with the more refined data now at our disposal, and when the great mass of additional observations made both before and since the date of Le Verrier's researches are included.

The circumstances connected with the discovery of Neptune have been so exhaustively recounted by a number of authors that it would be difficult to add anything not already familiar to astronomers without transcending our present limits. I shall therefore confine myself to such an account of previous researches on the theory of Uranus as may give an idea of their nature and extent, and facilitate their comparison with the methods and results of the present investigation.

The perturbations used by Bouvard in his tables are those of the *Mécanique Céleste*. Although not affected with any striking error, the numerical methods adopted in their computation are necessarily too rough to allow of much interest attaching to their comparison with the results of the more recent researches.

It is essential to a clear understanding of subsequent researches that we classify the methods which have been or may be adopted in the computation of the general perturbations of the planets. This computation comprises two distinct operations: (1) the development of the disturbing forces, or some quantities of which these forces are functions; (2) the integration of the equations of motion under the influence of these forces. In each of these operations three methods have been employed.

In developing the perturbative function, we have first the purely analytic method used by the great geometers of the last century. In this method this function is developed in powers of the eccentricities and mutual inclination of the orbits of the two planets, and the numerical coefficients are found by substituting the values of the elements in these expressions. It is only applicable when the eccentricities

and mutual inclination are small, and has for that reason fallen, of late, into a certain disrepute. The extended tables published by Le Verrier¹ have, however, added so much to its facility for use that it is not wholly unworthy of attention.

At the other extreme stands the purely mechanical method, in which special values of the disturbing force are computed for many combinations of the mean anomalies of the two planets, and the values of the coefficients in the general expression for the force thence deduced.

Between these two stands what I conceive we may designate as the Cauchy-Hansen method, in which the development is made mechanically with respect to the one planet, but the eccentric anomaly of the other is retained as an undetermined quantity. The germ of this method is found in several papers, by Cauchy, in the earlier volumes of the *Comptes Rendus* of the French Academy, which have since been combined into a homogeneous memoir by Puiseux.² The object had in view by these authors is only the computation of inequalities of long period. But Hansen has taken up the essential principle of the method, first, in his prize memoir on the perturbations of comets, crowned by the French Academy of Sciences, about 1848, and afterwards in his "*Auseinandersetzung einer zweckmässigen Methode zur Berechnung der Störungen der kleinen Planeten*,"³ and applied it to the general development of perturbations.

Among the three methods of integration, the first in point of analytical elegance and generality, but the last in order of convenience in use, is that of the variation of elements, a method with which the name of La Grange is inseparably associated.

In the second the direct integration of the differential equations which express the perturbations of longitude, latitude, and radius vector is effected by special devices.

In the first of these methods the problem is presented in this form: The equations of motion being completely integrated for the action of the principal forces only, how must the arbitrary constants of integration vary in order that the same expressions may represent the motion of the planet under the influence of the disturbing forces? In the second method, the same thing being presupposed, the question is, what expressions must be added to the integrals of undisturbed motion in order that the sum may represent the integrals of the disturbed motion?

The third is Hansen's method, in which the co-ordinates are partly expressed in terms of a certain function of the time known as the disturbed mean anomaly, determined by the condition that the true longitude in the disturbed orbit shall be the same function of the disturbed time that the longitude in the elliptic orbit is of the simple time.

Although the last two methods have a great advantage over the first in the computation of the periodic perturbations, I conceive the first to be best adapted to the computation of the secular variations, and perhaps, of terms of very long period in the mean longitude and the elements of the orbit.

¹ *Annales de l'Observatoire Impérial de Paris.* Tome I.

² *Annales de l'Observatoire Impérial de Paris.* Tome VII.

³ *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften.* Band V. VI, VII.

In his researches on the motion of Uranus, the first thing done by Le Verrier was to recompute the perturbations by Jupiter and Saturn. It will sufficiently describe his method of doing this to say that in the developments he used the purely mechanical method for the action of Saturn, and the algebraic development of the perturbative function for the action of Jupiter, while in the integration he used the method of the variation of elements. After completing the perturbations of the first order he made the earliest attempt at a complete determination of those of the second order. Beginning with the terms of this order which arise from the secular variations of the elements, he determines them by recomputing the terms of the first order for the epoch 2300, and assuming that the general term will then be given by interpolating between the two terms thus found, supposing them to increase uniformly with the time. This proceeding has the sanction of such high authority that it is worth while to call attention to its want of rigor. The differential coefficient of each element being given in the form

$$\frac{da}{dt} = k \cos bt,$$

k being a function of the elements, the perturbation of the first order will be

$$\delta a = \frac{k}{b} \sin bt.$$

When we take into account the variation of k , and suppose it of the form $k_0 + k't$, the process is equivalent to supposing that in this case

$$\delta a = \frac{k_0 + k't}{b} \sin bt,$$

whereas it really contains the additional term,

$$\frac{k'}{b^2} \cos bt,$$

which appears to be neglected in the process in question. It will be seen that the neglected coefficient is equal to the secular variation of the term during the time that its argument requires to increase by an amount equal to the unit radius. It is therefore the more important the longer the period of the inequality.

To obtain the periodic terms of the second order Le Verrier begins by determining the ten principal terms of the perturbations of the elements of Saturn produced by Jupiter. Next he takes up the terms in the mean longitude of Uranus which depend on the square of the mass of Saturn. The only sensible terms he finds are

$$\begin{aligned} & - 1''.17 \sin (\zeta^a - 3\zeta) - 0''.35 \cos (\zeta' - 3\zeta) \\ & + 0''.43 \sin (\zeta'' - 4\zeta'' + 4\zeta) - 0''.21 \cos (\zeta''' - 4\zeta'' + 4\zeta), \end{aligned}$$

ζ , ζ' , and ζ'' being the mean anomalies of Uranus, Saturn, and Jupiter, respectively. The terms depending on the product of the masses of Jupiter and Saturn are then taken up. Fifteen arguments are found the coefficients of which vary from a small fraction of a second to one or two seconds, while a single one of long period amounts to $32''$.

When the method of variation of elements is used, it is necessary not only to determine these variations to quantities of the second order, but, in the transforma-

tion of the perturbations of the elements into perturbations of the co-ordinates, to carry this transformation to terms of the second order also. This Le Verrier avoids by showing that the terms of the lowest order with respect to the eccentricities thus introduced are destroyed by certain terms in the perturbations of the elements, so that it is only necessary to omit both classes of terms. These terms are of that fictitious class which disappear of themselves by a simple change of elements. When, instead of the eccentricity and longitude of the perihelion, we take h and k , which represent the products of the eccentricity into the sine and cosine of this longitude respectively, these terms disappear of themselves both from the perturbations of the elements and of the co-ordinates. It is not likely that any of the neglected terms of this class exceed $0''.1$.

As soon as the elements of Neptune were known, the nature of its general action on Uranus became of interest. This subject was taken up by Prof. Peirce, whose results are found in the Proceedings of the American Academy of Arts and Sciences, Vol. I, pp. 334-337. This paper is accompanied with a comparison of his theory of Uranus with observations, to which similar comparisons of the theories of Adams and Le Verrier are added. This comparative exhibit is of sufficient interest to be given here. The numbers given are probably excesses of computed over observed longitudes.

| RESIDUAL DIFFERENCES BETWEEN THE THEORETICAL AND OBSERVED LONGITUDES OF URANUS,
FROM THE THEORIES OF PEIRCE, LE VERRIER, AND ADAMS. | | | | | | |
|--|--|---|--|---|--|--|
| Year. | From Le Verrier's best orbit of Uranus from the modern observations without any external planet. | From Le Verrier's original theory with his best orbit of hypothetical planet, of which the mass is $\frac{1}{5522}$. | From Adams's original theory with his second hypothetical planet of which mass is $\frac{1}{5555}$. | From Peirce's theory of Neptune adopting for its mass | | |
| | | | | That of Struve from his own observations of the satellite $\frac{1}{11155}$. | That deduced by Peirce from Bond's & Lassell's observations combined $\frac{1}{13780}$. | That deduced by Peirce from Bond's observations of Lassell's satellite $\frac{1}{19310}$. |
| 1690 | + 289.0 | - 19.9 | + 50.0 | - 124.7 | + 13.0 | + 0.8 |
| 1715 | + 279.6 | + 5.5 | - 6.6 | - 99.6 | + 10.0 | + 8.7 |
| 1756 | + 230.9 | - 4.0 | - 4.0 | - 102.4 | - 12.7 | + 4.0 |
| 1769 | + 123.3 | + 3.7 | + 1.8 | - 67.0 | - 16.0 | - 6.0 |
| 1782 | + 20.5 | + 2.3 | 0.0 | - 18.3 | - 5.6 | - 3.0 |
| 1787 | + 2.0 | - 1.2 | - 0.2 | - 4.7 | - 1.2 | - 0.5 |
| 1792 | - 7.8 | + 0.3 | - 1.1 | + 1.6 | + 0.5 | + 0.3 |
| 1797 | - 6.7 | - 1.0 | - 0.5 | + 3.3 | + 0.8 | + 0.3 |
| 1803 | - 3.4 | + 0.8 | + 1.6 | + 3.2 | + 1.2 | + 0.8 |
| 1808 | + 3.8 | + 0.8 | 0.0 | - 1.3 | - 0.6 | - 0.4 |
| 1813 | + 4.5 | + 0.9 | - 1.0 | - 2.3 | + 1.1 | - 0.3 |
| 1819 | + 3.8 | + 0.4 | - 2.2 | + 0.9 | + 0.7 | + 1.0 |
| 1824 | - 7.6 | - 5.4 | + 1.7 | - 1.6 | - 1.9 | - 2.0 |
| 1829 | - 7.8 | - 2.2 | + 2.0 | + 2.5 | + 1.3 | + 0.8 |
| 1835 | - 4.5 | - 0.8 | - 1.2 | + 3.9 | + 2.4 | + 2.0 |
| 1840 | + 0.7 | + 2.2 | + 1.3 | - 1.3 | - 1.3 | - 1.1 |
| 1845 | + 6.5 | - 0.3 | | - 2.8 | - 1.2 | - 0.9 |

In this paper Professor Peirce presents the results of a complete computation of the general perturbations of Uranus by Neptune in longitude and radius vector,

but without any details whatever of the investigation, or any statement of the methods employed. The minuteness of the residuals in the last column of the preceding table shows that employing these perturbations by Neptune, and those of Le Verrier by Jupiter and Saturn, we had a theory of Uranus from which quite accurate tables might have been constructed. But this never seems to have been done. The ephemeris of Uranus in the American Nautical Almanac was intended to be founded on this theory, but the proper definitive elements do not seem to have been adopted in the computations, as the ephemeris does not correspond with the theory.

Although twenty-five years have elapsed since the epoch of these researches, I am not aware of any published work of importance on the theory of Uranus during the interval. Mr. T. H. Safford has, however, made a very extended investigation of the subject, but has published nothing more than a brief general description of his work, which may be found in the Monthly Notices of the Royal Astronomical Society, Vol. 22. Like Professor Peirce, he took Le Verrier's perturbations by Jupiter and Saturn, but, instead of using general perturbations by Neptune, he computed the effect of the action of this planet by mechanical quadratures for the whole period of the observations of Uranus, and thus corrected the elements and the mass of Neptune from modern observations alone. The mass in question deduced was

$$\frac{1}{20039}$$

Mr. Safford does not give the representation of the modern observations, but presents the following comparison of the ancient ones, alongside which we place for comparison the corresponding numbers of Peirce's theory and those of the present investigation.

| EXCESS OF OBSERVATION OVER THEORY. | | | | |
|------------------------------------|-------------|----------|--------------------|----------|
| Date. | No. of obs. | Safford. | Peirce. | Newcomb. |
| 1690 | 1 | + 5".0 | — 0".8 | — 11" |
| 1715 | 3 | — 4.2 | — 8.7 | — 8 |
| 1750 | 2 | — 1.2 | }
..... } | + 2.9 |
| 1753 | 1 | — 0.2 | } | |
| 1756 | 1 | — 0.9 | — 4.0 | |
| 1764 | 1 | + 0.4 | | |
| 1769 | 8 | + 4.5 | + 6.0 | — 1.4 |

CHAPTER I.

METHOD OF DETERMINING THE PERTURBATIONS OF LONGITUDE, RADIUS VECTOR, AND LATITUDE OF A PLANET BY DIRECT INTEGRATION.

LET us conceive a plane determined by the condition that it shall pass through the sun and contain the tangent to the orbit of a planet at any moment. If the planet were acted on by the sun alone, the position of this plane would be invariable, but, under the influence of the disturbing forces of the other planets, it is subject, at each instant, to a motion of rotation around the radius vector of the planet. We may regard this as the instantaneous plane of the planet's orbit. The disturbing and the disturbed planet will each have its own instantaneous plane.

Let us now put:—

- v , the longitude of a planet counted from a determinate point in the instantaneous plane of its orbit.
- v , its distance from the node of intersection of its own orbit with that of another planet.
- γ , the mutual inclination of the two orbits.
- σ , $\sin \frac{1}{2} \gamma$.
- r , the radius vector of the planet.
- ρ , its logarithm.
- μ , the attractive force of the sun upon unit of matter at unit distance.
- a , the mean distance corresponding to the observed mean motion of the planet, determined by the condition

$$a^3 = \frac{\mu (1 + m)}{n^2},$$

m and n being as usual the mass and mean motion.

- a_0 , the value of a corrected for the constants introduced by the perturbations, so that, as in the elliptic motion, we have

$$\rho = \log a + f(l, e, \varpi),$$

we shall have in the disturbed motion

$$\rho = \log a_0 + f(l, e, \varpi) + \text{periodic terms only.}$$

- a_1 , the mean distance of an outer planet, whether it be a disturbing or disturbed planet.
- n , the logarithm of a .
- α , the ratio of two mean distances, taken less than unity.
- R , the perturbative function.

h , the coefficient of any term of $\frac{a_1}{m'} R$, so that we have

$$R = \sum \frac{m' h}{a_1} \cos N$$

m' being here the mass of the disturbing planet.

λ , the mean distance of the planet from the node, or the mean value of v .

ω , the distance of the perihelion from the node.

g , the mean anomaly.

l , the mean longitude, or the mean value of v .

ψ , the angle of eccentricity so that $e = \sin \psi$.

r_0 , the radius of the planet in the undisturbed ellipse.

r_1 , the quotient of r_0 divided by the mean distance, which is a function of the eccentricity and mean anomaly only.

T , the time after the epoch 1850, Jan. 0, Greenwich mean noon, counted in Julian centuries.

ν , the integrating factors of the periodic terms, or the ratio $\frac{n}{N}$, N being the change of the angle in unit of time.

u , the eccentric anomaly, and, in the tables, the argument of latitude.

We have for the value of R

$$R = \frac{m'}{\sqrt{r^2 - 2rr'(\cos v \cos v' + \sin v \sin v' \cos \gamma) + r'^2}} - \frac{m'r}{r'^2} (\cos v \cos v' + \sin v \sin v' \cos \gamma)$$

or, if we suppose r replaced by its value in ρ , namely

$$r = c^2$$

we shall have

$$R = m' f(v, v', \rho, \rho', \gamma).$$

With this value of R it is well known that the differential equations for the longitude and radius vector of a planet are

$$\begin{aligned} r \frac{d^2 r}{dt^2} - r^2 \frac{dv^2}{dt^2} + \frac{\mu(1+m)}{r} &= \mu \frac{\partial R}{\partial \rho}; \\ r^2 \frac{d^2 v}{dt^2} + 2r \frac{dr}{dt} \frac{dv}{dt} &= \mu \frac{\partial R}{\partial v}. \end{aligned} \quad (1)$$

If we multiply the first of these equations by $2 \frac{d\rho}{dt}$ and the second by $2 \frac{dv}{dt}$ and add them together, putting, for brevity,

$$\frac{\partial R}{\partial \rho} \frac{d\rho}{dt} + \frac{\partial R}{\partial v} \frac{dv}{dt} = D_t R, \quad (2)$$

and then integrate, we shall have

$$\frac{dr^2}{dt^2} + r^2 \frac{dv^2}{dt^2} - \frac{2\mu(1+m)}{r} = 2\mu (C + \int D_t R dt)$$

C being the arbitrary constant added to the integral. Adding this equation to the first of equations (1) we have

$$\frac{1}{2} \frac{d^2(r^2)}{dt^2} - \frac{\mu(1+m)}{r} = \mu \left(2C + 2 \int D_t R dt + \frac{\partial R}{\partial \rho} \right) \quad (3)$$

Let us now represent by r_0 that elliptic value of r which satisfies the equation

$$\frac{1}{2} \frac{d^2(r_0^2)}{dt^2} - \frac{\mu(1+m)}{r_0} = 2\mu C.$$

Subtracting this equation from the last we have

$$\frac{1}{2} \frac{d^2(r^2 - r_0^2)}{dt^2} - \mu(1+m) \left(\frac{1}{r} - \frac{1}{r_0} \right) = \mu \left(2 \int D_t R dt + \frac{\partial R}{\partial \rho} \right).$$

in which no constant is to be added to the integral, and both sides of the equation are of the order of the disturbing forces. As there is a decided advantage in taking the logarithm of the radius vector as the variable instead of r itself, we substitute for the latter its value

$$r = c^\rho, \quad r_0 = c^{\rho_0}$$

and put

$$\delta\rho = \rho - \rho_0.$$

Then

$$r^2 = c^{2\rho_0 + 2\delta\rho} = r_0^2 c^{2\delta\rho} = r_0^2 \left(1 + 2\delta\rho + \frac{2^2}{1.2} \delta\rho^2 + \text{etc.} \right)$$

$$\frac{1}{2} (r^2 - r_0^2) = r_0^2 \delta\rho + r_0^2 \delta\rho^2 + \text{etc.}$$

$$\frac{1}{r} - \frac{1}{r_0} = -\frac{\delta\rho}{r_0} + \frac{\delta\rho^2}{2r_0} + \text{etc.}$$

Substituting these values in the above equation, carrying the development only to terms of the second order, and transposing those terms to the right hand side of the equation, and putting $\mu' = \mu(1+m)$, we find

$$\frac{d^2(r_0^2 \delta\rho)}{dt^2} + \frac{\mu'}{r_0^3} (r_0^2 \delta\rho) = \mu \left(2 \int D_t R dt + \frac{\partial R}{\partial \rho} \right) - \frac{d^2(r_0^2 \delta\rho^2)}{dt^2} + \frac{\mu \delta\rho^2}{2r_0}, \quad (4)$$

an equation which gives the perturbations of radius vector.

The general mode of solving this equation by successive approximation is familiar. The principles on which the successive approximations are made being the same, we shall begin by assuming that we have obtained first approximations to the values of δv , $\delta v'$, $\delta\rho$, $\delta\rho'$, $\delta\gamma$, and that from these we wish to pass to a second approximation. We must first carry this approximation into the functions of R in the second member of (4). To effect this we must show how, from the development of R in terms of the elements and the time, we may form its successive derivatives with respect to the quantities which enter into it. R , while originally a function of v , v' , ρ , ρ' , and γ , is, in its developed form, a function of λ , λ' , ω , ω' , e , e' , v , v' and γ , the development being effected by substituting for the first set of quantities their values in terms of the second. The substitution is as follows:

$$\begin{aligned} v &= \lambda + Fg, \\ v' &= \lambda' + Fg', \\ \rho &= v + \phi g, \\ \rho' &= v' + \phi g', \end{aligned} \quad (5)$$

Fg being the equation of the centre, and ϕg the part of ρ depending on the eccentricity in the elliptic motion. It follows that if we express the developed expression for R as a function of $\lambda, \lambda', g, g', v, v'$, which we may do by putting

$$\begin{aligned} \omega &= \lambda - g, & \omega' &= \lambda' - g'; \\ a &= c^v, & a' &= c^{v'}; \end{aligned}$$

we shall have by successive differentiation

$$\begin{aligned} \frac{\partial R}{\partial \lambda} &= \frac{\partial R}{\partial v} \frac{\partial v}{\partial \lambda} = \frac{\partial R}{\partial v} \\ \frac{\partial^2 R}{\partial \lambda^2} &= \frac{\partial^2 R}{\partial v^2} \frac{\partial v}{\partial \lambda} = \frac{\partial^2 R}{\partial v^2} \\ \frac{\partial R}{\partial v} &= \frac{\partial R}{\partial \rho} \frac{\partial \rho}{\partial v} = \frac{\partial R}{\partial \rho} \\ \frac{\partial^2 R}{\partial v^2} &= \frac{\partial^2 R}{\partial \rho^2} \frac{\partial \rho}{\partial v} = \frac{\partial^2 R}{\partial \rho^2} \\ &\text{etc.} & & \text{etc.} & & \text{etc.} \end{aligned} \tag{6}$$

and in general

$$\frac{\partial^{m+n+m'+n'} R}{\partial \lambda^n \partial v^n \partial \lambda'^{m'} \partial v'^{n'}} = \frac{\partial^{m+n+m'+n'} R}{\partial v^m \partial \rho^n \partial v'^{m'} \partial \rho'^{n'}}$$

Thus, by expressing the developed R in the above form, we may find the derivative of any order with respect to v, v', ρ and ρ' , by taking the corresponding derivative with respect to λ, λ', v and v' .

The developed R is usually expressed in the form

$$R = \Sigma \frac{m'h}{a_1} \cos(i'\lambda' + i\lambda + j'\omega' + j\omega)$$

a_1 being the mean distance of the outer planet, whether disturbing or disturbed, and h a function of e, e', a , and γ . Substituting for ω its value in g , this equation will become

$$R = \Sigma \frac{m'h}{a_1} \cos((i' + j')\lambda' + (i + j)\lambda - j'g' - jg).$$

Putting for brevity

$$N = i'\lambda' + i\lambda + j'\omega' + j\omega,$$

the formulæ (6) give

$$\begin{aligned} \frac{\partial R}{\partial v} &= -\Sigma \frac{m'h}{a_1} (i + j) \sin N \\ \frac{\partial^2 R}{\partial v^2} &= -\Sigma \frac{m'h}{a_1} (i + j)^2 \cos N \\ \frac{\partial R}{\partial \rho} &= \Sigma m' \frac{\partial}{\partial v} \frac{h}{a_1} \cos N \end{aligned} \tag{7}$$

and in general

$$\frac{\partial^{u+u'+n+n'} R}{\partial v^u \partial v'^{u'} \partial \rho^n \partial \rho'^{n'}} = \Sigma \pm m' (i + j)^n (i' + j')^{u'} \frac{\partial^{n+n'} h}{a_1} \cos N$$

The formation of the derivatives in the second member of this equation demands attention. In the analytic development of the perturbative function each value of h is composed of a series of terms each of the form

$$E \times A,$$

E being a function of the eccentricities and mutual inclination, and A a function of α of the form

$$(0) \alpha^{s-\frac{1}{2}} b_s^{(i)} + (1) \alpha^{s+\frac{1}{2}} \frac{\partial b_s^{(i)}}{\partial \alpha} + (2) \alpha^{s+\frac{3}{2}} \frac{\partial^2 b_s^{(i)}}{\partial \alpha^2} + \text{etc.} + \alpha^{\frac{2s+2n-1}{2}} \frac{\partial^n b_s^{(i)}}{\partial \alpha^n}, \quad (8)$$

(0), (1), etc., being numerical coefficients connected with the coefficients $V^{(i)}$ tabulated by Le Verrier, in Tome I of his *Annales de l'Observatoire*, by the relation

$$(n) = \frac{V_n^{(i)}}{1.2.3 \dots n},$$

and $b_s^{(i)}$ being, as usual, the coefficient of $\cos i\phi$ in the development of

$$(1 - 2\alpha \cos \phi + \alpha^2)^{-s}$$

in multiples of $\cos \phi$, and $n - 1$ the sum of the exponents of the eccentricities in E .

It would have been much more convenient if in effecting this development the derivatives of $b_s^{(i)}$ had been taken with respect to x instead of α . In fact the derivative $\frac{\partial^n b_s^{(i)}}{\partial \alpha^n}$ when expressed in terms of the derivatives with respect to x is of the form

$$\alpha^n \frac{\partial^n b_s^{(i)}}{\partial \alpha^n} = n_1 \frac{\partial b_s^{(i)}}{\partial x} + n_2 \frac{\partial^2 b_s^{(i)}}{\partial x^2} + \text{etc.} + n_n \frac{\partial^n b_s^{(i)}}{\partial x^n}$$

Therefore, when expressed in terms of the derivatives with respect to x , A will be of the form

$$\alpha^{s-\frac{1}{2}} \left((0) b_s^{(i)} + (1) \frac{\partial b_s^{(i)}}{\partial x} + (2) \frac{\partial^2 b_s^{(i)}}{\partial x^2} + \text{etc.} \right),$$

from which the derivatives $\frac{\partial A}{\partial x}$, $\frac{\partial^2 A}{\partial x^2}$, etc., may be found with great facility.

As in the actual developments of R which we possess, the values of A are given in the form (8), we must find the expression for the first two derivatives of its several terms with respect to x , which we easily do by the application of the symbolic formulæ

$$\begin{aligned} D_x &= \alpha D_\alpha \\ D_x^2 &= \alpha (D_\alpha + \alpha D_\alpha^2). \end{aligned}$$

Beginning with the case of $s = \frac{1}{2}$, we have

$$\begin{aligned} \frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial x} &= \alpha \frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial \alpha} \\ \frac{\partial^2 b_{\frac{1}{2}}^{(i)}}{\partial x^2} &= \alpha \frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial \alpha} + \alpha^2 \frac{\partial^2 b_{\frac{1}{2}}^{(i)}}{\partial \alpha^2} \end{aligned}$$

$$\begin{aligned} \frac{\partial \left(\alpha^n \frac{\partial^n b_{\frac{1}{2}}^{(i)}}{\partial \alpha^n} \right)}{\partial x} &= n \alpha^n \frac{\partial^n b_{\frac{1}{2}}^{(i)}}{\partial \alpha^n} + \alpha^{n+1} \frac{\partial^{n+1} b_{\frac{1}{2}}^{(i)}}{\partial \alpha^{n+1}} \\ \frac{\partial^2 \left(\alpha^n \frac{\partial^n b_{\frac{1}{2}}^{(i)}}{\partial \alpha^n} \right)}{\partial x^2} &= n^2 \alpha^n \frac{\partial^n b_{\frac{1}{2}}^{(i)}}{\partial \alpha^n} + (2n+1) \alpha^{n+1} \frac{\partial^{n+1} b_{\frac{1}{2}}^{(i)}}{\partial \alpha^{n+1}} + \alpha^{n+2} \frac{\partial^{n+2} b_{\frac{1}{2}}^{(i)}}{\partial \alpha^{n+2}} \\ &= n \frac{\partial \left(\alpha^n \frac{\partial^n b_{\frac{1}{2}}^{(i)}}{\partial \alpha^n} \right)}{\partial x} + \frac{\partial \left(\alpha^{n+1} \frac{\partial^{n+1} b_{\frac{1}{2}}^{(i)}}{\partial \alpha^{n+1}} \right)}{\partial x} \end{aligned}$$

consequently we have for the derivatives of A from formulæ (8)

$$\begin{aligned} \frac{\partial A}{\partial x} &= (0) \alpha \frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial \alpha} + (1) \left(\alpha \frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial \alpha} + \alpha^2 \frac{\partial^2 b_{\frac{1}{2}}^{(i)}}{\partial \alpha^2} \right) + (2) \left(2\alpha^2 \frac{\partial^2 b_{\frac{1}{2}}^{(i)}}{\partial \alpha^2} + \alpha^3 \frac{\partial^3 b_{\frac{1}{2}}^{(i)}}{\partial \alpha^3} \right) + \text{etc.} \\ \frac{\partial^2 A}{\partial x^2} &= (0) \left(\alpha \frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial \alpha} + \alpha^2 \frac{\partial^2 b_{\frac{1}{2}}^{(i)}}{\partial \alpha^2} \right) + (1) \left(\alpha \frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial \alpha} + 3\alpha^2 \frac{\partial^2 b_{\frac{1}{2}}^{(i)}}{\partial \alpha^2} + \alpha^3 \frac{\partial^3 b_{\frac{1}{2}}^{(i)}}{\partial \alpha^3} \right) + \text{etc.} \end{aligned} \tag{9}$$

The derivatives of A being formed in this way, those of h are immediately deduced from the equations

$$\begin{aligned} \frac{\partial h}{\partial x} &= \Sigma E \frac{\partial A}{\partial x}, \\ \frac{\partial^2 h}{\partial x^2} &= \Sigma E \frac{\partial^2 A}{\partial x^2}. \end{aligned}$$

When s is equal to $\frac{3}{2}$, A is of the form

$$\alpha \left\{ (0) b_{\frac{3}{2}}^{(i)} + (1) \alpha \frac{\partial b_{\frac{3}{2}}^{(i)}}{\partial \alpha} + (2) \alpha^2 \frac{\partial^2 b_{\frac{3}{2}}^{(i)}}{\partial \alpha^2} + \text{etc.} \right\}$$

The quantity within parentheses is of the same form with A , in the case of $s = \frac{1}{2}$. If we represent it by A' we shall have

$$\begin{aligned} \frac{\partial A}{\partial x} &= \alpha \left(\frac{\partial A'}{\partial x} + A' \right) = \alpha \frac{\partial A'}{\partial x} + A \\ \frac{\partial^2 A}{\partial x^2} &= A + 2\alpha \frac{\partial A'}{\partial x} + \alpha \frac{\partial^2 A'}{\partial x^2} \end{aligned}$$

A' being the same form with A , the derivatives $\frac{\partial A'}{\partial x}$ and $\frac{\partial^2 A'}{\partial x^2}$ will be of the form (9), substituting $\frac{3}{2}$ for the index $\frac{1}{2}$, and (0)', (1)', etc., for (0), (1), etc.

In the case of $s = \frac{5}{2}$ the derivatives are obtained in the same way, which is too simple to need elucidation.

We have now to pass from the derivatives of h to those of $\frac{mh}{\alpha_1}$, the coefficients of the perturbative function. The form of these derivatives will depend not on whether the planet is disturbing or disturbed, but on whether it is an outer or

inner one. Let us then suppose for the present, that a and x refer to the inner planet, and put x_1 for the logarithm of the mean distance of the outer one. We then have for the derivatives relatively to x

$$\frac{\partial^n \frac{h}{a_1}}{\partial x^n} = \frac{1}{a_1} \frac{\partial^n h}{\partial x^n},$$

and for the first derivative relatively to x , using the symbolic notation,

$$\frac{\partial \frac{h}{a_1}}{\partial x_1} = \frac{1}{a_1} (D_{x_1} - 1) h.$$

The symbols in the second member being distributive, we have by successive differentiation

$$\frac{\partial^n \frac{h}{a_1}}{\partial x_1^n} = \frac{1}{a_1} (D_{x_1} - 1)^n h.$$

The quantity h is a function of α , the ratio of the mean distances or of $C^x - x_1$, C being the neperian base. Hence

$$D_{x_1} h = -D_x h,$$

which substituted in the last equation gives

$$\frac{\partial^n \frac{h}{a_1}}{\partial x_1^n} = \frac{(-1)^n}{a_1} (D_x + 1)^n h. \quad (10)$$

This formula gives for the first two derivatives

$$\begin{aligned} \frac{\partial \frac{h}{a_1}}{\partial x_1} &= -\frac{1}{a_1} \left(h + \frac{\partial h}{\partial x} \right) \\ \frac{\partial^2 \frac{h}{a_1}}{\partial x_1^2} &= \frac{1}{a_1} \left(h + 2 \frac{\partial h}{\partial x} + \frac{\partial^2 h}{\partial x^2} \right). \end{aligned}$$

Substituting in the general formulæ (7) these expressions for the derivatives relatively to x and x_1 we have expressions for the derivatives of R relatively to v , v' , ρ , ρ' , it being understood, however, that all the quantities are expressed in functions of the elements of elliptic motion.

In order to compute the perturbations of the second order we must carry R and such of its derivatives as enter into the differential equations (1) to quantities of the first order with respect to the perturbations. Let us then represent by v_0 , v'_0 , ρ_0 , ρ'_0 , γ_0 , the elliptic values of v , v' , ρ , ρ' , and γ , which we have assumed in the first approximation to the perturbations, and by δv , $\delta v'$, etc., the quantities to be

added to v_0, v'_0 , etc., to make the true values of v, v' , etc., whether perturbations or corrections of the elements. We shall then have

$$\begin{aligned} \delta R &= \frac{\partial R_0}{\partial v_0} \delta v + \frac{\partial R_0}{\partial v'_0} \delta v' + \frac{\partial R_0}{\partial \rho_0} \delta \rho + \frac{\partial R_0}{\partial \rho'_0} \delta \rho' + \frac{\partial R_0}{\partial \gamma_0} \delta \gamma \\ \delta \frac{\partial R}{\partial v} &= \frac{\partial^2 R_0}{\partial v_0^2} \delta v + \frac{\partial^2 R_0}{\partial v_0 \partial v'_0} \delta v' + \frac{\partial^2 R_0}{\partial v_0 \partial \rho_0} \delta \rho + \frac{\partial^2 R_0}{\partial v_0 \partial \rho'_0} \delta \rho' + \frac{\partial^2 R_0}{\partial v_0 \partial \gamma_0} \delta \gamma \\ \delta \frac{\partial R}{\partial \rho} &= \frac{\partial^2 R_0}{\partial \rho_0 \partial v_0} \delta v + \frac{\partial^2 R_0}{\partial \rho_0 \partial v'_0} \delta v' + \frac{\partial^2 R_0}{\partial \rho_0^2} \delta \rho + \frac{\partial^2 R_0}{\partial \rho_0 \partial \rho'_0} \delta \rho' + \frac{\partial^2 R_0}{\partial \rho_0 \partial \gamma_0} \delta \gamma \end{aligned} \quad (11)$$

The value of $D'_t R$ may be found either by equation (2), or by differentiating with respect to the time as introduced by the co-ordinates of the disturbed planet. When quantities of the first order only are considered the latter operation is very simple, but it is different when terms of the second order come in, because the true longitude of the planet is then expressed in terms not only of its own mean longitude, but also of the mean longitude of all the disturbing planets. The result can still be obtained in the same way by separating all the mean longitudes introduced by the co-ordinates of the disturbed planet from those introduced by the co-ordinates of the other until after the differentiation relatively to t .

Let us now resume the equation (4), representing its second member by μQ , so that it becomes

$$\frac{d^2 (r_0^2 \delta \rho)}{dt^2} + \frac{\mu (1+m)}{r_0^3} r_0^2 \delta \rho = \mu Q \quad (12)$$

where

$$Q = 2 \int D'_t R dt + \frac{\partial R}{\partial \rho} - \frac{1}{\mu} \frac{d^2 (r_0^2 \delta \rho^2)}{dt^2} + \frac{1}{2} \frac{\delta \rho^2}{r_0}$$

By the operations already given Q has become a known function of the time.

It is well known that the integration of (12) may be effected by finding two values of $r_0^2 \delta \rho$ which satisfy this equation when the second member is neglected, or, in other words, by finding two variables x and y which satisfy the equations

$$\begin{aligned} \frac{d^2 x}{dt^2} + \frac{\mu(1+m)}{r_0^3} x &= 0, \\ \frac{d^2 y}{dt^2} + \frac{\mu(1+m)}{r_0^3} y &= 0, \end{aligned}$$

when the required integral is

$$r_0^2 \delta \rho = \frac{\mu}{x \frac{dy}{dt} - y \frac{dx}{dt}} \left\{ y \int x Q dt - x \int y Q dt \right\}.$$

The above differential equations are satisfied by the rectangular co-ordinates of the planet in its assumed elliptic orbit. The position of the axes of co-ordinates being arbitrary we shall take the line of apsides for the axis of X , the perihelion being on the positive side. If we put

$$e_0 = \sin \psi,$$

we have

$$x \frac{dy}{dt} - y \frac{dx}{dt} = \sqrt{a\mu(1+m)} \cos \psi = \frac{\mu(1+m) \cos \psi}{an}$$

Let us, for convenience, replace x and y by two other variables ξ and η connected with them by the equations

$$\begin{aligned}x &= a\xi, \\y &= a\eta \cos \psi.\end{aligned}$$

ξ and η are then functions of the eccentricity and mean anomaly only, and may be developed according to the multiples of the latter. Substituting the last three expressions in the preceding value of $r_0^2 \delta\rho$ it becomes

$$r_0^2 \delta\rho = \frac{a^3 n}{1+m} \left\{ \eta \int \xi Q dt - \xi \int \eta Q dt \right\}.$$

If we put r_1^2 for the value of r_0 when the mean distance of the planet is put equal to unity, so that r_1 , like ξ and η contains only the eccentricity and mean anomaly, we shall have

$$\delta\rho = \frac{nr_1^{-2}}{1+m} \left\{ \eta \int \xi a Q dt - \xi \int \eta a Q dt \right\} \quad (13)$$

We must now express ξ and η in terms of the time, or of the mean anomaly. Putting for the present u for the eccentric and v for the true anomaly, we have, by the theory of the elliptic motion,

$$\begin{aligned}x &= r \cos v = a (\cos u - e), \\y &= r \sin v = a \cos \psi \sin u,\end{aligned}$$

from which follow

$$\begin{aligned}\xi &= \cos u - e, \\ \eta &= \sin u.\end{aligned}$$

As ξ and η are to be expressed in the form

$$\begin{aligned}\xi &= \frac{1}{2} \sum p_i \cos ig, \\ \eta &= \frac{1}{2} \sum q_i \sin ig,\end{aligned}$$

the finite integrals extending to all values of i from $-\infty$ to $+\infty$, we shall deduce general expressions from p_i and q_i arranged according to the power of the eccentricity. Since

$$u = g + e \sin u,$$

we have by Lagrange's theorem

$$\cos u = \cos g - e \sin^2 g - \frac{e^2}{2!} \frac{\partial \sin^3 g}{\partial g} - \frac{e^3}{3!} \frac{\partial^2 \sin^4 g}{\partial g^2} - \text{etc.};$$

or

$$\cos u = \sum_{n=0}^{n=\infty} \frac{e^n}{n!} \frac{\partial^{n-1} \sin^{n+1} g}{\partial g^{n-1}}$$

using the notation

$$n! = 1.2.3 \dots n = \Gamma(n+1).$$

We then have

$$0! = 1! = 1.$$

Substituting in the general term of the above series for $\sin g$ its value in imaginary exponential functions

$$2 \sin g = \sqrt{-1} (e^{-g\sqrt{-1}} - e^{g\sqrt{-1}})$$

we find by the binomial theorem, using the notation of combinations,

$$C_n^s = \frac{n(n-1)\dots(n-s+1)}{1.2.3\dots s} = \frac{n!}{s!(n-s)!}$$

$$2^{n+1} \sin^{n+1} g = (\sqrt{-1})^{n+1} \left\{ e^{-(n+1)g\sqrt{-1}} - C_{n+1}^1 e^{-(n-1)g\sqrt{-1}} + C_{n+1}^2 e^{-(n-3)g\sqrt{-1}} - \dots \right. \\ \left. + (-1)^n C_{n+1}^1 e^{(n-1)g\sqrt{-1}} + (-1)^{n+1} e^{(n+1)g\sqrt{-1}} \right\}$$

Differentiating $n-1$ times with respect to g , and putting together the first and last terms, the one after the first, and that before the last, and so on, we find

$$- 2^{n+1} \frac{\partial^{n-1} \sin^{n+1} g}{\partial g^{n-1}} = (n+1)^{n-1} (e^{(n+1)g\sqrt{-1}} + e^{-(n+1)g\sqrt{-1}}) \\ - C_{n+1}^1 (n-1)^{n-1} (e^{(n-1)g\sqrt{-1}} + e^{-(n-1)g\sqrt{-1}}) + \text{etc.}$$

Substituting for the exponentials their values in circular functions, and dividing by 2^{n+1} we have

$$\frac{\partial^{n-1} \sin^{n+1} g}{\partial g^{n-1}} = - \frac{1}{2^n} \left\{ (n+1)^{n-1} \cos(n+1)g - C_{n+1}^1 (n-1)^{n-1} \cos(n-1)g \right. \\ \left. + C_{n+1}^2 (n-3)^{n-1} \cos(n-3)g - \text{etc.} \right\}$$

the series terminating at the last positive coefficient of g . Substituting this last value in the general term of the series which gives $\cos u$, we have

$$\cos u = \sum_{n=0}^{n=\infty} \frac{e^n}{n! 2^n} \left\{ (n+1)^{n-1} \cos(n+1)g - C_{n+1}^1 (n-1)^{n-1} \cos(n-1)g + \text{etc.} \right\}$$

Let us now substitute for n another variable i , putting in the first term of the last factor $i = n+1$, in the second $i = n-1$, in the third $i = n-3$, etc. The limits of finite integration with respect to i will then be

$$\begin{array}{ll} \text{in the first} & \text{term, } +1 \text{ to } +\infty, \\ \text{in the second} & \text{term, } -1 \text{ to } +\infty, \\ \text{in the third} & \text{term, } -3 \text{ to } +\infty, \\ & \text{etc.} \qquad \qquad \text{etc.} \end{array}$$

But all the coefficients of g will then be i , and the formula supposes the factor of $\cos ig$ to vanish whenever i is zero or negative; whence, those elements of the finite integral in which i is negative must be omitted, and all the terms must be taken between the limits $+1$ and $+\infty$. Making the proposed substitution we have

$$\begin{aligned} \cos u &= \sum_{i=1}^{i=\infty} \left\{ \frac{i^{i-2}}{(i-1)! 2^{i-1}} e^{i-1} - \frac{i^i}{(i+1)! 2^{i+1}} C e^{i+1} + \frac{i^{i+2}}{(i+3)! 2^{i+3}} C^2 e^{i+3} - \text{etc.} \right\} \cos ig \\ &= \sum_{i=1}^{i=\infty} \frac{i^{i-2} e^{i-1}}{(i-1)! 2^{i-1}} \left\{ 1 - \frac{i^2 e^2}{2^2 i(i+1)} \frac{1}{C} + \frac{i^4 e^4}{2^4 i(i+1)(i+2)(i+3)} \frac{C^2}{C} - \text{e.c.} \right\} \cos ig \end{aligned}$$

We have, therefore, for all values of i different from zero

$$p_i = p_{-i} = \frac{i^{i-2} e^{i-1}}{(i-1)! 2^{i-1}} \left\{ 1 - \frac{1}{C} \frac{i e^2}{2^2 (i+1)} + \frac{C^2}{C} \frac{i^3 e^4}{2^4 (i+1)(i+2)(i+3)} - \text{etc.} \right\} \quad (14)$$

To obtain the value of p_0 we remark that the only constant term in $\cos u$ arises from the term $-e \sin^2 g$; its value is therefore $-\frac{1}{2}e$. The constant term in $\xi = \cos u - e$ is therefore $\frac{3}{2}e$, whence

$$p_0 = -3e. \quad (15)$$

The values of q_i may be obtained in a similar way by developing $\sin u$ by La Grange's theorem. But the development is rather more complex, and it is easier to derive them from p_i . Let us take up the equations

$$\begin{aligned} \xi &= \cos u - e \\ \eta &= \sin u \\ u - e \sin u &= g \end{aligned}$$

Considering u , like ξ and η , as a function of the independent variables e and g , we have by differentiation

$$\begin{aligned} \frac{\partial u}{\partial e} - \frac{\partial (e \sin u)}{\partial e} &= 0 \\ \therefore \frac{\partial u}{\partial e} &= \frac{\partial (e\eta)}{\partial e} = \frac{\sin u}{1 - e \cos u} \end{aligned} \quad (a)$$

$$\begin{aligned} \frac{\partial u}{\partial g} &= \frac{1}{1 - e \cos u} \\ \therefore \frac{\partial u}{\partial e} &= \sin u \frac{\partial u}{\partial g} = -\frac{\partial \xi}{\partial u} \frac{\partial u}{\partial g} = -\frac{\partial \xi}{\partial g} \end{aligned} \quad (b)$$

Comparing (a) and (b)

$$\frac{\partial \xi}{\partial g} = -\frac{\partial (e\eta)}{\partial e}$$

Putting in this equation for ξ and η their developed values this equation becomes

$$\sum i p_i \sin ig = \sum \frac{\partial (e q_i)}{\partial e} \sin ig$$

which gives by equating the coefficients of $\sin ig$

$$q_i = \frac{i}{e} \int p_i de. \quad (16)$$

The following are special values of p_i and q_i , developed to the sixth power of the eccentricities, as derived from the preceding formulæ :

$$\begin{aligned}
 p_0 &= -3e \\
 p_1 &= 1 - \frac{3}{8}e^2 + \frac{5}{192}e^4 - \frac{7}{9216}e^6 \\
 p_2 &= \frac{1}{2}e - \frac{1}{3}e^3 + \frac{1}{16}e^5 \\
 p_3 &= \frac{3}{8}e^2 - \frac{45}{128}e^4 + \frac{567}{5120}e^6 \\
 p_4 &= \frac{1}{3}e^3 - \frac{6}{15}e^5 \\
 p_5 &= \frac{125}{384}e^4 - \frac{4375}{9216}e^6 \\
 p_6 &= \frac{27}{80}e^5 \\
 p_7 &= \frac{16807}{46080}e^6 \\
 q_1 &= 1 - \frac{1}{8}e^2 + \frac{1}{192}e^4 - \frac{1}{9216}e^6 \\
 q_2 &= \frac{1}{2}e - \frac{1}{6}e^3 + \frac{1}{48}e^5 \\
 q_3 &= \frac{3}{8}e^2 - \frac{27}{128}e^4 + \frac{243}{5120}e^6 \\
 q_4 &= \frac{1}{3}e^3 - \frac{4}{15}e^5 \\
 q_5 &= \frac{125}{384}e^4 - \frac{3125}{9216}e^6 \\
 q_6 &= \frac{27}{80}e^5 \\
 q_7 &= \frac{16807}{46080}e^6
 \end{aligned} \tag{16}$$

Having the developed ξ and η in terms of time, let us resume the equation (13). As only purely linear operations are performed on Q in this equation, it follows that if we represent its several parts by Q_1, Q_2 , etc., and by $\delta\rho_1, \delta\rho_2$, etc., the values $\delta\rho$ obtained by putting $Q = Q_1, Q = Q_2$, etc., we shall have

$$\delta\rho = \delta\rho_1 + \delta\rho_2 + \text{etc.}$$

We have, therefore, only to find the separate values of $r_0^2\delta\rho$ corresponding to the different terms of Q , and to take their sum. Let us then represent, as before, by

$$\frac{m'}{a_1} h \cos (i'\lambda' + i\lambda + j'\omega' + j\omega)$$

any one term of R .

We then have, considering only terms of the first order with respect to the disturbing forces,

$$D_t R = -\frac{m'ihn}{a_1} \sin N, \quad (17)$$

$$\int D_t R = \frac{m'ihv}{a_1} \cos N;$$

$$\frac{\partial R}{\partial \rho} = m' \frac{\partial h}{\partial \rho} \cos N;$$

where we put for brevity,

$$v = \frac{n}{i'n' + in}$$

$$N = i'\lambda' + i\lambda + j'\omega' + j\omega.$$

Let us represent by Q_0 the terms in Q which are of the first order with respect to the disturbing forces, so that we have

$$Q_0 = 2 \int D_t R_0 + \frac{\partial R_0}{\partial \rho_0}.$$

The general term in R will then give rise in Q_0 to the term

$$m' \left[\frac{2ihv}{a_1} + \frac{\partial h}{\partial \rho} \right] \cos N.$$

In the case of the action of an outer on an inner planet this expression becomes

$$\frac{m'}{a_1} \left(2ivh + \frac{\partial h}{\partial \rho} \right) \cos N;$$

while in the contrary case it is

$$\frac{m'}{a_1} \left(2ivh - h - \frac{\partial h}{\partial \rho'} \right) \cos N,$$

both derivatives being taken with respect to the logarithm of the mean distance of the inner planet.

In the integration it will be more convenient to substitute for λ' and λ the mean longitudes counted from the perihelion of the disturbed planet. If we put

$$\lambda = g + \omega$$

$$\lambda' = l' + \omega$$

the angle N will become,

$$i'l' + ig + j'\omega' + (i + i' + j)\omega.$$

Since corresponding to each set of values of i' and i there are several values of j' and j , it will be convenient in the numerical computation to combine these different terms into a single one, because after forming the derivatives of R there is no need that ω , ω' and the other elements should appear in an analytical form. If we put

k for the coefficient of $\frac{m'}{a_1} \cos N$ in the preceding general term of Q_0 , this term will become

$$Q_0 = \frac{m'}{a_1} k \cos [j'\omega' + (i' + i + j)\omega] \cos [i'l + ig] \\ - \frac{m'}{a_1} k \sin [j'\omega' + (i' + i + j)\omega] \sin [i'l + ig]$$

If we put

$$k_c = \Sigma k \cos [j'\omega' + (i' + i + j)\omega], \\ k_s = \Sigma k \sin [j'\omega' + (i' + i + j)\omega],$$

the sign Σ being extended so as to include all values of j and j' which correspond to the given values of i and i' , we shall have for the general terms of Q_0

$$\frac{m'}{a_1} \left\{ k_c \cos (i'l + ig) + k_s \sin (i'l + ig) \right\},$$

or, when we represent the angle $i'l + ig$ by N_1

$$Q_0 = \frac{m'}{a_1} \left\{ k_c \cos N_1 + k_s \sin N_1 \right\}.$$

This we are to combine with the values of ξ and η

$$\xi = \frac{1}{2} \Sigma p_i \cos ig,^* \\ \eta = \frac{1}{2} \Sigma q_j \sin jg,$$

in the general integral formula (13). If we substitute them in this formula, and represent by μ the coefficient of t in the value of N we shall have to integrate differentials of the form

$$\frac{\sin (N_1 \pm ig)}{\cos (N_1 \pm ig)}.$$

in which the coefficient of the time t in the angle is $\mu + in$. Let us represent by v_i the integrating factor

$$\frac{n}{\mu + in}.$$

The formula (13) will become by these substitutions, which, though a little complex, offer no difficulty,

$$\delta\rho = \frac{1}{16} \frac{m'ar_1^{-2}}{a_1(1+m)} \sum_{i,j}^{\pm\infty} p_i q_j \times \\ \left\{ \begin{array}{l} \{v_{+j} - v_{+i}\} \{k_c \cos [N_1 + (i+j)g] + k_s \sin [N_1 + (i+j)g]\} \\ + \{v_{+i} - v_{-j}\} \{k_c \cos [N_1 + (i-j)g] + k_s \sin [N_1 + (i-j)g]\} \\ + \{v_{+j} - v_{-i}\} \{k_c \cos [N_1 - (i-j)g] + k_s \sin [N_1 - (i-j)g]\} \\ + \{v_{-i} - v_{-j}\} \{k_c \cos [N_1 - (i+j)g] + k_s \sin [N_1 - (i+j)g]\} \end{array} \right\}$$

The sign Σ of finite integration here includes the separate combination of every value of i with every value of j , except those combinations which make the

* The indices i and j , in these equations, are not to be confounded with the coefficients of λ and ω in the general terms of R and Q . We need not use the latter at present.

coefficient of the time under the sign *sin* or *cos* vanish, and so render the corresponding value of ν infinite. These cases have to be treated separately.

To find, from the expression, the coefficient of the sine or cosine of cosine $N_1 + ug$ in $r_1^2 \delta\rho$, we put, in the four lines of this equation, as follows:

$$\begin{aligned} \text{In the first,} & \quad i + j = u \quad \therefore j = u - i; \\ \text{" second,} & \quad i - j = u \quad \therefore j = i - u; \\ \text{" third,} & \quad -i + j = u \quad \therefore j = u + i; \\ \text{" fourth,} & \quad -i - j = u \quad \therefore j = -u - i. \end{aligned}$$

In the above expressions i and j being independent, and including all values from $-\infty$ to $+\infty$, i and u will also be independent, and include the same range of values. Substituting for j its value in u the coefficient of

$$\frac{1}{16 a_1 (1+m)} [k_c \cos(N_1 + ug) + k_s \sin(N_1 + ug)]$$

becomes

$$\Sigma \left\{ \begin{aligned} & p_i q_{(u-i)} (v_{(u-i)} - v_i) \\ & + p_i q_{(i-u)} (v_i - v_{(u-i)}) \\ & + p_i q_{(u+i)} (v_{(u+i)} - v_{-i}) \\ & + p_i q_{-(u+i)} (v_{-i} - v_{(u+i)}) \end{aligned} \right.$$

Since $q_j = -q_{-j}$ this expression reduces immediately to

$$2\Sigma \left\{ \begin{aligned} & p_i q_{(i-u)} (v_i - v_{(u-i)}) \\ & + p_i q_{(i+u)} (v_{(u+i)} - v_{-i}) \end{aligned} \right.$$

or, substituting $i - u$ for i in the second line

$$2\Sigma (p_i q_{(i-u)} + p_{(i-u)} q_i) (v_i - v_{u-i}).$$

Hence, writing N instead of N_1 ,

$$\delta\rho = \frac{1}{8} \frac{m' a r_1^{-2}}{a_1 (1+m)} \Sigma_{i,u}^2 (p_i q_{(i-u)} + p_{(i-u)} q_i) (v_i - v_{(u-i)}) [k_c \cos(N + ug) + k_s \sin(N + ug)] \quad (19)$$

This expression fails for the particular case $N = ug$, where the value of v_{-u} will be infinite. If we take each term of Q of the form

$$\frac{m'}{a_1} (k_c^{(u)} \cos ug + k_s^{(u)} \sin ug),$$

and substitute in the general expression (13) it will be found that the terms in $r_1^2 \delta\rho$ which have the infinite values of ν as a factor are to be omitted, and replaced by

$$r_1^2 \delta\rho = \frac{1}{2} \frac{m' a n t}{a_1 (1+m)} \{ \eta \Sigma p_u k_c^{(u)} - \xi \Sigma q_u k_s^{(u)} \} \quad (20)$$

The two parts of $r_1^2 \delta\rho$ thus found include all the terms of the first order with respect to the disturbing forces. But when terms of the second order are taken into account, we shall find terms in Q proceeding from secular variation in which the time appears as a factor, outside the signs *sin* and *cos*. Let us represent such of these terms as depend on any angle N by

$$Q = \frac{m' n t}{a_1} (k_c \cos N + k_s \sin N)$$

and use the symbol v , as before, to represent the ratio of the mean motion of the planet to the coefficient of t in the angle $N + ig$, so that if μ' represents the coefficients of t in N we have

$$v = \frac{n}{\mu'},$$

$$v_i = \frac{n}{\mu' + in} = \frac{1}{\frac{\mu'}{n} + i},$$

we find the expression

$$\int \xi Q dt = \frac{1}{4} \frac{m' \sum p_i}{a_1 n} \left\{ \begin{aligned} &(v_i^2 k_c - v_i k_s n t) \cos(N + ig) + (v_{-i}^2 k_c - v_{-i} k_s n t) \cos(N - ig) \\ &(v_i^2 k_s + v_i k_c n t) \sin(N + ig) + (v_{-i}^2 k_s + v_{-i} k_c n t) \sin(N - ig) \end{aligned} \right\}$$

$$\int \eta Q dt = \frac{1}{4} \frac{m' \sum q_i}{a_1 n} \left\{ \begin{aligned} &(v_j^2 k_c - v_j k_s n t) \sin(N + jg) - (v_{-j}^2 k_c - v_{-j} k_s n t) \sin(N - jg) \\ &-(v_j^2 k_s + v_j k_c n t) \cos(N + jg) + (v_{-j}^2 k_s + v_{-j} k_c n t) \cos(N - jg) \end{aligned} \right\}.$$

If we now put for brevity

$$v_i^2 k_s + v_i k_c n t = e_i,$$

$$v_i^2 k_c - v_i k_s n t = s_i,$$

the general value of $r_0^2 \delta \rho$ becomes

$$\delta \rho = \frac{1}{16} \frac{m' a r_1^{-2}}{a_1 (1+m)} \times$$

$$\sum^2 p_i q_j \left\{ \begin{aligned} &(e_j - e_i) \cos(N + (i+j)g) + (s_i - s_j) \sin(N + (i+j)g) \\ &+ (e_i - e_{-j}) \cos(N + (i-j)g) + (s_{-j} - s_i) \sin(N + (i-j)g) \\ &+ (e_j - e_{-i}) \cos(N - (i-j)g) + (s_{-i} - s_j) \sin(N - (i-j)g) \\ &+ (e_{-i} - e_{-j}) \cos(N - (i+j)g) + (s_{-j} - s_{-i}) \sin(N - (i+j)g) \end{aligned} \right\}$$

If, as before, we transform this expression by putting

$$\begin{aligned} \text{in the first line } & j = u - i; \\ \text{in the second } & \text{“ } j = i - u; \\ \text{in the third } & \text{“ } j = i + u; \\ \text{in the fourth } & \text{“ } j = i - u. \end{aligned}$$

the value of $r_1^2 \delta \rho$ reduces to

$$\frac{1}{8} \frac{m' a}{a_1 (1+m)} \times$$

$$\sum_{i,u}^2 \left\{ \begin{aligned} &p_i q_{(u-i)} (e_{(u-i)} - e_i) \cos(N + ug) + p_i q_{(u-i)} (s_i - s_{(u-i)}) \sin(N + ug) \\ &p_i q_{(u+i)} (e_{(u+i)} - e_{-i}) \cos(N + ug) + p_i q_{(u+i)} (s_i - s_{(u+i)}) \sin(N + ug) \end{aligned} \right\} \quad (21)$$

or, putting $i - u$ for i in the last line,

$$\delta \rho = \frac{1}{8} \frac{m' a r_1^{-2}}{a_1 (1+m)} \times$$

$$\sum_{u,i}^2 \{ p_i q_{(u-i)} - p_{(u-i)} q_i \} \{ (e_{(u-i)} - e_i) \cos(N + ug) + (s_i - s_{u-i}) \sin(N + ug) \};$$

to which expression is to be added, in lieu of the terms which will have infinite values of ν as a factor.

$$\frac{1}{4} \frac{m' a r_1^{-2} n^2 t^2}{a_1 (1+m)} \{ \eta \Sigma p_u k_c^{(u)} - \xi \Sigma q_u k_s^{(u)} \} \quad (22)$$

$k_c^{(u)}$ and $k_s^{(u)}$ being the factors of $\frac{m'}{a_1} n t \cos ug$ and $\frac{m'}{a_1} n t \sin ug$ in the expression for Q .

The formulæ 19, 20, 21, and 22 give the complete expressions for the perturbations of the logarithm of radius vector by successively substituting in it all the terms of Q .

Perturbations of Longitude.

We now pass to the perturbations of longitude. In the *Mécanique Céleste* (Première Partie, Liv. ii. Chap. vi.), Laplace gives an equation (Y) by which the perturbations of longitude, which are of the first order, may be derived from those of the radius vector without the formation of any other derivatives of R than those which enter into Q . But the formula does not seem easily adapted to the case in which the perturbations of the second order are taken into account, we shall therefore derive all the perturbations of longitude from the second of equations (1). By integration this equation gives

$$\frac{dv}{dt} = \frac{\mu}{r^2} \left\{ \int \frac{\partial R}{\partial v} dt + C \right\}$$

C being the arbitrary constant of the integral. Representing, as before, by subscript zeros the values of the co-ordinates corresponding to the ellipse to which the orbit is supposed to reduce itself when the disturbing forces vanish, we have

$$\frac{dv_0}{dt} = \frac{a^2 n \cos \psi}{r_0^2} = \frac{\mu C}{r_0^2},$$

because the constant to which the integral must reduce itself in the elliptic motion is $\frac{a^2 n \cos \psi}{\mu}$. Subtracting the last equation from the preceding, and putting $v - v_0 = \delta v$, we find

$$\frac{d\delta v}{dt} = \frac{\mu}{r^2} \int \frac{\partial R}{\partial v} dt + \left(\frac{1}{r^2} - \frac{1}{r_0^2} \right) a^2 n \cos \psi.$$

Developing $\frac{1}{r^2}$ to terms of the second order with respect to the disturbing force

$$\frac{1}{r^2} = \frac{1}{r_0^2} (1 - 2\delta\rho + 2\delta\rho^2 - \text{etc.}),$$

which, being substituted in the last equation by putting

$$\begin{aligned} \mu &= \frac{a^3 n^2}{1+m}, \\ r_0^2 &= a^2 r_1^2, \end{aligned}$$

gives

$$r_1^2 \frac{d\delta v}{dt} = \frac{a n^2}{1+m} (1 - 2\delta\rho) \int \frac{\partial R}{\partial v} dt - 2n \cos \psi (\delta\rho - \delta\rho^2), \quad (23)$$

which is rigorous to quantities of the second order.

The most convenient mode of making the numerical computation of the second order terms by means of this equation will depend upon circumstances. If the perturbations of longitude and radius vector of both planets are already known with a sufficient degree of approximation for the computation of formula (11), it will be more convenient to form at once the complete values of all the quantities which enter into the equations (12), (13), (19) to (22), and (23), so that no steps of the process shall have to be repeated. If such perturbations are not known, they must first be computed, and it will then be necessary to begin with the perturbations of the first order, and afterward add those of the second. There is, however, one class of terms of the second order which it will be most convenient to take account of from the beginning, namely, those arising from the constant term in $\delta\rho$ and $\delta\rho'$. This is effected by correcting the mean distances for an approximate value of these constants at the beginning of the computation, and then proceeding in the usual way. This is in fact what we have supposed to be done in the preceding investigation. The values of δv , $\delta v'$, $\delta\rho$, $\delta\rho'$ in formula (11) will then contain only periodic terms.

In computing the terms of the first order we determine the value of $\delta\rho$ from the equations (19) and (20), using the value of Q_0 in (18). Then those of δv are obtained by integrating the equation

$$\frac{d\delta v}{dt} = \frac{an^2r_1^{-2}}{1+m} \int \frac{\partial R_0}{\partial v} dt - 2n \cos \psi \frac{\delta\rho}{r_1^2}. \quad (24)$$

Having found the values of δv and $\delta\rho$ for both planets, they are to be substituted in (11), to obtain δR , $\delta \frac{\partial R}{\partial v}$ and $\delta \frac{\partial R}{\partial \rho}$. But, rigorously, δv and $\delta v'$ are not the same with δv and $\delta v'$, owing to the movement of the orbits of the planets, and the corrections for $\delta\gamma$ are also to be added. Considering, for the present, only the perturbations of the second order, which depend on δv , $\delta v'$, $\delta\rho$, and $\delta\rho'$, we may use the following equation for δR , and similar ones for its derivatives:

$$\delta R = \frac{\partial R}{\partial v} \delta v + \frac{\partial R}{\partial v'} \delta v' + \frac{\partial R}{\partial \rho} \delta\rho + \frac{\partial R}{\partial \rho'} \delta\rho'. \quad (25)$$

Having thus found δR , and hence $D_t \delta R$ by differentiation, and then $\delta \frac{\partial R}{\partial \rho}$, we form the quantity

$$\delta Q = 2 \int D_t \delta R dt + \delta \frac{\partial R}{\partial \rho} - \frac{1}{\mu} \frac{d^2(r_0^2 \delta\rho^2)}{dt^2} + \frac{1}{2} \frac{\delta\rho^2}{r_0} \quad (26)$$

which is the difference between the value of Q_0 in (18) and that of Q in (12). The terms in $\delta\rho$ arising from δQ are then to be computed by the formulæ (19), (20), (21), and (22), when we shall have $\delta\rho$ accurate to quantities of the second order. Let us represent these additional terms by $\delta^2\rho$. Subtracting (24) multiplied by r_1^2 from (23), recollecting that the $\delta\rho$ which appears in the second term of the former is really $\delta\rho - \delta^2\rho$, we find, neglecting quantities of the third order,

$$r_1^2 \frac{d\delta^2 v}{dt} = an^2 \left\{ \int \delta \frac{\partial R}{\partial v} dt - 2\delta\rho \int \frac{\partial R_0}{\partial v} dt \right\} - 2n \cos \psi (\delta^2\rho - \delta\rho^2)$$

from which the terms of δv of the second order are obtained by multiplying by r_1^{-2} and integrating.

Motion of the Orbital Planes.

The general theory of the motion of the planes of reference, especially of the motion of the instantaneous orbit, has been so often treated that I can scarcely hope to add anything essentially new to it. I shall, however, endeavor to present the differential equations of the motion in a simple and general form, and one in which the geometrical conceptions of the problem shall be made as clear as possible.

The orbital plane of each planet being at each moment osculatory to that part of the orbit which the planet is actually describing, its only motion is one of rotation around the radius vector of the planet as an instantaneous axis. This rotation may be resolved into two others around any pair of rectangular axes fixed in the moving plane. But the rotation produced by any one planet is most simply expressed when referred to axes, one of which coincides with the common node of the two orbits. The rotation produced by each separate planet must, therefore, be first referred to its node on the moving orbit, and then the combined rotations must be resolved into two around axes assumed at pleasure. To effect this, let us suppose positive rotation around an axis to be such that an observer looking from the origin along the positive direction of the axis sees the right hand side of the plane move downwards, and the left hand side upwards. Let us also denote the first axis in the order of longitude the principal axis, or that of X , and that 90° farther advanced the secondary axis, or that of Y . Let us now put

dq , the instantaneous rotation around the axis of X ;

$d\rho$, the instantaneous rotation around the axis of Y . Let us also put, relatively to any disturbing planet,

$d\eta$, the instantaneous rotation around the ascending node of the disturbing planet on the orbit of the disturbed one.

dk , that around the corresponding secondary axis.

Then, from the known equations for the perturbations of the inclination and node of an orbit, we find, that, if any term of the perturbative function be represented, as before, by

$$\frac{m'h}{a_1} \cos (i'\lambda' + i\lambda + j'\omega' + j\omega),$$

the differential rotations η and k will be given by the equations

$$\frac{d\eta}{dt} = -\frac{m'h}{a_1} \frac{an}{\cos \psi} \left\{ (i+j) \cot \gamma + (i'+j') \operatorname{cosec} \gamma \right\} \sin N$$

$$\frac{dk}{dt} = -\frac{m'an}{a_1 \cos \psi} \frac{\partial h}{\partial \gamma} \cos N.$$

As R is actually developed, the mutual inclination γ does not explicitly appear, but is replaced by

$$\sigma = \sin \frac{1}{2} \nu.$$

Making this substitution, and putting also

$$i'' + i + j'' + j = -i.$$

these equations become

$$\begin{aligned} \frac{d\eta}{dt} &= \frac{m'an}{a_1 \cos \psi \cos \frac{1}{2}\gamma} \left\{ \frac{dh}{2\sigma} + (i + j) \sigma h \right\} \sin N \\ \frac{dk}{dt} &= - \frac{m'an \cos \frac{1}{2}\gamma}{2a_1 \cos \psi} \frac{\partial h}{\partial \sigma} \cos N. \end{aligned} \tag{27}$$

To pass to the general rotations dp and dq , let us represent by θ_1, θ_2 , etc., the longitudes of the ascending nodes of the several orbits of the disturbing planets on that of the disturbed planet. We shall then have

$$\begin{aligned} \frac{dq}{dt} &= \Sigma \cos \theta_i \frac{d\eta_i}{dt} - \Sigma \sin \theta_i \frac{dk_i}{dt} \\ \frac{dp}{dt} &= \Sigma \cos \theta_i \frac{dk_i}{dt} + \Sigma \sin \theta_i \frac{d\eta_i}{dt}. \end{aligned} \tag{28}$$

These equations completely define the instantaneous motion of the orbital plane. They cannot, however, be rigorously integrated in their present form because p and q as integrals have no completely defined signification. To do this it is necessary to express the differential rotations dp, dq , etc., in terms of the differentials of any elements we may select to define the position of the orbital plane, and then to integrate the equations thus formed. But, for the purpose of constructing tables of the planets we may consider p, q , etc., to represent small rotations of the planes of which the powers and products may be neglected, and the integration is then quite simple.

Perturbations of the second order depending on the motion of the orbital planes.

R being a function of the five quantities of r, r', v, v' , and γ , the motion of the orbital planes introduces terms of the second order by changing the values of v, v' , and γ . These terms we have hitherto neglected. To investigate them let us refer the rotations of both planes as given by (28) to the node of the disturbing on the disturbed planet as the principal axis. If we represent by $d\eta, dk, d\eta',$ and dk' the rotations corresponding to this axis, and designate by the subscript 1, the quantities which refer to the disturbing planet whose action we are considering, and by 2, 3, etc., the other planets, the equations (28) will be replaced by these

$$\begin{aligned} \frac{d\eta}{dt} &= \frac{d\eta_1}{dt} + \Sigma \cos (\theta_i - \theta_1) \frac{d\eta_i}{dt} - \Sigma \sin (\theta_i - \theta_1) \frac{dk_i}{dt}, \\ \frac{dk}{dt} &= \frac{dk_1}{dt} + \Sigma \cos (\theta_i - \theta_1) \frac{dk_i}{dt} + \Sigma \sin (\theta_i - \theta_1) \frac{d\eta_i}{dt}, \end{aligned}$$

the summation commencing with $i = 2$.

By formulæ of the same kind we are to find the differential rotations $d\eta'$ and dk' of the orbit of the disturbing planet, produced by the action of all the planets.

These rotations will be around the same principal axis with the rotations $d\eta$ and dk , but around a secondary axis in the plane of the disturbing orbit, and therefore making an angle γ with the secondary axis of the disturbed orbit. A geometrical construction will now show quite simply that the infinitesimal rotations $\delta\eta$, δk , $\delta\eta'$, and $\delta k'$ will produce the following changes in v , v' , and γ .

$$\begin{aligned}\delta v &= \cot \gamma \delta k - \operatorname{cosec} \gamma \delta k' \\ \delta v' &= \operatorname{cosec} \gamma \delta k - \cot \gamma \delta k' \\ \delta \gamma &= \delta \eta' - \delta \eta\end{aligned}\tag{29}$$

If we substitute these values in the general formulæ (11) the terms of the second order added to δR will be

$$\begin{aligned}\delta R &= \left(\frac{\partial R}{\partial v} \cot \gamma + \frac{\partial R}{\partial v'} \operatorname{cosec} \gamma \right) \delta k \\ &\quad - \left(\frac{\partial R}{\partial v} \operatorname{cosec} \gamma + \frac{\partial R}{\partial v'} \cot \gamma \right) \delta k' \\ &\quad + \frac{\partial R}{\partial \gamma} (\delta \eta' - \delta \eta).\end{aligned}\tag{30}$$

The first two terms of this expression may be put into the form

$$\begin{aligned}&\left\{ \frac{1}{2} \left(\frac{\partial R}{\partial v} + \frac{\partial R}{\partial v'} \right) (\operatorname{cosec} \gamma + \cot \gamma) - \frac{1}{2} \left(\frac{\partial R}{\partial v} - \frac{\partial R}{\partial v'} \right) (\operatorname{cosec} \gamma - \cot \gamma) \right\} \delta k \\ &- \left\{ \frac{1}{2} \left(\frac{\partial R}{\partial v} + \frac{\partial R}{\partial v'} \right) (\operatorname{cosec} \gamma + \cot \gamma) + \frac{1}{2} \left(\frac{\partial R}{\partial v} - \frac{\partial R}{\partial v'} \right) (\operatorname{cosec} \gamma - \cot \gamma) \right\} \delta k' .\end{aligned}$$

But,

$$\operatorname{cosec} \gamma + \cot \gamma = \cot \frac{1}{2} \gamma = \frac{\cos \frac{1}{2} \gamma}{\sigma}.$$

$$\operatorname{cosec} \gamma - \cot \gamma = \tan \frac{1}{2} \gamma = \frac{\sigma}{\cos \frac{1}{2} \gamma}.$$

and in the general term of R , by (7)

$$\frac{\partial R}{\partial v} = -\frac{m'h}{a_1} (i+j) \sin N$$

$$\frac{\partial R}{\partial v'} = -\frac{m'h}{a_1} (i'+j') \sin N.$$

Making these substitutions, and putting, as before,

$$i+j+i'+j' = -\iota$$

the above value of δR reduces to

$$\begin{aligned}\delta R &= \frac{m'h}{2a_1} \{ \iota \cot \frac{1}{2} \gamma (\delta k - \delta k') + (i+j-i'-j') \tan \frac{1}{2} \gamma (\delta k + \delta k') \} \sin N \\ &\quad + \frac{m' \cos \frac{1}{2} \gamma}{2a_1} \frac{\partial h}{\partial \sigma} (\delta \eta' - \delta \eta) \cos N\end{aligned}\tag{31}$$

The corresponding terms of $\delta \frac{\partial R}{\partial v}$ and $\delta \frac{\partial R}{\partial \rho}$, and may be obtained in the same way by substituting $\frac{\partial R}{\partial v}$ and $\frac{\partial R}{\partial \rho}$ for R in (30) and continuing the corresponding substitutions of the general terms of the derivatives of R as given on page 9.

The equation (31), besides being of the second order with respect to the disturbing forces, is also of the second order with respect to the mutual inclinations. For δk , $\delta k'$, $\delta \eta$, and $\delta \eta'$ are of the first order with respect to both quantities, and, whenever ι is not zero, h is a quantity of the second order, containing σ^2 as a factor. It is, therefore, only in exceptional cases that the terms of the second order depending on the motion of the orbital planes can become sensible.

Reduction of the longitude in the orbit to longitude on the ecliptic.

The integration of (23) gives a value of δv , which, added to the longitude in orbit corresponding to the pure elliptic motion gives the longitude in the disturbed orbit, counted from a fixed point in the moving plane of that orbit. The position of this fixed point is completely determined by the condition that the instantaneous rotation of the plane in question around the axis perpendicular to itself is always zero, so that the motion of the point of reference is always perpendicular to the direction of the plane. But, although this instantaneous rotation is zero, the integrated rotation is not rigorously zero when we consider the terms of the second order. It follows that the value of v , the longitude in orbit, and the position of the plane of the orbit do not rigorously determine the position of the planet: we must also know how the fixed point of reference has changed its position in consequence of the motions which the plane has undergone. Let us consider the relative positions of this plane at two epochs. If the fixed point were equally distant from the common node of the two planes, the integrated rotation of the plane around its own axis would be zero. But, these distances not being equal, their difference is a correction to be applied to the longitude of the planet in its orbit. Suppose, now, that at the end of any time the inclination of the actual orbit to the primitive orbit is ϕ , and the distance of its ascending node from the present position of the moving axis of x is θ . A rotation around the line of nodes will not change the quantity sought. But, if we represent the infinitesimal rotation around an axis perpendicular to it by dr we shall have

$$\cos \theta dp - \sin \theta dq = dr,$$

dq and dk being the instantaneous rotations around the respective axes of x and γ . By this rotation it is easy to see that the relative distance of any two fixed points, one on each plane, from the node, will be altered by the quantity,

$$dr (\operatorname{cosec} \phi - \cot \phi) = dr \tan \frac{1}{2} \phi,$$

the relative longitude of the fixed point on the moving plane being increased by this amount. The correction to the longitude in orbit from this cause is, therefore,

$$dl = dr \tan \frac{1}{2} \phi = \tan \frac{1}{2} \phi (\cos \theta dp - \sin \theta dq).$$

Counting the integrated values of p and q in a direction perpendicular to the moving plane we have

$$\begin{aligned}\sin \theta &= \frac{\tan p}{\tan \phi} \\ \cos \theta &= \frac{\tan q}{\tan \phi}\end{aligned}$$

which, being substituted in the expression for dl , gives

$$dl = \frac{\cos \phi}{1 + \cos \phi} (\tan q dp - \tan p dq).$$

The approximate value of the integrated correction is therefore

$$\delta l = \frac{1}{2} \int (q dp - p dq). \quad (32)$$

For every pair of periodic terms in p and q , such as

$$q = s \sin \mu t, \quad p = s \cos \mu t,$$

δl will contain the secular term $-\frac{1}{2} s^2 \mu t$, which will be confounded with the mean motion, and, if it were not so confounded, would in few or none of the larger planets amount to a second in a thousand years. If the secular terms in p and q be

$$q = st; \quad p = s't$$

δl will vanish. We hence conclude that these terms are entirely unimportant in the present state of astronomy, and that, if we consider the positions of the plane of the orbit at two epochs, we may consider the points of departure in them to be equally distant from their common node.

We have therefore only to consider the motion of the inclination and node due to the change of the position of the orbit and of the ecliptic. If we put
 ϕ , the inclination of the orbit of the planet to the ecliptic,
 θ , the longitude of its node counted on the ecliptic,
 τ , the longitude of the same node counted from the same fixed point in the moving plane of the orbit from which v is counted,

Then, the longitude of the planet on the ecliptic, or L , will be given by the equation

$$\tan (L - \theta) = \cos \phi \tan (v - \tau),$$

or, when developed in powers of ϕ ,

$$L = v + \theta - \tau + D, \quad (33)$$

where D is the reduction to the ecliptic, the value of which is

$$D = -\tan^2 \frac{1}{2} \phi \sin 2(v - \tau) + \frac{1}{2} \tan^4 \frac{1}{2} \phi \sin 4(v - \tau) - \text{etc.}$$

Let us refer the instantaneous rotations of the orbit and of the ecliptic to the fixed points of reference in the two planes; q being the rotation around an axis passing through the sun and the fixed point, and p that around an axis in 90° greater longitude, and the accented quantities referring to the ecliptic. We then have

$$\begin{aligned}
\frac{d\phi}{dt} &= \cos \tau \frac{dq}{dt} + \sin \tau \frac{dp}{dt} \\
&\quad - \cos \theta \frac{dq'}{dt} - \sin \theta \frac{dp'}{dt} \\
\frac{d\theta}{dt} &= \operatorname{cosec} \phi \left(-\sin \tau \frac{dq}{dt} + \cos \tau \frac{dp}{dt} \right) \\
&\quad + \cot \phi \left(\sin \theta \frac{dq'}{dt} - \cos \theta \frac{dp'}{dt} \right) \\
\frac{d\tau}{dt} &= \cot \phi \left(-\sin \tau \frac{dq}{dt} + \cos \tau \frac{dp}{dt} \right) \\
&\quad + \operatorname{cosec} \phi \left(\sin \theta \frac{dq'}{dt} - \cos \theta \frac{dp'}{dt} \right)
\end{aligned} \tag{34}$$

If we differentiate (33) and substitute these values of $\frac{d\theta}{dt}$ and $\frac{d\tau}{dt}$, we shall have

$$\frac{dL}{dt} = \frac{dv}{dt} + \frac{dD}{dt} - \tan \frac{1}{2} \phi \left(\cos \tau \frac{dp}{dt} - \sin \tau \frac{dq}{dt} + \cos \theta \frac{dp'}{dt} - \sin \theta \frac{dq'}{dt} \right) \tag{35}$$

If we consider only quantities of the first order with respect to the disturbing forces, we may, in integrating, suppose τ and θ equal and constant, and ϕ constant. The integral will then be

$$L = v + D + \tan \frac{1}{2} \phi \{ \cos \theta (\delta k + \delta k') - \sin \theta (\delta \gamma + \delta \gamma') \} \tag{36}$$

In the case of Uranus, $\tan \phi$ is so small that this equation will be sufficient for a long time before and after our epoch.

In the application of the method to other planets the mode of operation must depend on the circumstances of each particular case. The differential equations (34) between θ , τ , and ϕ are rigorous, and their integrals may be approximated to in various ways, out of which that best applicable to the particular case must be selected.

Expressions for the latitude.

If the position of the orbital plane and of the ecliptic were each determined by the preceding formulæ, there would be no perturbations of the latitude, the latitude itself being given rigorously by the equation

$$\begin{aligned}
\sin \beta &= \sin \phi \sin (v - \tau). \\
&= \sin \phi \cos \tau \sin v - \sin \phi \sin \tau \cos v.
\end{aligned}$$

But the instantaneous values of ϕ and τ , or of $\sin \phi \cos \tau$ and $\sin \phi \sin \tau$, are troublesome to tabulate; it will therefore, in practice, be found more convenient to use only their mean values, and to consider their changes from this mean as perturbations of the latitude. Representing by the sign δ the deviations from the mean values, which are of course arbitrary, we have

$$\cos \beta \delta \beta = \cos \phi \sin (v - \tau) \delta \phi - \sin \phi \cos (v - \tau) \delta \tau.$$

Let us substitute for $\delta \phi$ and $\delta \tau$ their values given by the integration of (34) to

quantities of the first order, in which case θ and τ may be assumed equal. These values are

$$\begin{aligned}\delta\phi &= \sin\tau\delta p + \cos\tau\delta q \\ \sin\phi\delta\tau &= \cos\phi(\cos\tau\delta p - \sin\tau\delta q)\end{aligned}$$

the terms dependent on $\delta p'$ and $\delta q'$ being omitted because, being purely secular, they may be included in the mean values of ϕ and τ . Substituting in the expression for $\delta\beta$

$$\cos\beta\delta\beta = \cos\phi\{\sin v\delta q - \cos v\delta p\}. \quad (37)$$

In the case of all the larger planets both $\cos\beta$ and $\cos\phi$ may here be put equal to unity, when the expression for $\delta\beta$ will become

$$\delta\beta = \sin v\delta q - \cos v\delta p. \quad (38)$$

To develop this expression in purely periodic terms we must substitute for v its value in terms of the mean longitude or mean anomaly, namely,

$$v = l + 2e\sin g + \frac{5}{4}e^2\sin 2g + \text{etc.};$$

suppose the terms of δp and δq depending on any argument, N to be

$$\begin{aligned}\delta p &= -a_s\sin N - a_c\cos N \\ \delta q &= a'_s\sin N + a'_c\cos N\end{aligned} \quad (39)$$

and put π for the longitude of the perihelion, so that

$$l = \pi + g$$

then, to terms of the first order with respect to the eccentricities, we have

$$\begin{aligned}\delta\beta &= -e(a_s\cos\pi + a'_s\sin\pi)\sin N - e(a_c\cos\pi + a'_c\sin\pi)\cos N \\ &+ \frac{1}{2}\{(a_s + a'_c)\cos\pi + (a'_s - a_c)\sin\pi\}\sin(N + g) \\ &+ \frac{1}{2}\{(a_c - a'_s)\cos\pi + (a'_c + a_s)\sin\pi\}\cos(N + g) \\ &+ \frac{1}{2}\{(a_s - a'_c)\cos\pi + (a'_s + a_c)\sin\pi\}\sin(N - g) \\ &+ \frac{1}{2}\{(a_c + a'_s)\cos\pi + (a'_c - a_s)\sin\pi\}\cos(N - g) \\ &+ \frac{1}{2}e\{(a_s + a'_c)\cos\pi + (a'_s - a_c)\sin\pi\}\sin(N + 2g) \\ &+ \frac{1}{2}e\{(a_c - a'_s)\cos\pi + (a'_c + a_s)\sin\pi\}\cos(N + 2g) \\ &+ \frac{1}{2}e\{(a_s - a'_c)\cos\pi + (a'_s + a_c)\sin\pi\}\sin(N - 2g) \\ &+ \frac{1}{2}e\{(a_c + a'_s)\cos\pi + (a'_c - a_s)\sin\pi\}\cos(N - 2g)\end{aligned} \quad (40)$$

The point of the orbit from which π and v are counted is entirely arbitrary, and, in considering the action of but a single planet, it will be most convenient to count them from the common node, in which case π must be replaced by ω , and δp and δq by δk and $\delta\eta$. Thus, deducing the perturbations of the latitude immediately from the formulæ (27), we shall have

$$\delta\beta = \sin v\delta\eta - \cos v\delta k.$$

CHAPTER II.

APPLICATION OF THE PRECEDING METHOD TO THE COMPUTATION OF THE
PERTURBATIONS OF URANUS BY SATURN.*Data of Computation.*

THE elements of Uranus, adopted in this computation, were deduced from the comparison of nine normal heliocentric longitudes at intervals of 3697 days extending from 1781, December 26, to 1862, December 18, with corresponding provisional places derived from the elements given in the "Investigation of the Orbit of Neptune," with perturbations produced by Jupiter, Saturn, and Neptune. As the perturbations are to be entirely re-computed, and the elements to be re-corrected from more extended series of observations, all the details of this first approximation will be omitted. The resulting elements of Uranus are given in the following table, together with the adopted elements of Saturn, which are nearly the same as those employed in the theory of Neptune, except that the inclination and longitude of the node have been corrected to agree with observations:—

| | Elements II. of Uranus. | Elements I. of Saturn. |
|-----------------|-------------------------|------------------------|
| π | 168° 16' 31" | 90° 4' 0" |
| ε | 28 25 36.0 | 14 48 45.0 |
| θ | 73 11 58 | 112 20 0 |
| ϕ | 0 46 20 | 2 29 39.2 |
| e | .0469276 | .0560050 |
| e in seconds, | 9679.5 | 11551.9 |
| n | 15426.10 | 43996.13 |
| m | $\frac{1}{21000}$ | $\frac{1}{3501.6}$ |

In computing the perturbations of the radius vector, one of the largest terms will be a constant. To avoid the necessity of computing separately the perturbations of the second order, which depend on this constant, we shall include an approximate value of it in the mean distance. This approximate value is, in the action of an outer or an inner planet, $\delta \log a = -\frac{1}{6} m' M \alpha^2 D_\alpha b_\frac{1}{2}^{(0)}$. In the action of an inner or an outer planet, $\delta \log a' = +\frac{1}{6} m M (b_\frac{1}{2}^{(0)} + \alpha D_\alpha b_\frac{1}{2}^{(0)})$. M being the modulus of the system of logarithms.

Using the values of $b_\frac{1}{2}^{(0)}$ and $\alpha D_\alpha b_\frac{1}{2}^{(0)}$, which are found in different works relating to Celestial Mechanics, we find that the different planets produce the following changes in $6 \log a$, the units being those of the seventh place of decimals:—

| | On Saturn. | On Uranus. |
|------------------|------------|------------|
| Action of Venus, | + 22 | + 22 |
| “ Earth, | + 24 | + 25 |
| “ Mars, | + 3 | + 3 |
| “ Jupiter, | +10865 | +8780 |
| “ Saturn, | | +3081 |
| “ Uranus, | — 35 | |
| “ Neptune, | — 9 | — 119 |
| “ Sum, | + .0010870 | + .0011792 |
| $\delta \log a$ | + .0001812 | + .0001965 |

The uncorrected mean distance is deduced from the mean motion by the relation

$$a^3 = \frac{\mu(1+m)}{n^2}.$$

We thus have

| | Saturn. | Uranus. |
|------------------------------|----------|----------|
| Uncorrected mean dist. (log) | 0.979496 | 1.282901 |
| Action of the planets | + 181 | + 197 |
| Corrected log a | 0.979677 | 1.283098 |

The following functions of the elements are derived from the preceding elements by well known formulæ:—

| | |
|--|--------------|
| γ (mutual inclination) | 1° 57' 24.4" |
| $\log \sin \frac{1}{2} \gamma = \sigma$ | 8.232373 |
| $\log \cos \frac{1}{2} \gamma$ | 9.9999367 |
| τ (long. of ascending node of Saturn on Uranus) | 126° 44' 51" |
| ω | 41 31 40 |
| ω' | 323 18 21 |
| $\omega' - \omega = (\omega)$ | 281 46 41 |
| $\log \sin (\omega)$ | —9.990759 |
| $\log \cos (\omega)$ | +9.309888 |
| $\sin 2(\omega)$ | —0.39966 |
| $\cos 2(\omega)$ | —0.91667 |
| α | 0.497249 |
| $\frac{1}{\alpha^2}$ | 4.04438 |

The following functions of α , necessary in computing the coefficients h , are derived from Runkle's Tables, published by the Smithsonian Institution:—

Values of $\alpha^n D_\alpha^n b_{\frac{1}{2}}^{(i)}$.

| i | $b_{\frac{1}{2}}^{(0)}$ | $\alpha D_\alpha b_{\frac{1}{2}}^{(0)}$ | $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(0)}$ | $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(0)}$ | $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(0)}$ | $\alpha^5 D_\alpha^5 b_{\frac{1}{2}}^{(0)}$ |
|-----|-------------------------|---|---|---|---|---|
| 0 | 2.14447 | 0.33969 | 0.5878 | 1.081 | 3.44 | 13.6 |
| 1 | 0.55207 | .68314 | .4990 | 1.177 | 3.40 | 13.8 |
| 2 | 0.20836 | .47198 | .7396 | 1.152 | 3.59 | 13.9 |
| 3 | 0.08687 | .28491 | .7123 | 1.463 | 3.68 | 14.5 |
| 4 | 0.03793 | .16270 | .5632 | 1.596 | 4.30 | 15.1 |
| 5 | 0.01702 | .09010 | .3998 | 1.485 | 4.87 | 16.9 |
| 6 | 0.00777 | .04896 | .2653 | 1.231 | 4.98 | 19.1 |
| 7 | 0.00359 | .02624 | .1682 | 0.940 | 4.60 | 20.5 |
| 8 | 0.00168 | .01392 | .1022 | 0.675 | 3.91 | 20.4 |
| 9 | 0.00079 | 0.00733 | 0.0615 | 0.463 | 3.11 | 18.8 |

Derivatives with respect to $(\log a = v)$ of $\alpha^n D_\alpha^n b_{\frac{1}{2}}^{(i)}$.

| i | $n=$ | 1 | 2 | 3 | 4 |
|-----|---------|--|--|--|--|
| | 0 | $D_v(\alpha D_\alpha b_{\frac{1}{2}}^{(i)})$ | $D_v(\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)})$ | $D_v(\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)})$ | $D_v(\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)})$ |
| 0 | 0.33969 | 0.9275 | 2.257 | 6.68 | 27.4 |
| 1 | .68314 | 1.1821 | 2.175 | 6.93 | 27.4 |
| 2 | .47198 | 1.2116 | 2.631 | 7.05 | 28.3 |
| 3 | .28491 | 0.9972 | 2.888 | 8.07 | 29.2 |
| 4 | .16270 | 0.7259 | 2.722 | 9.09 | 32.3 |
| 5 | .09010 | 0.4899 | 2.285 | 9.33 | 36.4 |
| 6 | .04896 | 0.3143 | 1.762 | 8.67 | 39.0 |
| 7 | .02624 | 0.1944 | 1.276 | 7.41 | 38.9 |
| 8 | .01392 | 0.1161 | 0.879 | 5.93 | 36.0 |
| 9 | 0.00733 | 0.0688 | 0.586 | 4.50 | 31.2 |

Second derivatives.

| i | $D_v^2 b_{\frac{1}{2}}^{(0)}$ | $D_v^2(\alpha D_\alpha b_{\frac{1}{2}}^{(i)})$ | $D_v^2(\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)})$ | $D_v^2(\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)})$ |
|-----|-------------------------------|--|--|--|
| 0 | 0.9275 | 3.184 | 11.19 | 47.4 |
| 1 | 1.1821 | 3.357 | 11.28 | 48.2 |
| 2 | 1.2116 | 3.843 | 12.31 | 49.4 |
| 3 | 0.9972 | 3.885 | 13.85 | 53.4 |
| 4 | 0.7259 | 3.448 | 14.53 | 59.6 |
| 5 | 0.4899 | 2.775 | 13.90 | 64.4 |
| 6 | 0.3143 | 2.076 | 12.19 | 65.0 |

Values of $\alpha^{n+1} D_\alpha^n b_{\frac{1}{2}}^{(i)}$

| i | $\alpha b_{\frac{1}{2}}^{(0)}$ | $\alpha^2 D_\alpha b_{\frac{1}{2}}^{(0)}$ | $\alpha^3 D_\alpha^2 b_{\frac{1}{2}}^{(0)}$ | $\alpha^4 D_\alpha^3 b_{\frac{1}{2}}^{(0)}$ |
|-----|--------------------------------|---|---|---|
| 0 | 1.865 | 2.674 | 8.104 | 30.8 |
| 1 | 1.267 | 2.844 | 7.77 | 30.8 |
| 2 | 0.761 | 2.412 | 7.63 | 29.9 |
| 3 | 0.433 | 1.790 | 6.92 | 28.7 |
| 4 | 0.240 | 1.224 | 5.73 | 26.8 |
| 5 | 0.130 | 0.792 | 4.41 | 23.5 |
| 6 | 0.070 | 0.493 | 3.20 | 19.6 |

Derivatives with respect to (log a = v.)

| i | $D_v(ab_{\frac{1}{2}}^{(i)})$
$=a_1B_1^{(i)}$ | $D_v(a^2Dab_{\frac{1}{2}}^{(i)})$
$=2a_1B_2^{(i)}$ | $D_v(a^3Dab_{\frac{1}{2}}^{(i)})$
$=6a_1B_3^{(i)}$ |
|-----|--|---|---|
| 0 | 4.539 | 13.452 | 55.1 |
| 1 | 4.111 | 13.46 | 54.1 |
| 2 | 3.173 | 12.45 | 52.8 |
| 3 | 2.223 | 10.50 | 49.5 |
| 4 | 1.464 | 8.18 | 44.0 |
| 5 | 0.922 | 5.99 | 36.7 |
| 6 | 0.563 | 4.19 | 29.2 |

Second derivatives.

| i | $a_1D_vB_1^{(i)}$ | $a_1D_vB_2^{(i)}$ | $2a_1D_vB_3^{(i)}$ |
|-----|-------------------|-------------------|--------------------|
| 0 | 4.539 | 17.99 | 82.0 |
| 1 | 4.111 | 17.57 | 81.0 |
| 2 | 3.173 | 15.62 | 77.7 |
| 3 | 2.223 | 12.72 | 70.5 |
| 4 | 1.464 | 9.64 | 60.4 |
| 5 | 0.922 | 6.91 | 48.7 |
| 6 | 0.563 | 4.75 | 37.6 |

| | $a_1E^{(0)}$ | $a_1E_1^{(0)}$ | $2a_1E_2^{(0)}$ | $6a_1E_3^{(0)}$ | $a_1D_vE^{(0)}$ | $a_1D_vE_1^{(0)}$ | $2a_1D_vE_2^{(0)}$ |
|---|--------------|----------------|-----------------|-----------------|-----------------|-------------------|--------------------|
| 0 | 1.267 | 4.111 | 13.46 | 54.1 | 4.111 | 17.57 | 81.0 |
| 1 | 1.313 | 3.856 | 12.95 | 54.0 | 3.856 | 16.80 | 79.8 |
| 2 | 0.850 | 3.167 | 11.98 | 51.8 | 3.167 | 15.15 | 75.8 |
| 3 | 0.500 | 2.318 | 10.31 | 48.4 | 2.318 | 12.63 | 69.0 |
| 4 | 0.281 | 1.573 | 8.24 | 43.1 | 1.573 | 9.82 | 59.6 |
| 5 | 0.155 | 1.014 | 6.18 | 36.6 | 1.014 | 7.20 | 49.0 |

The notation $B_n^{(i)}$ and $E_n^{(i)}$ is that of Le Verrier in his development in the first volume of "*Annales de l'Observatoire Imperial de Paris.*"

Numeric expression of R and its derivatives.

We next proceed to the computation of the coefficients h and their derivatives. As an example of the most convenient form of computation we present in full that of the coefficient of $\frac{m'}{a_1} \cos (i\lambda' - (i-1)\lambda - \omega)$ in the expression of R for the action of Saturn on Uranus. In this computation I use the tables given by Le Verrier in his "*Annales de l'Observatoire,*" tome i, pages 358-383, comparing the development with that of Professor Peirce in the *Astronomical Journal*, vol. i, as a control.

$j=0; j=1.$

| i | -3 | -2 | -1 | 0 | +1 | 2 | 3 | 4 | 5 | 6 |
|--|--------------------|---------|----------|---------|----------|----------|---------|---------|---------|---------|
| i' | +4 | 3 | 2 | 1 | 0 | -1 | -2 | -3 | -4 | 5 |
| $(0) \times b_{\frac{1}{2}}^{(i)}$ | +0.5212 | +0.8334 | + 1.1041 | 0 | -1.10414 | -0.83344 | -0.5212 | -0.3034 | -0.1702 | -0.0929 |
| $(1) \times \alpha D \alpha b_{\frac{1}{2}}^{(i)}$ | -0.2849 | -0.4720 | - 0.6831 | -0.3397 | -0.68314 | -0.47198 | -0.2849 | -0.1627 | -0.0901 | -0.0490 |
| $\Delta \alpha_1(50)^{(i)}$ | | | -16.1775 | | | | | | | |
| $\alpha_1(50)^{(i)}$ | +0.2363 | +0.3614 | -15.7565 | -0.3397 | -0.78728 | -1.30542 | -0.8061 | -0.4661 | -0.2603 | -0.1419 |
| $(0) b_{\frac{1}{2}}^{(i)}$ | -13.55 | -11.25 | - 5.52 | 0.00 | 0.00 | + 2.92 | + 5.73 | + 6.83 | + 6.46 | |
| $(1) \times \alpha D \alpha$ | + 8.54 | + 7.78 | + 4.78 | +0.51 | 0.00 | + 1.18 | + 2.56 | + 3.25 | + 3.06 | |
| $(2) \times \alpha^2 D^2 \alpha$ | + 1.43 | + 0.74 | - 0.00 | -0.59 | - 1.00 | - 2.22 | - 2.85 | - 2.82 | - 2.40 | |
| $(3) \times \alpha^3 D^3 \alpha$ | - 0.73 | - 0.58 | - 0.59 | -0.54 | - 0.59 | - 0.58 | - 0.73 | - 0.80 | - 0.74 | |
| $\Delta \alpha_1(51)$ | | | + 48.53 | | | | | | | |
| $\alpha_1(51)$ | - 4.31 | - 3.31 | + 47.20 | -0.62 | - 1.59 | + 1.30 | + 4.71 | + 6.46 | + 6.38 | |
| $(0) \times b_{\frac{1}{2}}^{(i)}$ | -18.76 | -13.33 | - 4.42 | 0.00 | + 4.42 | +13.33 | +18.76 | +19.41 | +17.01 | |
| $(1) \times \alpha D \alpha$ | +13.10 | +10.38 | + 4.10 | -0.68 | - 1.37 | + 2.83 | + 6.27 | + 7.48 | + 7.02 | |
| $(2) \times \alpha^2 D^2 \alpha$ | + 1.43 | 0.00 | - 1.00 | -2.35 | - 3.00 | - 5.92 | - 7.13 | - 6.76 | - 5.60 | |
| $(3) \times \alpha^3 D^3 \alpha$ | - 1.46 | - 1.15 | - 1.18 | -1.08 | - 1.18 | - 1.15 | - 1.46 | - 1.60 | - 1.48 | |
| $\Delta \alpha_1(52)$ | | | + 32.36 | | | | | | | |
| $\alpha_1(52)$ | - 5.69 | - 4.10 | + 29.86 | -4.11 | - 1.13 | + 9.09 | +16.44 | +18.53 | +16.95 | |
| $(0) \times \alpha E_1^{(i)}$ | -3.00 | - 3.40 | - 2.63 | 0.00 | + 2.63 | + 3.40 | + 3.00 | + 2.25 | + 1.55 | |
| $(1) \times \alpha_1 E_1^{(i)}$ | +2.32 | + 3.17 | + 3.86 | +4.11 | + 3.86 | + 3.17 | + 2.32 | + 1.57 | + 1.01 | |
| $\Delta \alpha_1(60)$ | | | + 16.18 | | | | | | | |
| $\alpha_1(60)$ | -0.68 | - 0.23 | + 17.41 | +4.11 | + 6.49 | + 6.57 | + 5.32 | + 3.82 | + 2.56 | |
| $\frac{1}{2} e \times (50)$ | +6.61 ¹ | +10.12 | -441.22 | -9.51 | -50.049 | -36.556 | -22.573 | -13.05 | - 7.28 | -3.97 |
| $\frac{1}{2} e^3 \times (51)$ | -0.09 | - 0.07 | + 1.04 | -0.01 | - 0.035 | + 0.028 | + 0.103 | + 0.14 | + 0.14 | |
| $\frac{1}{8} e e^3 \times (52)$ | -0.08 | - 0.06 | + 0.45 | -0.06 | - 0.017 | + 0.138 | + 0.248 | + 0.28 | + 0.26 | |
| $\frac{1}{2} e e^2 \times (60)$ | -0.05 | - 0.02 | + 0.14 | +0.03 | + 0.051 | + 0.052 | + 0.043 | + 0.03 | + 0.02 | |
| h | +6.39 | + 9.97 | -439.59 | -9.55 | -50.050 | -36.338 | -22.179 | -12.60 | - 6.86 | -3.60 |

The derivatives of h with respect to $(\log. a = v)$ are computed in precisely the same way by simply substituting for $b_{\frac{1}{2}}^{(i)}, \alpha D \alpha b_{\frac{1}{2}}^{(i)}$, etc., their derivatives with respect to v as given in the above table of constants.

The quantities $\Delta \alpha_1(50)^{(i)}$, etc., which appear in the third series of terms above express that part of the perturbations of Uranus caused by the action of Saturn

¹ In units of the third place of decimals.

on the sun. They are each of the form $N \times \alpha^{-2}$, N being a numerical coefficient given by Le Verrier under the coefficient for each term. The derivative of this expression with respect to ν is $-2N \times \alpha^{-2}$, so that for the corresponding terms in $D_\nu h$ and $D_\nu^2 h$ we have

$$\begin{aligned}\Delta D_\nu h &= -2\Delta h \\ \Delta D_\nu^2 h &= +4\Delta h\end{aligned}$$

The values of h and its derivatives, corresponding to any one argument i' and i , are to be combined into two terms depending the one on the cosine, the other on the sine of the argument. Let us represent by g the mean anomaly of Uranus, and let us put l for the mean longitude of Saturn counted from the perihelion of Uranus, or, more exactly, for the arc $\lambda' - \omega$. Put also

$$\begin{aligned}N &= ig + i'l, \\ i + i' + j &= j'', \\ P &= j\omega + j'\omega', \\ P' &= j''\omega + j'\omega'.$$

Then, for each value of N there will be several values of P corresponding to different powers and products of the eccentricities and inclinations in h . Distinguishing these values and the corresponding values of h by subscript numerals, we shall have a series of terms of R of the following form—

$$R = \frac{m}{a_1} \left\{ \begin{array}{l} h_1 \cos(N + P'_1) \\ + h_2 \cos(N + P'_2) \\ + h_3 \cos(N + P'_3) \\ + \text{etc.} \quad \text{etc.} \end{array} \right\}$$

and by putting

$$\begin{aligned}h_c &= h_1 \cos P'_1 + h_2 \cos P'_2 + h_3 \cos P'_3 + \text{etc.} \\ h_s &= -h_1 \sin P'_1 - h_2 \sin P'_2 - h_3 \sin P'_3 - \text{etc.}\end{aligned}\tag{41}$$

The above terms may be condensed into

$$R = \frac{m}{a_1} h_c \cos N + \frac{m}{a_1} h_s \sin N,$$

which are of the form supposed in the preceding theory.

In order that the derivative of R , with respect to the true longitude of Uranus, may be expressed in the form

$$\frac{\partial R}{\partial \nu} = \frac{m}{a_1} v_s \sin N + \frac{m}{a_1} v_c \cos N$$

we must, by (7), put

$$\begin{aligned}v_s &= -(i + j_1) h_1 \cos P'_1 - (i + j_2) h_2 \cos P'_2 - \text{etc.} \\ v_c &= -(i + j_1) h_1 \sin P'_1 - (i + j_2) h_2 \sin P'_2 - \text{etc.}\end{aligned}\tag{42}$$

j_1, j_2 , representing the several values of j in the different terms which correspond to one and the same set of values of i and i' .

To obtain the derivative with respect to γ we notice that all the appreciable terms in the different values of h , which depend upon the mutual inclination, are of the form

$$h = \sigma^2 A,$$

where $\sigma = \sin \frac{1}{2} \gamma$. These equations give

$$\frac{\partial h}{\partial \gamma} = \frac{\partial h}{\partial \sigma} \frac{\partial \sigma}{\partial \gamma} = A \sigma \cos \frac{1}{2} \gamma = \frac{1}{2} A \sin \gamma.$$

Consequently

$$\frac{\partial R}{\partial \gamma} = \frac{1}{2} A \sin \gamma \cos (N + P').$$

and the various terms depending on the same argument (i', i) may be condensed into two, exactly as in the case of R itself.

The different co-efficients h and $D_i h$, computed in the way already described, are given *in extenso* in the following table. At the top of each individual column is given the value of P , or of $j\omega + j'\omega'$, corresponding to the values of h below, and immediately under P is given its modified value, or P' , to be used in condensing the terms, putting for brevity

$$(\omega) = \omega - \omega'.$$

P and P' are therefore regarded as constant angles the numerical values of the sines and cosines of which may be obtained from the values of ω and ω' already given.

The condensed h_c and h_s are given in the two right hand columns.

All the numbers are given in units of the third place of decimals.

| VALUES OF h . | | | | | | | | |
|-----------------|----------|--|--------------------|--|-----------|--|---------|--|
| P | 0 | | $\omega' - \omega$ | | | | | |
| P' | = 0 | | (ω) | | | | | |
| $i \quad i'$ | h | | h | | h_c | | h_s | |
| -4, +4 | | | + 0.62 | | | | | |
| -3, 3 | | | + 0.82 | | | | | |
| -2, 2 | | | + 0.88 | | | | | |
| -1, 1 | | | + 0.53 | | | | | |
| 0, 0 | +1072.90 | | - 0.4975 | | + 1072.80 | | | |
| +1, -1 | -3481.36 | | - 0.83 | | - 3481.42 | | - 1.33 | |
| 2, -2 | + 205.92 | | - 9.63 | | + 204.14 | | - 10.29 | |
| 3, -3 | + 84.27 | | + 1.62 | | + 84.77 | | + 0.79 | |
| 4, -4 | + 35.80 | | + 1.54 | | + 36.24 | | + 0.90 | |
| 5, -5 | + 15.48 | | + 1.2 | | + 15.81 | | + 0.81 | |

| P | $-\omega$ | $-\omega'$ | $\omega - 2\omega'$ | $\omega' - 2\omega$ | ω' | ω | | |
|--------------|-----------|---------------|---------------------|---------------------|--------------------|-------------|----------|----------|
| P' | 0 | -(ω) | -2(ω) | + (ω) | $\omega + \omega'$ | +2 ω | | |
| $i \quad i'$ | h | h | h | h | h | h | h_c | h_s |
| -3, +4 | - 2.31 | + 6.39 | 0 | 0 | 0 | 0 | - 1.01 | - 6.26 |
| -2, 3 | - 3.42 | + 9.97 | -0.95 | 0 | 0 | + .02 | - 0.51 | - 9.40 |
| -1, 2 | - 3.54 | -439.59 | 0 | 0 | -.03 | + .02 | - 93.30 | +430.35 |
| 0, 1 | +287.30 | - 9.55 | +0.13 | - .02 | -.04 | + .09 | +285.20 | + 9.19 |
| +1, 0 | + 58.38 | - 50.05 | -0.02 | - .05 | -.03 | 0 | + 48.15 | + 48.96 |
| 2, -1 | - 39.760 | - 36.338 | -0.074 | - .071 | -.016 | 0 | - 47.138 | + 35.531 |
| 3, -2 | + 35.13 | - 22.179 | -0.105 | - .313 | -.007 | 0 | + 30.63 | + 21.44 |
| 4, -3 | + 20.39 | - 12.60 | -0.11 | + .17 | 0 | 0 | + 17.96 | + 12.55 |
| 5, -4 | + 11.25 | - 6.86 | -0.10 | + .23 | 0 | 0 | + 9.98 | + 7.00 |
| 6, -5 | + 6.00 | - 3.6 | 0 | + .22 | 0 | 0 | + 5.38 | + 3.78 |

| VALUES OF h . | | | | | | | | |
|-----------------|-----------------|------------------------------------|-----------------------------|-----------------|--|--------|--------|--|
| P
P' | -2ω
0 | $-\omega' - \omega$
$-(\omega)$ | $-2\omega'$
$-2(\omega)$ | 0
$+2\omega$ | | | | |
| i i' | h | h | h | h | | h_c | h_s | |
| -1, 3 | +0.04 | -0.27 | -42.24 | +0.11 | | +38.72 | +17.03 | |
| 0, 2 | +0.06 | +31.65 | -0.04 | +0.18 | | +6.58 | -31.15 | |
| +1, 1 | -0.74 | -0.83 | -1.08 | -0.91 | | -0.03 | +2.15 | |
| 2, 0 | +3.09 | -5.19 | +1.89 | +0.18 | | +0.32 | +4.14 | |
| 3,-1 | +1.268 | -6.290 | +2.082 | +0.111 | | -1.912 | +5.216 | |
| 4,-2 | +4.16 | -5.458 | +1.767 | +0.061 | | +1.44 | +4.576 | |
| 5,-3 | +3.13 | -4.03 | +1.29 | +0.04 | | +1.13 | +3.39 | |
| 6,-4 | +2.11 | -2.74 | +0.87 | +0.02 | | +0.75 | +2.31 | |
| 7,-5 | +1.35 | -1.76 | +0.55 | +0.01 | | +0.49 | +1.49 | |

| P
P' | -3ω
0 | $-\omega' - 2\omega$
$-(\omega)$ | $-2\omega' - \omega$
$-2(\omega)$ | $-3\omega'$
$-3(\omega)$ | $-\omega'$
$3\omega - \omega'$ | $-\omega$
2ω | | |
|-------------|-----------------|-------------------------------------|--------------------------------------|-----------------------------|-----------------------------------|------------------------|-------|-------|
| i i' | h | h | h | h | h | h | h_c | h_s |
| +1,+2 | 0 | -.12 | 0 | -.11 | -.14 | +.02 | +.17 | +.06 |
| 2, 1 | +.01 | -.07 | +.03 | -.02 | -.02 | +.01 | 0 | +.04 |
| 3, 0 | +.18 | -.46 | +.34 | -.08 | -.02 | +.04 | -.15 | +.213 |
| 4,-1 | +.222 | -.760 | +.520 | -.117 | -.016 | +.029 | -.325 | +.416 |
| 5,-2 | +.424 | -.847 | +.558 | -.122 | -.012 | +.021 | -.176 | +.490 |
| 6,-3 | +.398 | -.773 | +.498 | -.107 | -.008 | +.014 | -.145 | +.460 |
| 7,-4 | +.327 | -.627 | +.395 | -.086 | -.005 | +.009 | -.107 | +.379 |
| 8,-5 | +.24 | -.47 | +.29 | -.06 | 0 | 0 | -.083 | +.289 |

| A
P' | -4ω
0 | $-\omega' - 3\omega$
$-(\omega)$ | $-2\omega' - 2\omega$
$-2(\omega)$ | $-3\omega' - \omega$
$-3(\omega)$ | $-4\omega'$
$-4(\omega)$ | | |
|-------------|-----------------|-------------------------------------|---------------------------------------|--------------------------------------|-----------------------------|-------|-------|
| i i' | h | h | h | h | h | h_s | h_c |
| 4, 0 | 0 | -.04 | +.04 | -.02 | 0 | -.04 | 0 |
| 5,-1 | +.02 | -.08 | +.08 | -.04 | +.01 | -.03 | +.003 |
| 6,-2 | +.039 | -.105 | +.106 | -.047 | +.008 | -.045 | +.028 |
| 7,-3 | +.043 | -.114 | +.112 | -.048 | +.008 | -.049 | +.034 |
| 8,-4 | +.042 | -.106 | +.103 | -.044 | +.007 | -.044 | +.032 |
| 9,-5 | +.035 | -.090 | +.086 | -.036 | +.006 | -.037 | +.029 |

| VALUES OF $D_{\omega}h$. | | | | |
|---------------------------|---------------|----------------------------------|-----------------|-----------------|
| P
P' | 0
0 | $\omega' - \omega$
(ω) | | |
| i i' | $D_{\omega}h$ | $D_{\omega}h$ | $D_{\omega}h_c$ | $D_{\omega}h_s$ |
| -4,+4 | | + 3.18 | | |
| -3, 3 | | + 3.24 | | |
| -2, 2 | | + 2.39 | | |
| -1, 1 | | + 0.40 | | |
| 0, 0 | + 171.93 | - 2.075 | + 171.51 | |
| +1,-1 | +8749.59 | - 2.69 | + 8749.12 | - 3.02 |
| 2,-2 | + 467.72 | + 20.83 | + 472.46 | + 18.05 |
| 3,-3 | + 277.00 | + 2.87 | + 278.25 | - 0.36 |
| 4,-4 | + 153.87 | + 4.62 | + 155.46 | + 1.41 |
| 5,-5 | + 82.12 | + 4.90 | + 83.54 | + 1.80 |

VALUES OF $D_{\nu}h$.

| P
P' | $-\omega$
0 | $-\omega'$
$-(\omega)$ | $\omega-2\omega$
$-2(\omega)$ | $\omega'-2\omega$
$+(\omega)$ | $+\omega$
$\omega+\omega'$ | ω
2ω | | |
|-------------|----------------|---------------------------|----------------------------------|----------------------------------|-------------------------------|-----------------------|--------------|--------------|
| i i' | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h_c$ | $D_{\nu}h_s$ |
| -3,+4 | - 9.28 | + 19.4 | 0 | 0 | 0 | 0 | - 5.32 | - 19.00 |
| -2, 3 | - 9.74 | + 18.5 | +2.05 | -.06 | -.12 | +.08 | - 7.97 | - 17.43 |
| -1, 2 | - 4.63 | +907.62 | + .05 | -.08 | -.13 | +.07 | +180.44 | -888.65 |
| -0, 1 | -556.40 | - 26.17 | -.10 | -.09 | -.14 | -.12 | -561.82 | + 25.60 |
| 1, 0 | + 30.16 | - 71.56 | + .04 | -.19 | -.11 | +.02 | + 15.37 | + 69.86 |
| 2,-1 | +265.17 | - 86.47 | -.19 | -.26 | -.07 | 0 | +247.58 | + 84.30 |
| 3,-2 | + 83.10 | - 74.60 | -.42 | +.63 | -.04 | 0 | + 68.34 | + 73.49 |
| 4,-3 | + 68.34 | - 54.9 | -.55 | +.29 | -.02 | 0 | + 57.70 | + 53.9 |
| 5,-4 | + 49.04 | - 36.9 | -.58 | +.68 | -.01 | 0 | + 42.19 | + 36.7 |
| 6,-5 | + 32.1 | - 23.2 | -.53 | +.90 | 0 | 0 | + 28.1 | + 23.4 |

| P
P' | -2ω
0 | $-\omega-\omega'$
$-(\omega)$ | $-2\omega'$
$-2(\omega)$ | 0
$+2\omega$ | | |
|-------------|-----------------|----------------------------------|-----------------------------|-----------------|--------------|--------------|
| i i' | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h_c$ | $D_{\nu}h_s$ |
| -1,+3 | + 0.16 | - 0.66 | +86.02 | +0.47 | -78.76 | -34.19 |
| 0, 2 | + 0.32 | -64.55 | + 0.15 | +0.60 | -12.92 | +62.54 |
| +1, 1 | + 3.27 | - 2.70 | + 4.69 | +3.02 | - 1.22 | - 2.26 |
| 2, 0 | + 2.56 | - 8.78 | + 5.00 | +0.60 | - 3.74 | + 6.00 |
| 3, 1 | +13.745 | -15.870 | + 7.232 | +0.463 | + 3.933 | +12.153 |
| 4,-2 | +10.25 | -18.93 | + 7.85 | +0.32 | - 0.77 | +15.07 |
| 5,-3 | +10.76 | -18.07 | + 6.99 | +0.21 | + 0.69 | +14.68 |
| 6,-4 | + 9.39 | -15.0 | + 5.59 | +0.13 | + 1.22 | +12.4 |
| 7,-5 | + 7.3 | -11.5 | + 4.21 | +0.09 | + 1.2 | + 9.6 |

| P
P' | -3ω
0 | $-2\omega-\omega'$
$-(\omega)$ | $-\omega-2\omega'$
$-2(\omega)$ | $-3\omega'$
$-3(\omega)$ | $-\omega'$
$3\omega-\omega'$ | $-\omega$
$+2\omega$ | | |
|-------------|-----------------|-----------------------------------|------------------------------------|-----------------------------|---------------------------------|-------------------------|--------------|--------------|
| i i' | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h_c$ | $D_{\nu}h_s$ |
| +1,+2 | +0.03 | +0.18 | +0.05 | +0.22 | +0.22 | +0.08 | -0.31 | -0.17 |
| 2, 1 | +0.12 | -0.26 | +0.33 | -0.09 | -.08 | +.17 | -0.08 | -0.09 |
| 3, 0 | +0.21 | -0.89 | +0.98 | -0.31 | -.09 | +.13 | -0.59 | +0.13 |
| 4,-1 | +0.89 | -2.02 | +1.89 | -0.54 | -.09 | +.13 | -0.84 | +0.69 |
| 5,-2 | +1.08 | -3.00 | +2.53 | -0.67 | -.08 | +.11 | -1.38 | +1.30 |
| 6,-3 | +1.40 | -3.47 | +2.73 | -0.69 | -.07 | +.09 | -1.33 | +1.68 |
| 7,-4 | +1.46 | -3.44 | +2.60 | -0.62 | -.05 | +.06 | -1.20 | +1.78 |
| 8,-5 | +1.32 | -3.00 | +2.20 | -0.51 | 0 | +.04 | -0.99 | +1.62 |

| P
P' | -4ω
0 | $-3\omega-\omega'$
$-(\omega)$ | $-2\omega-2\omega'$
$-2(\omega)$ | $-\omega-3\omega'$
$-3(\omega)$ | $-4\omega'$
$-4(\omega)$ | | |
|-------------|-----------------|-----------------------------------|-------------------------------------|------------------------------------|-----------------------------|--------------|--------------|
| i i' | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h$ | $D_{\nu}h_c$ | $D_{\nu}h_s$ |
| 4, 0 | +0.02 | -.09 | +0.13 | -.08 | +0.02 | -.06 | -.01 |
| 5,-1 | +0.05 | -.22 | +0.30 | -.18 | +0.04 | -.14 | -.02 |
| 6,-2 | +0.10 | -.38 | +0.48 | -.26 | +0.05 | -.24 | +0.01 |
| 7,-3 | +0.15 | -.52 | +0.62 | -.32 | +0.06 | -.30 | +0.04 |
| 8,-4 | +0.18 | -.59 | +0.66 | -.33 | +0.06 | -.32 | +0.09 |
| 9,-5 | +0.19 | -.59 | +0.65 | -.31 | +0.05 | -.31 | +0.11 |

The values of $D_{\nu}h$, needed in computing the perturbations of the second order with respect to the masses being obtained in the same way, by the simple substitution of the second derivatives of the functions $b_{,a}D_{ab}$, etc., for those functions themselves in the expressions for h , it is not necessary to present the details of the computation.

After obtaining h and its derivatives, it will be found convenient to change the arrangement of the terms. Hitherto we have kept in one series those in which the sum of the indices are a constant. Now, we shall put together all those in which

the index of the disturbing planet has the same value, arranging the individual terms of each series according to the index of the disturbed planet. Thus, the index of the product of any term, as $h \cos N$, by any multiple of the mean anomaly of the disturbed planet, as jg , will be found in the same series with that of N itself, and j lines above and below.

The next process will be the formations of the required functions of the mean anomaly of Uranus, $\frac{dv}{dt}$, $\frac{d\rho}{dt}$, $\frac{a^2}{j^2}$, $\log r$. Their values are as follows:—

| | | | |
|--------------------|---------------------|-------------------------|------------------------|
| $r_1^{-2} =$ | $\rho_0 = n$ | $\frac{d\rho_0}{ndt} =$ | $\frac{dv_0}{ndt} = 1$ |
| 1.001103 | +.0005507 | | |
| +.093933 $\cos g$ | -.0468889 $\cos g$ | +.0468889 $\sin g$ | +.0938294 $\cos g$ |
| +.005507 $\cos 2g$ | -.0016494 $\cos 2g$ | +.0032988 $\sin 2g$ | +.0055012 $\cos 2g$ |
| +.000336 $\cos 3g$ | -.0000732 $\cos 3g$ | +.0002196 $\sin 3g$ | +.0003357 $\cos 3g$ |
| +.000020 $\cos 4g$ | -.0000035 $\cos 4g$ | +.0000142 $\sin 4g$ | +.0000206 $\cos 4g$ |

Considering only those terms which are of the first order, the value of $D_t R$ may be found in two ways, the agreement of which will afford a check upon the entire development of the perturbative function, and upon the computations of R and $\frac{\partial R}{\partial v}$. These are (1) by direct differentiation, with respect to the time as contained in the mean anomaly of a single planet, whereby each term in R of the form

$$R = \frac{m}{a_1} h \cos N$$

will produce in $D_t R$ the term

$$D_t R = -\frac{m}{a_1} inh \sin N.$$

and (2) by forming the expression

$$D_t R = \frac{\partial R}{\partial v} \frac{dv_0}{dt} + \frac{\partial R}{\partial \rho} \frac{d\rho_0}{dt}.$$

As several "mechanical multiplications," like those indicated in this last expression, are to be performed, the following example of the form of computation is presented. It exhibits the formation of the product of those terms of $\frac{\partial R}{\partial v}$ in which $i' = -1$ by $\frac{dv_0}{dt}$.

| i' | -2 | -1 | 0 | +1 | 2 | 3 | 4 | 5 |
|--|--------|--------|---------|----------|---------|---------|-------|-------|
| $\frac{a_1}{m} \frac{\partial R}{\partial v} (\sin)$ | +.02 | + 1.63 | -287.42 | +3481.14 | + 54.42 | + 6.988 | +1.18 | +0.10 |
| $\times .046915$ | { 0 | 0 | + .08 | - 13.48 | +163.32 | + 2.553 | +0.33 | +0.06 |
| | { +.08 | -13.48 | +163.32 | + 2.55 | + 0.33 | + 0.056 | 0 | 0 |
| $\times .002750$ | { 0 | 0 | 0 | 0.00 | - 0.79 | + 9.576 | +0.15 | +.02 |
| | { -.79 | + 9.58 | + 0.15 | + .02 | 0 | 0 | 0 | 0 |
| $\times .000168$ | +.58 | + 0.01 | 0 | 0 | 0 | - .048 | + .58 | 0 |
| $\frac{a_1}{m'n} \frac{\partial R}{\partial v} \frac{dv}{dt} (\sin)$ | -.11 | - 2.26 | -123.87 | +3470.23 | +217.28 | +19.125 | +2.24 | +0.18 |

The multipliers on the left are each one-half the coefficient $\cos jg$ in the expression for $\frac{dv_0}{dt}$, and each product is placed in the two columns corresponding respectively to $N + jg$ and $N - jg$.

All the derivations of R_0 necessary in the computation of the perturbations of the first order are given in the following tables. First we have the values of $D_t R$ obtained by direct differentiation, as indicated in the preceding formulæ. Next we have $\frac{\partial R}{\partial v}$ and $\frac{\partial R}{\partial \rho}$, obtained by the formulæ (7) and (42). The products $\frac{\partial R}{\partial v}$ by $\frac{dv_0}{dt}$ and of $\frac{\partial R}{\partial \rho}$ by $\frac{\partial \rho}{dt}$, being formed in the simple way just pointed out, and with the values of the component factors just given, their sum is next shown. This sum should agree accurately with $D_t R$. The discrepancies are shown in the next two columns. The only apparently large discrepancy is found in the argument $5g' - 5l$. It probably arises from the incompleteness of the computation of R and $\frac{\partial R}{\partial v}$, so far as they depend on this argument. As the entire term does not amount to $0''.01$, I have not sought to correct it.

The great value of this check arises from the fact that it gives a complete control of the correctness of the development of the perturbative function, *ab initio*, since the two values of $D_t R$ are derived from different terms of that development.

It also controls all the computations except that of $\frac{\partial R}{\partial \rho}$. This quantity being multiplied by quantities of the order of the eccentricities in the second value of $D_t R$, an error in its value will produce a discrepancy of only $\frac{1}{400}$ its own amount in $D_t R$, and may therefore be overlooked. The derivative in question must therefore be checked by a complete duplicate computation.

In the column next following are given the integrating factors v , for which the expression is

$$v = \frac{n}{in' + in} = \frac{1}{i + i' \frac{n'}{n}}.$$

For each value of i' the values of v are therefore the reciprocals of a series of numbers in arithmetical progression, the common difference being unity.

| $g \quad \nu$ | $D'R = \frac{m'}{a_1} n \times$ | | $\frac{\partial R}{\partial v} = \frac{m'}{a_1} \times$ | | $\frac{\partial R}{\partial \rho} = \frac{m'}{a_1} \times$ | |
|---------------|---------------------------------|---------|---|---------|--|---------|
| | sin | cos | sin | cos | cos | sin |
| 0, 0 | 0 | 0 | | + 0.487 | -1244.31 | |
| 1, | - 48.15 | + 48.96 | + 10.20 | + 49.07 | - 63.52 | -118.82 |
| 2, | - 0.64 | + 8.28 | + 4.48 | + 3.20 | + 3.42 | - 10.14 |
| 3, | + 0.45 | + 0.64 | + 0.50 | - 0.08 | + 0.74 | + 0.34 |
| 4, | + 0.16 | 0 | + 0.06 | - 0.06 | + 0.10 | + 0.01 |
| -2,-1 | 0 | + 0.68 | + 0.02 | - 0.04 | + 0.08 | - 0.05 |
| -1, | - 0.03 | + 2.15 | + 1.63 | + 1.34 | + 1.25 | - 0.11 |
| 0, | 0 | 0 | - 287.42 | - 0.10 | + 276.62 | + 34.79 |
| 1, | +3481.42 | - 1.33 | +3481.14 | - 1.04 | -5267.70 | + 4.35 |
| 2, | + 94.28 | + 71.06 | + 54.42 | + 71.23 | - 200.44 | -119.83 |
| 3, | + 5.74 | + 15.65 | + 6.988 | + 9.490 | - 2.02 | - 17.37 |
| 4, | + 1.30 | + 1.66 | + 1.18 | + 0.42 | + 1.16 | - 1.11 |
| 5, | + 0.15 | + 0.15 | + 0.10 | 0 | + 0.17 | - 0.01 |
| -1,-2 | + 0.17 | + 0.06 | + 0.21 | - 0.16 | + 0.14 | - 0.11 |
| 0, | 0 | 0 | - 6.58 | + 30.99 | + 6.34 | + 31.39 |
| 1, | + 93.30 | -430.35 | + 96.81 | -430.37 | - 87.14 | -458.30 |
| 2, | - 408.28 | - 20.58 | - 410.42 | - 12.01 | - 676.60 | - 7.76 |
| 3, | - 91.89 | + 64.32 | - 57.01 | - 64.98 | - 98.97 | - 94.93 |
| 4, | - 5.76 | + 18.30 | + 1.45 | + 12.96 | - 0.67 | - 19.65 |
| 5, | + 0.88 | + 2.45 | + 1.30 | + 1.04 | + 1.56 | - 1.79 |
| 6, | + 0.27 | + 0.17 | + 0.20 | - 0.02 | + 0.28 | - 0.04 |
| 1,-3 | - 38.72 | - 17.03 | - 38.75 | - 17.29 | + 40.04 | - 17.16 |
| 2, | + 1.02 | + 18.80 | + 5.31 | + 19.14 | + 8.48 | - 26.83 |
| 3, | - 254.31 | + 2.37 | - 254.15 | - 0.02 | - 363.02 | - 0.43 |
| 4, | - 71.84 | + 50.20 | - 51.49 | + 49.90 | - 75.66 | - 66.45 |
| 5, | - 5.65 | + 16.95 | - 0.23 | + 13.0 | - 1.82 | - 18.07 |
| 6, | + 0.87 | + 2.76 | + 1.29 | + 1.5 | + 1.48 | - 2.14 |
| 7, | + 0.34 | + 0.24 | + 0.27 | 0 | + 0.35 | - 0.07 |
| 3,-4 | + 3.03 | + 18.78 | + 1.34 | + 18.78 | + 6.33 | - 25.26 |
| 4, | - 144.96 | + 3.60 | - 144.78 | + 1.5 | - 191.70 | - 2.31 |
| 5, | - 49.90 | + 35.00 | - 38.64 | + 34.58 | - 52.17 | - 43.7 |
| 6, | - 4.50 | + 13.86 | - 0.9 | + 11.2 | - 1.97 | - 14.7 |
| 7, | + 0.75 | + 2.65 | + 1.1 | + 1.6 | + 1.31 | - 2.2 |
| 8, | + 0.35 | + 0.26 | + 0.3 | + 0.1 | + 0.36 | - 0.1 |
| 5,-5 | - 79.05 | + 4.05 | - 78.9 | + 2.4 | - 99.3 | - 2.1 |
| 6, | - 32.3 | + 22.7 | - 26.3 | + 22.3 | - 33.5 | - 27.2 |
| 7, | - 3.4 | + 10.4 | - 1.1 | + 8.7 | - 1.7 | - 11.1 |
| 8, | + 0.7 | + 2.3 | + 0.9 | + 1.5 | + 1.1 | - 1.0 |
| 9, | + 0.3 | + 0.3 | + 0.3 | + 0.1 | + 0.3 | + 0.1 |

| <i>g</i> | <i>l'</i> | $\frac{\partial R}{\partial v} \frac{dv_0}{dt} + \frac{\partial R}{\partial v'} \frac{dv'_0}{dt}$
$= \frac{m'}{a_1} n \times$ | | Discrepancy. | | <i>v</i> | <i>k_c</i> | <i>k_s</i> |
|----------|-----------|--|---------|--------------|--------|-----------------|----------------------|----------------------|
| | | sin | cos | sin | cos | | | |
| 0, | 0 | 0 | — 0.01 | 0 | — .01 | | —1244.31 | |
| 1, | | — 48.05 | + 48.92 | + .10 | — .04 | + 1.0 | + 32.78 | — 20.90 |
| 2, | | — 0.63 | + 8.27 | + .01 | — .01 | + 0.5 | + 4.06 | — 1.86 |
| 3, | | + 0.45 | + 0.65 | 0 | + .01 | + $\frac{1}{3}$ | + 0.44 | + 0.08 |
| 4, | | + 0.11 | — 0.03 | — .05 | — .03 | + 0.25 | | |
| —2,—1 | | — 0.03 | + 0.02 | — .03 | — .06 | —0.206098 | + 0.08 | — 0.08 |
| —1, | | — 0.05 | + 2.16 | — .02 | + .01 | —0.259601 | + 1.23 | — 1.23 |
| 0, | | — 0.06 | + 0.09 | — .06 | + .09 | —0.350623 | + 276.62 | + 34.79 |
| +1 | | + 3481.41 | — 1.33 | — .01 | 0 | —0.539942 | —1508.18 | + 5.78 |
| 2, | | + 94.20 | + 71.06 | — .08 | 0 | —1.173630 | + 20.85 | —286.59 |
| 3, | | + 5.78 | + 15.63 | + .04 | — .02 | + 6.75940 | — 79.56 | + 194.17 |
| 4, | | + 1.27 | + 1.66 | — .03 | 0 | + 0.871126 | — 1.10 | + 1.79 |
| 5, | | + 0.14 | + 0.11 | — .01 | — .04 | + 0.4656 | + 0.03 | + 0.13 |
| —1,—2 | | + 0.16 | + 0.10 | — .01 | + .04 | —0.149162 | + 0.19 | — 0.13 |
| 0, | | + 0.01 | + 0.07 | + .01 | + .07 | —0.175312 | + 6.34 | + 31.39 |
| +1, | | + 93.24 | —430.37 | — .06 | — .02 | —0.212580 | — 47.47 | —275.33 |
| 2, | | — 408.28 | — 20.60 | 0 | — .02 | —0.269971 | — 897.05 | + 3.35 |
| 3, | | — 91.90 | + 64.33 | — .01 | + .01 | —0.369806 | — 166.93 | —142.50 |
| 4, | | — 5.77 | + 18.17 | — .01 | — .13 | —0.586813 | — 7.44 | — 41.13 |
| 5, | | + 0.91 | + 2.45 | + .03 | 0 | —1.42022 | + 4.05 | — 8.75 |
| 6, | | + 0.29 | + 0.15 | + .02 | — .02 | + 3.3797 | — 1.53 | + 1.09 |
| 1,—3 | | — 38.78 | — 17.03 | — .06 | 0 | —0.132342 | + 29.78 | — 12.65 |
| 2, | | + 1.02 | + 18.68 | 0 | — .12 | —0.152528 | + 8.79 | — 32.56 |
| 3, | | — 254.29 | + 2.37 | + .02 | 0 | —0.179981 | — 454.55 | — 1.28 |
| 4, | | — 71.88 | + 50.13 | — .04 | — .07 | —0.219482 | — 107.20 | — 88.50 |
| 5, | | — 5.69 | + 17.01 | — .04 | + .06 | —0.281202 | — 5.00 | — 27.61 |
| 6, | | + 0.91 | + 2.73 | + .04 | — .03 | —0.391210 | + 2.15 | — 4.30 |
| 7, | | + 0.35 | + 0.19 | + .01 | — .05 | —0.6426 | + 0.79 | — 0.38 |
| 3,—4 | | + 3.10 | + 18.78 | + .07 | 0 | + 0.11893 | + 7.05 | — 29.73 |
| 4, | | — 145.0 | + 3.51 | — .04 | — .09 | —0.13500 | — 230.83 | — 3.28 |
| 5, | | — 49.9 | + 34.85 | 0 | — .15 | —0.15605 | — 67.74 | — 54.60 |
| 6, | | — 4.7 | + 13.95 | — .20 | + .09 | —0.18490 | — 3.63 | — 19.80 |
| 7, | | + 0.8 | + 2.62 | + .05 | — .03 | —0.22685 | + 1.65 | — 3.36 |
| 8, | | + 0.3 | + 0.2 | — .05 | — .06 | —0.2934 | + 0.57 | — 0.27 |
| 5,—5 | | — 79.3 | + 2.7 | — .25 | — 1.35 | —0.1080 | — 116.4 | — 3.5 |
| 6, | | — 32.4 | + 22.7 | — .10 | 0 | —0.1211 | — 41.3 | — 32.7 |
| 7, | | — 3.5 | + 10.5 | — .10 | + .10 | —0.1377 | — 2.6 | — 13.9 |
| 8, | | + 0.8 | + 2.3 | + .10 | 0 | —0.1597 | + 1.3 | — 2.6 |
| 9, | | + 0.3 | + 0.2 | 0 | — .10 | —0.1901 | + 0.5 | — 0.2 |

The values of $\frac{a_1}{m} \int D_i R_0 dt$ are formed from $\frac{a_1}{m'n} D_i R$ by simple multiplication by v , and proper changes of sign. The values of k_c and k_s are then formed by adding the terms of $2 \frac{a_1}{m} \int D_i R_0 dt$ to the corresponding terms of $\frac{a_1}{m'} \frac{\partial R}{\partial \rho}$.

Perturbations of radius vector.

Let us now resume equation (19), and put for brevity

$$M = \frac{m'a}{a_1(1+m)}. \quad (44)$$

If we give to u the successive values 0, +1, -1, +2, -2, +3, -3, we have

$$r_1^2 \delta \rho = M \times \left\{ \begin{array}{l} \frac{1}{8} \sum_i p_i q_i (v_i - v_{-i}) \{k_c \cos N + k_s \sin N\} \\ + \frac{1}{8} \sum_i (p_i q_{(i-1)} + p_{(i-1)} q_i) (v_i - v_{(1-i)}) \{k_c \cos(N+g) + k_s \sin(N+g)\} \\ + \frac{1}{8} \sum_i (p_i q_{(i+1)} + p_{(i+1)} q_i) (v_i - v_{-(i+1)}) \{k_c \cos(N-g) + k_s \sin(N-g)\} \\ + \frac{1}{8} \sum_i (p_i q_{(i-2)} - p_{(i-2)} q_i) (v_i - v_{(2-i)}) \{k_c \cos(N+2g) + k_s \sin(N+2g)\} \\ + \quad \quad \quad \text{etc.} \quad \quad \quad \text{etc.} \quad \quad \quad \text{etc.} \quad \quad \quad \text{etc.} \end{array} \right\}$$

the finite integral being taken with respect to all values of i from $-\infty$ to $+\infty$, and the terms in which the angles $N \pm ug$ vanish being omitted. Proceeding farther to expand with respect to i , if we collect similar terms we shall find the individual terms in $r^2 \delta \rho$ to be as follows:

$$\begin{aligned} r_1^2 \delta \rho &= \frac{1}{2} M \left\{ \begin{array}{l} p_1 q_1 (v_1 - v_{-1}) \\ + p_2 q_2 (v_2 - v_{-2}) \end{array} \right\} \{k_c \cos N + k_s \sin N\} \\ &+ \frac{1}{4} M \left\{ \begin{array}{l} p_0 q_1 (v_1 - v_0) \\ + (p_1 q_2 + p_2 q_1) (v_2 - v_{-1}) \\ + (p_2 q_3 + p_3 q_2) (v_3 - v_{-2}) \\ + \quad \quad \quad \text{etc.} \quad \quad \quad \text{etc.} \end{array} \right\} \{k_c \cos(N+g) + k_s \sin(N+g)\} \\ &+ \frac{1}{4} M \left\{ \begin{array}{l} p_0 q_1 (v_0 - v_{-1}) \\ + (p_1 q_2 + p_2 q_1) (v_1 - v_{-2}) \\ + (p_2 q_3 + p_3 q_2) (v_2 - v_{-3}) \\ + \quad \quad \quad \text{etc.} \quad \quad \quad \text{etc.} \end{array} \right\} \{k_c \cos(N-g) + k_s \sin(N-g)\} \\ &+ \frac{1}{4} M \left\{ \begin{array}{l} p_0 q_2 (v_2 - v_0) \\ + (p_1 q_3 + p_3 q_1) (v_3 - v_{-1}) \\ + \quad \quad \quad \text{etc.} \quad \quad \quad \text{etc.} \end{array} \right\} \{k_c \cos(N+2g) + k_s \sin(N+2g)\} \\ &+ \frac{1}{4} M \left\{ \begin{array}{l} p_0 q_2 (v_0 - v_{-2}) \\ + (p_1 q_3 + p_3 q_1) (v_1 - v_{-3}) \\ + \quad \quad \quad \text{etc.} \quad \quad \quad \text{etc.} \end{array} \right\} \{k_c \cos(N-2g) + k_s \sin(N-2g)\} \\ &+ \frac{1}{4} M \left\{ \begin{array}{l} p_0 q_3 (v_3 - v_0) \\ + (p_1 q_4 + p_4 q_1) (v_4 - v_{-1}) \\ + (p_1 q_2 - p_2 q_1) (v_2 - v_1) \\ + \quad \quad \quad \text{etc.} \quad \quad \quad \text{etc.} \end{array} \right\} \{k_c \cos(N+3g) + k_s \sin(N+3g)\} \end{aligned}$$

$$+ \frac{1}{4} M \left\{ \begin{array}{l} p_0 q_3 (v_0 - v_{-3}) \\ + (p_1 q_4 + p_4 q_1) (v_2 - v_{-4}) \\ + (p_1 q_2 - p_2 q_1) (v_{-1} - v_{-2}) \\ + \quad \text{etc.} \quad \quad \quad \text{etc.} \end{array} \right\} \{k_c \cos (N - 3g) + k_s \sin (N - 3g)\}$$

A law of the factors of $k_c \cos (N + ug) + k_s \sin (N + ug)$ which will be noticed in the above expression, is this: Representing this factor by K_u , we have

$$K_u^i = K^{(i \pm u)},$$

the index i representing the coefficient of g in N , so that only half the values of K need be separately computed.

As the computation of $r_1^2 \delta \rho$ from these formulæ can be arranged in such a way as to be very simple, the computation of the terms in which the index i is -1 is here presented quite fully. The logarithms only are omitted, being used only in the cases in which they are more convenient than a table of products. In practice I find it convenient to write them in red ink immediately under the numbers which they represent.

First, to find M , it will be noticed that in the expression $\frac{m'a}{a_1(1+m)}$, the a in the numerator represents the mean distance of the *disturbed* planet, as deduced from the observed mean motion by the equation $a^3 n^2 = u(1+m)$ while a_1 represents the mean motion of the *outer* planet. When the outer planet is the disturbed one, the ratio $\frac{a}{a_1}$ would be unity, but that, to avoid a large class of second order terms, a_1 has been corrected for perturbations in the beginning (p. 32). In the case of Uranus disturbed by Saturn, we have in consequence

$$\log \frac{a}{a_1} = 9.999803.$$

Whence

$$M = 285.44$$

in units of the sixth place of decimals.

Computing the values of p_i and q_i from (16) we find, for Uranus,

| | | | |
|-------------------------------------|--------------|---|---|
| $\frac{1}{2} M p_1 q_1$ | $= + 142.56$ | } | In units of the sixth
place of decimals. |
| $\frac{1}{2} M p_2 q_2$ | $= + 0.0784$ | | |
| $\frac{1}{4} M p_0 q_1$ | $= - 10.044$ | | |
| $\frac{1}{4} M (p_1 q_2 + p_2 q_1)$ | $= + 3.3433$ | | |
| $\frac{1}{4} M (p_2 q_3 + p_3 q_2)$ | $= + 0.0028$ | | |
| $\frac{1}{4} M p_0 q_2$ | $= - 0.2358$ | | |
| $\frac{1}{4} M (p_1 q_3 + p_3 q_1)$ | $= + 0.118$ | | |
| $\frac{1}{4} M p_0 q_2$ | $= - 0.008$ | | |
| $\frac{1}{4} M (p_1 q_4 + p_4 q_1)$ | $= + 0.005$ | | |
| $\frac{1}{4} M (p_1 q_2 - p_2 q_1)$ | $= + 0.003$ | | |

In the computation the first three lines are copied from previous pages.

$$i' = -1$$

| <i>i</i> | -4 | -3 | -2 | -1 | 0 | +1 | 2 | 3 | 4 | 5 |
|----------|---------------------------------|---|--|---|--|---|--|---|--|--|
| <i>v</i> | -0.1459 | -0.1709 | -0.20610 | -0.25960 | -0.35062 | -0.53994 | -1.17363 | +6.75940 | +0.87113 | +0.4656 |
| | | <i>k_c</i>
<i>k_s</i> | +0.08
-0.08 | +1.23
-1.23 | +276.62
+ 34.79 | -1508.18
+5.79 | + 20.85
-286.59 | - 79.56
+194.17 | -1.10
+1.79 | +0.03
+0.13 |
| | +142.56
+ 0.0784 | <i>v₁-v₋₁</i>
<i>v₂-v₋₂</i> | -.0887
-.2047 | -.14452
-.3690 | -.28034
-.9675 | -.82301
+7.0190 | +7.29934
+1.2218 | +2.04476
+1.0055 | -6.2938
+1.4913 | -0.5535
-6.5186 |
| | -10.044
+ 3.3433
+ 0.0028 | <i>v₁-v₀</i>
<i>v₂-v₋₁</i>
<i>v₃-v₋₂</i> | -.0535
-.1797
-.39 | -.0910
-.3338
-1.00 | -.1893
-.9140
+6.96 | -.63368
+7.1100
+1.13 | +7.9330
+1.4110
+0.82 | -5.8883
+1.6392
+0.86 | -0.4056
-6.4417
+1.41 | |
| | -0.2358
+ 0.118 | <i>v₂-v₀</i>
<i>v₃-v₋₁</i> | -.144
-.369 | -.280
-.968 | -.823
+7.019 | +7.299
+1.222 | +2.045
+1.005 | -6.294
+1.490 | | |
| | -0.008
+ 0.005
+ 0.0003 | <i>v₃-v₀</i>
<i>v₄-v₋₁</i>
<i>v₂-v₋₁</i> | -.33
-1.00
-.09 | -.91
+6.97
-.19 |
.....
..... | +1.41
+0.82
+7.93 | +1.64
+0.86
-5.89 | | | |
| | 142.56
0.0784 | $\times (v_2-v_{-1})$
$\times (v_3-v_{-2})$
<i>K₀</i> | -12.35
- 0.02
-12.37 | -20.60
- 0.03
-20.63 | -39.965
- 0.075
-40.040 | -117.327
+0.550
-116.777 | +1040.59
+0.10
+1040.69 | +291.49
+0.08
+291.57 | -897.2
+0.1
-897.1 | -78.8
- 0.5
-79.3 |
| | -10.044
+ 3.3433
+ 0.0028 | $\times (v_1-v_0)$
$\times (v_2-v_{-1})$
$\times (v_3-v_{-2})$
<i>K₁</i>
<i>K₋₁</i> | +0.532
-0.601
-0.001
.....
..... | +0.914
-1.126
-0.003
-0.20
-0.07 | +1.901
-3.056
+0.020
-1.135
-0.205 | + 6.364
+23.770
+ 0.003
+30.137
- 1.135 | -79.68
+ 4.72
0
-74.96
+30.14 | +59.14
+ 5.48
0
+64.62
-74.96 | + 4.1
-21.5
0
-17.4
+64.6 | |
| | -2.358
+ .118 | $\times (v_2-v_0)$
$\times (v_3-v_{-1})$
<i>K₂</i>
<i>K₋₂</i> | +0.034
-0.043
.....
..... | +0.066
-0.114
.....
..... | +0.194
+0.828
+1.022
-0.009 | -1.721
+0.144
-1.577
-0.048 | -0.48
+0.12
-0.36
+1.02 | + 1.48
+ 0.18
+ 1.66
+ 1.58 | | |
| | - .008
+ .005 | $\times (v_1-v_0)$
$\times (v_4-v_1)$
<i>K₃</i>
<i>K₋₃</i> | + .003
- .005
.....
..... | + .007
+ .035
.....
..... |
.....
.....
..... | -.011
+.004
-.007
-.002 | -.013
+.004
-.009
+.042 | | | |
| | | <i>K₀ k_c</i>
<i>K₁ k_c</i>
<i>K₋₁ k_c</i>
<i>K₂ k_c</i>
<i>K₋₂ k_c</i>
<i>K₃ k_c</i>
<i>K₋₃ k_c</i>
<i>r₁² δ_p (cos)</i>
$\times .046915$
$\times .002751$
$\times .000168$
<i>cos δ δ_p (cos)</i> | - 1
0
0
0
- 3
0
+ 3
- 1
0
-26
+30
+ 3 | - 26
0
- 56
0
+ 72
0
+ 1
- 9
0
+485
- 3
+ 34 | -11076
0
+1712
0
+21
0
0
-9343
0
+8283
0
-48
-4
-1112 | + 176117
-314
+628
0
+126
0
0
+176557
-438
-821
0
-62
-1
+175235 | +21698
-45452
+5964
+ 282
0
0
0
-17508
+8283
-1053
-26
-11
0
-10315 | -23197
-1563
-71
+2378
0
0
0
-22453
-821
-195
+485
0
-2
-22986 | +991
-5142
0
-8
0
+8
0
-4151
-1053
-5
-48
0
+30
-5227 | -2
+19
0
-132
0
0
0
-115
-195
-62
0
-3
-375 |
| | | <i>K₀ k_s</i>
<i>K₁ k_s</i>
<i>K₋₁ k_s</i>
<i>K₂ k_s</i>
<i>K₋₂ k_s</i>
<i>K₃ k_s</i>
<i>K₋₃ k_s</i>
<i>r₁² δ_p (sin)</i>
$\times .046915$
$\times .002751$
$\times .000168$
<i>cos φ δ_p (sin)</i> | + 1
0
0
0
0
0
+ 1
0
-5
-5
- 9 | + 26
0
- 7
0
0
0
+ 8
0
-79
-27
-53
- 151 | -1393
0
-6
0
-293
0
-1692
0
-453
-860
+ 13
-2992 | -676
- 39
-8637
0
-306
0
-9658
-79
-14666
+215
+2
-24186 | -298254
+175
-14556
+36
- 1
0
-312600
-453
+3669
- 5
+30
0
-309359 | +56614
+21483
+116
-9
0
0
+78204
-14666
+518
-27
0
0
+64029 | -1606
+12548
-2
+104
0
0
+11044
+3669
+13
-860
-2
+13864 | -10
-30
0
+322
0
-1
+281
+518
0
+215
0
-3
+961 |

In forming the next ten lines, it will be noticed that the value of r_u corresponding to any vertical column is found u columns to the right. It is therefore necessary to extend the line v two columns at each end. The extension on the right is, however, omitted for want of space. In performing the subtractions it will be convenient to copy the v 's again on the lower edge of a horizontal strip of paper, and, in forming the differences $v_u - v'_u$ to lay the strip above the line of v 's, and $u - u'$ columns to the right.

On the left of each line of differences is written the factor by which that line is to be multiplied.

The mode of formation of the K 's is evident from the formula.

It will be seen that the same computation which gives K_u gives also K_{-u} , only the latter belongs u columns to the right.

Each k_c and k_s is multiplied in succession by all the K 's which lie below it in the same column, but the product by K_u is to be written u columns to the right, and that by K_{-u} u columns to the left. The sum of the products in any one column gives the coefficient of $\frac{\cos}{\sin} (ig + i'l)$ in the development of $r_1^2 \delta\rho$.

This quantity being multiplied by $\frac{dr_0}{u dt} = \frac{a^2 \cos \psi}{r_0^2}$ we have $\cos \psi \delta\rho$, which only needs to be multiplied by $\sec \psi = 1.001103$ to give $\delta\rho$. The units of $r_1^2 \delta\rho$ and $\delta\rho$ correspond to the ninth place of decimals.

All the periodic terms are to be treated in this manner, all the series of values of k_c and k_s , including the constant term, being subjected to the same process. But, when i' and i are both zero, v will be infinite. Here we simply omit the v , treating it as if it were zero. We thus obtain the complete value of the terms with constant coefficients in $r_1^2 \delta\rho$ and $\delta\rho$ which are given in the following table. The terms multiplied by the time are still to be computed. They are derived from (20), which may be put in the form

$$r_1^2 \delta\rho = \frac{1}{2} M \{ \gamma \sum p_u k_c^{(u)} - \xi \sum q_u k_s^{(u)} \} nt.$$

This expression is computed thus:

| u | p_u | $k_c^{(u)}$ | q_u | $k_s^{(u)}$ | $p_u k_c^{(u)}$ | $-q_u k_s^{(u)}$ |
|-----|----------|-------------|---------|-------------|-----------------|------------------|
| 0 | -.140783 | -1244.31 | 0 | 0 | +175.18 | 0 |
| 1 | +.99917 | + 32.78 | +.99972 | -20.90 | + 32.75 | +20.89 |
| 2 | +.0234 | + 4.06 | +.0234 | - 1.86 | + 0.09 | + 0.04 |

We have now

$$\begin{aligned} \sum p_u k_c^{(u)} &= + 208.62; & \frac{1}{2} M \sum p_u k_c^{(u)} &= + 29689 \\ -\sum q_u k_s^{(u)} &= + 20.93; & \frac{1}{2} M \sum q_u k_s^{(u)} &= + 2988 \end{aligned}$$

$$\begin{aligned} r_1^2 \delta\rho &= - 210 nt \\ &+ 2986 nt \cos g &+ 29681 nt \sin g \\ &+ 70 nt \cos 2g &+ 697 nt \sin 2g \\ &+ 2 nt \cos 3g &+ 24 nt \sin 3g, \end{aligned}$$

in units of the ninth place of decimals. The value of $\cos \psi \delta\rho$ is obtained from them by multiplying by r_1^{-2} , exactly as in the case of the constant terms.

| | | $r_1^2 \delta p$ | | $\cos \psi \delta p$ | | $\log M v$ | v_0 | v_1 |
|-----|------|------------------|-----------|----------------------|-----------|------------|----------|--------|
| g | l' | cos | sin | cos | sin | | | |
| 0, | 0 | -210 nt | | -70 nt | | | +139 n't | |
| 1, | | +2986 nt | +29681 nt | +2978 nt | +29633 nt | | | |
| 2, | | +70 nt | +697 nt | +209 nt | +2084 nt | | | |
| 3, | | +2 nt | +24 nt | +14 nt | +139 nt | | | |
| 0, | 0 | -355045 | | -354347 | | ∞ | 0 | |
| 1, | | +14875 | -1503 | -18411 | -1496 | 2.45551 | -2912 | +14206 |
| 2, | | -283 | +49 | -1536 | -21 | 2.25448 | -639 | +457 |
| 3, | | -20 | -1 | -112 | -2 | 1.97839 | -46 | -8 |
| -2, | -1 | -1 | +1 | +3 | -9 | 1.7696 | +1 | +2 |
| -1, | | -10 | +8 | +34 | -151 | 1.8698 | +12 | -99 |
| 0, | | -9343 | -1692 | -1112 | -2992 | 2.00035 | -2876 | +10 |
| +1, | | +176557 | -9658 | +175235 | -24186 | 2.18786 | +536512 | +160 |
| 2, | | -17508 | -312600 | -10315 | -309359 | 2.52504 | +1823 | -23860 |
| 3, | | -22453 | +78204 | -22986 | +64029 | 3.28542 | -13482 | +18310 |
| 4, | | -4151 | +11044 | -5227 | +13864 | 2.39559 | -293 | +104 |
| 5, | | -115 | +281 | -373 | +961 | 2.1235 | -13 | 0 |
| -1, | -2 | -1 | +1 | +1 | -2 | 1.6292 | +9 | +7 |
| 0, | | -56 | -276 | +32 | -100 | 1.6993 | -329 | -1551 |
| +1, | | +710 | +3723 | +1675 | +3728 | 1.78303 | +5874 | +26114 |
| 2, | | +20180 | -13 | +20583 | +484 | 1.88683 | -31660 | +926 |
| 3, | | +7797 | +6512 | +8824 | +6796 | 2.02348 | -6019 | -6860 |
| 4, | | +1534 | +6202 | +2054 | +6230 | 2.22401 | +243 | -2172 |
| 5, | | +2071 | -5954 | +2134 | -5610 | 2.60786 | +527 | -421 |
| 6, | | -653 | +755 | -550 | +494 | 2.98438 | -190 | -20 |
| 1, | -3 | -151 | +64 | -137 | +75 | 1.5772 | -1464 | +653 |
| 2, | | -52 | +221 | +149 | +229 | 1.63886 | +231 | -833 |
| 3, | | +4348 | +15 | +4420 | +88 | 1.71074 | -13050 | +1 |
| 4, | | +1564 | +1280 | +1774 | +1314 | 1.79691 | -3225 | -3126 |
| 5, | | +133 | +685 | +214 | +756 | 1.9045 | -18 | -1943 |
| 6, | | -111 | +232 | -109 | +272 | 2.0479 | +144 | -163 |
| 7, | | -163 | +87 | -168 | +100 | 2.2634 | +50 | -7 |
| 3, | -4 | -28 | +121 | +31 | +122 | 1.5408 | +181 | -638 |
| 4, | | +1223 | +17 | +1245 | +41 | 1.5858 | -5578 | -58 |
| 5, | | +485 | +389 | +545 | +400 | 1.6488 | -1721 | -1540 |
| 6, | | +38 | +201 | +63 | +221 | 1.7225 | -50 | -590 |
| 7, | | -25 | +53 | -23 | +63 | 1.8012 | +70 | -100 |
| 8, | | -15 | +7 | -16 | +9 | 1.9229 | | |
| 5, | -5 | +392 | +12 | +400 | +18 | 1.4889 | -2432 | -74 |
| 6, | | +175 | +138 | +194 | +142 | 1.5385 | -908 | -760 |
| 7, | | +14 | +77 | +22 | +84 | 1.5946 | -40 | -340 |
| 8, | | -10 | +20 | -9 | +24 | 1.6589 | +40 | -70 |
| 9, | | -5 | +3 | -5 | +4 | 1.7345 | | |

Perturbations of Longitude.

The perturbations of the longitude are now to be computed by formulæ (24). To do this in the most simple way we remark that the numbers given on page 42, under the heading $\frac{\partial R}{\partial v}$, are those represented in formula (42) by v_s and v_c . If we put

$$nt = t'$$

equation (24) may be put into the form

$$\frac{d^2 v}{dt^2} = r_1^{-2} \left\{ \frac{a}{1+m} \int \frac{\partial R}{\partial v} dt - 2 \cos \psi \delta \rho \right\};$$

but we have from (42)

$$\int \frac{\partial R}{\partial v} dt = \Sigma \left(\frac{m'v}{a_1} v_c \sin N - \frac{m'v}{a_1} v_s \cos N \right).$$

If now we represent the numerical values of $\cos \psi \delta \rho$, already found, by

$$\Sigma (\rho_s \sin N + \rho_c \cos N),$$

and if we substitute these expressions in the above value of $\frac{d^2 v}{dt^2}$, the latter will become

$$\frac{d^2 v}{dt^2} = r_1^{-2} \Sigma \{ (v_s - 2\rho_s) \sin N + (v_c - 2\rho_c) \cos N \},$$

where we put for brevity

$$\begin{aligned} v_s &= Mv_c, \\ v_c &= -Mv_s. \end{aligned}$$

The numerical expression for r_1^{-2} is given on page 40, and by multiplying the quantities within brackets by this expression, after the manner explained on pages 40 and 41, we form the terms of $\frac{d^2 v}{dt^2}$. Multiplying each of these terms by its corresponding value of v , changing *cos* to *sin* and *sin* to *cos*, we have the coefficients in the expressions for δv given on page 50.

As previously mentioned, before commencing the above computation, I had computed all the perturbations of Uranus by the method of "perturbations of the elements," using the formulæ developed in my Investigation of the Orbit of Neptune. The two results are here placed side by side, for the purpose of comparison. The discrepancies in the various coefficients, expressed in thousandths of a second, are shown in the sixth and seventh columns.

It will be seen that the largest discrepancies, and indeed the only ones (with a single exception) exceeding one-tenth of a second, occur in the coefficients of the terms $2j' - l$ and $3g' - l$. Here the errors are almost certainly in the computation from perturbations of the elements. Owing to the long period of the term $3g' - l$ they would not become sensible in the course of any one century.

PERTURBATIONS OF THE LONGITUDE OF URANUS PRODUCED BY THE ACTION OF SATURN, AND DEPENDING ON THE FIRST POWER OF THE DISTURBING FORCES.

| <i>g, l'</i> | From comp. preceding
δv | | From pert. of elements
δv | | Discrepancy. | | 0.434294 δp | |
|--------------|------------------------------------|--------------------|--------------------------------------|-------------------|----------------|----------------|---------------------|----------------|
| | sin
" | cos
" | sin
" | cos
" | sin
" | cos
" | cos | sin |
| 0, 0 | | + 10.9690 <i>t</i> | | +10.9645 <i>t</i> | | .0045 <i>t</i> | | |
| 1, | - 1.230 <i>nt</i> | + 12.231 <i>nt</i> | -1.228 <i>nt</i> | +12.271 <i>nt</i> | .002 <i>nt</i> | .040 <i>nt</i> | +13 <i>nt</i> | +123 <i>nt</i> |
| 2, | - 0.072 <i>nt</i> | + 0.717 <i>nt</i> | -0.072 <i>nt</i> | + 0.720 <i>nt</i> | 0 | .003 <i>nt</i> | +1 <i>nt</i> | +9 <i>nt</i> |
| 3, | - 0.004 <i>nt</i> | + 0.043 <i>nt</i> | -0.004 <i>nt</i> | + 0.043 <i>nt</i> | 0 | 0 | 0 | 0 |
| 0, 0 | | | | | | | -1541 | |
| 1, | + 8.545 | - 4.735 | + 2.844 | + 1.013 | | | - 80 | - 6 |
| 2, | + 0.461 | - 0.169 | + 0.133 | + 0.166 | | | - 7 | 0 |
| 3, | + 0.028 | - 0.005 | + 0.013 | + 0.014 | | | 0 | 0 |
| -1,-1 | + 0.036 | + 0.039 | + 0.032 | + 0.005 | 4 | 34 | 0 | - 1 |
| 0, | + 1.282 | + 0.718 | + 1.280 | + 0.719 | 2 | 1 | - 5 | - 13 |
| 1, | -20.817 | + 8.522 | - 20.873 | + 8.595 | 56 | 73 | +761 | - 106 |
| 2, | -11.890 | +143.463 | - 11.093 | +143.465 | 797 | 2 | - 45 | -1351 |
| 3, | + 49.30 | +115.86 | + 49.62 | +116.08 | 320 | 220 | -103 | + 280 |
| 4, | + 2.133 | + 5.616 | + 2.195 | + 5.621 | 62 | 5 | - 24 | + 61 |
| 5, | + 0.126 | + 0.329 | + 0.109 | + 0.331 | 17 | 2 | - 2 | + 4 |
| 0,-2 | + 0.017 | - 0.017 | + 0.025 | - 0.033 | 8 | 16 | 0 | 0 |
| 1, | + 0.042 | + 0.814 | + 0.034 | + 0.818 | 8 | 4 | + 7 | + 16 |
| 2, | + 4.110 | - 0.009 | + 4.103 | - 0.012 | 7 | 3 | + 89 | + 2 |
| 3, | + 2.079 | - 1.607 | + 2.106 | - 1.676 | 27 | 69 | + 38 | + 30 |
| 4, | + 0.648 | - 1.830 | + 0.643 | - 1.902 | 5 | 72 | + 9 | + 27 |
| 5, | + 1.163 | + 2.956 | + 1.274 | + 2.991 | 111 | 35 | + 9 | - 24 |
| 6, | + 0.503 | + 0.378 | + 0.556 | + 0.445 | 53 | 67 | - 2 | + 2 |
| 1,-3 | + 0.034 | + 0.012 | + 0.036 | + 0.015 | 2 | 3 | 0 | 0 |
| 2, | + 0.037 | - 0.041 | + 0.014 | - 0.050 | 23 | 9 | + 1 | + 1 |
| 3, | + 0.824 | - 0.019 | + 0.812 | - 0.017 | 12 | 2 | + 19 | 0 |
| 4, | + 0.355 | - 0.267 | + 0.351 | - 0.263 | 4 | 4 | + 8 | + 6 |
| 5, | + 0.047 | - 0.165 | + 0.039 | - 0.191 | 8 | 26 | + 1 | + 3 |
| 6, | - 0.026 | - 0.063 | - 0.028 | - 0.066 | 2 | 3 | 0 | + 1 |
| 7, | - 0.053 | - 0.032 | - 0.053 | - 0.018 | 0 | 14 | - 1 | 0 |
| 3,-4 | + 0.006 | - 0.022 | - 0.005 | - 0.023 | 11 | 1 | 0 | + 1 |
| 4, | + 0.228 | - 0.008 | + 0.221 | + 0.002 | 7 | 10 | + 5 | 0 |
| 5, | + 0.103 | - 0.077 | + 0.084 | - 0.075 | 19 | 2 | + 2 | + 2 |
| 6, | + 0.013 | - 0.044 | + 0.013 | - 0.057 | 0 | 13 | 0 | + 1 |
| 7, | - 0.005 | - 0.013 | - 0.001 | - 0.015 | 4 | 2 | 0 | 0 |
| 8, | - 0.003 | - 0.002 | 0 | + 0.002 | 3 | 4 | 0 | 0 |
| 5,-5 | + 0.074 | - 0.003 | + 0.071 | 0 | 3 | 3 | + 2 | 0 |
| 6, | + 0.038 | - 0.027 | + 0.023 | - 0.026 | 15 | 1 | + 1 | + 1 |
| 7, | + 0.005 | - 0.016 | + 0.005 | - 0.025 | 0 | 9 | 0 | 0 |
| 8, | - 0.002 | - 0.004 | 0 | - 0.003 | 2 | 1 | 0 | 0 |

Perturbations of Latitude.

These are computed from the formulæ (27) and (40), no reductions being made from δk and $\delta \eta$ to δp and δq , but the perturbations of the latitude being computed directly from the former by (40). We have only to represent the expressions for δk and $\delta \eta$ by

$$\begin{aligned}\delta k &= -\Sigma a_c \cos N - \Sigma a_s \sin N \\ \delta \eta &= \Sigma a'_c \cos N + \Sigma a'_s \sin N\end{aligned}$$

and substitute ω for π in the equations (40) from which $\delta \beta$ is computed.

The principal steps of the computation are shown quite fully in the following table. The values of

$$\frac{\partial h}{\partial \gamma} = \frac{1}{2} \cos \frac{1}{2} \gamma \frac{\partial h}{\partial \sigma}$$

are first formed from those terms of h , on pages 37 and 38, which contain σ as a coefficient. Then, having for each original term of R

$$\frac{\partial R}{\partial \gamma} = \frac{m'}{a_1} \frac{\partial h}{\partial \gamma} \cos N$$

all the terms which have the same coefficients of λ and λ' in N are combined into two depending on g and l as shown in the case of R on page 36. The coefficients of these terms, in units of the third place of decimals, are given in the columns headed $\frac{\partial R}{\partial \gamma}$.

The value of $\frac{\partial h}{2\sigma} \sin N$ being formed for each term of R , all the terms depending on the same multiples of λ and λ' are combined into two, of which the coefficients are given under the proper heading. The terms of $(i+j)\sigma h \sin N$ being formed in like manner, we have, by adding the last two expressions, all the quantities which enter into the formulæ (27). To integrate these equations thus forming the numerical values of δk and $\delta \eta$ we have only to multiply each term in the second, third, eighth, and ninth columns of the table by the corresponding values of $\frac{m' a \nu}{a_1 \cos \psi}$, for which we may use the value of $\frac{M \nu}{\sin 1''}$ already given.

The quantities given in the four columns under δk and $-\delta \eta$ show the values of $-a_s, -a_c, -a'_c, -a'_s$, corresponding to each argument. From these the terms of $\delta \beta$ are formed by equation (40) with the modification mentioned above.

PERTURBATIONS OF THE LATITUDE PRODUCED BY SATURN.

| $g \ v'$ | | $\frac{\partial R}{\partial \gamma} = \frac{m'}{a_1} \times$ | | $\Sigma \frac{ih}{2\sigma} \sin N$ | | $\Sigma (i+j)sh \sin N$ | | $\frac{dn}{dt} = Mn \times$ | | δk | | $-\delta \eta$ | | $\delta \beta$ | |
|----------|------|--|--------|------------------------------------|--------|-------------------------|--------|-----------------------------|--------|------------|--------|----------------|--------|----------------|--------|
| i | i' | cos | sin | sin | cos | sin | cos | sin | cos | sin | cos | cos | sin | sin | cos |
| 0, | 0 | -10.82 | 0 | 0 | 0 | 0 | +0.008 | 0.00 | + .008 | " | " | " | " | " | +0.192 |
| 1, | | - 3.45 | - 2.92 | -1.94 | - 0.11 | + 0.17 | +0.84 | - 1.77 | + 0.73 | +0.203 | -0.172 | -0.104 | -0.043 | +0.246 | +0.209 |
| 2, | | + 1.42 | -11.10 | +1.30 | +10.73 | + 0.07 | +0.05 | + 1.37 | +10.78 | -0.042 | -0.326 | +0.042 | -0.317 | -0.049 | +0.015 |
| 3, | | + 1.31 | - 1.71 | +1.31 | + 1.71 | + 0.01 | 0.00 | + 1.32 | + 1.71 | -0.026 | -0.033 | +0.026 | -0.033 | -0.008 | +0.003 |
| -2,-1 | | + 1.0 | + 0.1 | -1.0 | - 0.1 | 0.00 | 0.00 | - 1.0 | - 0.1 | +0.012 | -0.001 | +0.012 | -0.001 | 0.000 | 0.000 |
| -1, | | - 6.43 | -52.6 | +6.4 | -52.8 | + 0.03 | +0.02 | + 6.4 | -52.8 | -0.098 | +0.804 | -0.098 | -0.804 | -0.004 | +0.018 |
| 0, | | - 6.97 | + 7.06 | +1.53 | + 5.14 | - 4.91 | 0.00 | - 3.38 | + 5.14 | -0.144 | -0.145 | +0.070 | +0.106 | -0.661 | +0.576 |
| +1, | | +46.67 | 0 | 0.00 | 0.00 | +59.45 | -0.02 | +59.45 | - 0.02 | +1.482 | 0 | -1.887 | -0.001 | +0.084 | -0.026 |
| 2, | | - 1.84 | - 2.86 | -0.99 | - 0.20 | + 0.93 | +1.22 | - 0.06 | + 1.02 | -0.127 | +0.197 | +0.004 | +0.070 | +2.218 | +1.920 |
| 3, | | + 0.90 | - 6.98 | +0.78 | + 6.44 | + 0.12 | +0.16 | + 0.90 | + 6.60 | -0.358 | -2 781 | +0.358 | -2.630 | +0.091 | -0.071 |
| 4, | | + 1.09 | - 1.38 | +1.09 | + 1.38 | + 0.02 | +0.01 | + 1.11 | + 1.39 | -0.056 | -0.071 | +0.057 | -0.071 | -0.048 | +0.056 |
| -1,-2 | | + 7.6 | - 1.5 | -7.6 | + 1.5 | 0.00 | 0.00 | - 7.6 | + 1.5 | +0.067 | +0.013 | +0.067 | +0.013 | +0.003 | +0.004 |
| 0, | | + 1.3 | +10.9 | -1.3 | +10.7 | - 0.11 | +0.53 | - 1.4 | +11.2 | +0.013 | -0.112 | +0.014 | +0.116 | +0.008 | +0.034 |
| +1, | | - 0.27 | + 9.03 | +1.70 | + 0.91 | + 1.65 | -7.35 | + 3.35 | - 6.44 | -0.003 | -0.112 | -0.042 | -0.080 | +0.040 | -0.042 |
| 2, | | -14.51 | 0 | 0.00 | 0.00 | - 7.01 | -0.21 | - 7.01 | - 0.21 | -0.230 | 0 | +0.112 | -0.003 | 0 | -0.029 |
| 3, | | - 2.85 | - 2.29 | -0.42 | - 0.19 | - 0.97 | +1.11 | - 1.39 | + 0.92 | -0.062 | +0.050 | +0.030 | +0.020 | -0.063 | -0.054 |
| 4, | | + 0.43 | - 4.23 | +0.44 | + 3.67 | + 0.02 | +0.22 | + 0.46 | + 3.89 | +0.015 | +0.146 | -0.016 | +0.134 | -0.080 | -0.014 |
| 5, | | + 0.80 | - 0.99 | +0.80 | + 0.99 | + 0.02 | +0.02 | + 0.82 | + 1.01 | +0.007 | +0.083 | -0.068 | +0.084 | +0.008 | -0.004 |
| 2,-3 | | - 1.17 | + 0.86 | +1.22 | + 0.97 | + 0.09 | +0.33 | + 1.31 | + 1.30 | -0.010 | -0.008 | -0.011 | +0.012 | +0.050 | -0.046 |
| 3, | | - 8.54 | 0 | 0.00 | 0.00 | - 4.34 | 0 | - 4.34 | 0.00 | -0.090 | 0 | +0.046 | 0 | +0.004 | -0.010 |
| 4, | | - 2.08 | - 1.65 | -0.13 | - 0.15 | - 0.88 | +0.85 | - 1 01 | + 0.70 | -0.026 | +0.022 | +0.013 | +0.009 | -0.012 | -0.012 |
| 5, | | + 0.14 | - 2.50 | +0.24 | + 2.00 | 0.00 | +0.22 | + 0.24 | + 2.22 | +0.002 | +0.041 | -0.004 | +0.037 | -0.010 | -0.002 |
| 6, | | + 0.55 | - 0.66 | +0.55 | + 0.66 | + 0.02 | +0.02 | + 0.57 | + 0.68 | +0.013 | +0.015 | -0.013 | +0.016 | +0.004 | -0.001 |
| 3,-4 | | - 0.72 | + 0.47 | +0.82 | + 0.78 | + 0.09 | +0.32 | + 0.91 | + 1.10 | -0.005 | -0.003 | -0.006 | +0.008 | +0.022 | -0.020 |
| 4, | | - 4.80 | 0 | 0.00 | 0.00 | - 2.48 | -0.02 | - 2.48 | - 0.02 | -0.038 | 0 | +0.020 | 0 | +0.002 | -0.005 |
| 5, | | - 1.41 | - 1.09 | -0.01 | - 0.10 | - 0.66 | +0.59 | - 0.67 | + 0.49 | -0.013 | +0.010 | +0.006 | +0.004 | -0.004 | -0.003 |
| 6, | | + 0.12 | - 1.3 | +0.12 | + 1.0 | - 0.02 | +0.19 | + 0.10 | + 1.2 | +0.001 | +0.014 | -0.001 | +0.013 | +0.004 | +0.002 |
| 5,-5 | | - 2.64 | 0 | 0 | 0 | - 1.35 | +0.04 | - 1.35 | + 0.04 | -0.017 | 0 | +0.008 | 0 | | |

Secular terms $\left\{ \begin{array}{l} \delta k = + 4''.77 T \\ \delta \beta = - 4''.77 T \cos v. \end{array} \right.$

CHAPTER III.

PERTURBATIONS PRODUCED BY NEPTUNE AND JUPITER.

THE perturbations of Uranus by Neptune were originally computed with elements of both planets quite different from those finally adopted. But the last computations, on which the concluded values of the perturbations depend, were made with the concluded elements of Neptune found in my investigation of the orbit of that planet.¹ They are as follows:

| | ° | ' | " |
|-----------------|-----|----|-------------------|
| π , | 43 | 17 | 30 |
| θ , | 130 | 7 | 33 |
| ε , | 335 | 5 | 39 |
| ϕ , | 1 | 47 | 1.6 |
| n , | | | 7864.935 |
| e , | | | 0.0084962 |
| $\log a$, | | | 1.478141 |
| Mass, | | | $\frac{1}{17000}$ |

Hence follow the following functions of the elements of Neptune and Uranus:

$$\begin{aligned} \alpha &= 0.638195 \\ \omega &= 12^\circ 44' 58'' \\ \omega' &= 247 45 20 \\ \gamma &= 1 30 29.6 \\ \sigma = \sin \frac{1}{2} \gamma &= 0.013161 \\ M &= 37.522 \text{ (in units of 6th place of decimals).} \end{aligned}$$

From these values of the elements are obtained the following values of the various terms in the development of the perturbative function, and of ν . As the developments have been formed on the same principle as in the case of Saturn, it is deemed unnecessary to give the details of the process. It is only necessary to remark that the indices i and j are the coefficients of l and g respectively, the mean longitude of Neptune, or l , being counted from the perihelion of Uranus.

¹ Smithsonian Contributions to Knowledge, Vol. XV.

| | | ACTION OF NEPTUNE. | | | | | | | | | |
|------|-----|-----------------------------|---------|---|---------|---|--------|-----------------------------|---------|-----------|--|
| | | $R = \frac{m'}{a_1} \times$ | | $\frac{\partial R}{\partial v} = \frac{m'}{a_1} \times$ | | $\frac{\partial R}{\partial p} = \frac{m'}{a_1} \times$ | | $Q = \frac{m'}{a_1} \times$ | | v | |
| i' | i | h_c | h_s | r_s | r_c | cos | sin | k_c | k_s | | |
| 0, | 0 | +1135.63 | 0 | 0 | + 0.202 | +338.26 | 0 | +368.26 | 0 | + 10 | |
| | + 1 | - 18.07 | - 1.25 | + 1.01 | - 1.23 | - 64.80 | - 5.89 | -100.94 | - 8.39 | + 0.5 | |
| | + 2 | + 0.41 | - 0.03 | - 0.6 | - 0.1 | + 2.98 | + 0.12 | + 3.82 | + 0.06 | + 0.33333 | |
| | + 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | |
| 1, | - 3 | + 0.21 | - 0.01 | + 0.7 | - 0.2 | + 1.6 | + 0.2 | + 2.1 | + 0.1 | - 0.40158 | |
| | - 2 | - 5.21 | - 0.62 | - 4.90 | + 1.27 | - 29.5 | + 0.6 | - 43.5 | - 1.1 | - 0.67107 | |
| | - 1 | + 134.19 | + 0.30 | +133.98 | - 0.08 | +509.94 | + 0.77 | +1057.49 | + 1.99 | - 2.04023 | |
| | 0 | - 25.45 | -10.43 | -18.05 | + 0.02 | - 84.57 | -11.47 | - 84.57 | -11.47 | + 1.96137 | |
| | + 1 | + 1.12 | + 1.19 | + 0.5 | - 0.1 | + 5.83 | + 0.82 | + 7.31 | + 1.07 | + 0.66232 | |
| | + 2 | - 0.05 | + 0.01 | + 0.02 | + 0.02 | - 0.24 | 0 | - 0.32 | + 0.02 | + 0.39843 | |
| 2, | - 4 | + 0.85 | - 0.03 | + 2.1 | 0 | + 2.5 | + 0.1 | + 4.8 | 0 | - 0.33554 | |
| | - 3 | + 13.53 | - 1.01 | + 27.7 | + 3.1 | + 18.7 | - 2.1 | + 59.7 | - 5.5 | - 0.50497 | |
| | - 2 | + 375.92 | + 0.36 | +751.59 | - 0.73 | + 9 7.82 | + 1.25 | +2491.69 | + 2.72 | - 1.02009 | |
| | - 1 | - 60.090 | - 3.180 | -117.903 | + 3.203 | -171.79 | -13.13 | +5931.21 | +309.82 | +50.7820 | |
| | 0 | + 3.31 | + 0.46 | + 5.9 | - 0.4 | + 12.24 | + 1.84 | + 12.24 | + 1.84 | + 0.98069 | |
| | + 1 | - 0.09 | - 0.01 | - 0.09 | + 0.02 | - 0.51 | - 0.08 | - 0.60 | - 0.09 | + 0.49512 | |
| 3, | - 5 | + 0.83 | - 0.04 | + 2.7 | + 0.2 | + 3.0 | - 0.1 | + 5.5 | - 0.2 | - 0.28814 | |
| | - 4 | + 12.09 | - 0.9 | + 36.9 | + 3.8 | + 34.5 | - 3.6 | + 73.6 | - 6.7 | - 0.40478 | |
| | - 3 | + 200.37 | - 0.04 | +601.13 | - 0.42 | +717.94 | + 0.81 | +1535.51 | + 0.65 | - 0.68006 | |
| | - 2 | - 52.33 | - 9.88 | -150.02 | +19.81 | -190.33 | -27.51 | -635.23 | -111.69 | - 2.12559 | |
| | - 1 | + 5.05 | + 1.56 | + 13.8 | - 3.1 | + 20.12 | + 4.65 | + 1.0 | - 1.2 | + 1.88844 | |
| | 0 | - 0.24 | - 0.07 | - 0.58 | + 0.15 | - 1.11 | - 0.30 | - 1.11 | - 0.30 | + 0.65378 | |
| 4, | - 6 | + 0.75 | - 0.07 | + 3.2 | + 0.3 | + 3.4 | - 0.2 | + 5.7 | - 0.4 | - 0.25249 | |
| | - 5 | + 9.47 | - 0.79 | + 38.3 | + 4.0 | + 38.5 | - 3.9 | + 70.4 | - 6.6 | - 0.33777 | |
| | - 4 | + 111.16 | - 0.12 | +444.7 | - 0.18 | +512.22 | + 0.35 | +965.7 | - 0.1 | - 0.51004 | |
| | - 3 | - 38.58 | - 7.43 | -149.10 | +22.37 | -178.15 | -27.95 | -419.12 | -74.36 | - 1.04100 | |
| | - 2 | + 5.205 | + 1.896 | + 19.54 | - 5.50 | + 25.30 | + 7.19 | -503.3 | -185.4 | +25.3910 | |
| | - 1 | + 0.36 | - 0.17 | - 1.28 | + 1.49 | - 1.90 | - 0.70 | - 1.21 | - 0.37 | + 0.96210 | |
| 5, | - 7 | + 0.65 | - 0.07 | + 3.5 | + 0.4 | + 3.5 | - 0.3 | + 5.5 | - 0.5 | - 0.22468 | |
| | - 6 | + 6.97 | - 0.59 | + 35.1 | + 3.4 | + 36.7 | - 3.4 | + 60.9 | - 5.5 | - 0.28984 | |
| | - 5 | + 63.03 | - 0.16 | +315.3 | + 0.1 | +354.8 | - 0.1 | +612.0 | - 0.7 | - 0.40804 | |
| | - 4 | - 27.43 | - 5.32 | -133.4 | +21.4 | -153.6 | -25.3 | -304.8 | - 54.6 | - 0.68929 | |
| | - 3 | + 4.84 | + 1.81 | + 23.0 | - 7.0 | + 27.82 | + 8.57 | + 92.2 | + 32.7 | - 2.21843 | |
| | - 2 | - 0.45 | - 0.24 | - 2.09 | + 0.90 | - 2.77 | - 1.14 | + 0.48 | + 0.57 | + 1.82073 | |
| | - 1 | + 0.03 | + 0.02 | + 0.12 | - 0.06 | + 0.18 | + 0.09 | + 0.15 | + 0.07 | + 0.6455 | |
| 6, | - 7 | + 4.93 | - 0.41 | + 29.8 | + 2.7 | + 31.5 | - 2.9 | + 49.0 | - 4.4 | - 0.2538 | |
| | - 6 | + 36.16 | - 0.17 | +217.1 | + 0.4 | +239.9 | - 0.4 | +387.4 | - 1.1 | - 0.3400 | |
| | - 5 | - 19.04 | - 3.74 | -111.6 | +18.8 | -125.7 | -21.4 | -223.8 | - 40.6 | - 0.5152 | |
| | - 4 | + 4.14 | + 1.59 | + 23.9 | - 7.8 | + 27.76 | + 9.06 | + 63.0 | + 22.6 | - 1.0628 | |
| | - 3 | - 0.490 | - 0.271 | - 2.79 | + 1.30 | - 3.42 | - 1.55 | + 46.40 | + 26.01 | +16.9273 | |
| | - 2 | + 0.036 | + 0.025 | + 0.20 | - 0.12 | + 0.28 | + 0.16 | + 0.14 | + 0.06 | + 0.9443 | |
| 7, | - 8 | + 3.41 | - 0.28 | + 24.0 | + 2.1 | + 25.5 | - 2.5 | + 37.8 | - 3.5 | - 0.2257 | |
| | - 7 | + 20.90 | - 0.16 | +146.4 | + 0.4 | +160.0 | - 0.6 | +245.3 | - 1.3 | - 0.2914 | |
| | - 6 | - 12.99 | - 2.54 | - 89.2 | +15.4 | - 98.9 | -17.2 | -163.1 | - 29.8 | - 0.4113 | |
| | - 5 | + 3.40 | + 1.31 | + 23.0 | - 7.7 | + 26.0 | + 8.8 | + 49.8 | + 18.0 | - 0.6988 | |
| | - 4 | - 0.487 | - 0.276 | - 3.25 | + 1.61 | - 3.79 | - 1.87 | - 12.83 | - 6.99 | - 2.3198 | |
| | - 3 | + 0.044 | + 0.034 | + 0.28 | - 0.19 | + 0.37 | + 0.24 | - 0.09 | - 0.12 | + 1.7577 | |
| 8, | - 9 | + 2.35 | - 0.20 | + 18.8 | + 1.6 | + 19.5 | - 2.1 | + 28.1 | - 1.2 | - 0.2032 | |
| | - 8 | + 12.20 | - 0.13 | + 96.6 | + 0.3 | +105.0 | - 0.7 | +154.8 | - 1.3 | - 0.2550 | |
| | - 7 | - 8.8 | - 1.71 | - 68.9 | +12.1 | - 75.5 | -13.1 | -117.7 | - 21.3 | - 0.3423 | |
| | - 6 | + 2.69 | + 1.05 | + 21.0 | - 7.1 | + 23.2 | + 8.1 | + 40.0 | + 14.7 | - 0.5205 | |
| | - 5 | - 0.45 | - 0.25 | - 3.5 | + 1.8 | - 3.90 | - 1.99 | - 8.8 | - 4.8 | - 1.0855 | |
| | - 4 | + 0.049 | + 0.039 | + 0.37 | - 0.26 | + 0.45 | + 0.31 | - 4.50 | - 3.69 | +12.6955 | |
| 9, | - 9 | + 7.0 | - 0.1 | + 63.4 | + 0.1 | + 68.2 | - 0.6 | + 96.8 | - 1.0 | - 0.2267 | |
| | - 8 | - 5.9 | - 1.13 | - 51.9 | + 9.1 | - 56.3 | - 9.9 | - 84.0 | - 15.2 | - 0.2931 | |
| | - 7 | + 2.07 | + 0.81 | + 18.1 | - 6.4 | + 19.8 | + 7.0 | + 31.8 | + 11.7 | - 0.4147 | |
| | - 6 | - 0.39 | - 0.25 | - 3.4 | + 1.9 | - 3.78 | - 2.02 | - 7.1 | - 4.1 | - 0.7085 | |
| | - 5 | + 0.050 | + 0.042 | + 0.43 | - 0.32 | + 0.51 | + 0.36 | + 1.72 | + 1.38 | - 2.4308 | |
| 10, | -10 | + 4.1 | - 0.1 | + 43. | 0 | + 44.3 | - 0.5 | + 61. | - 0.8 | - 0.2040 | |
| | - 9 | - 3.9 | - 0.8 | - 40. | + 6. | - 42.0 | - 7.0 | - 60. | - 11. | - 0.2563 | |
| | - 8 | + 1.6 | + 0.6 | + 14.7 | - 5.6 | + 16.1 | + 6.0 | + 25.1 | + 9.3 | - 0.3446 | |
| | - 7 | - 0.3 | - 0.2 | - 3.1 | + 1.9 | - 3.5 | - 1.9 | - 5.8 | - 3.4 | - 0.5259 | |
| | - 6 | + 0.05 | + 0.04 | + 0.46 | - 0.37 | + 0.53 | + 0.40 | + 1.16 | + 0.96 | - 1.1092 | |

The term of Long Period.

From the expressions for the perturbations of Uranus, subsequently given, it will be seen that several of the terms have very large coefficients, that of $\sin (2l' - g)$ being nearly an entire degree. The magnitude of most of the terms in which l' is even arises from the near approach to commensurability in the mean motions of the two planets. Twice the mean annual motion of Neptune exceeds that of Uranus by only $303''.8$. The elements of the orbits of both planets will therefore, in consequence of their mutual action, be affected with a slow oscillation, having a period of about 4266 years. The employment of these large terms and the great inconveniences to which they will give rise, especially in the corrections of the elements of Uranus, may be avoided by the device employed in the theory of Neptune. The following are the essential features of this method:

First, all the perturbations arising from that portion of the perturbative function in which the coefficient of the time is $2n' - n$ or its multiples are considered and developed as perturbations of the elements.

Secondly, the arbitrary constants to be added to the integrals of these perturbations are so taken that the perturbations shall vanish at the epoch 1850.0.

In other words, the perturbations in question will be treated as producing secular variations of the elements of the orbit, only, instead of being developed in powers of the time, these variations will be retained in their rigorous form.

The formulæ for the computations of the perturbations in question, are as follows:

Let

$$\frac{m'}{a_1} h \cos (i' l' + i l + j' \omega' + j \omega) = \frac{m'}{a_1} \cos N$$

be any term of the perturbative function, h being a function of α , e , and σ .

$$\begin{aligned} \sin \psi &= e \\ g &= \cos \psi \tan \frac{1}{2} \psi \\ \iota &= -(i' + i + j' + j) \\ v &= \frac{n}{i'n' + i'n} \end{aligned}$$

For each such term, compute

$$\begin{aligned} A &= 2 i h \\ L &= -3 i h - 2 \frac{\partial h}{\partial v} + \frac{\partial h}{\partial e} \\ e W &= \cos \psi \frac{\partial h}{\partial e} \\ E &= -h(i g + j \cot \psi) \\ T &= \frac{1}{2} \frac{\partial h}{\partial \sigma} \\ I &= \frac{1}{2} \iota \frac{h}{\sigma} + (i + j) \sigma h. \end{aligned}$$

The corresponding perturbations of the elements may then be put into the form

$$\begin{aligned}\delta \log a &= MvA \cos N + \delta\alpha_0, \\ \delta l &= MvL \sin N + \delta l_0, \\ e\delta\pi &= MveW \sin N + e\delta\pi_0, \\ \delta e &= MvE \cos N + \delta e_0, \\ \delta\gamma &= MvI \cos N + \delta\gamma_0, \\ \tan\gamma \delta\tau &= MvT \sin N + \tan\gamma \delta\tau_0.\end{aligned}$$

Here, $\delta\alpha_0$, δl_0 , etc., are arbitrary constants so taken that $\delta \log a$, δl , etc., shall vanish at the fundamental epoch.

All the terms depending on the same values of i' and i are to be combined into a single one. And it will save labor to make this combination at as early a stage as possible in the computation; that is, to multiply the various values of h , $\frac{\partial h}{\partial v}$, $\frac{\partial h}{\partial e}$, $\frac{\partial h}{\partial \sigma}$, and E by the sines and cosines of $j'\omega' + j\omega$, and afterward proceed with the sums of the products according to the proper modification of the formulæ.

Thus are obtained the following long period perturbations of the elements of Uranus:

$$\begin{aligned}\delta l &= -3474.32 \sin(2l' - g) + 180.10 \cos(2l' - g) \\ &+ 146.72 \sin(4l' - 2g) - 54.10 \cos(4l' - 2g) \\ &- 8.97 \sin(6l' - 3g) + 5.03 \cos(6l' - 3g) \\ &+ 0.64 \sin(8l' - 4g) - 0.53 \cos(8l' - 4g) \\ &+ \text{constant} = 3320''.18.\end{aligned}$$

$$\begin{aligned}e\delta\pi &= -484.96 \sin(2l' - g) + 0.73 \cos(2l' - g) \\ &+ 38.06 \sin(4l' - 2g) - 7.06 \cos(4l' - 2g) \\ &- 3.61 \sin(6l' - 3g) + 1.38 \cos(6l' - 3g) \\ &+ 0.33 \sin(8l' - 4g) - 0.15 \cos(8l' - 4g) \\ &+ \text{constant} = 465''.23.\end{aligned}$$

$$\begin{aligned}&= -484.21 \cos(2l' - g) - 0.29 \sin(2l' - g) \\ &+ 38.21 \cos(4l' - 2g) + 7.16 \sin(4l' - 2g) \\ &- 3.61 \cos(6l' - 3g) - 1.40 \sin(6l' - 3g) \\ &+ 0.33 \cos(8l' - 4g) + 0.15 \sin(8l' - 4g) \\ &- \text{constant} = 158''.59.\end{aligned}$$

$$\begin{aligned}\delta\sigma &= +2277 \cos(2l' - g) + 120 \sin(2l' - g) \\ &- 198 \cos(4l' - 2g) - 78 \sin(4l' - 2g) \\ &+ 18 \cos(6l' - 3g) + 7 \sin(6l' - 3g) \\ &+ \text{constant} = 630.\end{aligned}$$

The variations of the elements which fix the position of the plane of the orbit are here omitted, because their nature is such that it is indifferent in which form they are developed.

These expressions are reduced to perturbations of the co-ordinates by the follow-

ing formulæ. Express the usual developments of the longitude and logarithm of radius vector in the form

$$v = l + \Sigma V_i \sin ig;$$

$$\rho = r + \Sigma R_i \cos ig.$$

Put also

$$V'_i = \frac{\partial V_i}{\partial e}$$

$$V''_i = \frac{i}{e} V_i$$

$$R'_i = \frac{\partial R_i}{\partial e}$$

$$R''_i = -\frac{i}{e} R_i.$$

Express any set of corresponding terms of the preceding perturbations in the form

$$\delta l = L_s \sin N + L_c \cos N;$$

$$e\delta l - e\delta\pi = F_s \sin N + F_c \cos N;$$

$$\delta e = E_c \cos N + E_s \sin N;$$

$$\delta r = A_s \cos N + A_c \sin N.$$

We shall then have

$$2\delta v = \Sigma (V''_i F_s + V'_i E_c) \sin (N + ig) + \Sigma (V''_i F_c - V'_i E_s) \sin (N - ig)$$

$$+ \Sigma (V''_i F_c - V'_i E_s) \cos (N + ig) + \Sigma (V''_i F_s + V'_i E_c) \cos (N - ig)$$

$$+ 2\delta l$$

$$2\delta\rho = \Sigma (R'_i E_c - R''_i F_s) \cos (N + ig) + \Sigma (R'_i E_s + R''_i F_c) \cos (N - ig)$$

$$+ \Sigma (R'_i E_s + R''_i F_c) \sin (N + ig) + \Sigma (R'_i E_c - R''_i F_s) \sin (N - ig)$$

$$+ 2\delta r$$

The numerical values of V' , V'' , R' , and R'' are as follows:

| | | | |
|----------|-----------|-----------|-----------|
| $V'_1 =$ | 1.99835 | $V''_1 =$ | 1.99945 |
| $V'_2 =$ | 0.11713 | $V''_2 =$ | 0.11722 |
| $V'_3 =$ | 0.00714 | $V''_3 =$ | 0.00714 |
| $V'_4 =$ | 0.00044 | $V''_4 =$ | 0.00044 |
| $R'_0 =$ | + 0.02348 | | |
| $R'_1 =$ | - 0.99753 | $R''_1 =$ | + 0.99917 |
| $R'_2 =$ | - 0.07020 | $R''_2 =$ | + 0.07030 |
| $R'_3 =$ | - 0.00466 | $R''_3 =$ | + 0.00467 |

The final results of the entire computations are given in the following table: In the columns δv_1 , we have the complete perturbations of the longitude computed by the direct method from the values of $\frac{\partial R}{\partial v}$, $\frac{\partial R}{\partial \rho}$, Q , etc., already given. Next we have, under the caption δv_2 , the perturbations of the true longitude deduced from the long period perturbations of the elements, as set forth in the last paragraph, omitting the constants added to the perturbations. Under δv_3 we have the

perturbations of the longitude deduced from all the remaining terms of the perturbations of the elements. The sum of the columns δv_2 and δv_3 shows the entire perturbations computed by the method of variation of elements. Thus, in δv_1 and $\delta v_2 + \delta v_3$ we have two complete sets of perturbations computed by methods entirely independent. The differences of the results, expressed in thousandths of a second, are given in the last two columns of the table.

This comparison gives rise to remarks similar to those suggested by the perturbations of Saturn computed by the same methods. The only terms in which the difference of results amounts to as much as one-tenth of a second are those of very long period, and those very nearly the period of Uranus, where a more accurate value is not at present of great importance, because the error will be compensated by the corrections of the element during several centuries.

| PERTURBATIONS OF THE LONGITUDE OF URANUS PRODUCED BY NEPTUNE. | | | | | | | | | |
|---|-----|--------------|-------------|--------------|---------|--------------|--------|--------------|-----|
| ν | g | δv_1 | | δv_2 | | δv_3 | | Discrepancy. | |
| | | sin | cos | sin | cos | sin | cos | sin | cos |
| 0, | 0 | | —0.4262 t | | | | | | |
| | —1 | — .065 nt | —1.181 nt | | | | | | |
| | —2 | — .004 nt | —0.069 nt | | | | | | |
| 0, | 0 | | | | | | | | |
| | —1 | + 0.697 | — 0.088 | | | | | | |
| | —2 | + 0.046 | — 0.005 | | | | | | |
| | —3 | + 0.003 | | | | | | | |
| 1, | —3 | + 0.147 | — 0.001 | | | + 0.146 | —0.002 | 1 | 1 |
| | —2 | + 2.509 | — 0.019 | | | + 2.509 | —0.010 | 0 | 9 |
| | —1 | + 39.658 | — 0.080 | | | +39.673 | —0.081 | 15 | 1 |
| | 0 | + 4.257 | — 0.511 | | | + 4.249 | —0.478 | 8 | 33 |
| | —1 | + 0.280 | — 0.032 | | | + 0.275 | —0.032 | 5 | 0 |
| | —2 | + 0.017 | — 0.002 | | | | | | |
| 2, | —4 | + 2.978 | + 0.027 | + 2.89 | + 0.03 | + 0.098 | —0.002 | 10 | 1 |
| | —3 | + 49.015 | + 0.413 | + 47.22 | + 0.44 | + 1.797 | —0.021 | 2 | 6 |
| | —2 | + 840.93 | + 7.388 | + 805.64 | + 7.43 | +35.355 | —0.028 | 65 | 14 |
| | —1 | —3475.4 | + 180.36 | —3474.32 | +180.10 | — 0.700 | +0.095 | 380 | 165 |
| | 0 | — 162.07 | + 8.01 | — 161.96 | + 8.01 | — 0.067 | +0.015 | 43 | 15 |
| | —1 | — 9.447 | + 0.468 | — 9.50 | + 0.47 | | | | |
| 3, | —5 | — 0.076 | 0.000 | | | — 0.077 | —0.003 | 1 | 3 |
| | —4 | — 1.162 | 0.000 | | | — 1.153 | —0.011 | 9 | 11 |
| | —3 | — 17.286 | + 0.228 | | | —17.285 | +0.229 | 1 | 1 |
| | —2 | — 22.085 | + 4.037 | | | —22.077 | +4.020 | 8 | 17 |
| | —1 | — 0.673 | + 0.082 | | | — 0.682 | +0.079 | 9 | 3 |
| | 0 | — 0.037 | + 0.006 | | | | | | |
| 4, | —6 | — 0.036 | + 0.002 | — 0.015 | + 0.002 | — 0.027 | —0.003 | 6 | 3 |
| | —5 | — 0.558 | + 0.037 | — 0.25 | + 0.04 | — 0.315 | —0.007 | 7 | 4 |
| | —4 | — 7.968 | + 0.750 | — 4.08 | + 0.68 | — 3.908 | +0.059 | 20 | 11 |
| | —3 | — 75.00 | + 12.832 | — 69.55 | +11.67 | — 5.733 | +1.067 | 283 | 95 |
| | —2 | + 146.78 | — 54.218 | +146.72 | —54.10 | + 0.126 | —0.079 | 66 | 39 |
| | —1 | + 6.960 | — 2.579 | + 6.81 | — 2.63 | | | | |

PERTURBATIONS OF THE LONGITUDE—*Continued.*

| <i>U</i> <i>g</i> | δv_1 | | δv_2 | | δv_3 | | Discrepancy. | |
|-------------------|--------------|--------|--------------|--------|--------------|--------|--------------|-----|
| | sin | cos | sin | cos | sin | cos | sin | cos |
| 5,— 7 | —0.009 | 0.000 | | | —0.015 | —0.002 | 6 | 2 |
| — 6 | —0.103 | +0.006 | | | —0.113 | —0.004 | 10 | 10 |
| — 5 | —0.986 | —0.042 | | | —0.994 | —0.042 | 8 | 0 |
| — 4 | +3.366 | —0.670 | | | +3.370 | —0.662 | 4 | 8 |
| — 3 | +3.210 | —1.169 | | | +3.227 | —1.186 | 17 | 17 |
| — 2 | +0.077 | —0.017 | | | +0.075 | —0.002 | 2 | 15 |
| — 1 | +0.005 | —0.001 | | | | | | |
| 6,— 7 | —0.050 | —0.004 | 0.00 | 0.00 | —0.054 | —0.002 | 4 | 2 |
| — 6 | —0.387 | —0.020 | +0.02 | —0.01 | —0.423 | —0.013 | 16 | 3 |
| — 5 | +1.261 | —0.320 | +0.40 | —0.14 | +0.855 | —0.169 | 6 | 11 |
| — 4 | +7.781 | —2.825 | +6.80 | —2.54 | +0.857 | —0.318 | 124 | 33 |
| — 3 | —9.025 | +5.073 | —8.97 | +5.03 | +0.017 | —0.022 | 72 | 65 |
| — 2 | —0.437 | +0.246 | —0.42 | +0.26 | | | | |
| 7,— 8 | —0.027 | —0.002 | | | | | | |
| — 7 | —0.186 | —0.003 | | | —0.189 | —0.009 | 3 | 6 |
| — 6 | +0.272 | —0.046 | | | +0.260 | —0.047 | 12 | 1 |
| — 5 | —0.571 | +0.217 | | | —0.538 | +0.202 | 33 | 15 |
| — 4 | —0.459 | +0.250 | | | —0.419 | +0.255 | 40 | 5 |
| — 3 | —0.010 | +0.005 | | | | | | |
| 8,— 9 | —0.013 | 0.000 | | | | | | |
| — 8 | —0.084 | —0.002 | | | —0.093 | —0.007 | 9 | 5 |
| — 7 | +0.125 | —0.022 | | | +0.115 | —0.024 | 10 | 2 |
| — 6 | —0.194 | +0.082 | —0.038 | +0.017 | —0.145 | +0.055 | 11 | 10 |
| — 5 | —0.839 | +0.463 | —0.66 | +0.30 | —0.110 | +0.067 | 69 | 96 |
| — 4 | +0.647 | —0.501 | +0.64 | —0.53 | | | | |
| 9,— 9 | —0.040 | —0.001 | | | —0.048 | —0.005 | 8 | 4 |
| — 8 | +0.063 | —0.011 | | | +0.002 | —0.011 | 61 | 0 |
| — 7 | —0.055 | +0.020 | | | —0.059 | +0.025 | 4 | 5 |
| — 6 | +0.080 | —0.049 | | | +0.083 | —0.054 | 3 | 5 |
| — 5 | +0.059 | —0.050 | | | | | | |
| 10—10 | —0.020 | 0.000 | | | —0.025 | —0.005 | 5 | 5 |
| — 9 | +0.035 | —0.005 | | | | | | |
| — 8 | —0.027 | —0.010 | | | | | | |
| — 7 | +0.026 | —0.017 | | | | | | |
| — 6 | +0.092 | —0.079 | | | | | | |

The perturbations of the logarithms of the radius vector are given in a form similar to those of the longitude. Under $\delta\rho_1$ we have the complete perturbation. Under $\delta\rho_2$ the effect of the perturbations of long period. But under $\delta\rho_3$ we have only the difference between $\delta\rho_1$ and $\delta\rho_2$, it being deemed unnecessary to present in full the perturbations of the radius vector as computed by the other method. $\delta\rho_1$ being employed in computing δv_1 may, in fact, be regarded as completely checked by its affording a correct value of the latter.

In the last two columns $\delta\rho_3$ is reduced to common logarithms by multiplying the coefficients by the modulus 0.434294.

PERTURBATIONS OF THE LOGARITHM OF THE RADIUS VECTOR OF URANUS PRODUCED
BY NEPTUNE.

| $l' g$ | $\delta\rho_1$ | | $\delta\rho_2$ | | $\delta\rho_3$ | | $M\delta\rho$ | |
|--------|----------------|------|----------------|------|----------------|------|---------------|------|
| | cos | sin | cos | sin | cos | sin | cos | sin |
| 0, 0 | +138 | | | | | | + 60 | 0 |
| —1 | | | | | | | | |
| —2 | | | | | | | | |
| 1,—3 | + 4 | 0 | | | | | + 2 | 0 |
| —2 | + 68 | 0 | | | | | + 30 | 0 |
| —1 | +523 | + 1 | | | | | +227 | 0 |
| 0 | — 68 | — 6 | | | | | — 30 | — 3 |
| +1 | — 8 | — 1 | | | | | — 3 | 0 |
| 2,—4 | + 94 | — 1 | +92 | — 1 | + 2 | 0 | + 1 | 0 |
| —3 | +1416 | — 12 | + 1374 | — 12 | + 42 | 0 | + 18 | 0 |
| —2 | +20025 | —179 | +19490 | —180 | +535 | + 1 | +232 | 0 |
| —1 | +1663 | +116 | + 1726 | +120 | — 63 | — 4 | — 27 | — 2 |
| 0 | +3912 | +194 | + 3927 | +194 | — 15 | 0 | — 7 | 0 |
| 3,—5 | — 3 | 0 | | | | | — 1 | 0 |
| —4 | — 38 | 0 | | | | | — 17 | 0 |
| —3 | —527 | — 6 | | | | | —229 | — 3 |
| —2 | —284 | —54 | | | | | —124 | —23 |
| —1 | + 16 | + 2 | | | | | + 7 | + 1 |
| 4,—6 | — 1 | 0 | | | | | | |
| —5 | — 19 | — 1 | — 8 | | — 11 | — 1 | — 5 | 0 |
| —4 | —254 | — 22 | — 118 | — 20 | —136 | — 2 | — 59 | — 1 |
| —3 | —1759 | —300 | —1681 | —282 | — 78 | —18 | — 34 | — 8 |
| —2 | —143 | — 61 | — 154 | — 70 | + 11 | + 9 | + 5 | + 4 |
| —1 | —167 | — 62 | — 165 | — 64 | — 2 | + 2 | — 1 | + 1 |
| 5,—6 | — 4 | 0 | | | | | — 2 | 0 |
| —5 | — 40 | + 1 | | | | | — 17 | 0 |
| —4 | +103 | +20 | | | | | + 45 | + 9 |
| —3 | + 39 | +15 | | | | | + 17 | + 6 |
| 6,—7 | — 2 | 0 | | | — 2 | 0 | — 1 | 0 |
| —6 | — 17 | + 1 | | | — 17 | + 1 | — 7 | 0 |
| —5 | + 42 | +10 | + 10 | + 4 | + 32 | + 6 | + 14 | + 3 |
| —4 | +180 | +65 | +164 | + 61 | + 16 | + 4 | + 7 | + 2 |
| —3 | + 13 | + 8 | + 14 | + 5 | — 1 | + 3 | 0 | + 1 |
| 7,—7 | — 8 | 0 | | | | | — 3 | 0 |
| —6 | +11 | + 2 | | | | | + 5 | + 1 |
| —5 | —17 | — 7 | | | | | — 7 | — 3 |
| —4 | — 5 | — 3 | | | | | — 2 | — 1 |
| 8,—8 | — 4 | 0 | | | | | — 2 | 0 |
| —7 | + 5 | + 1 | | | | | + 2 | 0 |
| —6 | — 7 | — 3 | | | | | — 3 | — 1 |
| —5 | —19 | —11 | | | | | — 8 | — 5 |
| 9,—9 | — 2 | 0 | | | | | — 1 | 0 |
| —8 | + 3 | 0 | | | | | + 1 | 0 |
| —7 | — 2 | — 1 | | | | | — 1 | 0 |
| —6 | + 2 | + 2 | | | | | + 1 | + 1 |

PERTURBATIONS OF THE LATITUDE PRODUCED BY NEPTUNE.

| <i>i'</i> | <i>i</i> | $\frac{\partial R}{\partial \gamma} = \frac{m'}{a_1} \times$ | | $\Sigma \frac{ih}{2\sigma} \sin N$ | | $\Sigma \frac{(i+j)\sigma}{h \sin N}$ | | $\frac{d\eta}{dt} = m' an \times$ | | δk | | $\delta \eta$ | | $\delta \beta$ | |
|-----------|----------|--|-------|------------------------------------|-------|---------------------------------------|-------|-----------------------------------|-------|------------|----------|---------------|----------|----------------|--------|
| | | cos | sin | sin | cos | sin | cos | sin | cos | sin | cos | cos | sin | sin | cos |
| 0 | 0 | -21.65 | 0 | 0 | 0 | 0 | 0.00 | 0 | 0 | '' | '' | '' | '' | '' | '' |
| | 1 | +1.08 | +2.73 | -4.00 | -1.66 | + .01 | -0.01 | -3.99 | -1.67 | -0.008 | +0.021 | +0.030 | -0.012 | + .080 | + .020 |
| | 2 | +19.32 | -9.39 | +19.32 | +9.39 | - .01 | 0 | +19.31 | +9.39 | -0.073 | -0.036 | -0.073 | +0.036 | + .016 | - .002 |
| 1, | -3 | +14.58 | +7.0 | -14.7 | +7.0 | 0 | 0 | -14.7 | +7.0 | +0.045 | -0.022 | -0.045 | -0.022 | + .007 | - .002 |
| | -2 | -1.16 | -2.65 | +4.04 | -2.40 | -0.07 | +0.02 | +3.97 | -2.4 | -0.006 | +0.014 | +0.020 | +0.012 | + .201 | - .046 |
| | -1 | -34.26 | 0 | 0 | 0 | +1.76 | 0 | +1.76 | 0 | -0.540 | 0 | +0.028 | 0 | - .008 | - .037 |
| | 0 | +3.74 | +2.32 | -2.80 | -1.48 | -0.24 | 0 | -3.04 | -1.48 | -0.057 | +0.035 | +0.046 | -0.022 | + .364 | + .084 |
| | +1 | +15.77 | -7.87 | +16.25 | +7.75 | 0 | 0 | +16.25 | +7.75 | -0.081 | -0.040 | -0.083 | +0.040 | + .060 | + .007 |
| 2, | -4 | +10.52 | +5.06 | -10.6 | +5.0 | 0 | 0 | -10.6 | +5.0 | +0.027 | -0.013 | -0.027 | -0.013 | + .002 | - .001 |
| | -3 | -2.60 | -2.38 | +3.65 | -2.28 | +0.36 | +0.04 | +4.01 | -2.24 | -0.010 | +0.009 | +0.016 | +0.009 | + .062 | - .014 |
| | -2 | -33.38 | 0 | 0 | 0 | +9.88 | -0.01 | +9.88 | -0.01 | -0.264 | 0 | +0.078 | 0 | + .008 | - .004 |
| | -1 | +5.81 | +2.04 | -2.12 | -1.04 | -1.53 | +0.04 | -3.65 | -1.00 | [-2.283] | [+0.802] | [+1.431] | [-0.393] | + .326 | + .075 |
| | 0 | +18.79 | -9.52 | +19.56 | +9.32 | +0.08 | 0 | +19.64 | +9.32 | -0.143 | -0.072 | -0.049 | +0.071 | + .002 | 0 |
| 3, | -5 | +7.4 | +3.6 | -7.4 | +3.5 | 0 | 0 | -7.4 | +3.5 | +0.017 | -0.008 | -0.017 | -0.008 | + .002 | 0 |
| | -4 | -3.06 | -2.00 | +3.12 | -2.0 | +0.48 | +0.05 | +3.60 | -2.0 | -0.010 | +0.006 | +0.011 | +0.006 | + .023 | - .004 |
| | -3 | -24.4 | 0 | 0 | 0 | +7.87 | 0 | +7.87 | 0 | -0.128 | 0 | +0.041 | 0 | - .004 | + .041 |
| | -2 | +6.58 | +1.73 | -1.20 | -0.54 | -1.98 | +0.26 | -3.18 | -0.28 | +0.108 | -0.029 | -0.053 | +0.004 | + .303 | + .074 |
| | -1 | +13.66 | -7.28 | +14.68 | +7.00 | +0.18 | -0.04 | -14.86 | +6.96 | -0.199 | -0.106 | -0.217 | +0.101 | - .090 | - .026 |
| | 0 | -1.51 | +0.22 | -1.51 | -0.22 | 0 | 0 | -1.51 | -0.22 | +0.008 | +0.001 | +0.008 | -0.001 | - .013 | 0 |
| 4, | -5 | -2.96 | -1.63 | +2.53 | -1.7 | +0.5 | 0 | +3.0 | -1.7 | -0.008 | +0.004 | +0.008 | +0.004 | + .022 | - .004 |
| | -4 | -17.4 | 0 | 0 | 0 | +5.8 | 0 | +5.8 | 0 | -0.070 | 0 | +0.023 | 0 | - .008 | + .006 |
| | -3 | +6.34 | +1.34 | -0.59 | -0.22 | -1.97 | +0.30 | -2.56 | +0.08 | +0.051 | -0.011 | -0.020 | -0.001 | + .048 | + .010 |
| | -2 | +9.42 | -5.37 | +10.57 | +5.04 | +0.25 | -0.07 | +10.82 | +4.97 | [-1.852] | [-1.056] | [-2.127] | [+0.977] | - .047 | 0 |
| | -1 | -1.72 | +0.32 | -1.72 | -0.32 | +0.02 | +0.02 | -1.74 | -0.30 | +0.013 | +0.002 | +0.013 | -0.002 | - .001 | 0 |
| 5, | -6 | -2.4 | -1.40 | +1.8 | -1.4 | +0.5 | 0 | +2.3 | -1.4 | -0.005 | +0.003 | +0.005 | +0.003 | + .012 | - .003 |
| | -5 | -12.0 | 0 | 0 | 0 | +4.0 | 0 | +4.0 | 0 | -0.038 | 0 | +0.013 | 0 | - .012 | 0 |
| | -4 | +5.58 | +1.01 | -0.22 | -0.04 | -1.75 | +0.28 | -1.97 | +0.24 | +0.030 | -0.005 | -0.010 | -0.001 | - .109 | - .028 |
| | -3 | +6.25 | -3.89 | +7.43 | +3.54 | +0.30 | -0.09 | +7.73 | +3.45 | +0.107 | +0.067 | +0.131 | -0.059 | - .038 | + .001 |
| | -2 | -1.66 | +0.33 | -1.66 | -0.33 | -0.03 | +0.01 | -1.69 | -0.32 | +0.024 | +0.005 | +0.025 | -0.004 | + .018 | 0 |
| 6, | -6 | -8.4 | 0 | 0 | 0 | +2.8 | 0 | +2.80 | 0 | -0.022 | 0 | +0.007 | 0 | - .010 | + .002 |
| | -5 | +4.63 | +0.76 | -0.03 | +0.04 | -1.46 | +0.24 | -1.49 | +0.28 | +0.019 | -0.003 | -0.006 | -0.001 | - .028 | - .008 |
| | -4 | +4.03 | -2.79 | +5.15 | +2.45 | +0.31 | -0.10 | +5.46 | +2.35 | +0.033 | +0.022 | +0.045 | -0.019 | - .010 | 0 |
| | -3 | -1.45 | +0.31 | -1.45 | -0.31 | -0.04 | +0.02 | -1.49 | -0.29 | [+0.190] | [+0.041] | [+0.195] | [-0.038] | + .015 | 0 |
| 7, | -7 | -5.6 | 0 | 0 | 0 | +1.9 | 0 | +1.9 | 0 | -0.012 | 0 | +0.004 | 0 | - .006 | + .001 |
| | -6 | +3.63 | +0.61 | 0 | 0 | -1.17 | +0.20 | -1.17 | +0.20 | +0.012 | -0.002 | -0.004 | -0.001 | - .008 | - .004 |
| | -5 | +2.49 | -2.00 | +3.52 | +1.68 | +0.30 | -0.10 | +3.82 | +1.58 | +0.013 | +0.011 | +0.020 | -0.008 | + .014 | 0 |
| | -4 | -1.19 | +0.27 | -1.19 | -0.27 | -0.04 | +0.02 | -1.23 | -0.25 | -0.021 | -0.005 | -0.022 | +0.004 | + .002 | 0 |
| 8, | -8 | -4.0 | 0 | 0 | 0 | +1.3 | 0 | +1.3 | 0 | -0.008 | 0 | +0.003 | 0 | - .003 | + .002 |
| | -7 | +2.76 | +0.50 | 0 | 0 | -0.91 | +0.16 | -0.91 | +0.16 | +0.007 | -0.002 | +0.002 | 0 | - .003 | - .002 |
| | -6 | +1.49 | -1.41 | +2.38 | +1.14 | +0.27 | -0.09 | +2.65 | +1.05 | +0.006 | +0.006 | +0.011 | -0.004 | + .004 | 0 |
| | -5 | -0.97 | +0.22 | -0.97 | -0.22 | 0 | +0.02 | -0.97 | -0.20 | -0.008 | -0.002 | -0.008 | +0.002 | + .002 | 0 |

Secular term, $\delta k = + 1''.25 T$
 $\delta \beta = - 1''.25 T \cos v$

The terms δk and $\delta \eta$, which are inclosed in brackets, are of very long period, and are therefore omitted in forming the values of $\delta \beta$ in the last two columns

Perturbations produced by Jupiter.

The series in which these perturbations are expressed converge so rapidly that I deem it unnecessary to present the details of the computation. They have been computed by both methods, and the separate and independent results are given in the following table, where δv_1 represents the perturbations computed by the method developed in Chapter I, and δv_2 those computed by the method of variation of elements.

The apparently large discrepancy between the coefficients multiplied by the time arises from the circumstances that in the form of development the mean motion, and hence the mean anomaly, appears affected by the perturbation $31''.2t$. Accordingly when we enter the table which gives the true longitude in terms of the mean anomaly in the form

$$v = l + 2e \sin(l - \pi) + \text{etc.},$$

we may consider this quantity $31''.2t$ as a secular variation of $l - \pi$ producing in v the term

$$\delta v = 62''.4et \cos(l - \pi).$$

In δv_1 this term is left in its primitive form, while in δv_2 the value of l is supposed to include this term, and the secular terms are only those which arise from the secular variation of the eccentricity and perihelion.

It is also to be remarked that the terms which are independent of the mean longitude of Jupiter, or those in which $i = 0$, are not comparable, as they correspond to slightly different elliptic elements in the two theories.

| PERTURBATIONS OF URANUS BY JUPITER. | | | | | | | | | | | |
|-------------------------------------|-----------|------------------|-------------------|-------------------|-------------------|-------|-------|-------------------------------|-------|-----------------|-------|
| <i>g</i> | <i>l'</i> | δv_1 | | δv_2 | | Diff. | | $\cos \downarrow \delta \rho$ | | $M \delta \rho$ | |
| | | sin | cos | sin | cos | sin | cos | cos | sin | cos | sin |
| 0, 0 | | | +31.2116 <i>t</i> | | +31.1982 <i>t</i> | | | | | | |
| 1 | | -0.160 <i>nt</i> | +37.585 <i>nt</i> | -0.1622 <i>nt</i> | -1.5406 <i>nt</i> | 2 | | | | | |
| 2 | | -0.010 <i>nt</i> | +2.207 <i>nt</i> | -0.0095 <i>nt</i> | -0.0899 <i>nt</i> | 0 | | | | | |
| 3 | | | +0.135 <i>nt</i> | -0.0006 <i>nt</i> | -0.0054 <i>nt</i> | 0 | | | | | |
| 0, 0 | | | | | | | | -10089 | | -4387. | 0 |
| 1 | | +25.657 | -1.859 | -1.346 | +1.361 | | | -491.5 | -1.9 | -213.7 | -0.8 |
| 2 | | +1.397 | -0.087 | -0.030 | +0.086 | | | -32.8 | +1.1 | -14.2 | +0.5 |
| 3 | | +0.072 | -0.005 | | | | | -1.8 | 0 | -0.8 | 0 |
| -1,-1 | | +0.027 | +0.009 | +0.017 | +0.011 | 10 | 2 | 0.0 | -0.5 | 0.0 | -0.2 |
| 0 | | +1.269 | +0.002 | +1.232 | +0.001 | 37 | 1 | -58.8 | -0.5 | -25.6 | -0.2 |
| 1 | | -53.064 | -0.004 | -53.084 | -0.002 | 20 | 2 | +2585.7 | +0.1 | +1127.2 | 0 |
| 2 | | -3.495 | -0.092 | -3.565 | -0.082 | 70 | 10 | +195.3 | +5.0 | +82.8 | +2.2 |
| 3 | | -0.148 | -0.047 | -0.164 | -0.050 | 16 | 3 | +17.1 | +2.3 | +7.4 | +1.0 |
| 0,-2 | | -0.027 | -0.011 | -0.031 | -0.014 | 4 | 3 | +1.4 | -0.6 | +0.6 | -0.3 |
| 1 | | +1.182 | +0.515 | +1.176 | +0.515 | 6 | 0 | -56.5 | +24.9 | -24.6 | +10.7 |
| 2 | | +0.277 | +0.036 | +0.263 | +0.037 | 14 | 1 | +8.5 | +1.8 | +3.7 | +1.6 |
| 3 | | +0.074 | -0.005 | +0.083 | -0.003 | 9 | 3 | +4.2 | +0.5 | +1.8 | +0.8 |
| 4 | | +0.015 | -0.003 | +0.014 | -0.003 | 1 | 0 | +0.9 | +0.2 | +0.4 | +0.2 |
| 1,-3 | | -0.032 | -0.034 | -0.025 | -0.025 | 7 | 9 | | | | |
| 2 | | -0.005 | 0 | -0.005 | -0.001 | 0 | 1 | | | | |
| 3 | | +0.025 | 0 | +0.015 | 0 | 10 | 0 | | | | |
| 4 | | +0.011 | -0.001 | +0.037 | -0.010 | 26 | 9 | | | | |
| 5 | | +0.003 | 0 | -0.017 | +0.004 | 20 | 4 | | | | |

| PERTURBATIONS OF THE LATITUDE. | | | | | | | |
|--------------------------------|-----------|------------|--------|---------------|--------|----------------|--------|
| <i>g</i> | <i>l'</i> | δk | | $\delta \eta$ | | $\delta \beta$ | |
| <i>i</i> | <i>i'</i> | sin | cos | cos | sin | cos | sin |
| 0 | 0 | | | | | | +0.065 |
| 1 | | +0.071 | -0.041 | +0.030 | +0.036 | +0.060 | +0.047 |
| 2 | | -0.012 | -0.075 | -0.012 | +0.075 | -0.010 | -0.007 |
| 3 | | -0.003 | -0.012 | -0.003 | +0.012 | -0.002 | 0 |
| -2 | -1 | -0.006 | +0.042 | +0.006 | +0.042 | +0.005 | -0.004 |
| -1 | | -0.297 | +1.949 | +0.297 | +1.949 | +0.001 | -0.003 |
| 0 | | -0.161 | -0.191 | -0.189 | -0.190 | -0.494 | +0.420 |
| 1 | | +2.611 | 0 | +2.628 | 0 | -0.024 | -0.013 |
| 2 | | +0.070 | +0.021 | +0.066 | -0.002 | +0.002 | +0.019 |
| -3 | -2 | +0.055 | -0.088 | -0.055 | -0.088 | +0.010 | -0.003 |
| 0 | | +0.002 | -0.011 | -0.002 | -0.011 | +0.017 | -0.004 |
| 1 | | -0.110 | -0.050 | -0.109 | +0.046 | -0.009 | +0.002 |
| 2 | | -0.014 | 0 | -0.011 | 0 | -0.002 | 0 |

//

Secular terms $\begin{cases} \delta k = 1.14 T \\ \delta \beta = -1.14 T \cos v \end{cases}$

CHAPTER IV.

TERMS OF THE SECOND ORDER PRODUCED BY THE ACTION OF SATURN.

Preliminary Investigation of the Orbit of Saturn.

For the accurate determination of the perturbations of a planet it is essential that the functions of the time which are substituted for the co-ordinates of each planet in the expression of the disturbing forces should approximately represent the true places of the planet. The difference between the true place and that implicitly assumed in the investigation should be so small and of such a character that, when multiplied by the mass of the disturbing planet, and by the factors introduced by the process of integration, the result shall be insensible. If one of these factors is so large as to make a perturbation of an order of magnitude approximating that of the inequality which gives rise to it, it will represent an inequality of very long period in the elements, which, though apparently sensible, may be neglected for a great length of time.

The perturbations hitherto found have been computed on the hypothesis that the disturbing action of Saturn on Uranus is the same as if both planets moved in the elliptic orbits corresponding to the adopted elements. We have given formulæ for the computation of the corrected perturbations when, to the co-ordinates of the two planets corresponding to the adopted ellipse, we add corrections represented by δv , $\delta v'$, $\delta \rho$, etc. These corrections are now to be taken of such magnitude that when thus added they shall very nearly represent the actual motions of the planets.

Generally, it is considered sufficient to take for these corrections the perturbations of the first order. But this presupposes that the elliptic elements are nearly correct, which does not hold true in the case of the old elements of the outer planets. Bouvard's Tables of Saturn, the elements of which have been adopted, are subject to recurring errors amounting to 30" or more. Moreover, when we substitute the new and more accurate perturbations for the old and imperfect ones adopted in the tables, the chances are that the errors will be increased. Desiring that the theory shall be as far as possible free from doubt, we begin with a preliminary investigation of the orbit of Saturn, the design of which will be to give the co-ordinates of that body in terms of the time with sufficient certainty and accuracy to serve for computing the perturbations both of Jupiter and Uranus. As usual, the first step in this investigation will be the determinations of the perturbations of the planet.

General Perturbations of Saturn.

The perturbations produced by Jupiter will be taken from the exhaustive prize memoir of Hansen.¹ As the perturbations required are those of the co-ordinates, it will be necessary to transform those of Hansen into the usual form. Hansen gives the true anomaly v in the form

$$v = g + n\delta z + e_1 \sin(g + n\delta z) + e_2 \sin 2(g + n\delta z) + \text{etc.},$$

e_1, e_2 , etc., being the coefficients of the multiples of the mean anomaly in the usual development of the elliptic true anomaly. Whence, neglecting the second power of $n\delta z$,

$$\delta v = n\delta z (1 + e_1 \cos g + 2e_2 \cos 2g + \text{etc.}).$$

To make the development sufficiently rigorous it is only necessary to increase g by $\frac{1}{2}n\delta z$ in this expression. In the same way, we have for the perturbations of $\log r$,

$$\delta \rho = \delta \rho_0 + n\delta z (e^{(1)} \sin g + e^{(2)} \sin 2g + \text{etc.})$$

$\delta \rho_0$ being Hansen's perturbation, and $e^{(i)}$ the negative coefficient of $\cos ig$ in the development of the elliptic $\log r$.

Hansen having adopted $\frac{1}{100} \frac{1}{70.5}$ as the mass of Jupiter, it will be necessary to multiply his perturbations by 1.0216 to reduce them to Bessel's mass. Thus the perturbations by Jupiter hereafter given have been obtained.

The perturbations by Uranus and Neptune have been computed by the preceding general method, and are given in the following table. In the table l' is the mean longitude of the disturbing planet, Uranus or Neptune, counted from the perihelion of Saturn. $\delta \rho$ is the perturbation of the Napierian logarithm, in units of the seventh place of decimals.

| GENERAL PERTURBATIONS OF THE LONGITUDE IN ORBIT AND THE LOGARITHM OF THE RADIUS VECTOR OF SATURN. | | | | | | | | | | | |
|---|-----|------------|--------|---------------|--------------------|------|-------|------------|-------|---------------|------|
| Action of Uranus. | | | | | Action of Neptune. | | | | | | |
| l' | g | δv | | $\delta \rho$ | | l' | g | δv | | $\delta \rho$ | |
| | | sin | cos | cos | sin | | | sin | cos | cos | sin |
| 1, | 0 | 0.88 | 0.92 | -12.7 | +7.9 | 1,— | 0 | +0.23 | -0.05 | -2.2 | -0.2 |
| | -1 | +8.60 | +0.21 | +146.6 | -3.5 | | -1 | +1.93 | 0.00 | +39.9 | 0.0 |
| 2, | 0 | -0.42 | -0.17 | +10.2 | -3.7 | 2, 0 | +0.44 | +0.02 | -3.0 | +0.1 | |
| | -1 | -8.49 | -2.53 | -66.8 | +21.4 | | -1 | -1.11 | +0.02 | -18.5 | -0.4 |
| | -2 | -13.39 | -0.25 | -392.5 | +6.6 | | -2 | -1.27 | 0.00 | -41.1 | 0.0 |
| 3, | 0 | +0.06 | +1.62 | +0.5 | +39.1 | 3,— | +0.02 | -0.03 | +0.2 | +0.4 | |
| | -1 | -2.9 | +28.1 | +10.8 | +33.0 | | -2 | +0.10 | -0.04 | +3.2 | +1.3 |
| | -2 | -10.61 | -20.88 | -247. | +487. | | -3 | -0.10 | 0.00 | -3.9 | 0.0 |
| | -3 | -2.05 | -1.47 | -67.5 | +42.8 | | | | | | |
| 4,— | -1 | +0.06 | +0.05 | -0.1 | +0.6 | | | | | | |
| | -2 | -0.62 | +0.80 | -10.5 | -12.5 | | | | | | |
| | -3 | +0.30 | +0.73 | +10.5 | -23.4 | | | | | | |
| | -4 | -0.26 | +0.03 | -10.6 | -0.8 | | | | | | |

¹ Untersuchungen über die gegenseitigen Störungen des Jupiters und Saturns. Von P. A. Hansen. Berlin, 1831.

I have submitted these perturbations to such duplicate computations and other checks as lead me to believe that none of the terms can be in error by more than a small fraction of a second, but, as they are not intended to form the basis of a definitive theory of Saturn, I do not vouch for their absolute precision.

In this provisional correction of the orbit of Saturn only heliocentric longitudes have been employed. These were derived for a series of dates from Airy's reduction of the Greenwich observations, the modern Greenwich observations, and the Washington observations.

For these dates the value of $n^{\circ}z$ for Saturn was computed from the formulæ found on pages 189 and 190 of the work of Hansen, already quoted, omitting all terms less than 1", and including only tenths of seconds in the results. The dates, the resulting values of $n^{\circ}z$, of the factor $e_1 \cos (g + \frac{1}{2} n^{\circ}z) + 2e_2 \cos 2(g + \frac{1}{2} n^{\circ}z)$, and of the concluded δv are as follows. The formulæ for δv is

$$\delta v = 1.0216 n^{\circ}z \{1 + e_1 \cos (g + \frac{1}{2} n^{\circ}z) + 2e_2 \cos 2(g + \frac{1}{2} n^{\circ}z)\}.$$

| Date | $n^{\circ}z$ | Factor. | δv |
|----------------|--------------|---------|------------|
| Gr. Mean Noon. | " | " | " |
| 1751 May 31 | -1947.7 | -.0990 | -1792.7 |
| 1757 Aug. 7 | -2134.2 | -.0652 | -2038.2 |
| 1758 Aug. 27 | -2212.5 | -.0474 | -2153.2 |
| 1761 Oct. 6 | -2546.2 | +.0244 | -2664.5 |
| 1763 Nov. 1 | -2880.3 | +.0729 | -3157.1 |
| 1765 Nov. 23 | -3095.1 | +.1082 | -3504.0 |
| 1773 Feb. 26 | -3342.0 | +.0419 | -3557.2 |
| 1780 May 24 | -2858.2 | -.0956 | -2640.7 |
| 1794 Nov. 16 | -3321.1 | +.1017 | -3737.9 |
| 1802 Feb. 23 | -3184.7 | +.0529 | -3425.5 |
| 1823 Nov. 13 | -2716.3 | +.0944 | -3036.8 |
| 1831 Feb. 18 | -3378.5 | +.0639 | -3671.7 |
| 1838 May 19 | -2976.7 | -.0866 | -2777.9 |
| 1845 Aug. 17 | -2342.7 | -.0721 | -2220.9 |
| 1852 Nov. 15 | -2847.0 | +.0863 | -3159.3 |
| 1860 Feb. 14 | -3161.4 | +.0740 | -3468.3 |
| 1867 May 15 | -2373.1 | -.0812 | -2227.6 |

The perturbations by Uranus and Neptune were computed from the values of their terms just given. The principal terms, the sum of which make up the heliocentric longitude resulting from the adopted elements, are shown in the first of the following tables.

In the next table we have after the date the heliocentric longitude from Bouvard's Tables, as deduced from the longitudes given in Airy's reductions of the Greenwich Observations, from the *Astronomisches Jahrbuch* for 1831, and from the Nautical Almanac. Then follow the corrections, roughly deduced from observations made near the opposition. Adding these columns, we have the longitude

from observation. To the right of these are the equations of condition for the correction of the elements.

| Long. of Perihelion. | Mean anomaly. | Equation of centre. | Perturbations by | | | Red. to Ecliptic. | Nuta-tion. | True longitude. |
|----------------------|---------------|---------------------|------------------|---------|-----------|-------------------|------------|-----------------|
| | | | Jupiter. | Uranus. | Nep-tune. | | | |
| ° ' " | ° ' " | ° ' " | ' " | " " | " " | ' " | " " | ° ' " |
| 88 41 27.5 | 159 55 50.1 | +2 3 56.6 | -29 52.7 | -45.7 | -2.7 | +1 36.9 | +15.8 | 250 12 25.8 |
| 88 46 41.2 | 236 13 7.1 | -5 7 38.1 | -33 58.2 | -36.6 | +1.0 | -1 20.5 | -11.8 | 319 16 4.1 |
| 88 47 31.4 | 248 25 53.1 | -5 48 30.4 | -35 53.2 | -35.2 | +1.7 | -1 35.8 | -15.0 | 330 46 36.6 |
| 88 50 7.6 | 286 26 29.8 | -6 16 1.3 | -44 24.6 | - 3.3 | +2.9 | -0 43.5 | -14.1 | 8 15 13.5 |
| 88 51 51.6 | 311 44 13.6 | -5 0 58.2 | -52 37.1 | +11.9 | +2.4 | +0 43.3 | - 4.9 | 34 43 22.6 |
| 88 53 35.1 | 336 55 56.1 | -2 41 10.6 | -58 24.0 | +19.2 | +1.9 | +1 36.6 | + 6.7 | 62 12 1.0 |
| 88 59 39.6 | 65 40 2.3 | +6 0 35.9 | -59 17.2 | -32.9 | -0.8 | -1 37.1 | + 4.8 | 159 38 54.6 |
| 89 5 43.4 | 154 8 4.8 | +2 37 57.7 | -44 0.7 | -53.8 | -1.7 | +1 37.5 | -14.5 | 245 8 13.2 |
| 89 17 50.7 | 331 6 10.4 | -3 18 5.2 | -62 17.9 | + 9.6 | +3.5 | +1 30.9 | -14.1 | 56 5 7.9 |
| 89 23 55.7 | 59 56 17.9 | +5 44 46.6 | -57 5.5 | -18.6 | +2.5 | -1 37.3 | + 1.4 | 154 6 2.7 |
| 89 42 7.0 | 325 22 26.0 | -3 51 58.0 | -50 36.8 | -27.9 | -3.0 | +1 21.6 | +14.7 | 50 23 3.6 |
| 89 48 11.9 | 54 10 33.1 | +5 25 2.6 | -61 11.7 | +13.3 | +2.5 | -1 33.2 | - 7.4 | 148 21 11.1 |
| 89 54 16.1 | 142 44 37.0 | +3 40 38.1 | -46 17.9 | - 3.4 | +2.1 | +1 29.7 | - 4.3 | 235 34 37.4 |
| 90 0 20.4 | 231 18 40.9 | -4 47 36.6 | -37 0.9 | -35.5 | +0.9 | -1 11.8 | +14.8 | 315 52 52.2 |
| 90 6 24.6 | 319 52 44.8 | -4 21 53.0 | -52 39.3 | + 4.7 | -0.9 | +1 8.8 | -18.4 | 44 45 31.3 |
| 90 12 28.7 | 48 26 48.7 | +5 1 46.4 | -57 48.3 | +30.3 | -3.4 | -1 25.1 | +14.1 | 142 42 31.4 |
| 90 18 32.8 | 137 0 52.6 | +4 9 31.2 | -37 7.6 | - 8.2 | +1.3 | +1 21.6 | - 4.1 | 230 52 59.7 |

| Date. | Tabular longitude. | Obs. cor. | Long. from observation. | EQUATIONS OF CONDITION. |
|--------------|--------------------|-----------|-------------------------|---|
| 1751 May 31 | 250 13 38.0 | - 8.5 | 250 13 29.5 | 63.7 = +0.90δ _ε -44δ _n +0.60δ _e +1.82eδ _ω |
| 1757 Aug. 27 | 319 17 8.5 | -18.0 | 319 16 50.5 | 46.4 0.94 -39 -1.53 +1.14 |
| 1758 Aug. 27 | 330 47 37.9 | -14.3 | 330 47 23.6 | 47.0 0.96 -40 -1.76 +0.80 |
| 1761 Oct. 6 | 8 15 13.8 | + 0.2 | 8 15 14.0 | 0.5 1.03 -39 -1.99 -0.50 |
| 1763 Nov. 1 | 34 43 42.3 | +15.3 | 34 43 57.6 | 35.0 1.07 -39 -1.63 -1.31 |
| 1765 Nov. 23 | 62 12 20.7 | +20.4 | 62 12 41.1 | 40.1 1.11 -38 -0.88 -1.88 |
| 1773 Feb. 26 | 159 40 3.8 | +18.8 | 159 40 22.6 | 88.0 1.04 -28 +1.93 -0.79 |
| 1780 May 24 | 245 9 17.6 | + 7.8 | 245 9 25.4 | 72.2 0.90 -18 +0.76 +1.74 |
| 1794 Nov. 16 | 56 5 38.8 | +14.7 | 56 5 53.5 | 45.6 1.10 - 6 -1.08 -1.78 |
| 1802 Feb. 23 | 154 7 2.7 | +10.8 | 154 7 13.5 | 70.8 1.05 + 2 +1.85 -0.98 |
| 1823 Nov. 13 | 50 23 34.5 | +21.3 | 50 23 55.8 | 52.2 1.09 +26 -1.27 -1.66 |
| 1831 Feb. 18 | 148 22 38.5 | + 3.0 | 148 22 41.5 | 90.4 1.07 +33 +1.75 -1.16 |
| 1838 May 19 | 235 36 11.2 | + 3.4 | 235 36 14.6 | 97.2 0.91 +35 +1.08 +1.56 |
| 1845 Aug. 17 | 315 53 42.2 | +15.7 | 315 53 57.9 | 65.7 0.93 +42 -1.43 +1.28 |
| 1852 Nov. 15 | 44 46 15.4 | + 5.9 | 44 46 21.3 | 50.0 1.09 +57 -1.42 -1.53 |
| 1860 Feb. 14 | 142 44 16.2 | -13.7 | 142 44 2.5 | 91.1 1.08 +65 +1.63 -1.35 |
| 1867 May 15 | 230 54 47.8 | + 5.1 | 230 54 52.9 | 113.2 +0.92 +62 +1.23 +1.45 |

A normal equation for δ_ε is obtained by taking the sum of all the equations. That for δ_n is formed by subtracting the sum of the first seven from the sum of the last seven, and those for δ_e and eδ_ω by taking the sum of the equations in which the coefficients of δ_e or eδ_ω are greater than unity, after changing the signs of the equations in which they are negative. The normals thus obtained are

$$\begin{array}{rccccr}
 17.19\delta\varepsilon + 31\delta n - 2.16\delta e - 3.15e\delta\omega & = & +1069.3 & & '' \\
 0.04 & +587 & + 6.83 & - 0.69 & + 239.3 \\
 - 2.14 & +207 & +21.58 & + 2.19 & + 208.1 \\
 - 2.11 & - 60 & + 3.61 & +19.66 & + 53.8
 \end{array}$$

These equations give

$$\begin{array}{l}
 \delta\varepsilon = + 64.8 \text{ (Epoch, 1800.)} \\
 \delta n = + 0.268 \\
 \delta e = + 12.6 \\
 e\delta\omega = + 8.2
 \end{array}$$

—Substituting these values in the seventeen equations of condition we have the following residuals, or excesses of theoretical over observed longitudes:

| | | | | | |
|----|---|------|----|---|------|
| 1 | + | 7.2 | 10 | + | 10.7 |
| 2* | - | 4.3 | 11 | - | 6.4 |
| 3 | - | 10.2 | 12 | - | 1.1 |
| 4 | + | 21.7 | 13 | - | 0.4 |
| 5 | - | 9.0 | 14 | | 0.0 |
| 6 | - | 7.1 | 15 | + | 3.7 |
| 7 | - | 11.3 | 16 | + | 4.1 |
| 8 | + | 8.2 | 17 | - | 7.7 |
| 9 | - | 6.4 | | | |

These residuals are much larger than they should be, and I scarcely know to what cause to attribute their magnitude. The results are however amply reliable for the purposes of the investigation, and lead to the following elements of Saturn:

$$\begin{array}{l}
 \pi, \quad \overset{\circ}{90} \quad \overset{''}{6} \quad \overset{''}{26} \\
 \varepsilon, \quad 14 \quad 50 \quad 3.2 \\
 \theta, \quad 112 \quad 20 \quad 0 \\
 \phi, \quad 2 \quad 29 \quad 39.2 \\
 n, \quad 43996.395 \\
 e, \quad .0560660 \\
 \log(a + \delta a), \quad 0.979676 \\
 \text{Epoch, 1850, Jan. 0, Greenwich mean noon.}
 \end{array}$$

It will be seen that the adopted position of the plane of Saturn's orbit is retained. It was corrected from observations before the perturbations were finally computed.

Of the above corrections, those of the epoch and mean motion need not be taken account of in the corrections of the co-ordinates, since the mean longitude remains in the formulæ as an arbitrary quantity to the end. The effect of the correction of the mean distance is insensible. The corrections of eccentricity and perihelion are therefore alone to be retained. They are allowed for by adding to δv and $\delta \phi$ the terms

$$\begin{aligned} \delta v &= 2\delta e \sin g - e\delta\omega \cos g \\ &= + 25''.2 \sin g - 16''.4 \cos g; \\ \delta\rho &= -\delta e \cos g - e\delta\omega \sin g \\ &= - 12''.6 \cos g - 8''.2 \sin g. \end{aligned}$$

Perturbations of Saturn and Uranus.

The following expressions include, with these corrections, all the perturbations of Saturn and Uranus which can produce any appreciable perturbations of the second order in their mutual action. In these expressions the initial letter of each planet is put for its mean longitude counted from the perihelion of Uranus.

PERTURBATIONS OF SATURN.

| Argument. | $\delta v'$ | | $\delta\rho'$ | |
|-------------------------|-----------------|-----------------|-----------------|-----------------|
| | sin
" | cos
" | cos
" | sin
" |
| <i>S</i> | + 25.2 | + 21.4 | - 2.7 | + 9.0 |
| <i>S</i> | + 1.64 <i>t</i> | - 1.48 <i>t</i> | - 0.80 <i>t</i> | - 0.74 <i>t</i> |
| <i>S</i> - <i>J</i> | - 26.1 | + 6.2 | + 176.7 | - 15.9 |
| 2 <i>S</i> - <i>J</i> | - 421.6 | - 8.8 | - 115.8 | - 9.1 |
| 3 <i>S</i> - <i>J</i> | - 54.7 | - 12.4 | + 12.3 | - 16.6 |
| 2 <i>S</i> - 2 <i>J</i> | + 32.6 | - 1.0 | + 29.5 | - 1.1 |
| 3 <i>S</i> - 2 <i>J</i> | + 27.7 | - 21.3 | - 15.4 | - 12.3 |
| 4 <i>S</i> - 2 <i>J</i> | + 216.9 | - 769.2 | + 11.4 | + 403.0 |
| 5 <i>S</i> - 2 <i>J</i> | + 2392. | - 1692. | - 58.8 | - 34.4 |
| 6 <i>S</i> - 2 <i>J</i> | + 133.0 | - 93.6 | - 58.1 | + 56.9 |
| <i>U</i> - <i>S</i> | + 8.6 | + 0.2 | + 3.0 | - 0.1 |
| 2 <i>U</i> - <i>S</i> | + 0.8 | - 8.8 | + 0.1 | + 1.4 |
| 2 <i>U</i> - 2 <i>S</i> | - 13.4 | + 0.2 | - 8.0 | + 0.1 |
| 3 <i>U</i> - <i>S</i> | - 8.4 | - 27.0 | + 0.1 | - 0.7 |
| 3 <i>U</i> - 2 <i>S</i> | + 18.3 | - 14.6 | + 8.8 | + 7.0 |

PERTURBATIONS OF URANUS.

| Argument. | δv | | $\delta\rho$ | |
|-------------------------|-----------------|-----------------|-----------------|-----------------|
| | sin
" | cos
" | cos
" | sin
" |
| <i>U</i> | - 0.11 <i>t</i> | - 0.28 <i>t</i> | + 0.05 <i>t</i> | - 0.14 <i>t</i> |
| <i>U</i> - <i>S</i> | - 20.8 | + 8.5 | + 36.0 | - 4.9 |
| 2 <i>U</i> - <i>S</i> | - 11.9 | + 143.5 | - 2.0 | - 63.8 |
| 3 <i>U</i> - <i>S</i> | + 49.3 | + 115.9 | - 4.7 | + 13.2 |
| 2 <i>U</i> - 2 <i>S</i> | + 4.1 | 0.0 | + 4.2 | + 0.1 |
| 3 <i>U</i> - 2 <i>S</i> | + 2.1 | - 1.6 | + 1.8 | + 1.4 |
| 4 <i>U</i> - 2 <i>S</i> | + 0.6 | - 1.8 | + 0.4 | + 1.3 |
| 5 <i>U</i> - 2 <i>S</i> | + 1.2 | + 3.0 | + 0.4 | - 1.2 |

Let us now resume the equation

$$\delta Q = 2 \int D_t \delta R dt + \delta \frac{\partial R}{\partial \rho} - \frac{1}{\mu} D_t^2 (r_0^2 \delta \rho^2) + \frac{1}{2} \frac{\delta \rho^2}{r_0}.$$

Beginning with the last two terms of this expression, it may be shown at the outset that they are quite insensible. The effect of the constant terms in $\delta\rho$ and $\delta\rho'$ has already been included by correcting the logarithm of the mean distance by their amount; they are therefore omitted. The largest remaining term is $64''$, the square of which is only $0''.02$. In the product $r_1^2\delta\rho^2$ the largest terms are

$$\begin{aligned} &+ 0.014 \\ &- 0.013 \sin g \\ &- 0.011 \sin (3g - 2l) \\ &- 0.011 \cos (4g - 2l) \end{aligned}$$

which may be entirely neglected.

We shall therefore only consider in δQ the terms

$$2 \int D_t \delta R dt + \delta \frac{\partial R}{\partial \rho}$$

As already remarked, R is rigorously a function only of V , ρ , and ρ' , V being the angle made by the radii vectores of the two planets. But, in the analytical development of R , the quantity V is considered as a function of v , v' , and γ , so that we have

$$R = f(\rho, \rho', v, v', \gamma).$$

In the previous computation of the perturbations of Uranus, we have supposed R to be a function of ρ_0, ρ'_0 , etc. The corrections to R and its derivatives with respect to v and ρ are now given by the equations (11), with the modifications shown on pages 24 to 27. The derivatives of R_0 which enter into these equations are formed as follows: If, in the value of R produced by the action of Saturn on Uranus, we consider any term of the form

$$\frac{m'h}{a_1} \cos N$$

where

$$N = i\lambda + i'\lambda' + j\omega + j'\omega'$$

the accented quantities always referring to Saturn, but a_1 being the corrected mean distance of Uranus, then we shall have the following terms in the derivatives of R .

$$\begin{aligned} \frac{\partial R}{\partial v} &= -\frac{m'h}{a_1} (i + j) \sin N \\ \frac{\partial R}{\partial v'} &= -\frac{m'h}{a_1} (i' + j') \sin N \\ \frac{\partial R}{\partial \rho} &= -\frac{m'}{a_1} \left(h + \frac{\partial h}{\partial v} \right) \cos N \\ \frac{\partial R}{\partial \rho'} &= -\frac{m'}{a_1} \frac{\partial h}{\partial v'} \cos N \\ \frac{\partial^2 R}{\partial v^2} &= -\frac{m'h}{a_1} (i + j)^2 \cos N \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 R}{\partial v \partial v'} &= -\frac{m'h}{a_1} (i+j) (i'+j') \cos N \\ \frac{\partial^2 R}{\partial v \partial \rho} &= \frac{m'}{a_1} \left(h + \frac{\partial h}{\partial x} \right) (i+j) \sin N \\ &= -\frac{\partial R}{\partial v} - \frac{\partial^2 R}{\partial v \partial \rho'} \\ \frac{\partial^2 R}{\partial v \partial \rho'} &= -\frac{m'}{a_1} \frac{\partial h}{\partial x} (i+j) \sin N \\ \frac{\partial^2 R}{\partial \rho \partial v'} &= \frac{m'}{a_1} \left(h + \frac{\partial h}{\partial x} \right) (i'+j') \sin N \\ \frac{\partial^2 R}{\partial \rho^2} &= \frac{m'}{a_1} \left(h + 2 \frac{\partial h}{\partial x} + \frac{\partial^2 h}{\partial x^2} \right) \cos N \\ &= -\frac{\partial R}{\partial \rho} - \frac{\partial^2 R}{\partial \rho \partial \rho'} \\ \frac{\partial^3 R}{\partial \rho \partial \rho'} &= -\frac{m'}{a_1} \left(\frac{\partial h}{\partial x} + \frac{\partial^2 h}{\partial x^2} \right) \cos N \end{aligned}$$

All the numerical data necessary for the computation of these derivatives have been given in Chapter II. Combining the terms having the same argument, we find the following values, omitting those given in Chapter II, and those which are derived from the others by mere addition. The terms of $\frac{\partial^2 R}{\partial v^2}$ are also omitted, because they are sensibly the same with those of $\frac{\partial^2 R}{\partial v \partial v'}$, changing the algebraic sign.

| <i>g</i> <i>l'</i> | $\frac{a_1}{m'} \frac{\partial R}{\partial v'}$ | | $\frac{a_1}{m'} \frac{\partial^2 R}{\partial v \partial v'}$ | | $\frac{a_1}{m'} \frac{\partial^2 R}{\partial \rho \partial v'}$ | |
|--------------------|---|---------|--|---------|---|---------|
| | sin | cos | sin | cos | sin | cos |
| 0,—1 | +0.2874 | 0 | 0 | +0.2872 | +0.2691 | 0 |
| —2 | +0.0066 | —0.0310 | +0.0310 | +0.0067 | +0.0060 | —0.0322 |
| 1, 0 | —0.0102 | —0.0491 | +0.0490 | —0.0101 | +0.0249 | +0.1194 |
| —1 | —3.4811 | —0.0010 | —0.0021 | —3.4810 | —5.2684 | 0 |
| —2 | —0.0968 | +0.4304 | —0.4304 | —0.1039 | —0.0792 | +0.4583 |
| —3 | +0.0388 | +0.0173 | —0.0179 | +0.0389 | +0.0398 | +0.0163 |
| 2, 0 | —0.0045 | —0.0035 | +0.0021 | —0.0080 | +0.0155 | +0.0066 |
| —1 | —0.0544 | —0.0712 | +0.1426 | —0.0689 | —0.1760 | +0.2403 |
| —2 | +0.4104 | +0.0120 | —0.0171 | +0.8233 | —1.3515 | +0.0014 |
| —3 | —0.0053 | —0.0191 | +0.0390 | —0.0226 | +0.0288 | +0.0547 |
| 3,—1 | —0.0070 | —0.0093 | +0.0171 | —0.0210 | +0.0197 | +0.0304 |
| —2 | +0.0570 | —0.0650 | +0.1957 | +0.1013 | —0.1791 | +0.2840 |
| —3 | +0.2542 | 0 | —0.0065 | +0.7624 | —1.0884 | —0.0071 |
| 4,—2 | —0.0014 | —0.0130 | +0.0368 | —0.0193 | +0.0213 | +0.0547 |
| —3 | +0.0515 | —0.0499 | +0.1987 | +0.1450 | —0.2144 | +0.2647 |
| —4 | +0.1448 | —0.0015 | —0.0016 | +0.5789 | —0.7644 | —0.0005 |

| $g' \quad l$ | $\frac{a_1}{m'} \frac{\partial R}{\partial \rho'}$ | | $\frac{a_1}{m'} \frac{\partial^2 R}{\partial v \partial \rho'}$ | | $\frac{a_1}{m'} \frac{\partial^2 R}{\partial v \partial \rho'}$ | |
|--------------|--|--------|---|--------|---|--------|
| | sin | cos | sin | cos | sin | cos |
| 0, 0 | | +0.172 | | +0.002 | | -0.645 |
| —1 | -0.026 | -0.562 | +0.556 | 0 | +0.113 | -0.610 |
| 1, 0 | +0.070 | +0.015 | +0.015 | +0.070 | -0.227 | -0.079 |
| —1 | -0.003 | +8.749 | -8.750 | -0.001 | +0.011 | +6.189 |
| —2 | +0.889 | +0.180 | -0.176 | +0.889 | +0.912 | +0.190 |
| 2,—1 | +0.084 | +0.248 | -0.230 | +0.169 | -0.322 | +0.020 |
| —2 | +0.018 | +0.472 | -0.941 | +0.013 | -0.006 | -1.685 |
| 3,—1 | +0.012 | +0.004 | +0.013 | +0.021 | -0.049 | +0.028 |
| —2 | +0.073 | +0.068 | -0.122 | +0.219 | -0.344 | -0.244 |
| —3 | 0 | +0.278 | -0.834 | -0.007 | +0.010 | -1.260 |
| 4,—2 | +0.015 | -0.001 | +0.020 | +0.042 | -0.071 | +0.019 |
| —4 | +0.001 | +0.155 | -0.619 | 0 | +0.001 | -0.854 |

The derivations with respect to γ and the node have been omitted because they are quite insensible. The terms of δR depending on these derivatives are given by equation (31). In the case of Uranus disturbed by Saturn the largest values of the coefficients

$$\frac{1}{2} h \cot \frac{1}{2} \gamma; \frac{1}{2} h \tan \frac{1}{2} \gamma, \frac{1}{2} \frac{\partial h}{\partial \sigma}$$

are only about .05, while the largest coefficients in δk , $\delta k'$, and $\delta \gamma$ are less than $10''$. Hence the largest terms in (31) will be of the order of magnitude $0''.5$ multiplied by the mass of Saturn, and may therefore be omitted entirely. Omitting them the values of δR , $\delta \frac{\partial R}{\partial v}$ and $\delta \frac{\partial R}{\partial \rho}$ become

$$\begin{aligned} \delta R &= \frac{\partial R}{\partial v} \delta v + \frac{\partial R}{\partial v'} \delta v' + \frac{\partial R}{\partial \rho} \delta \rho + \frac{\partial R}{\partial \rho'} \delta \rho' \\ \delta \frac{\partial R}{\partial v} &= \frac{\partial^2 R}{\partial v^2} \delta v + \frac{\partial^2 R}{\partial v \partial v'} \delta v' + \frac{\partial^2 R}{\partial v \partial \rho} \delta \rho + \frac{\partial^2 R}{\partial v \partial \rho'} \delta \rho' \\ \delta \frac{\partial R}{\partial \rho} &= \frac{\partial^2 R}{\partial v \partial \rho} \delta v + \frac{\partial^2 R}{\partial v' \partial \rho} \delta v' + \frac{\partial^2 R}{\partial \rho^2} \delta \rho + \frac{\partial^2 R}{\partial \rho \partial \rho'} \delta \rho' \end{aligned}$$

All the separate factors from which the second members of these equations are formed have already been given. Forming their products in the way described in Chapter II, we have the result given in the following tables.

The expressions for δR are arranged so that the value of $D' \delta R$ can be obtained from them by direct differentiation. This is done by distinguishing the time introduced into R by the co-ordinates of Uranus from that introduced by the co-ordinates of Saturn.

| | | $\frac{2a_1}{m'} \left\{ \frac{\partial R}{\partial v'} \delta v' + \frac{\partial R}{\partial p'} \delta p' \right\}$ | | | | $\frac{2a_1}{m'} \left\{ \frac{\partial R}{\partial v'} \delta v' + \frac{\partial R}{\partial p'} \delta p' \right\}$ | | | |
|---------------|-----------------|--|--------------|-------|--------|--|--|--|--|
| <i>U S</i> | cos | sin | <i>U S J</i> | " | " | | | | |
| 1, 0 | - 1.40 <i>t</i> | - 1.33 <i>t</i> | +1, 3-2 | +375 | +7336 | | | | |
| 2, 0 | + 0.06 <i>t</i> | - 0.28 <i>t</i> | +2, | -990 | -558 | | | | |
| | | | +3, | - 73 | -261 | | | | |
| 0,-1 | - 0.24 <i>t</i> | + 0.20 <i>t</i> | +4, | + 23 | - 42 | | | | |
| 1,-1 | + 0.03 <i>t</i> | + 0.04 <i>t</i> | | | | | | | |
| 2,-1 | - 1.05 <i>t</i> | - 0.95 <i>t</i> | 0, -2 | -647 | -319 | | | | |
| 3,-1 | + 0.02 <i>t</i> | - 0.30 <i>t</i> | 1, | +7737 | +5616 | | | | |
| | | | 2, | +153 | -103 | | | | |
| 0,-2 | + 0.93 <i>t</i> | - 0.83 <i>t</i> | 3, | + 24 | - 26 | | | | |
| 1,-2 | -12.69 <i>t</i> | +11.60 <i>t</i> | 4, | 0 | 0 | | | | |
| 2,-2 | - 0.24 <i>t</i> | + 0.30 <i>t</i> | | | | | | | |
| 3,-2 | - 0.63 <i>t</i> | - 0.59 <i>t</i> | -2, 5-2 | + 90 | + 60 | | | | |
| | | | -1, | -625 | +739 | | | | |
| 1,-3 | - 1.60 <i>t</i> | - 1.14 <i>t</i> | 0, | + 46 | - 76 | | | | |
| 2,-3 | + 0.25 <i>t</i> | - 0.29 <i>t</i> | 1, | + 61 | +719 | | | | |
| 3,-3 | + 0.08 <i>t</i> | + 0.02 <i>t</i> | 2, | - 20 | + 6 | | | | |
| 4,-3 | - 0.36 <i>t</i> | - 0.34 <i>t</i> | | | | | | | |
| | | | -4, 6-2 | +27.5 | - 3.9 | | | | |
| 3,-4 | + 0.19 <i>t</i> | - 0.17 <i>t</i> | -3, | +94.1 | + 24.4 | | | | |
| 4,-4 | + 0.15 <i>t</i> | 0.0 | -2, | + 66 | +243 | | | | |
| | | | -1, | -8823 | -6114 | | | | |
| | | | 0, | +699 | +522 | | | | |
| <i>U U' S</i> | | | | | | | | | |
| 1, 0 0 | + 38'' | + 4'' | | | | | | | |
| 1, 0-1 | + 2 | + 6 | -4, 7-2 | + 70 | - 2 | | | | |
| 1, 0-2 | -112 | -153 | -3, | +299 | +147 | | | | |
| 1, 0-3 | + 14 | - 18 | -2, | +937 | +706 | | | | |
| 1,+1-1 | + 56 | - 2 | -1, | -1978 | +1117 | | | | |
| 1,-1+1 | - 4 | 0 | | | | | | | |
| 1,+2-1 | + 3 | + 43 | | | | | | | |
| 1,-2+1 | - 1 | + 19 | | | | | | | |
| 1,+2-2 | -117 | + 2 | | | | | | | |
| 1,-2+2 | - 23 | 0 | | | | | | | |
| 1,+3-1 | - 28 | + 88 | | | | | | | |
| 1,-3+1 | + 30 | +100 | | | | | | | |
| 1,+3-2 | +140 | +112 | | | | | | | |
| 1,-3+2 | + 14 | - 10 | | | | | | | |
| <i>U S J</i> | | | | | | | | | |
| 0, 1-1 | +246 | - 1 | | | | | | | |
| 1, | -2469 | - 50 | | | | | | | |
| 2, | - 22 | + 4 | | | | | | | |
| 3, | + 3 | + 6 | | | | | | | |
| | | | | | | | | | |
| -2, 2-1 | + 42 | - 16 | | | | | | | |
| -1, | +1638 | - 88 | | | | | | | |
| 0, | -136 | + 12 | | | | | | | |
| 1, | - 89 | - 89 | | | | | | | |
| 2, | + 2 | + 4 | | | | | | | |
| | | | | | | | | | |
| -3, 3-1 | + 6 | - 8 | | | | | | | |
| -2, | + 66 | + 24 | | | | | | | |
| -1, | +478 | -280 | | | | | | | |
| 0, | | | | | | | | | |
| | | | | | | | | | |
| -3, 4-1 | + 9 | + 28 | | | | | | | |
| -2, | -221 | - 5 | | | | | | | |
| -1, | +297 | -286 | | | | | | | |

| | | $\frac{2a_1}{m'} \left\{ \frac{\partial R}{\partial v'} \delta v' + \frac{\partial R}{\partial p'} \delta p' \right\}$ | | |
|--------------|------|--|-----------------|--|
| <i>U S S</i> | cos | sin | Fact. <i>nt</i> | |
| | " | " | " | |
| 2,-1-1 | -120 | - 3 | -0.852 | |
| 3, | + 59 | +801 | +0.148 | |
| 4, | -139 | +353 | +1.448 | |
| | | | | |
| 3,-1-2 | -122 | + 26 | +0.148 | |
| 4, | - 60 | - 29 | +1.148 | |
| 5, | + 24 | - 60 | +2.148 | |
| | | | | |
| -0,-1+1 | -250 | 0 | -2.852 | |
| +1, | - 4 | -170 | -1.852 | |
| 2, | +197 | -450 | -0.852 | |
| 3, | - 15 | + 38 | +0.148 | |
| | | | | |
| -1,-1+2 | - 8 | + 19 | -3.852 | |
| 0, | - 11 | +134 | -2.852 | |
| 1, | - 80 | + 33 | -1.852 | |
| | | | | |
| 1,-1 0 | - 90 | + 12 | -1.852 | |
| 2, | + 6 | +158 | -0.852 | |
| 3, | + 12 | - 32 | +0.148 | |

| U S | $2 \frac{a_1}{m'n} \delta D'_1 R$ | | $2 \frac{a_1}{m'} \delta \frac{\partial R}{\partial v}$ | | $2 \frac{a_1}{m'} \delta \frac{\partial R}{\partial \rho}$ | |
|-------|-----------------------------------|----------|---|----------|--|----------|
| | sin | cos | sin | cos | cos | sin |
| | " | " | " | " | " | " |
| 0, 0 | | | 0 | 0 | + 0.14t | |
| 1, 0 | + 1.40t | - 1.33t | + 1.40t | - 1.35t | + 3.54t | + 3.21t |
| 2, 0 | - 0.12t | - 0.56t | - 0.27t | - 0.43t | - 0.34t | + 0.90t |
| 0,-1 | 0 | 0 | - 0.04t | - 0.06t | + 1.06t | - 1.02t |
| 1, 1 | - 0.03t | + 0.04t | - 0.06t | + 0.03t | + 0.07t | - 0.12t |
| 2, 2 | + 2.10t | - 1.90t | + 2.10t | - 1.89t | + 3.54t | + 3.22t |
| 3, 3 | - 0.06t | - 0.90t | - 0.19t | - 0.73t | - 0.19t | + 1.18t |
| 0,-2 | 0 | 0 | - 0.91t | - 0.83t | + 0.84t | - 0.94t |
| 1, 1 | + 12.69t | + 11.60t | + 12.69t | + 11.62t | - 13.52t | + 12.34t |
| 2, 2 | + 0.48t | + 0.60t | + 0.21t | + 0.37t | - 0.42t | + 0.14t |
| 3, 3 | + 1.89t | - 1.77t | + 1.89t | - 1.75t | + 2.78t | + 2.52t |
| 1,-3 | + 1.60t | - 1.14t | + 1.61t | - 1.11t | - 1.64t | - 1.21t |
| 2, 2 | - 0.50t | - 0.58t | - 0.53t | - 0.56t | - 0.83t | + 0.75t |
| 3, 3 | - 0.24t | + 0.06t | - 0.19t | + 0.07t | - 0.25t | - 0.10t |
| 4, 4 | + 1.44t | - 1.36t | + 1.43t | - 1.32t | + 1.93t | + 1.76t |
| 3,-4 | - 0.57t | - 0.51t | - 0.55t | - 0.53t | - 0.74t | + 0.68t |
| 4, 4 | - 0.60t | 0 | - 0.52t | + 0.10t | - 0.73t | - 0.10t |
| 0, 0 | | | | + 5 | - 192 | |
| 1, 0 | - 45 | + 319 | + 9 | + 160 | - 49 | - 709 |
| 2, 2 | + 168 | + 383 | + 199 | + 449 | + 278 | - 591 |
| 3, 3 | + 2 | + 5 | | | | |
| -1,-1 | + 148 | - 61 | + 70 | - 71 | + 18 | + 125 |
| 0, 0 | + 27 | - 381 | - 16 | - 247 | - 49 | + 347 |
| 1, 1 | - 139 | - 70 | - 4 | - 32 | + 103 | + 47 |
| 2, 2 | - 21 | - 36 | + 3 | - 12 | + 1 | - 215 |
| 3, 3 | - 4 | + 38 | + 1 | - 1 | - 10 | + 38 |
| 4, 4 | + 28 | + 88 | - 8 | + 4 | - 19 | - 9 |
| 1,-2 | + 87 | - 153 | - 14 | - 119 | - 10 | + 251 |
| 2, 2 | - 88 | - 7 | + 145 | - 13 | + 84 | + 7 |
| 3, 3 | + 108 | + 121 | - 60 | + 796 | + 83 | + 816 |
| 4, 4 | + 20 | + 517 | + 141 | + 352 | - 262 | + 617 |
| 5,-2 | | | - 18 | + 14 | - 45 | + 2 |
| 3,-3 | + 18 | + 4 | + 208 | + 16 | + 17 | + 34 |
| 4, 4 | + 69 | - 33 | + 82 | - 67 | - 35 | + 44 |
| 5, 5 | - 52 | - 128 | - 49 | - 130 | - 92 | + 181 |
| 4,-4 | | | + 55 | - 1 | + 110 | + 8 |
| 5, 5 | | | + 8 | - 104 | + 12 | + 52 |
| 6, 6 | | | - 42 | - 103 | - 61 | + 148 |

| U S J | $2 \frac{a_1}{m'n} \delta D_i R$ | | $2 \frac{a_1}{m'} \delta \frac{\partial R}{\partial v}$ | | $2 \frac{a_1}{m'} \delta \frac{\partial R}{\partial p}$ | |
|---------|----------------------------------|--------|---|--------|---|--------|
| | sin | cos | sin | cos | cos | sin |
| | " | " | " | " | " | " |
| 0, 1-1 | 0 | 0 | -185 | + 2 | - 84 | + 13 |
| 1, | +2469 | - 50 | +2477 | - 50 | -2947 | - 69 |
| 2, | + 44 | + 8 | - 7 | + 13 | -176 | - 17 |
| 3, | - 9 | + 18 | - 6 | + 16 | - 17 | - 20 |
| -2, 2-1 | + 84 | + 32 | + 42 | + 29 | + 9 | + 49 |
| -1, | +1638 | + 88 | +1642 | + 88 | +1231 | -143 |
| 0, | 0 | 0 | -114 | - 2 | + 41 | + 11 |
| +1, | + 89 | - 89 | + 83 | - 89 | -196 | - 62 |
| +2, | - 4 | + 8 | - 1 | + 8 | - 11 | - 18 |
| -3, 3-1 | + 18 | + 24 | + 25 | + 28 | - 42 | + 41 |
| -2, | +132 | - 48 | +143 | - 54 | -191 | -104 |
| -1, | +478 | +280 | +477 | +279 | +1529 | -279 |
| 0, | 0 | 0 | - 57 | - 8 | - 60 | + 44 |
| -3, 4-1 | + 27 | - 84 | + 71 | - 91 | - 89 | -134 |
| -2, | -442 | + 10 | -449 | + 7 | +776 | - 8 |
| -1, | +297 | +286 | +310 | +270 | +362 | -258 |
| 1, 3-2 | -375 | +7336 | -393 | +7343 | + 629 | +7722 |
| 2, | +1980 | -1116 | +1987 | -1271 | +3304 | +2544 |
| 3, | +219 | -783 | + 51 | -672 | +151 | +1041 |
| 4, | - 92 | -168 | -104 | -102 | -136 | +158 |
| 0, 4-2 | 0 | 0 | +669 | -485 | -601 | -966 |
| 1, | -7737 | +5616 | -7733 | +5606 | +12136 | +8698 |
| 2, | -306 | -206 | -235 | -258 | +273 | +910 |
| 3, | - 72 | - 78 | + 59 | - 52 | - 60 | +109 |
| -2, 5-2 | +180 | -120 | +159 | + 6 | -337 | - 37 |
| -1, | -625 | -739 | -620 | -740 | -1222 | -1217 |
| 0, | 0 | 0 | + 62 | - 56 | +167 | - 60 |
| +1, | - 61 | +719 | - 63 | +725 | + 72 | +1109 |
| +2, | + 40 | + 12 | - 13 | - 6 | - 17 | + 67 |
| -4, 6-2 | +110.0 | + 15.6 | + 86.1 | + 32.9 | -127.5 | + 39.4 |
| -3, | +282.3 | - 73.2 | +248.8 | - 14.5 | -413.6 | - 75.7 |
| -2, | +132 | -486 | +237 | -557 | -1131 | -1461 |
| -1, | -8823 | +6114 | -8829 | +6121 | -12973 | -9020 |
| 0, | 0 | 0 | +746 | -493 | +774 | +406 |
| -4, 7-2 | +280 | + 8 | +207 | + 45 | -216 | + 48 |
| -3, | +897 | -441 | +727 | -632 | -1128 | -977 |
| -2, | +1874 | -1412 | +1913 | -1374 | -3236 | -2195 |
| -1, | -1978 | -1117 | -1995 | -1097 | -2071 | +904 |
| 0, | 0 | 0 | +139 | + 90 | +144 | - 71 |

In the terms of δR introduced by the perturbations of Saturn, namely, $\frac{\partial R}{\partial v'} \delta v' + \frac{\partial R}{\partial \rho'} \delta \rho'$, the differentiation represented by D'_t should be performed by considering $\delta v'$ and $\delta \rho'$ as constant, although they are expressed as a function of the mean longitude of Uranus, as well as of Saturn. The mean longitude of Uranus thus introduced is therefore represented by U' , which is regarded as constant in taking $D'_t R$, and U only supposed to vary.

Again, in the terms $\frac{\partial R}{\partial v} \delta v + \frac{\partial R}{\partial \rho} \delta \rho$, since δv and $\delta \rho$ represent perturbations of Uranus, their complete derivatives, with respect to the time, are to be taken. But their expressions contain the mean longitude of Saturn as well as Uranus. The mean longitude of Saturn thus introduced is represented by S'' , and is to be considered variable in obtaining $D'_t \delta R$, while S is considered constant. The ratio of the coefficient of t to n in the various terms of this part of δR is given to the right of each corresponding term.

The value of $D'_t \delta R$ being once obtained, there is no longer any distinction necessary between U , U' , or between S and S'' . The similar terms are therefore combined by putting $S'' = S$; $U' = U$.

From the above values of $2\delta D'_t R$ and $2\delta \frac{\partial R}{\partial \rho}$, we form the following value of

$$\frac{a_1}{m} \delta Q = \frac{2a_1}{m} \int \delta D'_t R dt + \frac{a_1}{m} \delta \frac{\partial R}{\partial \rho}$$

and of the other quantities which enter the perturbations of the co-ordinates. We shall begin with those terms which depend only on the mutual action of Saturn and Uranus, because they are few and small, and the only terms which are sensible are those in which the coefficient of the mean longitude of Saturn is -1 . We shall therefore confine ourselves to these. And, instead of employing the condensed formulæ, we shall make the computation in full by (13).

| $\frac{a_1}{m'} \delta Q = 2 \frac{a_1}{m} \int \delta D'_t R dt + \delta \frac{\partial R}{\partial \rho}$ | | | $\frac{a_1}{m'} \xi \delta Q$ | | $\frac{a_1}{m'} \eta \delta Q$ | | |
|---|------|------------|-------------------------------|------------|--------------------------------|------------|------------|
| g | l' | | cos | sin | sin | cos | |
| | | | " " | " " | " " | " " | |
| -1,-1 | | + 47 | + 78 | - 10 | +148 | + 7 | +153 |
| 0 | | - 14+0.53t | +307-0.51t | + 13+0.02t | + 49+0.05t | + 36-0.06t | - 8+0.01t |
| 1 | | - 24+0.02t | + 62-0.08t | - 41+2.37t | +138+1.62t | + 29-1.85t | -164+2.11t |
| 2 | | - 60+4.23t | - 28+3.84t | -270-0.13t | +157-3.05t | +250-0.13t | + 85-2.70t |
| 3 | | -525+0.31t | +240-5.49t | + 7+2.09t | - 31+2.30t | - 30+2.11t | + 14-1.92t |
| 4 | | | | -262+0.15t | +120-2.74t | -262+0.15t | -120+2.74t |

| | | cos $\downarrow \delta \rho$ | | δv | |
|------|------|------------------------------|-----------------|----------------|----------------|
| g | l' | cos | sin | sin | cos |
| | | " " | " " | " " | " " |
| 0,-1 | | + .001- .00001t | - .023+ .00002t | +0.002+0.0000t | +0.021+0.0000t |
| 1, | | + .024+ .00033t | - .036+ .00033t | +0.043+0.0006t | +0.058+0.0000t |
| 2, | | + .333+ .00437t | - .406+ .00432t | +0.620+0.0098t | +0.803-0.0097t |
| 3, | | - .142- .00002t | + .079- .00169t | +2.69 -0.0011t | +0.830-0.0153t |
| 4, | | - .071 | + .019 | +0.132 | 0t |
| | | | | | +0.040+0.0000t |

The computation of these terms being extremely complex, a check upon their accuracy is desirable. In the case of the secular variations of the coefficients, the coefficients of the time are easily obtained by substituting in the integrated perturbations the variations of the eccentricity and perihelion of Saturn. Thus I have found

$$\begin{aligned} \delta v = & +0.0103 t \sin (2g - l) - 0.0094 t \cos (2g - l) \\ & + 0.0027 t \sin (3g - l) - 0.0138 t \cos (3g - l) \end{aligned}$$

The greatest discrepancy is found in the coefficient of $\sin (3g - l)$, and it amounts to $0''.0038t$, or about $0''.4$ in a century. But, owing to the great period of this term, nearly 600 years, this difference, during any one century, will be nearly eliminated through the mean longitude and mean motion.

It may also be remarked that in this case the terms derived from the perturbations of the elements are undoubtedly the correct ones, and will therefore be employed.

The terms which the preceding integration fails to give, owing to the constant terms introduced into $\xi\delta Q$ and $\eta\delta Q$, are found by (22).

We thus have

$$\begin{aligned} n\Sigma p_a k_e^{(n)} &= + 0''.36 \\ n\Sigma q_a k_s^{(n)} &= + 0.27 \\ r_1^2 \delta \rho &= \frac{1}{4} Mut^2 \{0''.36 \sin g - 0''.27 \cos g\} \\ \delta v &= \frac{1}{2} Mut^2 \{0''.36 \cos g + 0''.27 \sin g\} \\ &= t^2 \{0''.0000038 \cos g + 0''.0000029 \sin g\}. \end{aligned}$$

The greatest effect of these terms amounts to less than one-twentieth of a second in a century. They may therefore be neglected in the present theory. The other terms containing the square of the time are yet smaller.

Applying the terms of the second order thus found to the terms of the first order depending on the corresponding arguments, the perturbations of Uranus by Saturn become

| $g \quad l'$ | δv | | $\cos \delta \rho$ | |
|--------------|--------------------|---------------------|--------------------|-------|
| | sin | cos | cos | sin |
| -1,-1 | + 0.036 | + 0.039 | + 0 | - 2 |
| 0,-1 | + 1.284 | + 0.739 | - 11 | - 31 |
| 1,-1 | -20.774 + 0.06 T | + 8.580 | +1753 | -244 |
| 2,-1 | -11.270 + 1.03 T | +144.265 - 0.94 T | - 87 | -3114 |
| 3,-1 | +51.99 + 0.27 T | +116.69 - 1.38 T | -237 | +644 |
| 4,-1 | + 2.265 | + 5.656 | - 55 | +140 |
| 5,-1 | + 0.126 | + 0.329 | - 4 | + 10 |

T here represents the time counted in centuries from 1850.0.

The other terms remain the same as given on page 50.

Perturbations depending on the product of the masses of Jupiter and Saturn.

The values of $\delta D'R$, $\delta \frac{\partial R}{\partial v}$, and $\delta \frac{\partial R}{\partial \rho}$, depending on the products of the masses

of Jupiter and Saturn, are given on page 74. The computation from these data being conducted in the same way as in the case of the terms of the first order, it is not necessary to give much more than the results. These are shown in the following table. The indices to the left represent the coefficients of the mean longitudes of Uranus, Saturn, and Jupiter, all counted from the perihelion of Uranus. Column ν gives the ratio of the mean motion of Uranus to the coefficient of the time in each argument. The perturbations of the common logarithm of the radius vector are expressed in units of the seventh place of decimals.

| <i>U S J</i> | ν | δv | | 0.4343 δp | |
|--------------|----------|------------|----------|-------------------|-----|
| | | sin
" | cos
" | cos | sin |
| 0, 1,—1 | —0.2364 | +0.002 | 0 | 0 | 0 |
| 1, | —0.3095 | —0.020 | 0 | 0 | 0 |
| 3, | —0.4480 | +0.004 | —0.003 | 0 | 0 |
| 4, | —0.8127 | +0.016 | —0.024 | 0 | 0 |
| —2, 2,—1 | —0.2960 | —0.007 | —0.001 | 0 | 0 |
| —1, | —0.4204 | —0.108 | —0.007 | — 2 | 0 |
| 0, | —0.7254 | —0.014 | —0.012 | 0 | 0 |
| 1, | —2.6420 | +0.164 | —0.267 | + 1 | + 1 |
| 2, | +1.6090 | +0.005 | —0.005 | 0 | 0 |
| —3, 3,—1 | —0.6551 | +0.012 | —0.002 | 0 | 0 |
| —2, | —1.8997 | +0.175 | —0.015 | + 1 | 0 |
| —1, | +2.1115 | +0.078 | +0.512 | — 2 | + 3 |
| 0, | +0.6786 | +0.007 | +0.024 | 0 | 0 |
| —3, 4,—1 | +0.754 | —0.030 | +0.081 | 0 | + 1 |
| —2, | +0.430 | +0.043 | +0.005 | — 1 | 0 |
| —1, | +0.301 | +0.001 | —0.003 | 0 | 0 |
| 1, 3,—2 | —0.2170 | —0.002 | —0.032 | 0 | — 1 |
| 2, | —0.2771 | —0.051 | +0.035 | — 1 | — 1 |
| 3, | —0.3833 | —0.010 | +0.050 | 0 | — 1 |
| 4, | —0.6215 | +0.032 | +0.052 | 0 | — 1 |
| 0, 4,—2 | —0.3627 | —0.010 | —0.004 | 0 | 0 |
| 1, | —0.5692 | +0.075 | —0.154 | — 5 | — 2 |
| 2, | —1.3210 | —0.349 | —1.297 | — 3 | +11 |
| 3, | +4.1150 | —0.510 | —0.453 | + 2 | — 2 |
| —2, 5,—2 | —0.5250 | —0.253 | —0.034 | — 3 | 0 |
| —1, | —1.1051 | —4.433 | —0.617 | —43 | + 7 |
| 0, | +10.5152 | —0.546 | —0.032 | + 2 | 0 |
| 1, | +0.9132 | +0.206 | —3.254 | — 2 | —36 |
| 2, | +0.4773 | +0.012 | —0.192 | 0 | — 2 |
| —4, 6,—2 | —0.9497 | +1.824 | —0.519 | +19 | + 6 |
| —3, | —18.9250 | +40.650 | —10.500 | +32 | +10 |
| —2, | +1.0558 | +6.237 | —7.866 | —63 | —79 |
| —1, | +0.5136 | +0.467 | —0.539 | 0 | — 2 |
| 0, | +0.3393 | +0.007 | —0.017 | 0 | 0 |
| —4, 7,—2 | +0.5558 | —0.050 | —0.003 | + 1 | 0 |
| —3, | +0.3573 | —0.046 | +0.032 | + 1 | + 1 |
| —2, | +0.2632 | —0.045 | +0.032 | + 1 | + 1 |
| —1, | +0.2084 | +0.006 | +0.007 | 0 | 0 |
| —0, | +0.1724 | 0 | 0 | 0 | 0 |

CHAPTER V.

COLLECTION AND TRANSFORMATION OF THE PRECEDING PERTURBATIONS OF URANUS.

THE terms of the perturbations which neither contain the elements of the disturbing planets, nor depend on the secular variations of the eccentricity and perihelion, admit of being greatly simplified by a slight change in the arbitrary elements. These terms are as follows:

(1) In the longitude of Uranus

| | | | | | |
|--------------------|-------------------|----------------------|-----------------------|---------------------|-----------------------|
| | " | " | " | " | " |
| Action of Jupiter, | +31.2116 <i>t</i> | +25.657 sin <i>g</i> | +1.397 sin 2 <i>g</i> | -1.859 cos <i>g</i> | -0.087 cos 2 <i>g</i> |
| Action of Saturn, | +10.9690 <i>t</i> | + 8.545 sin <i>g</i> | +0.461 sin 2 <i>g</i> | -4.735 cos <i>g</i> | -0.169 cos 2 <i>g</i> |
| Action of Neptune, | - 0.4262 <i>t</i> | + 0.697 sin <i>g</i> | +0.046 sin 2 <i>g</i> | -0.088 cos <i>g</i> | -0.005 cos 2 <i>g</i> |
| Total, | +41.7544 <i>t</i> | +34.899 sin <i>g</i> | +1.904 sin 2 <i>g</i> | -6.682 cos <i>g</i> | -0.261 cos 2 <i>g</i> |

(2) In the value of $\cos \psi \delta \rho$, units of 7th place of decimals.

| | | | | | |
|--------------------|--------|-------------------|--------------------|------------------|-------------------|
| Action of Jupiter, | -10089 | -492 cos <i>g</i> | -33 cos 2 <i>g</i> | - 2 sin <i>g</i> | +1 sin 2 <i>g</i> |
| Action of Saturn, | - 3543 | -184 cos <i>g</i> | -15 cos 2 <i>g</i> | -15 sin <i>g</i> | |
| Action of Neptune, | + 138 | - 1 cos <i>g</i> | - 1 cos 2 <i>g</i> | + 1 sin <i>g</i> | |
| Total, | -13494 | -677 cos <i>g</i> | -49 cos 2 <i>g</i> | -16 sin <i>g</i> | +1 sin 2 <i>g</i> |

Let us first consider the first or constant term in the perturbation of each co-ordinate. If we suppose a change of δn in the mean motion of a planet, the corresponding change in $\delta \rho$ will be

$$\delta \rho = -\frac{2 \delta n}{3 n}.$$

If, then, we increase the mean motion of Uranus by 41".754, the corresponding change in $\delta \rho$ will be -18045, and in $\cos \psi \delta \rho$, -18025. Subtracting these from the above perturbations, the secular term in the mean motion will disappear, and we shall have for the constant term of $\cos \psi \delta \rho$

$$+4531$$

This same change in the mean motion will produce a secular term in the equation of the centre of the same nature with that produced by the secular variation of the perihelion. The differences of the values of the secular terms, found by the two methods employed in Chapters II. and III., proceeds from the fact that in the one case the effect of the above term in the mean motion is included, and in the other excluded.

If we subduct the effect in question when necessary, the remainder will be the effect of the secular variation of the longitude of the perihelion of Uranus, to which we shall revert presently.

Let us next introduce such a change in the eccentricity of Uranus as shall produce the term $34''.899 \sin g$, and ascertain its effect on the other terms. For this purpose we must determine δe by the condition

$$(2 - \frac{3}{4}e^2) \delta e = 34''.899$$

which gives

$$\delta e = 17''.464 = .0000847.$$

A change of this amount in δe will introduce the following terms in δv and $\delta \rho$

$$\begin{aligned} \delta v &= 34''.899 \sin g + 2''.048 \sin 2g \\ \cos \psi \delta \rho &= 20 - 844 \cos g - 59 \cos 2g. \end{aligned}$$

Subtracting these terms from the expressions previously found we have

$$\begin{aligned} \delta v &= -0''.144 \sin 2g - 6''.682 \cos g - 0''.261 \cos 2g. \\ \cos \psi \delta \rho &= +4511 + 167 \cos g + 10 \cos 2g - 16 \sin g + 1 \sin 2g. \end{aligned}$$

Again, let us put

$$e\delta\pi = 3''.342 = .0000162,$$

we shall have the elliptic terms

$$\begin{aligned} \delta v &= -6''.682 \cos g - 0''.391 \cos 2g \\ \cos \psi \delta \rho &= -162 \sin g - 11 \sin 2g. \end{aligned}$$

Subtracting these expressions the constant terms, independent of the mean longitude of the disturbing planets, are reduced to

$$\begin{aligned} \delta v &= -0''.144 \sin 2g + 0''.130 \cos 2g. \\ \cos \psi \delta \rho &= 4511 + 167 \cos g + 10 \cos 2g + 146 \sin g + 12 \sin 2g. \\ 0.43429 \delta \rho &= 1969 + 73 \cos g + 4 \cos 2g + 63 \sin g + 5 \sin 2g. \end{aligned}$$

In the last equation we have introduced the constant $+.0000008$ produced in $\delta \rho$ by the combined action of Venus, the Earth, and Mars. The effect of each planet is computed by the approximate formula

$$\delta \rho = \frac{1}{6} m'(b_1^{(0)} + \alpha D \alpha b_1^{(0)}).$$

Secular Variations.

The following inequalities result from the secular variations of the eccentricity and longitude of perihelion produced by each of the disturbing planets, T being the time expressed in centuries.

From the variation of the eccentricity

| | | | | |
|--------------------|--------------|-------------------|--------------------|--------------------|
| | $\delta v =$ | $''$ | $''$ | $''$ |
| Action of Jupiter, | $\delta v =$ | $-1.216 T \sin g$ | $-0.072 T \sin 2g$ | $-0.005 T \sin 3g$ |
| Action of Saturn, | | $-9.182 T \sin g$ | $-0.538 T \sin 2g$ | $-0.032 T \sin 3g$ |
| Action of Neptune, | | $-0.502 T \sin g$ | $-0.030 T \sin 2g$ | $-0.002 T \sin 3g$ |

| | | |
|--------------------|------------------------------|----------------|
| Action of Jupiter, | $M\delta\rho = +13 T \cos g$ | $+1 T \cos 2g$ |
| Action of Saturn, | $+98 T \cos g$ | $+7 T \cos 2g$ |
| Action of Neptune, | $+6 T \cos g$ | |

The secular variation of the longitude of the perihelion is

| | |
|--------------------|------------------------|
| Action of Jupiter, | $+122.1 T$ |
| Action of Saturn, | $+118.4 T$ |
| Action of Neptune, | $+51.1 T$ |
| Total, | $\delta\pi = +291.6 T$ |

The effect of this secular variation on the longitude and radius vector is

| | | | |
|--------------------|------------------------------|--------------------|--------------------|
| Action of Jupiter, | $\delta v = -11.46 T \cos g$ | $-0.671 T \cos 2g$ | $-0.047 T \cos 3g$ |
| Action of Saturn, | $-11.11 T \cos g$ | $-0.651 T \cos 2g$ | $-0.039 T \cos 3g$ |
| Action of Neptune, | $-4.80 T \cos g$ | $-0.281 T \cos 2g$ | $-0.016 T \cos 3g$ |
| Total, | $-27.37 T \cos g$ | $-1.603 T \cos 2g$ | $-0.102 T \cos 3g$ |

| | | |
|--------------------|-------------------------------|----------------|
| Action of Jupiter, | $M\delta\rho = -120 T \sin g$ | $-8 T \sin 2g$ |
| Action of Saturn, | $-117 T \sin g$ | $-8 T \sin 2g$ |
| Action of Neptune, | $-50 T \sin g$ | $-3 T \sin 2g$ |

For the purpose of conveniently tabulating the perturbations, we shall express them in a form similar to that adopted in the theory of Neptune. Let us select, from the terms of the periodic perturbations produced by any planet, all those in which the difference between the indices i and i' is the same. For example, in the perturbations of the longitude produced by Jupiter, let us consider the terms

$$\begin{aligned} \delta v = & + 1.269 \sin (\quad - l) & + 0.002 \cos (\quad - l) \\ & - 3.495 \sin (2g - l) & - 0.092 \cos (2g - l) \\ & + 1.182 \sin (g - 2l) & + 0.515 \cos (g - 2l) \\ & + 0.074 \sin (3g - 2l) & - 0.005 \cos (3g - 2l) \\ & - 0.005 \sin (2g - 3l) & \\ & + 0.011 \sin (4g - 3l) & - 0.001 \cos (4g - 3l) \end{aligned}$$

These terms may be expressed in the form

$$\begin{aligned} \delta v = \sin g \times & \left\{ \begin{array}{ll} + 0.094 \sin (g - l) & - 4.764 \cos (g - l) \\ + 0.520 \sin 2(g - l) & - 1.108 \cos 2(g - l) \\ & + 0.016 \cos 3(g - l) \end{array} \right\} \\ + \cos g \times & \left\{ \begin{array}{ll} - 2.226 \sin (g - l) & - 0.090 \cos (g - l) \\ + 1.256 \sin 2(g - l) & + 0.510 \cos 2(g - l) \\ + 0.006 \sin 3(g - l) & \end{array} \right\} \end{aligned}$$

In general, a series of terms of the form

$$\begin{aligned} & \Sigma a_i \sin(iA + sg) + \Sigma b_i \cos(iA + sg) \\ & + \Sigma a'_i \sin(iA - sg) + \Sigma b'_i \cos(iA - sg), \end{aligned}$$

may be put in the form

$$\begin{aligned} & \{ \Sigma (a_i - a'_i) \cos iA - \Sigma (b_i - b'_i) \sin iA \} \sin sg \\ & + \{ \Sigma (a_i + a'_i) \sin iA + \Sigma (b_i + b'_i) \cos iA \} \cos sg. \end{aligned}$$

All the periodic terms containing only g and l in the arguments may be put into this form by taking

$$A = g - l,$$

so that the coefficients of $\sin sg$ and $\cos sg$ may all be expressed as a function of the single variable argument A .

The perturbations of the elements may be reduced to perturbations of the co-ordinates expressed as the sum of several products of slowly varying functions into the sines and cosines of the multiples of g . We have, in fact,

$$\begin{aligned} \delta v = \delta l & \\ & + \left(2 - \frac{3}{4} e^2 \right) \delta e \times \sin g \quad + \left(2 - \frac{1}{4} e^2 \right) e \delta g \times \cos g \\ & + \left(\frac{5}{2} e - \frac{11}{6} e^3 \right) \delta e \times \sin 2g \quad + \left(\frac{5}{2} e - \frac{11}{12} e^3 \right) e \delta g \times \cos 2g. \\ & + \text{etc.} \quad + \text{etc.} \end{aligned}$$

It appears, therefore, that all the perturbations in which the arguments contain the mean longitudes of only two planets may be put in the form

$$\begin{aligned} \delta v &= (v.c.0) + (v.c.1) \cos g + (v.c.2) \cos 2g + \text{etc.} \\ & \quad + (v.s.1) \sin g + (v.s.2) \sin 2g + \text{etc.} \\ M\delta\rho &= (\rho.c.0) + (\rho.c.1) \cos g + (\rho.c.2) \cos 2g + \text{etc.} \\ & \quad + (\rho.s.1) \sin g + (\rho.s.2) \sin 2g + \text{etc.} \end{aligned}$$

We have next to reduce to the same form those terms which contain the mean longitudes of both Jupiter and Saturn, and which are given on page 78. We have here twenty-four terms, each greater than $0''.04$. As most of these terms depend on three independent arguments, they cannot be included in a double entry table, while, if we include them as perturbations of the longitude in tables of single entry, we shall have to enter twenty-two tables with as many different arguments. But, by taking, for the argument A , the middle one in each series of arguments which depend on the same multiples of Jupiter and Saturn, and expressing the terms above and below it in each series as coefficients of $\sin g$, $\cos g$, $\sin 2g$, and

$\cos 2g$, we may reduce the number of arguments to eight, and the number of tables to seventeen. Consider, for instance, the terms of the second series,

$$\begin{aligned} & -0.108 \sin(-g + 2S - J) & -0.007 \cos(-g + 2S - J) \\ & -0.014 \sin(2S - J) & -0.012 \cos(2S - J) \\ & +0.164 \sin(g + 2S - J) & -0.267 \cos(g + 2S - J). \end{aligned}$$

These terms may be allowed for by adding to $(v.c.0)$, $(v.s.1)$, $(v.c.1)$, the terms

$$\begin{aligned} (v.c.0) &= -0.014 \sin(2S - J) - 0.012 \cos(2S - J) \\ (v.s.1) &= +0.260 \sin(2S - J) + 0.272 \cos(2S - J) \\ (v.c.1) &= +0.056 \sin(2S - J) - 0.274 \cos(2S - J). \end{aligned}$$

From the perturbations of longitude and radius vector already given, we readily find the following values of $(v.c.0)$, $(v.s.1)$, etc.

Action of Jupiter.

$$A_1 = l' - g$$

$$\begin{aligned} (v.c.0) &= +53.064 \sin A_1 - 0.004 \cos A_1 \\ &\quad - 0.277 \sin 2A_1 + 0.036 \cos 2A_1 \\ &\quad - 0.025 \sin 3A_1 \\ (v.c.1) &= + 2.226 \sin A_1 - 0.090 \cos A_1 \\ &\quad - 1.256 \sin 2A_1 + 0.510 \cos 2A_1 \\ &\quad - 0.006 \sin 3A_1 \\ &\quad - 11''.46T \\ (v.c.2) &= + 0.121 \sin A_1 - 0.038 \cos A_1 \\ &\quad + 0.012 \sin 2A_1 - 0.014 \cos 2A_1 \\ &\quad + 0.029 \sin 3A_1 - 0.034 \cos 3A_1 \\ &\quad - 0''.67T \\ (v.c.3) &= - 0.04T \\ (\rho.c.0) &= +1127 \cos A_1 \\ &\quad + 4 \cos 2A_1 \\ (\rho.c.1) &= - 2 \sin A_1 + 57 \cos A_1 \\ &\quad + 10 \sin 2A_1 - 23 \cos 2A_1 \\ &\quad + 13T \\ (\rho.c.2) &= + 7 \cos A_1 + 1T \\ (v.s.1) &= -0.094 \sin A_1 - 4.764 \cos A_1 \\ &\quad - 0.520 \sin 2A_1 - 1.108 \cos 2A_1 \\ &\quad + 0.016 \cos 3A_1 \\ &\quad - 1''.22T \\ (v.s.2) &= -0.056 \sin A_1 - 0.175 \cos A_1 \\ &\quad + 0.008 \sin 2A_1 + 0.042 \cos 2A_1 \\ &\quad + 0.034 \sin 3A_1 + 0.035 \cos 3A_1 \\ &\quad - 0''.07T \\ (v.s.3) &= -0.005T \\ (\rho.s.1) &= +108 \sin A_1 + 2 \cos A_1 \\ &\quad + 26 \sin 2A_1 + 12 \cos 2A_1 \\ &\quad - 120T \\ (\rho.s.2) &= + 7 \sin A_1 - 8T \end{aligned}$$

Action of Saturn.

$$A_2 = l - g$$

| | |
|--|--|
| $(v.c.0) = \begin{pmatrix} +20.774 \\ -0.06T \end{pmatrix} \sin A_2 \quad +8.580 \cos A_2$ $\begin{aligned} & -4.110 \sin 2A_2 & -0.009 \cos 2A_2 \\ & -0.824 \sin 3A_2 & -0.019 \cos 3A_2 \\ & -0.228 \sin 4A_2 & -0.008 \cos 4A_2 \\ & -0.074 \sin 5A_2 & -0.003 \cos 5A_2 \\ & -0.025 \sin 6A_2 & \end{aligned}$ | $(v.s.1) = \begin{pmatrix} +143.52 \\ -0.94T \end{pmatrix} \sin A_2 + \begin{pmatrix} -12.554 \\ +1.03T \end{pmatrix} \cos A_2$ $\begin{aligned} & -2.421 \sin 2A_2 & +2.037 \cos 2A_2 \\ & -0.226 \sin 3A_2 & +0.318 \cos 3A_2 \\ & -0.055 \sin 4A_2 & +0.097 \cos 4A_2 \\ & -0.027 \sin 5A_2 & +0.038 \cos 5A_2 \\ & -11''.11T & -9''.18T \end{aligned}$ |
| $(v.c.2) = \begin{pmatrix} -52.026 \\ -0.27T \end{pmatrix} \sin A_2 + \begin{pmatrix} +116.73 \\ -1.38T \end{pmatrix} \cos A_2$ $\begin{aligned} & -0.665 \sin 2A_2 & -1.847 \cos 2A_2 \\ & -0.081 \sin 3A_2 & -0.153 \cos 3A_2 \\ & -0.013 \sin 4A_2 & -0.044 \cos 4A_2 \\ & -0.005 \sin 5A_2 & -0.016 \cos 5A_2 \\ & -0''.65T & \end{aligned}$ | $(v.s.2) = \begin{pmatrix} +116.650 \\ -1.38T \end{pmatrix} \sin A_2 + \begin{pmatrix} 51.954 \\ +0.27T \end{pmatrix} \cos A_2$ $\begin{aligned} & -1.813 \sin 2A_2 & +0.631 \cos 2A_2 \\ & -0.177 \sin 3A_2 & +0.013 \cos 3A_2 \\ & -0.044 \sin 4A_2 & +0.013 \cos 4A_2 \\ & -0.016 \sin 5A_2 & +0.005 \cos 5A_2 \\ & -0''.54T & \end{aligned}$ |
| $(v.c.3) = \begin{aligned} & -2.265 \sin A_2 & +5.656 \cos A_2 \\ & -1.163 \sin 2A_2 & +2.956 \cos 2A_2 \\ & +0.026 \sin 3A_2 & -0.063 \cos 3A_2 \\ & +0.005 \sin 4A_2 & -0.013 \cos 4A_2 \\ & +0.002 \sin 5A_2 & -0.004 \cos 5A_2 \\ & -0''.04T & \end{aligned}$ | $(v.s.3) = \begin{aligned} & +5.656 \sin A_2 & +2.265 \cos A_2 \\ & +2.956 \sin 2A_2 & +1.163 \cos 2A_2 \\ & -0.063 \sin 3A_2 & -0.026 \cos 3A_2 \\ & -0.013 \sin 4A_2 & -0.005 \cos 4A_2 \\ & -0.004 \sin 5A_2 & -0.002 \cos 5A_2 \\ & -0''.03T & \end{aligned}$ |
| $(v.c.4) = \begin{aligned} & -0.126 \sin A_2 & +0.329 \cos A_2 \\ & -0.503 \sin 2A_2 & +0.378 \cos 2A_2 \\ & +0.053 \sin 3A_2 & -0.032 \cos 3A_2 \end{aligned}$ | $(v.s.4) = \begin{aligned} & +0.329 \sin A_2 & +0.126 \cos A_2 \\ & +0.378 \sin 2A_2 & +0.503 \cos 2A_2 \\ & -0.032 \sin 3A_2 & -0.053 \cos 3A_2 \end{aligned}$ |
| $(\rho.c.0) = \begin{aligned} & +106 \sin A_2 & +761 \cos A_2 \\ & -2 \sin 2A_2 & +89 \cos 2A_2 \\ & & +19 \cos 3A_2 \\ & & +5 \cos 4A_2 \end{aligned}$ | $(\rho.s.1) = \begin{aligned} & -33 \sin A_2 & -1338 \cos A_2 \\ & +31 \sin 2A_2 & +14 \cos 2A_2 \\ & +7 \sin 3A_2 & +5 \cos 3A_2 \\ & +2 \sin 4A_2 & +1 \cos 4A_2 \\ & & -117T \end{aligned}$ |
| $(\rho.c.1) = \begin{aligned} & +1364 \sin A_2 & -43 \cos A_2 \\ & -46 \sin 2A_2 & +45 \cos 2A_2 \\ & -7 \sin 3A_2 & +9 \cos 3A_2 \\ & -3 \sin 4A_2 & +2 \cos 4A_2 \\ & & +98T \end{aligned}$ | $(\rho.s.2) = \begin{aligned} & -103 \sin A_2 & +281 \cos A_2 \\ & +9 \sin 2A_2 & +27 \cos 2A_2 \\ & & +3 \cos 3A_2 \\ & & -8T \end{aligned}$ |
| $(\rho.c.2) = \begin{aligned} & -279 \sin A_2 & -103 \cos A_2 \\ & -27 \sin 2A_2 & +9 \cos 2A_2 \\ & -3 \sin 3A_2 & \\ & & +7T \end{aligned}$ | $(\rho.s.3) = \begin{aligned} & -24 \sin A_2 & +61 \cos A_2 \\ & +9 \sin 2A_2 & -24 \cos 2A_2 \end{aligned}$ |
| $(\rho.c.3) = \begin{aligned} & -61 \sin A_2 & -24 \cos A_2 \\ & +24 \sin 2A_2 & +9 \cos 2A_2 \end{aligned}$ | $(\rho.s.3) = \begin{aligned} & -24 \sin A_2 & +61 \cos A_2 \\ & +9 \sin 2A_2 & -24 \cos 2A_2 \end{aligned}$ |

Action of Neptune.

$$A_3 = g - l'$$

$$\begin{aligned} (v.c.0) = & -39.66 \sin A_3 - 0.08 \cos A_3 \\ & -35.36 \sin 2A_3 - 0.03 \cos 2A_3 \\ & +17.29 \sin 3A_3 + 0.23 \cos 3A_3 \\ & + 3.91 \sin 4A_3 + 0.06 \cos 4A_3 \\ & + 0.99 \sin 5A_3 - 0.04 \cos 5A_3 \\ & + 0.42 \sin 6A_3 - 0.01 \cos 6A_3 \\ & + 0.19 \sin 7A_3 \\ & + 0.09 \sin 8A_3 \\ & + 0.02 \sin 9A_3 \\ & + \delta l \end{aligned}$$

$$\begin{aligned} (v.c.1) = & -6.77 \sin A_3 - 0.53 \cos A_3 \\ & - 1.10 \sin 2A_3 + 0.07 \cos 2A_3 \\ & +23.25 \sin 3A_3 + 4.04 \cos 3A_3 \\ & + 6.05 \sin 4A_3 + 1.06 \cos 4A_3 \\ & - 3.26 \sin 5A_3 - 0.66 \cos 5A_3 \\ & - 0.80 \sin 6A_3 - 0.17 \cos 6A_3 \\ & - 0.24 \sin 7A_3 - 0.05 \cos 7A_3 \\ & - 0.12 \sin 8A_3 - 0.02 \cos 8A_3 \\ & - 0.06 \sin 9A_3 - 0.01 \cos 9A_3 \\ & - 0.04 \sin 10A_3 \\ & + 1.99945e\delta g \end{aligned} \quad \begin{aligned} (v.s.1) = & -0.49 \sin A_3 + 1.75 \cos A_3 \\ & + 0.12 \sin 2A_3 - 2.48 \cos 2A_3 \\ & + 4.04 \sin 3A_3 - 20.92 \cos 3A_3 \\ & + 1.07 \sin 4A_3 - 5.42 \cos 4A_3 \\ & - 0.68 \sin 5A_3 + 3.47 \cos 5A_3 \\ & - 0.17 \sin 6A_3 + 0.91 \cos 6A_3 \\ & - 0.04 \sin 7A_3 + 0.30 \cos 7A_3 \\ & - 0.02 \sin 8A_3 + 0.12 \cos 8A_3 \\ & - 0.01 \sin 9A_3 + 0.06 \cos 9A_3 \\ & + 0.04 \cos 10A_3 \\ & + 1.99835\delta e \end{aligned}$$

$$\begin{aligned} (v.c.2) = & -0.43 \sin A_3 - 0.03 \cos A_3 \\ & -0.03 \sin 2A_3 + 0.01 \cos 2A_3 \\ & +0.75 \sin 3A_3 + 0.08 \cos 3A_3 \\ & -0.10 \sin 4A_3 - 0.08 \cos 4A_3 \\ & -3.20 \sin 5A_3 - 1.17 \cos 5A_3 \\ & -0.83 \sin 6A_3 - 0.32 \cos 6A_3 \\ & +0.57 \sin 7A_3 + 0.22 \cos 7A_3 \\ & +0.14 \sin 8A_3 + 0.06 \cos 8A_3 \\ & +0.06 \sin 9A_3 + 0.02 \cos 9A_3 \\ & +0.03 \sin 10A_3 - 0.01 \cos 10A_3 \\ & + 0.11722e\delta g \end{aligned} \quad \begin{aligned} (v.s.2) = & -0.03 \sin A_3 + 0.13 \cos A_3 \\ & +0.02 \sin 2A_3 - 0.16 \cos 2A_3 \\ & +0.08 \sin 3A_3 - 0.60 \cos 3A_3 \\ & -0.08 \sin 4A_3 + 0.15 \cos 4A_3 \\ & -1.17 \sin 5A_3 + 3.22 \cos 5A_3 \\ & -0.32 \sin 6A_3 + 0.86 \cos 6A_3 \\ & +0.22 \sin 7A_3 - 0.57 \cos 7A_3 \\ & +0.06 \sin 8A_3 - 0.14 \cos 8A_3 \\ & +0.02 \sin 9A_3 - 0.06 \cos 9A_3 \\ & -0.01 \sin 10A_3 - 0.03 \cos 10A_3 \\ & + 0.11713\delta e \end{aligned}$$

$$\begin{aligned} (v.c.3) = & -0.02 \sin A_3 \\ & +0.04 \sin 3A_3 + 0.01 \cos 3A_3 \\ & -0.15 \sin 4A_3 - 0.05 \cos 4A_3 \\ & -0.08 \sin 5A_3 - 0.02 \cos 5A_3 \\ & -0.02 \sin 6A_3 - 0.02 \cos 6A_3 \\ & +0.46 \sin 7A_3 + 0.25 \cos 7A_3 \\ & +0.11 \sin 8A_3 + 0.07 \cos 8A_3 \\ & -0.08 \sin 9A_3 - 0.05 \cos 9A_3 \\ & -0.03 \sin 10A_3 - 0.02 \cos 10A_3 \\ & + 0.00714e\delta g \end{aligned} \quad \begin{aligned} (v.s.3) = & +0.02 \cos A_3 \\ & +0.01 \sin 3A_3 - 0.04 \cos 3A_3 \\ & -0.05 \sin 4A_3 + 0.15 \cos 4A_3 \\ & -0.02 \sin 5A_3 + 0.08 \cos 5A_3 \\ & -0.02 \sin 6A_3 + 0.02 \cos 6A_3 \\ & +0.25 \sin 7A_3 - 0.46 \cos 7A_3 \\ & +0.07 \sin 8A_3 - 0.11 \cos 8A_3 \\ & -0.05 \sin 9A_3 + 0.08 \cos 9A_3 \\ & -0.02 \sin 10A_3 + 0.03 \cos 10A_3 \\ & + 0.00714\delta e \end{aligned}$$

Action of Neptune.—Continued.

$$A_3 = g - l'$$

$$(v.c.4) = \begin{array}{l} \text{''} \\ -0.06 \sin 9A_3 - 0.05 \cos 9A_3 \\ -0.09 \sin 10A_3 - 0.08 \cos 10A_3 \\ + 0.00044e^\delta g \end{array} \quad (v.s.4) = \begin{array}{l} \text{''} \\ -0.05 \sin 9A_3 + 0.06 \cos 9A_3 \\ -0.08 \sin 10A_3 + 0.09 \cos 10A_3 \\ + 0.00044\delta e \end{array}$$

$$(\rho.c.0) = \begin{array}{l} + 227 \cos A_3 \\ + 232 \cos 2A_3 \\ + 3 \sin 3A_3 - 229 \cos 3A_3 \\ - 59 \cos 4A_3 \\ - 17 \cos 5A_3 \\ - 7 \cos 6A_3 \\ - 3 \cos 7A_3 \\ + 0.01018\delta e \end{array}$$

$$(\rho.c.1) = \begin{array}{l} + 3 \sin A_3 \\ + 2 \sin 2A_3 - 9 \cos 2A_3 \\ + 23 \sin 3A_3 - 141 \cos 3A_3 \\ + 8 \sin 4A_3 - 39 \cos 4A_3 \\ - 9 \sin 5A_3 + 43 \cos 5A_3 \\ - 3 \sin 6A_3 + 13 \cos 6A_3 \\ - 1 \sin 7A_3 + 5 \cos 7A_3 \\ - 0.43322^\delta e \end{array}$$

$$(\rho.s.1) = \begin{array}{l} - 60 \sin A_3 - 3 \cos A_3 \\ - 45 \sin 2A_3 - 2 \cos 2A_3 \\ - 107 \sin 3A_3 - 23 \cos 3A_3 \\ - 29 \sin 4A_3 - 8 \cos 4A_3 \\ + 47 \sin 5A_3 + 9 \cos 5A_3 \\ + 15 \sin 6A_3 + 3 \cos 6A_3 \\ + 5 \sin 7A_3 + 1 \cos 7A_3 \\ + 0.43394e^\delta g \end{array}$$

$$(\rho.c.2) = \begin{array}{l} - 1 \cos A_3 \\ - 6 \cos 2A_3 \\ - 1 \sin 3A_3 + 6 \cos 3A_3 \\ - 4 \sin 4A_3 + 5 \cos 4A_3 \\ - 6 \sin 5A_3 + 17 \cos 5A_3 \\ - 2 \sin 6A_3 + 7 \cos 6A_3 \\ + 3 \sin 7A_3 - 7 \cos 7A_3 \\ - 0.03048^\delta e \end{array}$$

$$(\rho.s.2) = \begin{array}{l} - 5 \sin A_3 \\ - 8 \sin 2A_3 \\ + 8 \sin 3A_3 + 1 \cos 3A_3 \\ + 5 \sin 4A_3 + 4 \cos 4A_3 \\ + 17 \sin 5A_3 + 6 \cos 5A_3 \\ + 7 \sin 6A_3 + 2 \cos 6A_3 \\ - 7 \sin 7A_3 - 3 \cos 7A_3 \\ + 0.03053e^\delta g \end{array}$$

$$(\rho.c.3) = -0.000202^\delta e$$

$$(\rho.s.3) = +0.00203e^\delta g$$

PERTURBATIONS OF THE LATITUDE.

(The secular terms being omitted.)

Action of Jupiter.

$$\begin{array}{l} (b.c.0) = \begin{array}{l} \text{''} \\ 0.024 \sin A_1 - 0.013 \cos A_1 \end{array} \\ (b.s.1) = -0.420 \sin A_1 + 0.494 \cos A_1 \\ (b.c.1) = +0.494 \sin A_1 + 0.420 \cos A_1 \\ (b.s.2) = +0.004 \sin 2A_1 - 0.017 \cos 2A_1 \\ (b.c.2) = -0.017 \sin 2A_1 - 0.004 \cos 2A_1 \end{array}$$

Action of Saturn.

| | | | |
|----------|---------------------------|---------------------------|---|
| | | " | " |
| | | (b.c.0)= | -0.08 sin A ₂ -0.03 cos A ₂ |
| | | | -0.03 cos 2A ₂ |
| | | | -0.01 cos 3A ₂ |
| " | " | (b.c.1)= | -1.56 sin A ₂ +2.50 cos A ₂ |
| (b.s.1)= | +1.34 sin A ₂ | +2.88 cos A ₂ | +2.50 cos A ₂ |
| | -0.01 sin 2A ₂ | -0.10 cos 2A ₂ | -0.10 cos 2A ₂ |
| | +0.03 sin 3A ₂ | -0.06 cos 3A ₂ | -0.06 cos 3A ₂ |
| | +0.02 sin 4A ₂ | -0.03 cos 4A ₂ | -0.02 cos 4A ₂ |
| (b.s.2)= | -0.09 sin A ₂ | +0.10 cos A ₂ | -0.09 sin A ₂ -0.05 cos A ₂ |
| | -0.05 sin 2A ₂ | -0.09 cos 2A ₂ | +0.07 sin 2A ₂ +0.02 cos 2A ₂ |
| | | -0.01 cos 3A ₂ | +0.01 sin 3A ₂ |

Action of Neptune.

| | | | |
|----------|---------------------------|---------------------------|---|
| | | " | " |
| | | (b.c.0)= | +0.01 sin A ₃ -0.04 cos A ₃ |
| | | | -0.01 sin 2A ₃ +0.00 cos 2A ₃ |
| | | | -0.01 sin 3A ₃ +0.04 cos 3A ₃ |
| | | | +0.01 sin 4A ₃ +0.01 cos 4A ₃ |
| | | | +0.01 sin 5A ₃ |
| | | | +0.01 sin 6A ₃ |
| " | " | (b.c.1)= | -0.57 sin A ₃ +0.04 cos A ₃ |
| (b.s.1)= | +0.13 sin A ₃ | +0.16 cos A ₃ | +0.04 cos A ₃ |
| | +0.09 sin 2A ₃ | +0.26 cos 2A ₃ | +0.06 cos 2A ₃ |
| | +0.08 sin 3A ₃ | +0.28 cos 3A ₃ | +0.07 cos 3A ₃ |
| | +0.01 sin 4A ₃ | +0.03 cos 4A ₃ | +0.01 cos 4A ₃ |
| | -0.02 sin 5A ₃ | -0.12 cos 5A ₃ | -0.03 cos 5A ₃ |
| | -0.01 sin 6A ₃ | -0.03 cos 6A ₃ | +0.10 sin 5A ₃ +0.03 sin 6A ₃ |
| | | -0.01 cos 7A ₃ | +0.01 sin 7A ₃ |
| (b.s.2)= | +0.01 sin A ₃ | +0.05 cos A ₃ | -0.07 sin A ₃ -0.03 cos 3A ₃ |
| | -0.03 sin 3A ₃ | -0.09 cos 3A ₃ | +0.09 sin 3A ₃ +0.05 sin 4A ₃ |
| | | -0.05 cos 4A ₃ | +0.05 sin 4A ₃ |
| | | -0.04 cos 5A ₃ | +0.04 sin 5A ₃ |
| | | +0.01 cos 7A ₃ | +0.01 sin 6A ₃ |
| | | | -0.01 sin 7A ₃ |

Action of Jupiter and Saturn.

(Terms multiplied by the product of their masses.)

$$\begin{aligned}
 N_1 &= 2S - J \\
 N_2 &= -U + 3S - J \\
 N_3 &= -2U + 4S - J \\
 N_4 &= 3U + 3S - 2J \\
 N_5 &= 2U + 4S - 2J \\
 N_6 &= 5S - 2J \\
 N_7 &= -3U + 6S - 2J \\
 N_8 &= -3U + 7S - 2J
 \end{aligned}$$

Action of Jupiter and Saturn —Continued.

(Terms multiplied by the product of their masses.)

$$\begin{aligned}
 (v.c.0) &= + \overset{''}{0.08} \sin N_2 + \overset{''}{0.51} \cos N_2 \\
 &\quad + 0.04 \sin N_3 + 0.01 \cos N_3 \\
 &\quad - 0.01 \sin N_4 + 0.05 \cos N_4 \\
 &\quad - 0.35 \sin N_5 - 1.30 \cos N_5 \\
 &\quad \left\{ \begin{array}{l} - 0.55 \sin N_6 - 0.03 \cos N_6 \\ + 40.65 \sin N_7 - 10.50 \cos N_7 \end{array} \right\} \\
 &\quad - 0.05 \sin N_8 + 0.03 \cos N_8 \\
 \\
 (v.c.1) &= + 0.06 \sin N_1 - 0.27 \cos N_1 \\
 &\quad + 0.18 \sin N_2 + 0.01 \cos N_2 \\
 &\quad - 0.03 \sin N_3 + 0.08 \cos N_3 \\
 &\quad - 0.02 \sin N_4 + 0.09 \cos N_4 \\
 &\quad - 0.44 \sin N_5 - 0.61 \cos N_5 \\
 &\quad \left\{ \begin{array}{l} - 4.23 \sin N_6 - 3.87 \cos N_6 \\ + 8.06 \sin N_7 - 8.38 \cos N_7 \end{array} \right\} \\
 &\quad - 0.10 \sin N_8 + 0.03 \cos N_8 \\
 \\
 (v.s.1) &= + \overset{''}{0.26} \sin N_1 + \overset{''}{0.27} \cos N_1 \\
 &\quad - 0.04 \sin N_2 - 0.17 \cos N_2 \\
 &\quad + 0.08 \sin N_3 + 0.03 \cos N_3 \\
 &\quad - 0.02 \sin N_4 + 0.08 \cos N_4 \\
 &\quad + 0.30 \sin N_5 - 0.58 \cos N_5 \\
 &\quad \left\{ \begin{array}{l} + 2.64 \sin N_6 + 4.64 \cos N_6 \\ + 7.35 \sin N_7 + 4.41 \cos N_7 \end{array} \right\} \\
 &\quad - 0.04 \sin N_8 + 0.00 \cos N_8 \\
 \\
 (v.c.2) &= \left\{ \begin{array}{l} - 0.24 \sin N_6 - 0.22 \cos N_6 \\ + 0.47 \sin N_7 - 0.54 \cos N_7 \end{array} \right\} \\
 (v.s.2) &= \left\{ \begin{array}{l} + 0.16 \sin N_6 + 0.26 \cos N_6 \\ + 0.54 \sin N_7 + 0.47 \cos N_7 \end{array} \right\} \\
 \\
 (\rho.c.0) &= + 11 \sin N_5 - 3 \cos N_5
 \end{aligned}$$

Two of these arguments, namely, $5S - 2J$, and $-3g + 6S - 2J$, are of very long period, that of the first being about 880, and that of the second about 1590 years. It will, therefore, be convenient to tabulate them both as functions of the time for the time during which the theory is to be used. To make their effect as small as possible during the period for which the provisional ephemeris is to be computed, we shall suppose the longitude of epoch, mean motion, and longitude of the perigee to be affected with the negative of the following corrections:

$$\begin{aligned}
 \delta\varepsilon &= + \overset{''}{27.27}, \\
 \delta\pi &= + \overset{''}{27.27}, \\
 \delta n &= - 0.1172.
 \end{aligned}$$

Reducing these corrections to corrections of the co-ordinates, and adding them to the terms of long period in the true longitude and logarithm of radius vector, we shall have for these terms,

$$\begin{aligned}
 (v.c.0) &= - \overset{''}{0.546} \sin N_6 - \overset{''}{0.032} \cos N_6 \\
 &\quad + 40.650 \sin N_7 - 10.500 \cos N_7 + 27.27 - 11.72 T \\
 (v.s.1) &= \overset{''}{2.63} \sin N_6 + \overset{''}{4.64} \cos N_6 + \overset{''}{7.35} \sin N_7 + \overset{''}{4.42} \cos N_7 \\
 (v.c.1) &= - 4.22 \sin N_6 - 3.87 \cos N_6 + 8.06 \sin N_7 - 8.39 \cos N_7 - 1.10 T
 \end{aligned}$$

$$\begin{aligned}
 (v.s.2) &= + 0.16 \sin N_6 + 0.26 \cos N_6 + 0.54 \sin N_7 + 0.47 \cos N_7 \\
 (v.c.2) &= - 0.24 \sin N_6 - 0.22 \cos N_6 + 0.47 \sin N_7 - 0.54 \cos N_7 - 0''.07 T \\
 (\rho.c.0) &= + 2 \cos N_6 + 10 \sin N_7 + 32 \cos N_7 + 22 \\
 (\rho.s.1) &= - 41 \sin N_6 - 43 \cos N_6 + 82 \sin N_7 - 85 \cos N_7 - 12 T \\
 (\rho.c.1) &= - 29 \sin N_6 - 45 \cos N_6 - 73 \sin N_7 - 44 \cos N_7
 \end{aligned}$$

The values of these and of the other secular terms and terms of long period for the period during which Uranus has been observed, are given in the following table:

| (v.c.0) | | | |
|---------|------------------------|-----------------------------------|--------|
| | Neptune
(long per.) | Jupiter and Saturn
(long per.) | Sum. |
| | " | " | " |
| 1700 | +85.54 | +4.86 | +90.40 |
| 1750 | +38.45 | +1.90 | +40.35 |
| 1760 | 31.25 | 1.48 | 32.73 |
| 1770 | 24.80 | 1.11 | 25.91 |
| 1780 | 19.09 | 0.80 | 19.89 |
| 1790 | 14.12 | 0.54 | 14.66 |
| 1800 | 9.89 | +0.33 | 10.22 |
| 1810 | 6.42 | 0.17 | 6.59 |
| 1820 | 3.69 | +0.07 | 3.76 |
| 1830 | 1.71 | 0.00 | 1.71 |
| 1840 | + 0.48 | -0.02 | + 0.46 |
| 1850 | 0.00 | 0.00 | 0.00 |
| 1860 | + 0.27 | +0.06 | + 0.33 |
| 1870 | 1.29 | +0.16 | 1.45 |
| 1880 | + 3.06 | +0.30 | + 3.36 |

| VALUES OF (v.s.1) | | | | | | |
|-------------------|--------------------------|-------------------------|--------------------------|-------------------------------|--|---------|
| | (1)
Jupiter
(sec.) | (2)
Saturn
(sec.) | (3)
Neptune
(sec.) | (4)
Neptune
(long per.) | (5)
Jupiter & Saturn
(long per.) | Sum. |
| | " | " | " | " | " | " |
| 1700 | +1.82 | +13.77 | +0.75 | -212.73 | -10.18 | -206.57 |
| 1750 | +1.22 | + 9.18 | +0.50 | -141.97 | - 9.11 | -140.18 |
| 1760 | 1.10 | 8.26 | 0.45 | -127.76 | - 8.83 | -126.78 |
| 1770 | 0.97 | 7.34 | 0.40 | -113.55 | - 8.54 | -113.38 |
| 1780 | 0.85 | 6.43 | 0.35 | - 99.33 | - 8.24 | - 99.94 |
| 1790 | 0.73 | 5.51 | 0.30 | - 85.11 | - 7.93 | - 86.50 |
| 1800 | 0.61 | 4.59 | 0.25 | - 70.90 | - 7.61 | - 73.06 |
| 1810 | 0.49 | 3.67 | 0.20 | - 56.69 | - 7.28 | - 59.61 |
| 1820 | 0.36 | 2.76 | 0.15 | - 42.49 | - 6.94 | - 46.16 |
| 1830 | 0.24 | 1.84 | 0.10 | - 28.31 | - 6.60 | - 32.73 |
| 1840 | +0.12 | + 0.92 | +0.05 | - 14.14 | - 6.26 | - 19.31 |
| 1850 | 0.00 | 0.00 | 0.00 | 0.00 | - 5.91 | - 5.91 |
| 1860 | -0.12 | - 0.92 | -0.05 | + 14.12 | - 5.57 | + 7.46 |
| 1870 | -0.24 | - 1.84 | -0.10 | 28.23 | - 5.23 | 20.82 |
| 1880 | -0.36 | - 2.76 | -0.15 | + 42.32 | - 4.89 | + 34.16 |

| VALUES OF (v.c.1) | | | | | | |
|--------------------------|--------------------------|-------------------------|--------------------------|-------------------------------|--|--------|
| | (1)
Jupiter
(sec.) | (2)
Saturn
(sec.) | (3)
Neptune
(sec.) | (4)
Neptune
(long per.) | (5)
Jupiter & Saturn
(long per.) | Sum. |
| | " | " | " | " | " | " |
| 1700 | +17.19 | +16.67 | +7.20 | +37.63 | -1.98 | +76.71 |
| 1750 | +11.46 | +11.11 | +4.80 | +28.67 | -2.28 | +53.76 |
| 1760 | 10.31 | 10.00 | 4.32 | 26.46 | -2.36 | 48.73 |
| 1770 | 9.17 | 8.89 | 3.84 | 24.10 | -2.45 | 43.55 |
| 1780 | 8.02 | 7.78 | 3.36 | 21.60 | -2.53 | 38.23 |
| 1790 | 6.88 | 6.67 | 2.88 | 18.95 | -2.61 | 32.77 |
| 1800 | 5.73 | 5.56 | 2.40 | 16.16 | -2.69 | 27.16 |
| 1810 | 4.58 | 4.44 | 1.92 | 13.23 | -2.76 | 21.41 |
| 1820 | 3.44 | 3.33 | 1.44 | 10.15 | -2.83 | 15.53 |
| 1830 | 2.29 | 2.22 | 0.96 | 6.92 | -2.90 | 9.49 |
| 1840 | + 1.15 | + 1.11 | +0.48 | + 3.53 | -2.97 | + 3.30 |
| 1850 | 0.00 | 0.00 | 0.00 | 0.00 | -3.03 | - 3.03 |
| 1860 | - 1.15 | - 1.11 | -0.48 | - 3.68 | -3.08 | - 9.50 |
| 1870 | - 2.29 | - 2.22 | -0.96 | - 7.51 | -3.13 | -16.11 |
| 1880 | - 3.44 | - 3.33 | -1.44 | -11.49 | -3.16 | -22.86 |
| (v.c.2) const. = -0".14. | | | | | | |
| | (1) | (2) | (3) | (4) | (5) | Sum. |
| | " | " | " | " | " | " |
| 1700 | +0.11 | +0.81 | +0.04 | -12.46 | -0.72 | -12.36 |
| 1750 | +0.07 | +0.54 | +0.03 | -8.31 | -0.70 | -8.51 |
| 1760 | 0.06 | 0.49 | 0.03 | -7.48 | -0.69 | -7.73 |
| 1770 | 0.06 | 0.43 | 0.02 | -6.64 | -0.68 | -6.95 |
| 1780 | 0.05 | 0.38 | 0.02 | -5.81 | -0.66 | -6.16 |
| 1790 | 0.04 | 0.32 | 0.02 | -4.98 | -0.65 | -5.39 |
| 1800 | 0.04 | 0.27 | 0.02 | -4.15 | -0.64 | -4.60 |
| 1810 | 0.03 | 0.22 | 0.01 | -3.32 | -0.63 | -3.83 |
| 1820 | 0.02 | 0.16 | 0.01 | -2.49 | -0.61 | -3.05 |
| 1830 | 0.01 | 0.11 | +0.01 | -1.66 | -0.60 | -2.27 |
| 1840 | +0.01 | +0.05 | 0.00 | -0.83 | -0.58 | -1.49 |
| 1850 | 0.00 | 0.00 | 0.00 | 0.00 | -0.57 | -0.71 |
| 1860 | -0.01 | -0.05 | 0.00 | +0.82 | -0.56 | +0.06 |
| 1870 | -0.01 | -0.11 | -0.01 | 1.65 | -0.54 | 0.84 |
| 1880 | -0.02 | -0.16 | -0.01 | +2.48 | -0.53 | +1.62 |
| (v.c.2) const. = +0".13. | | | | | | |
| | (1) | (2) | (3) | (4) | (5) | Sum. |
| | " | " | " | " | " | " |
| 1700 | +1.00 | +0.93 | +0.42 | +2.20 | -0.11 | +4.62 |
| 1750 | +0.67 | +0.65 | +0.28 | +1.68 | -0.12 | +3.29 |
| 1760 | +0.60 | 0.59 | 0.25 | 1.55 | -0.12 | 3.00 |
| 1770 | +0.54 | 0.52 | 0.22 | 1.41 | -0.12 | 2.70 |
| 1780 | +0.47 | 0.46 | 0.20 | 1.26 | -0.13 | 2.39 |
| 1790 | +0.40 | 0.39 | 0.17 | 1.11 | -0.13 | 2.07 |
| 1800 | +0.34 | 0.33 | 0.14 | 0.94 | -0.13 | 1.75 |
| 1810 | +0.27 | 0.26 | 0.11 | 0.77 | -0.14 | 1.40 |
| 1820 | +0.20 | 0.20 | 0.08 | 0.59 | -0.14 | 1.06 |
| 1830 | +0.13 | 0.13 | 0.06 | 0.39 | -0.14 | 0.70 |
| 1840 | +0.07 | +0.07 | +0.03 | +0.20 | -0.15 | +0.35 |
| 1850 | 0.00 | 0.00 | 0.00 | 0.00 | -0.15 | -0.02 |
| 1860 | -0.07 | -0.07 | -0.03 | -0.21 | -0.15 | -0.40 |
| 1870 | -0.13 | -0.13 | -0.06 | -0.43 | -0.16 | -0.78 |
| 1880 | -0.20 | -0.20 | -0.08 | -0.68 | -0.16 | -1.19 |

(v.s.3)

| | (1)
Jupiter
(sec.) | (2)
Saturn
(sec.) | (3)
Neptune
(sec.) | (4)
Neptune
(long per.) | (5)
Jupiter & Saturn
(long per.) | Sum. |
|------|--------------------------|-------------------------|--------------------------|-------------------------------|--|-------|
| | " | " | " | " | " | " |
| 1700 | +0.01 | +0.05 | 0.00 | -0.76 | 0 | -0.70 |
| 1750 | 0.00 | +0.03 | 0 | -0.51 | 0 | -0.48 |
| 1760 | 0 | 0.03 | 0 | -0.46 | 0 | -0.43 |
| 1770 | 0 | 0.03 | 0 | -0.40 | 0 | -0.37 |
| 1780 | 0 | 0.02 | 0 | -0.35 | 0 | -0.33 |
| 1790 | 0 | 0.02 | 0 | -0.30 | 0 | -0.28 |
| 1800 | 0 | 0.02 | 0 | -0.25 | 0 | -0.23 |
| 1810 | 0 | 0.01 | 0 | -0.20 | 0 | -0.19 |
| 1820 | 0 | 0.01 | 0 | -0.15 | 0 | -0.14 |
| 1830 | 0 | +0.01 | 0 | -0.10 | 0 | -0.09 |
| 1840 | 0 | 0.00 | 0 | -0.05 | 0 | -0.05 |
| 1850 | 0 | 0.00 | 0 | 0.00 | 0 | 0.00 |
| 1860 | 0 | 0.00 | 0 | +0.05 | 0 | +0.05 |
| 1870 | 0 | -0.01 | 0 | +0.10 | 0 | +0.09 |
| 1880 | 0 | -0.01 | 0 | +0.15 | 0 | +0.14 |

(v.c.3)

| | (1) | (2) | (3) | (4) | Sum. |
|------|-------|-------|-------|-------|-------|
| | " | " | " | " | " |
| 1700 | +0.06 | +0.06 | +0.02 | +0.13 | +0.27 |
| 1750 | +0.04 | +0.04 | +0.02 | +0.10 | +0.20 |
| 1760 | 0.04 | 0.04 | 0.01 | +0.09 | +0.18 |
| 1770 | 0.03 | 0.03 | 0.01 | +0.08 | +0.15 |
| 1780 | 0.03 | 0.03 | 0.01 | +0.07 | +0.14 |
| 1790 | 0.02 | 0.02 | 0.01 | +0.06 | +0.12 |
| 1800 | 0.02 | 0.02 | 0.01 | +0.06 | +0.11 |
| 1810 | 0.02 | 0.02 | +0.01 | +0.05 | +0.09 |
| 1820 | 0.01 | 0.01 | 0.00 | +0.04 | +0.06 |
| 1830 | +0.01 | +0.01 | 0.00 | +0.02 | +0.04 |
| 1840 | 0.00 | 0.00 | 0.00 | +0.01 | +0.02 |
| 1850 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1860 | 0.00 | 0.00 | 0.00 | -0.01 | -0.02 |
| 1870 | -0.01 | -0.01 | 0.00 | -0.02 | -0.04 |
| 1880 | -0.01 | -0.01 | 0.00 | -0.04 | -0.06 |

FOR THE RADIUS VECTOR. VALUES OF (p.c.0)

| | (1) | (2) | (3) | (4) | (5) | Sum. |
|------|------|------|------|------|-----|------|
| | " | " | " | " | " | " |
| 1700 | +0.4 | +3.5 | -0.2 | +164 | +13 | +181 |
| 1750 | +0.3 | +2.3 | -0.1 | +110 | 7 | +120 |
| 1760 | 0.3 | 2.1 | 0 | 99 | 6 | 107 |
| 1770 | 0.2 | 1.9 | 0 | 88 | 5 | 95 |
| 1780 | 0.2 | 1.6 | 0 | 77 | 4 | 83 |
| 1790 | 0.2 | 1.4 | 0 | 67 | 3 | 72 |
| 1800 | 0.2 | 1.2 | 0 | 56 | 2 | 60 |
| 1810 | 0.1 | 0.9 | 0 | 45 | 2 | 48 |
| 1820 | 0.1 | 0.7 | 0 | 34 | + 1 | 36 |
| 1830 | +0.1 | 0.5 | 0 | 22 | 0 | 23 |
| 1840 | .0 | +0.2 | 0 | + 11 | 0 | + 11 |
| 1850 | .0 | 0.0 | 0 | 0 | - 1 | - 1 |
| 1860 | .0 | -0.2 | 0 | - 11 | - 2 | - 13 |
| 1870 | -0.1 | -0.5 | 0 | 22 | - 2 | - 25 |
| 1880 | -0.1 | -0.7 | 0 | - 33 | - 3 | - 37 |

| (p.s.1) const. = +63. | | | | | | |
|-----------------------|--------------------------|-------------------------|--------------------------|-------------------------------|--|-------|
| | (1)
Jupiter
(sec.) | (2)
Saturn
(sec.) | (3)
Neptune
(sec.) | (4)
Neptune
(long per.) | (5)
Jupiter & Saturn
(long per.) | Sum. |
| | " | " | " | " | " | " |
| 1700 | +180 | +176 | +75 | +396 | -21 | +869 |
| 1750 | +120 | +117 | +50 | +302 | -25 | +627 |
| 1760 | 108 | 105 | 45 | 280 | 26 | 575 |
| 1770 | 96 | 94 | 40 | 255 | 28 | 520 |
| 1780 | 84 | 82 | 35 | 228 | 29 | 463 |
| 1790 | 72 | 70 | 30 | 200 | 30 | 405 |
| 1800 | 60 | 59 | 25 | 170 | 31 | 346 |
| 1810 | 48 | 47 | 20 | 139 | 32 | 285 |
| 1820 | 36 | 35 | 15 | 106 | 33 | 222 |
| 1830 | 24 | 23 | 10 | 72 | 34 | 158 |
| 1840 | + 12 | + 12 | + 5 | + 37 | 34 | 95 |
| 1850 | 0 | 0 | 0 | 0 | 35 | + 28 |
| 1860 | - 12 | - 12 | - 5 | - 39 | 36 | - 41 |
| 1870 | 24 | 23 | 10 | 80 | 36 | -110 |
| 1880 | - 36 | - 35 | -15 | -122 | -37 | -182 |
| (p.c.1) const. = +73. | | | | | | |
| | (1) | (2) | (3) | (4) | (5) | Sum. |
| | " | " | " | " | " | " |
| 1700 | -20 | -147 | -9 | +2241 | +104 | +2242 |
| 1750 | -13 | - 98 | -6 | +1496 | + 94 | +1546 |
| 1760 | -12 | - 88 | -5 | 1346 | 91 | 1405 |
| 1770 | -10 | - 78 | -5 | 1197 | 88 | 1265 |
| 1780 | - 9 | - 69 | -4 | 1047 | 85 | 1123 |
| 1790 | - 8 | - 59 | -4 | 897 | 82 | 981 |
| 1800 | - 6 | - 49 | -3 | 747 | 78 | 840 |
| 1810 | - 5 | - 39 | -2 | 598 | 74 | 699 |
| 1820 | - 4 | - 29 | -2 | 448 | 71 | 557 |
| 1830 | - 3 | - 20 | -1 | 299 | 68 | 416 |
| 1840 | - 1 | - 10 | -1 | + 149 | 64 | 274 |
| 1850 | 0 | 0 | 0 | 0 | 61 | + 134 |
| 1860 | + 1 | + 10 | +1 | - 149 | 58 | - 6 |
| 1870 | 3 | 20 | 1 | 298 | 54 | - 147 |
| 1880 | 4 | 29 | 2 | - 447 | + 51 | - 288 |
| (p.s.2) const. = +5. | | | | | | |
| | (1) | (2) | (3) | (4) | (5) | Sum. |
| | " | " | " | " | " | " |
| 1700 | +12 | +12 | +4 | +28 | 0 | +61 |
| 1750 | + 8 | + 8 | +6 | +21 | 0 | +48 |
| 1760 | 7 | 7 | 5 | 19 | 0 | 43 |
| 1770 | 6 | 6 | 5 | 17 | 0 | 39 |
| 1780 | 6 | 6 | 4 | 15 | 0 | 36 |
| 1790 | 5 | 5 | 4 | 13 | 0 | 32 |
| 1800 | 4 | 4 | 3 | 11 | 0 | 27 |
| 1810 | 3 | 3 | 2 | 8 | 0 | 21 |
| 1820 | 2 | 2 | 2 | 6 | 0 | 17 |
| 1830 | 2 | 2 | 1 | 4 | 0 | 14 |
| 1840 | + 1 | + 1 | +1 | + 2 | 0 | 10 |
| 1850 | 0 | 0 | 0 | 0 | 0 | + 5 |
| 1860 | - 1 | - 1 | -1 | - 2 | 0 | 0 |
| 1870 | 2 | - 2 | -1 | 4 | 0 | - 4 |
| 1880 | - 2 | - 2 | -2 | - 6 | 0 | - 7 |

| (p.c.2) const. = + 4 | | | | | | |
|----------------------|--------------------------|-------------------------|--------------------------|-------------------------------|--|------|
| | (1)
Jupiter
(sec.) | (2)
Saturn
(sec.) | (3)
Neptune
(sec.) | (4)
Neptune
(long per.) | (5)
Jupiter & Saturn
(long per.) | Sum. |
| | " | " | " | " | " | " |
| 1700 | -1 | -10 | 0 | +157 | 0 | +150 |
| 1750 | -1 | - 7 | 0 | +105 | 0 | +101 |
| 1760 | -1 | - 6 | 0 | 95 | 0 | 92 |
| 1770 | -1 | - 6 | 0 | 84 | 0 | 81 |
| 1780 | -1 | - 5 | 0 | 74 | 0 | 72 |
| 1790 | -1 | - 4 | 0 | 63 | 0 | 62 |
| 1800 | 0 | - 3 | 0 | 52 | 0 | 53 |
| 1810 | 0 | - 3 | 0 | 42 | 0 | 43 |
| 1820 | 0 | - 2 | 0 | 31 | 0 | 33 |
| 1830 | 0 | - 1 | 0 | 21 | 0 | 24 |
| 1840 | 0 | - 1 | 0 | + 10 | 0 | 13 |
| 1850 | 0 | 0 | 0 | 0 | 0 | + 4 |
| 1860 | 0 | + 1 | 0 | - 10 | 0 | - 5 |
| 1870 | 0 | 1 | 0 | 21 | 0 | -16 |
| 1880 | 0 | + 2 | 0 | - 31 | 0 | -25 |

Reduced Expressions for the Latitude of Uranus.

If we represent by V_1, V_2, V_3 the distances of Uranus from its descending nodes on the respective orbits of Jupiter, Saturn, and Uranus, we find the following perturbations of the latitude, which are independent of the mean longitude of the disturbing planets.

$$\begin{aligned} \delta\beta = & -0.0114t \cos V_1 \\ & -0.0477t \cos V_2 \\ & -0.0125t \cos V_3 \\ & + 0.245 \\ & + 0.386 \sin g + 0.266 \cos g \\ & - 0.043 \sin 2g + 0.006 \cos 2g. \end{aligned}$$

To find how far the last five terms may be represented by simple corrections to the elliptic elements, we first represent the effect of minute corrections to the inclination and node of Uranus as a function of its mean anomaly. Putting u for the argument of latitude of Uranus, we have to a sufficient degree of approximation

$$\begin{aligned} \delta\beta = & \sin u \delta\phi - \sin \phi \cos u \delta\theta \\ u = & g + \omega + 2e \sin g \\ \sin u = & -e \sin \omega \\ & + \cos \omega \sin g + \sin \omega \cos g \\ & + e \cos \omega \sin 2g + e \sin \omega \cos 2g \end{aligned}$$

$$\begin{aligned}\cos u &= -e \cos \omega \\ &+ \cos \omega \cos g - \sin \omega \sin g \\ &+ e \cos \omega \cos 2g - e \sin \omega \sin 2g.\end{aligned}$$

Substituting these values of $\sin u$ and $\cos u$ in the expression for $\delta\beta$, and putting $\sin \phi \delta\theta = \delta'\theta$, we have

$$\begin{aligned}\delta\beta &= e \cos \omega \delta'\theta - e \sin \omega \delta\phi \\ &+ (\cos \omega \delta\phi + \sin \omega \delta'\theta) \sin g + (\sin \omega \delta\phi - \cos \omega \delta'\theta) \cos g \\ &+ (e \cos \omega \delta\phi + e \sin \omega \delta'\theta) \sin 2g + (e \sin \omega \delta\phi - e \cos \omega \delta'\theta) \cos 2g.\end{aligned}$$

To represent the numerical coefficients of $\sin g$ and $\cos g$ in $\delta\beta$ we must put

$$\begin{aligned}\cos \omega \delta\phi + \sin \omega \delta'\theta &= 0''.386 \\ \sin \omega \delta\phi - \cos \omega \delta'\theta &= 0.266.\end{aligned}$$

Since $\omega = 95^\circ 3'$, this gives

$$\begin{aligned}\delta\phi &= 0.231; \\ \delta'\theta &= 0.409; \\ \delta\beta &= -0.013 \\ &+ 0.386 \sin g + 0.266 \cos g \\ &+ 0.018 \sin 2g + 0.013 \cos 2g\end{aligned}$$

Subtracting this expression from the corresponding terms of $\delta\beta$, we have left

$$\delta\beta = +0''.258 - 0''.061 \sin 2g - 0''.007 \cos 2g.$$

The first term of this expression shows that the mean orbit of Uranus at the present time is a small circle of the sphere one-quarter of a second north of its parallel great circle.

If we put

v = longitude of Uranus in its orbit, referred to the equinox and ecliptic of 1850, we have

$$\begin{aligned}V_1 &= v - 127^\circ 37' \\ V_2 &= v - 126 \quad 45 \\ V_3 &= v - 155 \quad 32\end{aligned}$$

Substituting these values in the first three terms of $\delta\beta$, and multiplying the last term by the factor $(1 + \mu)$ by which the adopted mass of Neptune, $\frac{1}{17000}$, must be multiplied to obtain the true mass, we find

$$\delta\beta = (4''.69 + 1''.14\mu) T \cos v - (5''.24 + 0''.52\mu) T \sin v.$$

To these terms must be added those which arise from the motion of the ecliptic.

In the absence of any exhaustive investigation of the obliquity and motion of the ecliptic, I adopt the elements of Hansen, employed in his "*Tables du Soleil*," because they are a mean between the results of others, and are very accordant with recent observations. The secular motion of the obliquity there employed is

$$-46''.78.$$

Hansen mentions $-5''.39$ as the corresponding motion at the equinox of 1850, found by Olufsen, but I cannot reproduce this result from the secular diminution with any masses of Mercury, Venus, and Mars, which seem to me probable. The expressions in terms of the masses given by Le Verrier are (*Annales de l'Observatoire Imperial de Paris*, tome ii, p. 101),

$$\begin{aligned} \text{Secular change} &= -47.59'' - 0.52\nu'' - 28.90\nu' - 0.83\nu''' \\ \text{Mot. at equinox} &= +5.89'' + 0.62\nu'' + 7.57\nu' + 0.73\nu''' \end{aligned}$$

In this expression the masses of Mercury, Venus, and Mars are represented by $\frac{1+\nu}{3,000,000}$, $\frac{1+\nu'}{401,847}$, and $\frac{1+\nu''}{2,680,337}$, respectively. The influence of admissible changes in the masses of the other planets is insensible.

From the researches of Le Verrier on the motions of the four inner planets I conclude that the following are about the most probable distribution of the corrections of the masses necessary to produce the motion of the obliquity given by Hansen, namely,

$$\begin{aligned} \nu &= -\frac{2}{5} \\ \nu' &= -.018 \\ \nu'' &= -\frac{1}{10} \end{aligned}$$

These values give for the motion at the equinox of 1850

$$+5''.43$$

Introducing the secular variation of these motions we have, for the change in the latitude of any celestial body near the ecliptic, arising from motion of the ecliptic,

$$\delta\beta = (5''.43T + 0''.19T^2) \cos v + (46''.78T - 0''.06T^2) \sin v.$$

Combining this with the change arising from the motion of the orbit of Uranus, we find

$$\begin{aligned} \delta\beta &= \{(10''.12 + 1''.14u)T + 0''.19T^2\} \cos v \\ &+ \{(41''.54 - 0''.52u)T - 0''.06T^2\} \sin v. \end{aligned}$$

We may represent these expressions in the usual way by secular variations of the inclination and node of Uranus. But, owing to the small inclination, and consequent rapid motion of the node, it will be necessary to include the coefficients of the second power of the time. On the other hand, no distinction between τ and θ is necessary. Putting ϕ for the inclination of the orbit, θ for the longitude of the node referred to the equinox of 1850, and

$$\begin{aligned} p &= \sin \phi \sin \theta, \\ q &= \sin \phi \cos \theta; \end{aligned}$$

we have

$$\begin{aligned} \sin \beta &= -p \cos v + q \sin v \\ \cos \beta \delta\beta &= -\delta p \cos v + \delta q \sin v. \end{aligned}$$

From the expressions for p and q we obtain

$$\begin{aligned}\cos \phi D_t \phi &= \sin \theta D_t p + \cos \theta D_t q; \\ \sin \phi D_t \theta &= \cos \theta D_t p - \sin \theta D_t q.\end{aligned}$$

And, neglecting $(D_t \phi)^2 \times \sin \phi$, we have farther,

$$\begin{aligned}\cos \phi D_t^2 \phi &= \sin \phi (D_t \theta)^2 + \sin \theta D_t^2 p + \cos \theta D_t^2 q; \\ \sin \phi D_t^2 \theta &= -2 \cos \phi D_t \theta D_t \phi + \cos \theta D_t^2 p - \sin \theta D_t^2 q.\end{aligned}$$

Since ϕ is only $46'$ we may put $\cos \phi$ and $\cos \beta$ both equal to unity in these expressions, while we have, for 1850,

$$\begin{aligned}\sin \theta &= 0.9573 \\ \cos \theta &= 0.2890 \\ D_t p &= -10''.12 - 1''.14\mu \\ D_t q &= +41.54 - 0.52\mu \\ L_t^2 p &= -0.38 \\ L_t^2 q &= -0.12 \\ \log \sin \phi &= 8.129606.\end{aligned}$$

The above formulæ then give

$$\begin{aligned}D_t \phi &= +2''.31 - 1''.24\mu \\ D_t \theta &= -3167''.5 + 12''.6\mu \\ D_t^2 \phi &= +0''.26 \\ D_t^2 \theta &= +5''.6\end{aligned}$$

$$\begin{aligned}\phi &= \phi_0 + (2''.31 - 1''.24\mu) T + 0''.13 T^2 \\ \theta &= \theta_0 - (3167''.5 - 12''.6\mu) T + 2.8 T^2,\end{aligned}$$

or, adding Struve's precession, we have when θ is counted from the mean equinox of date,

$$\theta = \theta_0 + (1857''.7 + 12''.6\mu) T + 3''.9 T^2.$$

Using the values of ϕ and θ given by these expressions, the latitude, secular variation included, will be given by the expression

$$\sin \beta = \sin \phi \sin (v - \theta).$$

If we take from a table, as the principal term of the latitude, the value of $\sin \phi_0 \sin (v - \theta)$, the secular term to be added will be

$$\{(2''.31 - 1''.24\mu) T + 0''.13 T^2\} \sin (v - \theta).$$

If we represent, as before, by ω the variable distance of the perihelion from the node, this term will be allowed for by adding to (*b.s.1*), (*b.c.1*), etc., the terms

$$\begin{aligned}
 (b.c.0) &= -e \sin \omega \delta \phi, \\
 (b.s.1) &= \cos \omega \delta \phi, \\
 (b.c.1) &= \sin \omega \delta \phi, \\
 (b.s.2) &= e \cos \omega \delta \phi, \\
 (b.c.2) &= e \sin \omega \delta \phi;
 \end{aligned}$$

where

$$\delta \phi = (2''.31 - 1''.24t) T + 0''.13 T^2$$

Putting in the above expressions

$$\begin{aligned}
 \omega &= 95^\circ 3' + 3459'' T, \\
 \cos \omega &= -.0880 - .0167 T, \\
 \sin \omega &= +.9961 - .0015 T,
 \end{aligned}$$

we find

$$\begin{aligned}
 (b.c.0) &= -(0''.11 - 0''.06t) T \\
 (b.s.1) &= -(0.20 - 0.11t) T - 0''.05 T^2 \\
 (b.c.1) &= (2.30 - 1.24t) T + 0.12 T^2 \\
 (b.s.2) &= -0.01 T \\
 (b.c.2) &= (0.11 - 0.06t) T.
 \end{aligned}$$

We have, finally, to consider the terms of long period in $\delta \gamma$ and δk which have been omitted from the periodic perturbations produced by Neptune, in computing the terms of $\delta \beta$ on page 61, and which are as follows:

$$\begin{aligned}
 \delta \gamma &= 1''.43 \cos (2l - g) - 0''.39 \sin (2l - g) \\
 &\quad - 2.12 \cos (4l - 2g) + 1.00 \sin (4l - 2g) \\
 &\quad + 0.20 \cos (6l - 3g) - 0.04 \sin (6l - 3g) \\
 &\quad + \text{constant} = 0''.00
 \end{aligned}$$

$$\begin{aligned}
 \delta k &= 0''.80 \cos (2l - g) - 2''.28 \sin (2l - g) \\
 &\quad - 1.06 \cos (4l - 2g) - 1.85 \sin (4l - 2g) \\
 &\quad + 0.04 \cos (6l - 3g) + 0.19 \sin (6l - 3g) \\
 &\quad + \text{constant} = 0''.364.
 \end{aligned}$$

For the period during which Uranus has been observed, these values of $\delta \gamma$ and δk may be replaced by the following:

$$\begin{aligned}
 \delta \gamma &= -0''.80 T \\
 \delta k &= +0.27 T
 \end{aligned}$$

which are to be multiplied by the factor $1 + \mu$. The corresponding perturbation of the latitude will be

$$\delta \beta = \sin v \delta \gamma - \cos v \delta k.$$

Putting for v its approximate value

$$v = g + \omega + 2e \sin g$$

and developing to quantities of the first order with respect to the eccentricities, we have

$$\begin{aligned}
 \sin v &= \sin (g + \omega) + e \sin (2g + \omega) - e \sin \omega \\
 \cos v &= \cos (g + \omega) + e \cos (2g + \omega) - e \cos \omega.
 \end{aligned}$$

Substituting for ω its value, $12^{\circ}45'$, and for $\delta\eta$ and δk their above values in the expression for $\delta\beta$, we find that the terms of $\delta\beta$ in question will add the following terms to (b.c.0), (b.s.1), etc.

$$\begin{aligned}(b.c.0) &= - .010 \delta\eta + .046 \delta k = + 0''.02T(1 + \mu) \\(b.s.1) &= + .975 \delta\eta + .221 \delta k = - 0.72T(1 + \mu) \\(b.c.1) &= + .221 \delta\eta - .975 \delta k = - 0.44T(1 + \mu) \\(b.s.2) &= + .046 \delta\eta + .011 \delta k = - 0.04T(1 + \mu) \\(b.c.2) &= - (b.c.0) = - 0.02T(1 + \mu)\end{aligned}$$

These values will be employed in the construction of the provisional ephemeris, but not in the tables.

Collecting all three classes of terms discussed in this section, we have the following constant and secular terms in (b.c.0), (b.s.1), etc.

$$\begin{aligned}(b.c.0) &= + 0''.26 + (-0''.09 + 0.08\mu)T \\(b.s.1) &= (-0''.92 - 0''.61\mu)T - 0''.05T^2 \\(b.c.1) &= (+ 1.86 - 1.68\mu)T + 0.12T^2 \\(b.s.2) &= - 0.06 - 0.05T \\(b.c.2) &= - 0.01 + (0.09 - .08\mu)T\end{aligned}$$

Positions of Uranus resulting from the preceding theory.

The next step in order is the preparation of an ephemeris of the planet for comparison with observations. As this provisional theory is, for future use, superseded by the tables appended to the present work, it seems unnecessary to enter very fully into the details of the computation of the ephemeris. The perturbations of the longitude, logarithm of radius vector, and latitude, were first computed by the formulæ already given.

$$\begin{aligned}\delta v &= (v.c.0) + (v.c.1) \cos g + (v.c.2) \cos 2g + \text{etc.}, \\&\quad + (v.s.1) \sin g + (v.s.2) \sin 2g + \text{etc.}, \\M\delta\rho &= (\rho.c.0) + (\rho.c.1) \cos g + (\rho.c.2) \cos 2g + \text{etc.}, \\&\quad + (\rho.s.1) \sin g + (\rho.s.2) \sin 2g + \text{etc.}, \\\delta\beta &= (b.c.0) + (b.c.1) \cos g + (b.s.1) \sin g.\end{aligned}$$

Each coefficient (v.c.0), (v.c.1), etc., is composed at most of the following quantities:

1. The five classes of secular, long period, or constant terms, the separate values of which, with the sum of all, are given on pages 89 to 93.
2. Periodic terms due to the action of Jupiter, Saturn, and Neptune, given on pages 83 to 87.
3. Terms depending on the product of the masses of Jupiter and Saturn, given on page 88, omitting those depending on N_6 and N_7 , because they are given in column 5 of the terms of the first class.

The sum of the perturbations thus computed is given in the third column of the following ephemeris.

An approximate value of the perturbations produced by Neptune alone is independently computed for every fourth date, and the result is given in the fourth

column. The secular and long period terms are here taken from columns (3) and (4) of the tables on pages 89 to 93.

The elliptic co-ordinates were then derived from the following elements, which are a little different from those employed in the computation of the perturbations.

Elements III. of Uranus.

| | |
|-------------------|-----------------------------|
| π , | 168° 15' 12".0 |
| ϵ , | 28 25 29 .5 |
| θ , | 73 11 58 .0 |
| ϕ , | 0 46 20 .0 |
| e , | .0469436 |
| e , (in sec.) | 9682".81 |
| n , | 15426.196 |
| $\log a$, | 1.2828989 |
| Red. to Ecliptic, | $-9".37 \sin 2(v - \theta)$ |

The longitudes thus found are corrected for lunar, but not for solar nutation, and the results are given in the fifth column.

The column "correction" arises in this way: after the comparison of the ephemeris with observations was nearly completed, it was found that some errors had crept into the former, the most important of which was the employment of a mean anomaly, g , corrected for secular variation of the perihelion in the computation of the perturbations from the preceding formulæ. As a large portion of the computations on the provisional ephemeris had been made by assistants furnished by the Smithsonian Institution and Nautical Almanac, I deemed it prudent to make a careful recomputation of the perturbations for every sixth date during the entire period of the modern observations. The longitudes actually printed in the fifth column are the results of the original incorrect computation, while the numbers in the next column show the several corrections to be applied to obtain the results of my final revised computation.

During the period of the modern observations the ephemeris is computed for intervals of 120 days, and the selected dates are all exact multiples of that interval before or after the fundamental epoch, 1850, Jan. 0, Greenwich mean noon. For convenience of reference the dates are numbered from an epoch earlier by 212 intervals, and the number is given in the second column.

Between 1796 and 1801 no observations worth using were made on Uranus, the ephemeris has, therefore, not been extended over this interval.

HELIOCENTRIC EPHEMERIS OF URANUS FROM THE PRECEDING PROVISIONAL THEORY.

[The longitudes are corrected for lunar but not for solar nutation.]

| Date.
Greenwich
mean noon. | No. | Sum of
perturba-
tions. | Approximate
perturbations
produced by
Neptune. | Longitude. | Correction. | Latitude. | Logarithm
Radius
vector. |
|----------------------------------|-----|-------------------------------|---|--------------|-------------|-----------|--------------------------------|
| | | ' " | " | ° ' " | " | ' " | |
| 1690, Dec. 23 | | +4 28.5 | +236.1 | 59 40 35.2 | | -10 7.8 | 1.2876828 |
| Dec. 24 | | +4 28.5 | | 59 41 16.5 | | -10 7.3 | 1.2876789 |
| 1712, April 2 | | +8 37.9 | | 155 24 42.6 | | +45 58.7 | 1.2626650 |
| April 3 | | +8 37.9 | | 155 25 29.0 | | +45 58.8 | 1.2626640 |
| 1715, Mar. 4 | | +6 58.3 | +136.2 | 169 10 21.4 | | +46 2.1 | 1.2623890 |
| Mar. 5 | | +6 58.2 | | 169 11 7.7 | | +46 2.1 | 1.2623893 |
| Mar. 10 | | +6 57.5 | | 169 15 0.0 | | +46 1.7 | 1.2623911 |
| April 29 | | +6 51.1 | +134.8 | 169 53 42.2 | | +45 58.0 | 1.2624066 |
| April 30 | | +6 51.1 | | 169 54 28.8 | | +45 57.9 | 1.2624102 |
| 1748, Oct. 21 | | -0 28.9 | - 65.0 | 316 13 19.3 | | -41 27.6 | 1.3001855 |
| Oct. 22 | | -0 28.8 | | 316 13 58.8 | | -41 27.9 | 1.3001875 |
| 1750, Sept. 13 | | +0 6.3 | - 51.3 | 323 44 14.9 | | -43 47.2 | 1.3013714 |
| Sept. 14 | | +0 6.3 | | 323 44 54.0 | | -43 47.3 | 1.3013731 |
| Oct. 14 | | +0 6.6 | - 50.8 | 324 4 22.1 | | -43 52.4 | 1.3014165 |
| Oct. 15 | | +0 6.6 | | 324 5 1.0 | | -43 52.5 | 1.3014181 |
| Dec. 3 | | +0 8.0 | - 49.8 | 324 36 49.5 | | -44 0.6 | 1.3014856 |
| Dec. 4 | | +0 8.0 | | 324 37 28.5 | | -44 0.8 | 1.3014872 |
| 1753, Dec. 3 | | -0 5.6 | - 25.6 | 336 25 0.6 | | -46 1.1 | 1.3025789 |
| Dec. 4 | | -0 5.6 | | 336 25 39.3 | | -46 1.2 | 1.3025796 |
| 1756, Sept. 25 | | -0 49.5 | - 0.4 | 347 25 34.9 | | -46 9.2 | 1.3029363 |
| Sept. 26 | | -0 49.5 | | 347 26 13.6 | | -46 9.1 | 1.3029363 |
| 1764, Jan. 15 | | +1 24.0 | | 16 13 35.0 | | -38 36.9 | 1.3003508 |
| Jan. 16 | | +1 24.0 | | 16 14 14.2 | | -38 36.7 | 1.3003486 |
| 1768, Dec. 27 | | +1 33.1 | | 36 3 35.2 | | -27 42.4 | 1.2955331 |
| Dec. 29 | | +1 33.1 | +106.9 | 36 4 44.1 | | -27 41.5 | 1.2955274 |
| Dec. 31 | | +1 33.1 | | 36 6 5.0 | | -27 40.7 | 1.2955216 |
| 1769, Jan. 15 | | +1 33.5 | | 36 16 4.6 | | -27 34.2 | 1.2954753 |
| Jan. 18 | | +1 33.5 | | 36 18 4.5 | | -27 33.0 | 1.2954659 |
| Jan. 21 | | +1 33.5 | +106.9 | 36 20 4.5 | | -27 31.6 | 1.2954566 |
| Jan. 24 | | +1 33.5 | | 36 22 4.4 | | -27 30.3 | 1.2954476 |
| 1781, Jan. 1 | 1 | +3 57.46 | +178.9 | 86 36 13.23 | +0.16 | +11 3.63 | 1.2785972 |
| May 1 | 2 | +3 53.41 | | 88 2 34.70 | | +12 11.14 | 1.2780883 |
| Aug. 29 | 3 | +3 48.58 | | 89 29 7.41 | | +13 18.33 | 1.2775834 |
| Dec. 27 | 4 | +3 43.09 | | 90 55 51.40 | | +14 25.14 | 1.2770827 |
| 1782, April 26 | 5 | +3 37.18 | +180.4 | 92 22 46.89 | | +15 31.53 | 1.2765870 |
| Aug. 24 | 6 | +3 30.97 | | 93 49 53.89 | | +16 37.47 | 1.2760968 |
| Dec. 22 | 7 | +3 24.52 | | 95 17 12.51 | +0.18 | +17 42.88 | 1.2756124 |
| 1783, April 21 | 8 | +3 18.03 | | 96 44 42.73 | | +18 47.73 | 1.2751345 |
| Aug. 19 | 9 | +3 11.87 | +180.1 | 98 12 24.96 | | +19 51.92 | 1.2746633 |
| Dec. 17 | 10 | +3 6.01 | | 99 40 19.00 | | +20 55.67 | 1.2742012 |
| 1784, April 15 | 11 | +3 0.78 | | 101 8 25.13 | | +21 58.69 | 1.2737412 |
| Aug. 13 | 12 | +2 56.24 | | 102 36 43.29 | | +23 0.92 | 1.2732907 |
| Dec. 11 | 13 | +2 52.82 | +178.6 | 104 5 13.79 | +0.23 | +24 2.46 | 1.2728469 |
| 1785, April 10 | 14 | +2 50.58 | | 105 33 56.61 | | +25 3.14 | 1.2724105 |
| Aug. 8 | 15 | +2 49.29 | | 107 2 51.36 | | +26 3.05 | 1.2719806 |
| Dec. 6 | 16 | +2 49.35 | | 108 31 58.18 | | +27 1.98 | 1.2715564 |
| 1786, April 5 | 17 | +2 50.53 | +175.5 | 110 1 16.88 | | +27 59.97 | 1.2711405 |
| Aug. 3 | 18 | +2 53.29 | | 111 30 47.68 | | +28 57.02 | 1.2707299 |
| Dec. 1 | 19 | +2 57.31 | | 113 0 30.11 | +0.31 | +29 53.03 | 1.2703265 |
| 1787, Mar. 31 | 20 | +3 2.38 | | 114 30 23.81 | | +30 47.92 | 1.2699294 |
| July 29 | 21 | +3 8.51 | +171.0 | 116 0 28.62 | | +31 41.72 | 1.2695394 |
| Nov. 26 | 22 | +3 15.48 | | 117 30 44.11 | | +32 34.33 | 1.2691551 |

HELIOCENTRIC EPHEMERIS OF URANUS.—Continued.

| Date.
Greenwich
mean noon. | No. | Sum of
perturba-
tions. | Approximate
perturbations
produced by
Neptune. | Longitude. | | Correction. | Latitude. | | Logarithm
Radius
vector. |
|----------------------------------|-----|-------------------------------|---|------------|----------|-------------|-----------|-----------|--------------------------------|
| | | | | ° | ' " | | ' " | | |
| 1788, Mar. 25 | 23 | +3 23.02 | " | 119 | 1 9.79 | | +33 25.72 | 1.2687775 | |
| July 23 | 24 | 3 31.36 | | 120 | 31 45.79 | | 34 15.81 | 1.2684065 | |
| Nov. 20 | 25 | 3 39.82 | +165.1 | 122 | 2 31.13 | +0.28 | 35 4.58 | 1.2680423 | |
| 1789, Mar. 20 | 26 | 3 38.67 | | 123 | 33 25.96 | | 35 51.99 | 1.2676852 | |
| July 18 | 27 | 3 57.40 | | 125 | 4 29.49 | | 36 38.01 | 1.2673353 | |
| Nov. 15 | 28 | 4 5.97 | | 126 | 35 41.50 | | 37 22.50 | 1.2669932 | |
| 1790, Mar. 15 | 29 | 4 14.35 | +157.8 | 128 | 7 1.70 | | 38 5.50 | 1.2666592 | |
| July 13 | 30 | 4 22.48 | | 129 | 38 29.88 | | 38 47.00 | 1.2663336 | |
| Nov. 10 | 31 | 4 30.00 | | 131 | 10 5.37 | +0.25 | 39 26.85 | 1.2660167 | |
| 1791, Mar. 10 | 32 | 4 36.95 | | 132 | 41 47.99 | | 40 5.04 | 1.2657087 | |
| July 8 | 33 | 4 42.95 | +149.1 | 134 | 13 37.15 | | 40 41.58 | 1.2654082 | |
| Nov. 5 | 34 | 4 48.11 | | 135 | 45 32.62 | | 41 16.44 | 1.2651215 | |
| 1792, Mar. 4 | 35 | 4 52.33 | | 137 | 17 34.16 | | 41 49.53 | 1.2648434 | |
| July 2 | 36 | 4 56.03 | | 138 | 49 41.78 | | 42 20.89 | 1.2645770 | |
| Oct. 30 | 37 | 4 58.32 | +139.3 | 140 | 21 54.47 | +0.46 | 42 50.35 | 1.2643210 | |
| 1793, Feb. 27 | 38 | 4 59.50 | | 141 | 54 12.11 | | 43 18.00 | 1.2640766 | |
| June 27 | 39 | 4 59.85 | | 143 | 26 34.79 | | 43 43.81 | 1.2638448 | |
| Oct. 25 | 40 | 4 58.84 | | 144 | 59 1.64 | | 44 7.75 | 1.2636267 | |
| 1794, Feb. 22 | 41 | 4 56.80 | +128.6 | 146 | 31 32.77 | | 44 29.74 | 1.2634226 | |
| June 22 | 42 | 4 53.48 | | 148 | 4 7.47 | | 44 49.85 | 1.2632327 | |
| Oct. 20 | 43 | 4 49.13 | +122.9 | 149 | 36 45.84 | +0.47 | 45 7.99 | 1.2630577 | |
| 1795, Feb. 17 | 44 | 4 44.20 | | 151 | 9 27.73 | | 45 24.24 | | |
| June 17 | 45 | 4 38.00 | | 152 | 42 12.55 | | 45 38.44 | | |
| Oct. 15 | 46 | 4 31.04 | | 154 | 15 0.15 | | 45 50.64 | | |
| 1801, Jan. 17 | 62 | +3 22.92 | | 179 | 3 50.23 | | +44 34.51 | 1.2627599 | |
| Mar. 17 | 63 | 3 26.59 | + 57.3 | 180 | 36 54.94 | | 44 12.86 | 1.2628920 | |
| Sept. 14 | 64 | 3 31.06 | | 182 | 9 57.40 | | 43 49.24 | 1.2630362 | |
| 1802, Jan. 12 | 65 | 3 36.26 | | 183 | 42 57.23 | | 43 23.66 | 1.2631922 | |
| Mar. 12 | 66 | 3 41.88 | | 185 | 15 54.08 | | 42 56.34 | 1.2633599 | |
| Sept. 9 | 67 | 3 47.78 | + 43.7 | 186 | 48 46.54 | +0.35 | 42 26.97 | 1.2635387 | |
| 1803, Jan. 7 | 68 | 3 54.01 | | 188 | 21 35.11 | | 41 55.86 | 1.2637281 | |
| Mar. 7 | 69 | 4 0.29 | | 189 | 54 18.55 | | 41 22.99 | 1.2639280 | |
| Sept. 4 | 70 | 4 6.43 | | 191 | 26 56.80 | | 40 48.28 | 1.2641386 | |
| 1804, Jan. 2 | 71 | 4 12.95 | + 30.7 | 192 | 59 30.15 | | 40 11.79 | 1.2643591 | |
| Mar. 1 | 72 | 4 18.74 | | 194 | 31 56.83 | | 39 33.63 | 1.2645891 | |
| Aug. 29 | 73 | 4 24.38 | | 196 | 4 17.13 | +0.14 | 38 53.72 | 1.2648287 | |
| Dec. 27 | 74 | 4 29.34 | | 197 | 36 30.24 | | 38 12.21 | 1.2650777 | |
| 1805, April 26 | 75 | 4 33.60 | + 18.0 | 199 | 8 35.97 | | 37 29.07 | 1.2653360 | |
| Aug. 24 | 76 | 4 37.42 | | 200 | 40 33.91 | | 36 44.40 | 1.2656037 | |
| Dec. 22 | 77 | 4 40.11 | | 202 | 12 23.39 | | 35 58.15 | 1.2658808 | |
| 1806, April 21 | 78 | 4 41.83 | | 203 | 44 4.25 | | 35 10.48 | 1.2661674 | |
| Aug. 19 | 79 | 4 42.55 | + 5.9 | 205 | 15 36.11 | +0.21 | 34 21.34 | 1.2664631 | |
| Dec. 17 | 80 | 4 42.19 | | 206 | 46 58.67 | | 33 30.88 | 1.2667689 | |
| 1807, April 16 | 81 | 4 40.52 | | 208 | 18 11.39 | | 32 39.05 | 1.2670847 | |
| Aug. 14 | 82 | 4 37.43 | | 209 | 49 13.98 | | 31 45.96 | 1.2674103 | |
| Dec. 12 | 83 | 4 33.71 | — 5.0 | 211 | 20 6.96 | | 30 51.63 | 1.2677459 | |
| 1808, April 10 | 84 | 4 28.30 | | 212 | 50 49.02 | | 29 56.12 | 1.2680922 | |
| Aug. 8 | 85 | 4 21.63 | | 214 | 21 20.46 | +0.25 | 28 59.51 | 1.2684491 | |
| Dec. 6 | 86 | 4 14.32 | | 215 | 51 41.72 | | 28 1.80 | 1.2688166 | |
| 1809, April 5 | 87 | 4 5.84 | — 15.4 | 217 | 21 51.97 | | 27 3.03 | 1.2691945 | |
| Aug. 3 | 88 | 3 56.63 | | 218 | 51 51.53 | | 26 3.28 | 1.2695829 | |
| Dec. 1 | 89 | 3 47.07 | | 220 | 21 40.62 | | 25 2.66 | 1.2699816 | |
| 1810, Mar. 31 | 90 | 3 37.16 | | 221 | 51 19.10 | | 24 1.10 | 1.2703905 | |
| July 29 | 91 | 3 27.15 | — 24.3 | 223 | 20 47.07 | +0.23 | 22 58.68 | 1.2708094 | |
| Nov. 26 | 92 | +3 17.52 | | 224 | 50 4.83 | | +21 55.38 | 1.2712377 | |

| HELIOCENTRIC EPIHEMERIS OF URANUS.—Continued. | | | | | | | |
|---|-----|-------------------------------|---|--------------|-------------|-----------|--------------------------------|
| Date.
Greenwich
mean noon. | No. | Sum of
perturba-
tions. | Approximate
perturbations
produced by
Neptune. | Longitude. | Correction. | Latitude. | Logarithm
Radius
vector. |
| | | ' " | " | ° ' " | " | ' " | |
| 1811, Mar. 26 | 93 | +3 7.99 | " | 226 19 12.04 | " | +20 51.52 | 1.2716752 |
| July 24 | 94 | 2 59.12 | | 227 48 9.06 | | 19 46.95 | 1.2721215 |
| Nov. 21 | 95 | 2 50.91 | —32.1 | 229 16 55.87 | | 18 41.69 | 1.2725760 |
| 1812, Mar. 20 | 96 | 2 43.45 | | 230 45 32.41 | | 17 35.80 | 1.2730381 |
| July 18 | 97 | 2 37.26 | | 232 13 59.13 | +0.11 | 16 29.34 | 1.2735072 |
| Nov. 15 | 98 | 2 31.98 | | 233 42 15.55 | | 15 22.35 | 1.2739838 |
| 1813, Mar. 15 | 99 | 2 27.86 | —39.1 | 235 10 21.88 | | 14 14.92 | 1.2744651 |
| July 13 | 100 | 2 25.05 | | 236 38 18.26 | | 13 7.03 | 1.2749507 |
| Nov. 10 | 101 | 2 22.98 | | 238 6 3.95 | | 11 58.79 | 1.2754409 |
| 1814, Mar. 10 | 102 | 2 22.35 | | 239 33 39.61 | | 10 50.21 | 1.2759367 |
| July 8 | 103 | 2 22.54 | —44.6 | 241 1 4.63 | +0.22 | 9 41.32 | 1.2764360 |
| Nov. 5 | 104 | 2 24.24 | | 242 28 19.59 | | 8 32.19 | 1.2769372 |
| 1815, Mar. 5 | 105 | 2 26.58 | | 243 55 23.67 | | 7 22.86 | 1.2774403 |
| July 3 | 106 | 2 30.00 | | 245 22 17.17 | | 6 13.39 | 1.2779459 |
| Oct. 31 | 107 | 2 34.17 | —49.3 | 246 48 59.79 | | 5 3.80 | 1.2784533 |
| 1816, Feb. 28 | 108 | 2 39.06 | | 248 15 31.47 | | 3 54.09 | 1.2789600 |
| June 27 | 109 | 2 44.51 | | 249 41 52.08 | —0.01 | 2 44.46 | 1.2794688 |
| Oct. 25 | 110 | 2 50.43 | | 251 8 1.42 | | 1 34.82 | 1.2799725 |
| 1817, Feb. 22 | 111 | 2 56.75 | —54.4 | 252 33 59.49 | | + 0 25.25 | 1.2804769 |
| June 22 | 112 | 3 3.22 | | 253 59 46.00 | | — 0 44.19 | 1.2809792 |
| Oct. 20 | 113 | 3 10.03 | | 255 25 21.18 | | 1 53.44 | 1.2814788 |
| 1818, Feb. 17 | 114 | 3 16.63 | | 256 50 44.48 | | 3 2.48 | 1.2819759 |
| June 17 | 115 | 3 22.96 | —54.7 | 258 15 55.79 | 0.00 | 4 11.23 | 1.2824701 |
| Oct. 15 | 116 | 3 28.90 | | 259 40 55.14 | | 5 19.73 | 1.2829599 |
| 1819, Feb. 12 | 117 | 3 34.28 | | 261 5 42.25 | | 6 27.83 | 1.2834467 |
| June 12 | 118 | 3 38.77 | | 262 30 17.04 | | 7 35.52 | 1.2839317 |
| Oct. 10 | 119 | 3 42.48 | —56.1 | 263 54 39.42 | | 8 42.77 | 1.2844143 |
| 1820, Feb. 7 | 120 | 3 45.03 | | 265 18 49.19 | | 9 49.52 | 1.2848927 |
| June 6 | 121 | 3 46.37 | | 266 42 46.29 | —0.01 | 10 55.76 | 1.2853670 |
| Oct. 4 | 122 | 3 46.38 | | 268 6 30.67 | | 12 1.40 | 1.2858391 |
| 1821, Feb. 1 | 123 | 3 45.07 | —56.4 | 269 30 2.43 | | 13 6.44 | 1.2863087 |
| June 1 | 124 | 3 42.35 | | 270 53 21.55 | | 14 10.88 | 1.2867757 |
| Sept. 29 | 125 | 3 38.08 | | 272 16 27.96 | | 15 14.57 | 1.2872402 |
| 1822, Jan. 27 | 126 | 3 32.86 | | 273 39 22.29 | | 16 17.58 | 1.2877030 |
| May 27 | 127 | 3 26.18 | —56.2 | 275 2 4.25 | —0.10 | 17 19.84 | 1.2881632 |
| Sept. 24 | 128 | 3 18.32 | | 276 24 34.06 | | 18 21.32 | 1.2886214 |
| 1823, Jan. 22 | 129 | 3 9.68 | | 277 46 52.38 | | 19 22.01 | 1.2890772 |
| May 22 | 130 | 3 0.24 | | 279 8 59.05 | | 20 21.89 | 1.2895311 |
| Sept. 19 | 131 | 2 50.36 | —56.1 | 280 30 54.82 | | 21 20.91 | 1.2899826 |
| 1824, Jan. 17 | 132 | 2 40.10 | | 281 52 39.71 | | 22 19.10 | 1.2904315 |
| May 16 | 133 | 2 29.74 | | 283 14 14.16 | —0.07 | 23 16.40 | 1.2908776 |
| Sept. 13 | 134 | 2 19.49 | | 284 35 38.60 | | 24 12.78 | 1.2913208 |
| 1825, Jan. 11 | 135 | 2 9.25 | —55.5 | 285 56 52.92 | | 25 8.22 | 1.2917606 |
| May 11 | 136 | 1 59.46 | | 287 17 57.80 | | 26 2.71 | 1.2921967 |
| Sept. 3 | 137 | 1 50.21 | | 288 38 53.43 | | 26 56.22 | 1.2926290 |
| 1826, Jan. 6 | 138 | 1 41.44 | | 289 59 40.04 | | 27 48.71 | 1.2930570 |
| May 6 | 139 | 1 33.58 | —54.5 | 291 20 18.03 | —0.09 | 28 40.26 | 1.2934804 |
| Sept. 3 | 140 | 1 26.44 | | 292 40 47.50 | | 29 30.76 | 1.2938987 |
| 1827, Jan. 1 | 141 | 1 20.21 | | 294 1 8.83 | | 30 20.16 | 1.2943116 |
| May 1 | 142 | 1 14.89 | | 295 21 22.15 | | 31 8.60 | 1.2947186 |
| Aug. 29 | 143 | 1 10.52 | —54.0 | 296 41 27.72 | | 31 55.92 | 1.2951196 |
| Dec. 27 | 144 | 1 7.22 | | 298 1 25.72 | | 32 42.14 | 1.2955136 |
| 1828, April 25 | 145 | 1 4.79 | | 299 21 16.45 | —0.07 | 33 27.24 | 1.2959002 |
| Aug. 23 | 146 | 1 3.51 | | 300 41 0.14 | | 34 11.25 | 1.2962790 |
| Dec. 21 | 147 | +1 3.11 | —52.9 | 302 0 36.75 | | —34 54.11 | 1.2966496 |

HELIOCENTRIC EPHEMERIS OF URANUS.—Continued.

| Date.
Greenwich
mean noon. | No. | Sum of
perturba-
tions. | | Approximate
perturbations
produced by
Neptune. | Longitude. | | | Correction. | Latitude. | | Logarithm
Radius
vector. |
|----------------------------------|-----|-------------------------------|-------|---|------------|----|-------|-------------|-----------|-------|--------------------------------|
| | | ' | " | | ° | ' | " | | ' | " | |
| 1829, April 20 | 148 | +1 | 3.68 | " | 303 | 20 | 6.73 | " | -35 | 35.77 | 1.2970111 |
| Aug. 18 | 149 | 1 | 5.15 | | 304 | 39 | 30.08 | | 36 | 16.26 | 1.2973630 |
| Dec. 16 | 150 | 1 | 7.37 | | 305 | 58 | 46.96 | | 36 | 55.60 | 1.2977052 |
| 1830, April 15 | 151 | 1 | 10.14 | -52.3 | 307 | 17 | 57.34 | +0.08 | 37 | 33.68 | 1.2980368 |
| Aug. 13 | 152 | 1 | 13.74 | | 308 | 37 | 1.72 | | 38 | 10.56 | 1.2983571 |
| Dec. 11 | 153 | 1 | 17.63 | | 309 | 55 | 59.75 | | 38 | 46.13 | 1.2986661 |
| 1831, April 10 | 154 | 1 | 21.69 | | 311 | 14 | 51.68 | | 39 | 20.49 | 1.2989638 |
| Aug. 8 | 155 | 1 | 26.06 | -52.2 | 312 | 33 | 37.76 | | 39 | 53.54 | 1.2992501 |
| Dec. 6 | 156 | 1 | 30.26 | | 313 | 52 | 17.74 | | 40 | 25.29 | 1.2995247 |
| 1832, April 4 | 157 | 1 | 34.13 | | 315 | 10 | 51.63 | +0.05 | 40 | 55.74 | 1.2997873 |
| Aug. 2 | 158 | 1 | 37.50 | | 316 | 29 | 19.67 | | 41 | 24.90 | 1.3000379 |
| Nov. 30 | 159 | 1 | 40.39 | -52.2 | 317 | 47 | 41.94 | | 41 | 52.65 | 1.3002768 |
| 1833, Mar. 30 | 160 | 1 | 42.02 | | 319 | 5 | 57.95 | | 42 | 19.10 | 1.3005047 |
| July 28 | 161 | 1 | 42.86 | | 320 | 24 | 8.31 | | 42 | 44.17 | 1.3007216 |
| Nov. 25 | 162 | 1 | 42.45 | | 321 | 42 | 12.89 | | 43 | 7.88 | 1.3009280 |
| 1834, Mar. 25 | 163 | 1 | 40.92 | -52.5 | 323 | 0 | 11.92 | +0.15 | 43 | 30.24 | 1.3011231 |
| July 23 | 164 | 1 | 38.01 | | 324 | 18 | 5.47 | | 43 | 51.18 | 1.3013090 |
| Nov. 20 | 165 | 1 | 34.01 | | 325 | 35 | 53.99 | | 44 | 10.74 | 1.3014851 |
| 1835, Mar. 20 | 166 | 1 | 28.76 | | 326 | 53 | 37.59 | | 44 | 28.92 | 1.3016515 |
| July 18 | 167 | 1 | 22.19 | -52.9 | 328 | 11 | 16.27 | | 44 | 45.69 | 1.3018084 |
| Nov. 15 | 168 | 1 | 14.89 | | 329 | 28 | 50.92 | | 45 | 1.14 | 1.3019567 |
| 1836, Mar. 14 | 169 | 1 | 6.50 | | 330 | 46 | 21.41 | +0.19 | 45 | 15.12 | 1.3020960 |
| July 12 | 170 | 0 | 57.31 | | 332 | 3 | 48.20 | | 45 | 27.71 | 1.3022265 |
| Nov. 9 | 171 | 0 | 47.51 | -53.9 | 333 | 21 | 11.76 | | 45 | 38.98 | 1.3023482 |
| 1837, Mar. 9 | 172 | 0 | 37.27 | | 334 | 38 | 32.35 | | 45 | 48.84 | 1.3024612 |
| July 7 | 173 | 0 | 26.69 | | 315 | 55 | 50.40 | | 45 | 57.28 | 1.3025656 |
| Nov. 4 | 174 | 0 | 15.92 | | 337 | 13 | 6.16 | | 46 | 4.45 | 1.3026622 |
| 1838, Mar. 4 | 175 | +0 | 5.23 | -55.1 | 338 | 30 | 20.17 | +0.17 | 46 | 10.04 | 1.3027503 |
| July 2 | 176 | -0 | 5.21 | | 339 | 47 | 32.76 | | 46 | 14.36 | 1.3028297 |
| Oct. 30 | 177 | 0 | 15.76 | | 341 | 4 | 43.81 | | 46 | 17.29 | 1.3029008 |
| 1839, Feb. 27 | 178 | 0 | 25.58 | | 342 | 21 | 54.35 | | 46 | 18.83 | 1.3029634 |
| June 27 | 179 | 0 | 34.82 | -56.9 | 243 | 39 | 4.45 | | 46 | 19.00 | 1.3030176 |
| Oct. 25 | 180 | 0 | 43.68 | | 344 | 56 | 14.08 | | 46 | 17.84 | 1.3030630 |
| 1840, Feb. 22 | 181 | 0 | 51.42 | | 346 | 13 | 24.23 | +0.17 | 46 | 15.28 | 1.3030996 |
| June 21 | 182 | 0 | 58.36 | | 347 | 30 | 34.70 | | 46 | 11.29 | 1.3031269 |
| Oct. 19 | 183 | 1 | 4.34 | -58.4 | 348 | 47 | 46.09 | | 46 | 6.01 | 1.3031447 |
| 1841, Feb. 16 | 184 | 1 | 9.47 | | 350 | 4 | 58.23 | | 45 | 59.35 | 1.3031528 |
| June 16 | 185 | 1 | 13.36 | | 351 | 22 | 11.79 | | 45 | 51.33 | 1.3031509 |
| Oct. 14 | 186 | 1 | 16.11 | | 352 | 39 | 26.97 | | 45 | 41.91 | 1.3031392 |
| 1842, Feb. 11 | 187 | 1 | 17.73 | -60.5 | 353 | 56 | 43.91 | +0.15 | 45 | 31.07 | 1.3031164 |
| June 11 | 188 | 1 | 18.13 | | 355 | 14 | 2.89 | | 45 | 18.90 | 1.3030817 |
| Oct. 9 | 189 | 1 | 17.35 | | 256 | 31 | 24.13 | | 45 | 5.38 | 1.3030351 |
| 1843, Feb. 6 | 190 | 1 | 15.50 | | 357 | 48 | 47.74 | | 44 | 50.50 | 1.3029767 |
| June 6 | 191 | 1 | 12.83 | -66.7 | 359 | 6 | 13.65 | | 44 | 34.22 | 1.3029060 |
| Oct. 4 | 192 | 1 | 9.27 | | 0 | 23 | 42.19 | | 44 | 16.64 | 1.3028228 |
| 1844, Feb. 1 | 193 | 1 | 4.76 | | 1 | 41 | 13.59 | +0.07 | 43 | 57.59 | 1.3027267 |
| May 31 | 194 | 0 | 59.91 | | 2 | 58 | 47.55 | | 43 | 37.23 | 1.3026173 |
| Sept. 28 | 195 | 0 | 54.61 | -64.9 | 4 | 16 | 24.34 | | 43 | 15.53 | 1.3024947 |
| 1845, Jan. 26 | 196 | 0 | 49.19 | | 5 | 34 | 3.83 | | 42 | 52.43 | 1.3023588 |
| May 26 | 197 | 0 | 43.90 | | 6 | 51 | 46.06 | | 42 | 27.99 | 1.3022095 |
| Sept. 23 | 198 | 0 | 38.92 | | 8 | 9 | 31.09 | | 42 | 2.22 | 1.3020472 |
| 1846, Jan. 21 | 199 | 0 | 34.37 | -67.7 | 9 | 27 | 19.04 | +0.15 | 41 | 35.11 | 1.3018722 |
| May 21 | 200 | 0 | 30.55 | | 10 | 45 | 9.82 | | 41 | 6.76 | 1.3016851 |
| Sept. 18 | 201 | -0 | 27.21 | | 12 | 3 | 3.39 | | -40 | 37.01 | 1.3014861 |

| HELIOCENTRIC EPHEMERIS OF URANUS.—Continued. | | | | | | | |
|--|-----|-------------------------------|---|-------------|------------|-----------|--------------------------------|
| Date.
Greenwich
mean noon. | No. | Sum of
perturba-
tions. | Approximate
perturbations
produced by
Neptune. | Longitude. | Correction | Latitude. | Logarithm
Radius
vector. |
| | | ' " | " | ° " | " | ' " | |
| 1847, Jan. 16 | 202 | —0 25.64 | | 13 21 0.27 | | —40 6.03 | 1.3012758 |
| May 16 | 203 | 0 24.73 | —70.2 | 14 39 0.21 | | 39 33.75 | 1.3010544 |
| Sept. 13 | 204 | 0 24.95 | | 15 57 3.61 | | 39 0.23 | 1.3008219 |
| 1848, Jan. 11 | 205 | 0 26.43 | | 17 15 10.46 | +0.06 | 38 25.40 | 1.3005790 |
| May 10 | 206 | 0 28.95 | | 18 33 21.30 | | 37 49.42 | 1.3003263 |
| Sept. 7 | 207 | 0 32.75 | —72.8 | 19 51 36.15 | | 37 12.18 | 1.3000640 |
| 1849, Jan. 5 | 208 | 0 37.59 | | 21 9 55.34 | | 36 33.73 | 1.2997923 |
| May 5 | 209 | 0 43.47 | | 22 28 19.02 | | 35 54.16 | 1.2995115 |
| Sept. 2 | 210 | 0 50.27 | | 23 46 48.13 | | 35 13.36 | 1.2992224 |
| Dec. 31 | 211 | 0 57.74 | —75.3 | 25 5 22.56 | —0.08 | 34 31.47 | 1.2989252 |
| 1850, April 30 | 212 | 1 6.18 | | 26 24 2.38 | | 33 48.40 | 1.2986203 |
| Aug. 28 | 213 | 1 15.11 | | 27 42 48.39 | | 33 4.37 | 1.2983078 |
| Dec. 26 | 214 | 1 24.38 | | 29 1 40.87 | | 32 19.06 | 1.2979881 |
| 1851, April 25 | 215 | 1 34.33 | —77.9 | 30 20 39.78 | | 31 32.79 | 1.2976615 |
| Aug. 23 | 216 | 1 44.28 | | 31 39 45.91 | | 30 45.46 | 1.2973287 |
| Dec. 21 | 217 | 1 54.21 | | 32 58 59.55 | —0.09 | 29 57.09 | 1.2969894 |
| 1852, April 19 | 218 | 2 4.12 | | 34 18 20.89 | | 29 7.71 | 1.2966436 |
| Aug. 17 | 219 | 2 13.61 | —80.5 | 35 37 50.43 | | 28 17.33 | 1.2962916 |
| Dec. 15 | 220 | 2 23.00 | | 36 57 28.19 | | 27 25.95 | 1.2959338 |
| 1853, April 14 | 221 | 2 31.54 | | 38 17 15.06 | | 26 33.62 | 1.2955701 |
| Aug. 12 | 222 | 2 39.58 | | 39 37 10.77 | | 25 40.35 | 1.2952015 |
| Dec. 10 | 223 | 2 46.62 | —82.6 | 40 57 16.17 | —0.07 | 24 46.17 | 1.2948270 |
| 1854, April 9 | 224 | 2 52.64 | | 42 17 31.26 | | 23 51.08 | 1.2944458 |
| Aug. 7 | 225 | 2 57.77 | | 43 37 56.24 | | 22 55.08 | 1.2940583 |
| Dec. 5 | 226 | 3 1.60 | | 44 58 31.59 | | 21 58.25 | 1.2936645 |
| 1855, April 4 | 227 | 3 4.03 | —84.6 | 46 19 17.49 | | 21 0.51 | 1.2932645 |
| Aug. 2 | 228 | 3 5.35 | | 47 40 13.93 | | 20 1.97 | 1.2928582 |
| Nov. 30 | 229 | 3 5.32 | | 49 1 21.26 | —0.11 | 19 2.66 | 1.2924449 |
| 1856, Mar. 29 | 230 | 3 3.95 | | 50 22 39.57 | | 18 2.55 | 1.2920239 |
| July 27 | 231 | 3 1.24 | —86.1 | 51 44 9.08 | | 17 1.67 | 1.2915954 |
| Nov. 24 | 232 | 2 57.62 | | 53 5 49.32 | | 16 0.03 | 1.2911592 |
| 1857, Mar. 24 | 233 | 2 52.95 | | 54 27 40.66 | | 14 57.72 | 1.2907155 |
| July 22 | 234 | 2 47.58 | | 55 49 42.98 | | 13 54.71 | 1.2902643 |
| Nov. 19 | 235 | 2 41.65 | —87.9 | 57 11 56.10 | —0.10 | 12 51.08 | 1.2898057 |
| 1858, Mar. 19 | 236 | 2 35.18 | | 58 34 20.16 | | 11 46.89 | 1.2893395 |
| July 17 | 237 | 2 28.82 | | 59 56 54.59 | | 10 41.96 | 1.2888659 |
| Nov. 14 | 238 | 2 22.46 | | 61 19 39.71 | | 9 36.91 | 1.2883853 |
| 1859, Mar. 14 | 239 | 2 16.35 | —88.6 | 62 42 35.14 | | 8 30.87 | 1.2878981 |
| July 12 | 240 | 2 10.33 | | 64 5 41.38 | | 7 24.61 | 1.2874049 |
| Nov. 9 | 241 | 2 5.17 | | 65 28 57.59 | —0.13 | 6 17.90 | 1.2869061 |
| 1860, Mar. 8 | 242 | 2 0.67 | | 66 52 24.15 | | 5 10.84 | 1.2864023 |
| July 6 | 243 | 1 57.10 | —89.1 | 68 16 0.90 | | 4 3.42 | 1.2858941 |
| Nov. 3 | 244 | 1 54.37 | | 69 39 47.78 | | 2 55.74 | 1.2853816 |
| 1861, Mar. 3 | 245 | 1 52.61 | | 71 3 44.98 | | 1 47.83 | 1.2848658 |
| July 1 | 246 | 1 51.92 | | 72 27 52.38 | | —0 39.69 | 1.2843468 |
| Oct. 29 | 247 | 1 52.43 | —89.3 | 73 52 10.31 | —0.19 | +0 28.60 | 1.2838257 |
| 1862, Feb. 26 | 248 | 1 53.51 | | 75 16 38.24 | | 1 37.00 | 1.2833029 |
| June 26 | 249 | 1 55.99 | | 76 41 16.65 | | 2 45.46 | 1.2827788 |
| Oct. 24 | 250 | 1 59.54 | | 78 6 5.46 | | 3 53.95 | 1.2822542 |
| 1863, Feb. 21 | 251 | 2 3.98 | —88.5 | 79 31 4.97 | | 5 2.40 | 1.2817290 |
| June 21 | 252 | 2 9.43 | | 80 56 15.07 | | 6 10.80 | 1.2812047 |
| Oct. 19 | 253 | 2 15.63 | | 82 21 36.01 | —0.04 | 7 19.09 | 1.2806819 |
| 1864, Feb. 16 | 254 | 2 22.52 | | 83 47 7.95 | | 8 27.25 | 1.2801609 |
| June 15 | 255 | 2 30.20 | —87.2 | 85 12 50.83 | | 9 35.22 | 1.2796421 |
| Oct. 13 | 256 | —2 38.39 | | 86 38 44.88 | | +8 42.95 | 1.2791263 |

HELIOCENTRIC EPHEMERIS OF URANUS.—*Concluded.*

| Date.
Greenwich
mean noon. | No. | Sum of
perturba-
tions. | Approximate
perturbations
produced by
Neptune. | Longitude. | Correction. | Latitude. | Logarithm
Radius
vector. |
|----------------------------------|-----|-------------------------------|---|--------------|-------------|-----------|--------------------------------|
| | | ' " | " | ° ' " | " | ' " | |
| 1865, Feb. 10 | 257 | —2 47.06 | | 88 4 50.26 | | +11 52.20 | 1.2786142 |
| June 10 | 258 | 2 55.93 | | 89 31 7.17 | | 12 57.54 | 1.2781063 |
| Oct. 8 | 259 | 3 5.01 | —85.2 | 90 57 35.61 | —0.69 | 14 4.34 | 1.2776029 |
| 1866, Feb. 5 | 260 | 3 13.91 | | 92 24 16.00 | | 15 10.74 | 1.2771044 |
| June 5 | 261 | 3 22.61 | | 93 51 8.39 | | 16 16.70 | 1.2766112 |
| Oct. 3 | 262 | 3 30.79 | | 95 18 13.06 | | 17 22.18 | 1.2761237 |
| 1867, Jan. 31 | 263 | 3 38.49 | —82.5 | 96 45 29.91 | | 18 27.15 | 1.2756419 |
| May 31 | 264 | 3 45.25 | | 98 12 59.44 | | 19 31.54 | 1.2751656 |
| Sept. 23 | 265 | 3 50.96 | | 99 40 41.71 | —0.08 | 20 35.37 | 1.2746948 |
| 1868, Jan. 26 | 266 | 3 55.67 | | 101 8 36.54 | | 21 38.51 | 1.2742299 |
| May 25 | 267 | 3 59.06 | —79.1 | 102 36 44.31 | | 22 40.96 | 1.2737709 |
| Sept. 22 | 268 | 4 1.02 | | 104 5 5.00 | | 23 42.75 | 1.2733175 |
| 1869, Jan. 20 | 269 | 4 1.71 | | 105 33 38.41 | | 24 43.69 | 1.2728693 |
| May 20 | 270 | 4 0.81 | | 107 2 24.67 | | 25 43.81 | 1.2724264 |
| Sept. 17 | 271 | 3 58.59 | —75.1 | 108 31 23.55 | —0.11 | 26 43.07 | 1.2719888 |
| 1870, Jan. 15 | 272 | 3 54.96 | | 110 0 34.97 | | 27 41.44 | 1.2715563 |
| May 15 | 273 | 3 50.23 | | 111 29 58.50 | | 28 38.77 | 1.2711290 |
| Sept. 12 | 274 | 3 44.34 | | 112 59 33.91 | | 29 35.16 | 1.2707070 |
| 1871, Jan. 10 | 275 | 3 37.90 | —69.8 | 114 29 20.79 | | 30 30.41 | 1.2702903 |
| May 10 | 276 | 3 30.56 | | 115 59 19.11 | | 31 24.57 | 1.2698791 |
| Sept. 7 | 277 | 3 22.61 | | 117 29 28.39 | —0.18 | 32 17.57 | 1.2694735 |
| 1872, Jan. 5 | 278 | 3 14.50 | | 118 59 48.05 | | 33 9.31 | 1.2690738 |
| May 4 | 279 | 3 6.19 | —64.2 | 120 30 17.95 | | 33 59.79 | 1.2686804 |
| Sept. 1 | 280 | —2 57.65 | | 122 0 57.87 | | +34 48.97 | 1.2682934 |

The next operation would be to interpolate these co-ordinates to intervals of time suitable for the computation of a geocentric ephemeris, to correct the longitudes for solar nutation, and then to compute the geocentric right ascension and declination. This operation has not, however, been completely carried out except for most of the observations before 1830, and for three of the oppositions observed since, the latter being computed only as a check upon the accuracy of the comparisons. As a general rule, it may be said that wherever a complete geocentric ephemeris, with the heliocentric ephemeris from which it was computed, were available, these ephemerides were made use of in a manner which will be more fully described hereafter, while, in all other cases, the geocentric places were computed directly.

It may also be stated here that Hansen's *Tables du Soleil* have been adopted as giving the places of the sun to be used in computing the geocentric places.

CHAPTER VI.

REDUCTION OF THE OBSERVATIONS OF URANUS, AND THEIR COMPARISON WITH THE PRECEDING THEORY.

THE observations of Uranus naturally divide themselves into two distinct classes. (1) The purely accidental ones, made previous to the recognition of the planet by Herschel in 1781, and therefore without any suspicion on the part of the observers that the object was not a fixed star, and (2) the systematic observations made since.

The first class are nearly all so uncertain in comparison with the second that I have hesitated over the question of employing them at all. If nothing but a determination of the elements of Uranus were called for, they would certainly not be worth using, since these elements may be determined with entire certainty from the observations which have been made during the entire revolution of the planet since 1781. But the mass of Neptune is also to be determined, and it is at least possible that these observations, uncertain though they are, may add materially to the weight of this determination. I have, therefore, determined to include them all, re-reducing them when there seemed to be good reason so to do.

The earliest observations are those of Flamsteed, published in the *Historiæ Cœlestis*. The observations themselves, as printed, together with the principal elements for reduction, are given in the following tables.

The first column of the table gives the name of the star. The second gives the clock time of transit over the wire of the quadrant as given by Flamsteed. The time, it will be seen, is only given to entire seconds. We must, therefore, expect to find a probable error, of which the mathematical minimum is $0^{\text{s}}.25$, and of which the minimum we can reasonably expect is much greater.

Next we have the apparent right ascensions of the stars as computed. For these data I am indebted to Prof. Coffin, Superintendent of the American Ephemeris. The mean places are mostly derived from the "Star Tables of the American Ephemeris," and from the two Greenwich Seven Year Catalogues, while the reduction to apparent place is made with the modern constants.

The fourth column gives the apparent clock correction for sidereal time, in which is included the effect of deviation of the instrument from the meridian.

The clock keeping mean time, the errors are in the next column reduced to those of sidereal time at the moment of the transit of Uranus.

The next two columns give the corrections for clock rate, and for deviation of the instrument from the meridian, as inferred from the observations themselves, both being referred to the time and position of the transit of Uranus.

In the last column we have the seconds of concluded correction for clock and instrument to be applied to the observed time of transit of Uranus.

1690, *December 23. Right Ascension.*

| Star. | Time of Tr. | | | R. A. of star. | | | Clock. | | | C'. | | R. | Dev. | C''. |
|---|-------------|----|------------------|----------------|----|------|--------|----|------|-----|------|------|------|------|
| | h. | m. | s. | h. | m. | s. | h. | m. | s. | m. | s. | | | |
| α Arietis, | 7 | 48 | 40 | 1 | 49 | 52.1 | -5 | 58 | 47.9 | 58 | 29.3 | -1.1 | -1.2 | 31.6 |
| π Arietis, | 8 | 30 | 54 | 2 | 32 | 8.4 | -5 | 58 | 45.6 | 58 | 33.9 | -0.7 | +1.7 | 32.9 |
| σ Arietis, | 8 | 33 | 17 | 2 | 34 | 31.0 | -5 | 58 | 46.0 | 58 | 34.7 | -0.7 | +2.9 | 32.5 |
| ε Arietis, | 8 | 40 | 21 | 2 | 41 | 38.0 | -5 | 58 | 43.0 | 58 | 33.0 | -0.6 | -0.2 | 33.8 |
| δ Arietis, | 8 | 52 | 44 | 2 | 54 | 3.0 | -5 | 58 | 41.0 | 58 | 33.0 | -0.5 | +0.5 | 33.0 |
| η Tauri, | 9 | 27 | 47 | 3 | 29 | 13.6 | -5 | 58 | 33.4 | 58 | 31.1 | -0.1 | -1.3 | 32.5 |
| Uranus, | 9 | 41 | 49 | | | | | | | | | | | |
| A Tauri, | 9 | 45 | 3 | 3 | 46 | 31.1 | -5 | 58 | 31.9 | 58 | 32.4 | 0.0 | -0.8 | 33.2 |
| α Virginis, | 19 | 4 | 15 | 13 | 8 | 58.2 | -5 | 55 | 16.8 | 56 | 49.3 | | | |
| α Bootis, | 19 | 58 | 31 $\frac{1}{2}$ | 14 | 1 | 34.0 | -5 | 56 | 57.5 | 58 | 39.0 | | | |
| Hourly rate of clock, | | | | | | | | | | | | | | -0.6 |
| Deviation of instrument for each degree of Z. D., | | | | | | | | | | | | | | -0.5 |
| Transit of Uranus, | | | | | | | | | | | | h. | m. | s. |
| Correction for clock and instrument (mean), | | | | | | | | | | | | 9 | 41 | 49.0 |
| Observed R. A. of the planet, | | | | | | | | | | | | -5 | 58 | 32.8 |
| | | | | | | | | | | | | 3 | 43 | 16.2 |

1690, *December 23. Declination.*

| | Z. D. observed. | Refraction. | Declination. | Eq. point. |
|--|-----------------|-------------|--------------|-------------|
| α Arietis, | 29° 29' 10" | +0' 33" | 21° 58' 53" | 51° 28' 36" |
| π Arietis, | 35 18 55 | +0 41 | 16 9 4 | 51 28 40 |
| σ Arietis, | 37 41 0 | +0 44 | 13 46 58 | 51 28 42 |
| ε Arietis, | 31 23 15 | +0 35 | 20 4 37 | 51 28 27 |
| δ Arietis, | 32 55 55 | +0 37 | 18 31 40 | 51 28 12 |
| η Tauri, | 28 20 55 | +0 31 | 23 6 53 | 51 28 19 |
| Uranus, | 31 52 35 | +0 36 | | |
| A Tauri, | 30 15 55 | +0 34 | 21 12 3 | 51 28 32 |
| Circle reading for Uranus, corrected for refraction, | | | | 31° 53' 11" |
| Equatorial point on circle, | | | | 51 28 30 |
| Declination of Uranus, from observation, | | | | +19 35 19 |

1712, *April 2. Right Ascension.*

| | h. | m. | s. | h. | m. | s. | m. | s. |
|-------------------------|----|----|----|----|----|------|-----|------|
| Uranus, | 9 | 35 | 19 | | | | | |
| ε Virginis, | 12 | 0 | 19 | 12 | 47 | 52.4 | +47 | 33.4 |
| e Virginis, | 12 | 14 | 51 | 13 | 2 | 31.5 | +47 | 40.5 |
| ζ Virginis, | 12 | 32 | 11 | 13 | 20 | 5.7 | +47 | 54.7 |

The discordance of clock errors, and the time which intervened between the transit of the planet and that of the first star, seem to render an accurate reduction impossible.

1715, *March 4. Right Ascension.*

| | T. | | | R. A. | | | C. | | | C''. |
|---|----|----|----|-------|----|------|----|----|------|------|
| | h. | m. | s. | h. | m. | s. | h. | m. | s. | s. |
| <i>d</i> Leonis, | 11 | 50 | 19 | 10 | 45 | 52.5 | -1 | 4 | 26.5 | 20.5 |
| Uranus, | 12 | 27 | 1 | | | | | | | |
| <i>b</i> Virginis, | 12 | 49 | 41 | 11 | 45 | 23.6 | -1 | 4 | 17.4 | 21.1 |
| Clock time of transit of Uranus, | | | | | | | | 12 | 27 | 1 |
| Correction for clock and instrument, | | | | | | | | -1 | 4 | 20.8 |
| Right ascension of Uranus from observation, | | | | | | | | 11 | 22 | 40.2 |

1715, *March 4. Declination.*

| | Z. D. | | | R. | Dec. | | | Eq. point. | | | |
|---------------------------------|-------|-----|-----|-----|------|-----|----|------------|-----|-----|-----|
| | ° | ' | " | ' | ° | ' | " | ° | ' | " | |
| <i>d</i> Leonis, | 46° | 19' | 40" | +1' | 0" | +5° | 8' | 6" | 51° | 28' | 46" |
| Uranus, | 46 | 33 | 10 | +1 | 1 | | | | | | |
| <i>b</i> Virginis, | 46 | 13 | 20 | +1 | 0 | 5 | 14 | 17 | 51 | 28 | 37 |
| Circle reading for Uranus, | | | | | | | | | 46° | 34' | 11" |
| Equatorial point, | | | | | | | | | 51 | 28 | 42 |
| Observed declination of Uranus, | | | | | | | | | +4 | 54 | 31 |

1715, *March 5. Right Ascension.*

| | T. | | | R. A. | | | C. | | | C' |
|--------------------------------------|----|----|------|-------|----|------|----|----|------|------|
| | h. | m. | s. | h. | m. | s. | h. | m. | s. | s. |
| <i>d</i> Leonis, | 11 | 46 | 24 | 10 | 45 | 52.5 | -1 | 0 | 31.5 | 25.5 |
| Uranus, | 12 | 22 | 59.: | | | | | | | |
| <i>b</i> Virginis, | 12 | 45 | 49 | 11 | 45 | 23.6 | -1 | 0 | 25.4 | 29.1 |
| Transit of Uranus, | | | | | | | | 12 | 22 | 59 |
| Correction for clock and instrument, | | | | | | | | -1 | 0 | 27.7 |
| Observed right ascension of Uranus, | | | | | | | | 11 | 22 | 31.5 |

The large apparent clock rate, and the colons after the time of transit, both throw doubt on this observation.

Declination.

The circle readings for the stars are the same as on the day preceding, while that for Uranus is 50" less. The declination is therefore 50" greater, or +4° 55' 21".

1715, *March 10. Right Ascension.*

| | T. | | | R. A. | | | C. | | | C' | |
|--------------------------------------|----|----|----|-------|----|------|----|----|------|------|------|
| | h. | m. | s. | h. | m. | s. | h. | m. | s. | s. | |
| <i>d</i> Leonis, | 11 | 25 | 58 | 10 | 45 | 52.5 | -0 | 40 | 5.5 | 59.5 | |
| <i>p</i> ³ Leonis, | 11 | 32 | 28 | 10 | 52 | 25.1 | -0 | 40 | 2.9 | 58.1 | |
| Uranus, | 12 | 1 | 42 | | | | | | | | |
| <i>b</i> Virginis, | 12 | 25 | 18 | 11 | 45 | 23.6 | -0 | 39 | 54.4 | 58.2 | |
| Clock time of transit of Uranus, | | | | | | | | | 12 | 1 | 42 |
| Correction for clock and instrument, | | | | | | | | | -0 | 39 | 58.6 |
| Observed right ascension of Uranus, | | | | | | | | | 11 | 21 | 43.4 |

Declination.

| | Z. D. | R. | Dec. | Eq. Pt. |
|---------------------------------|-------------|--------|----------|-------------|
| <i>d</i> Leonis, | 46° 19' 35" | +1' 1" | 5° 8' 6" | 51° 28' 42" |
| <i>p</i> ³ Leonis, | 47 58 35 | +1 3 | 3 29 25 | 51 28 63 |
| Uranus, | 46 27 0 | +1 1 | | |
| <i>b</i> Virginis, | 46 13 25 | +1 0 | 5 14 17 | 51 28 42 |
| Circle reading for Uranus, | | | | 46 28 1 |
| Equatorial point on circle, | | | | 51 28 49 |
| Observed declination of Uranus, | | | | +5 0 48 |

1715. April 29. *Right Ascension.*

| | T. | R. A. | C. | C'. |
|--------------------|-----------|------------|------------|------|
| | h. m. s. | h. m. s. | h. m. s. | s. |
| σ Leonis, | 8 42 11 | 11 6 28.3 | +2 24 17.3 | 18.7 |
| Uranus, | 8 50 44 | | | |
| ν Virginis, | 9 6 55 | 11 31 14.1 | +2 24 19.1 | 16.4 |
| 17 Virginis, | 9 43 38.: | 12 8 6.5 | +2 24 28.5 | 19.8 |
| α Virginis, | 11 32 48 | 13 57 47.6 | +2 24 59.6 | 33.1 |

The discordance of the clock corrections makes a satisfactory determination of the right ascension very difficult. I deem it best to reject the doubtful observation of 17 Virginis, and the discordant one of α Virginis. The result will then be

| | | |
|--------------------------------------|-----------|---------|
| Observed transit of Uranus, | h. m. s. | 8 50 44 |
| Correction for clock and instrument, | 2 24 17.6 | |
| Observed right ascension of Uranus, | 11 15 1.6 | |

Declination.

| | Z. D. | R. | Dec. | Eq. Pt. |
|---------------------------------|-------------|---------|------------|-------------|
| σ Leonis, | 43° 52' 40" | +0' 55" | 7° 34' 51" | 51° 28' 26" |
| Uranus, | 45 45 30 | +0 59 | | |
| ν Virginis, | 43 20 20 | +0 54 | 8 7 11 | 51 28 25 |
| 17 Virginis, | 44 34 10 | +0 56 | 6 53 33 | 51 28 39 |
| α Virginis, | 60 23 5 | +1 41 | -8 55 42 | 51 28 64 |
| Circle reading for Uranus, | | | | 45° 46' 29" |
| Mean equatorial point, | | | | 51 28 38 |
| Observed declination of Uranus, | | | | +5 42 9 |

The next observations in the order of time are two by Bradley, discovered by Mr. Hugh Breen, but still unpublished. The following are the results as given by Mr. Breen in the *Astronomische Nachrichten*, No. 1463.

| Mean Time. | R. A. | N. P. D. | |
|---------------------|-----------|------------|-------------|
| h. m. s. | h. m. s. | ° ' " | |
| 1748, October 21, | 7 6 18.4 | 21 4 37.93 | 107 29 |
| 1750, September 13, | 10 8 57.8 | 21 40 0.23 | 104 42 33.9 |

Mr. Breen remarks: "The right ascensions are very accurate. It has been assumed that the N. P. D., on 1750, September 13, is identical with μ Capricorni, with which it was compared. The first observation was by the transit instrument, and the second by the quadrant."

No ground is given for the above assumption respecting the N. P. D. for the second observation; it may, therefore, be omitted as valueless.

In the year 1750 we have also two observations by Le Monnier at Paris. For these, and all the other observations by the same observer, I shall adopt the results given by Bouvard in the *Connaissance des Temps*, for 1821, p. 341, with the corrections indicated by Le Verrier, in *Connaissance des Temps*, for 1849, pp. 125 and 126. The necessary uncertainty of the observations is such that, considering that Bouvard reduced them with the star positions of the "*Fundamenta*," scarcely anything will be gained by a new reduction.

1753, December 3, we have another observation of right ascension by Bradley. I adopt the result kindly communicated by my distinguished friend, Dr. Auwers.

$$1753, \text{ December } 3, \quad \begin{matrix} \text{h.} & \text{m.} \\ 5 & 33. \end{matrix} \quad \text{R. A.} = \begin{matrix} \text{h.} & \text{m.} & \text{s.} \\ 22 & 23 & 21.59 \end{matrix}$$

1756, September 25. Observation by Mayer, at Gottingen. I adopt the result given by Bessel, in *Fundamenta Astronomiae*, p. 284.

$$1756, \text{ September } 25, \quad \begin{matrix} \text{h.} & \text{m.} \\ 10 & 12. \end{matrix} \quad \text{R. A.} = \begin{matrix} \circ & ' & '' \\ 348 & 0 & 54.5 \end{matrix}$$

$$\text{Dec.} = \begin{matrix} -6 & 1 & 49.4 \end{matrix}$$

The following is a tabular summary of the preceding results, with their comparison with the provisional theory. In the computation of the geocentric place the places of the sun were derived from Hansen's Tables. I am indebted to Professor Coffin for a duplicate computation of the geocentric places from the provisional ephemeris, which was executed by Mr. Joseph A. Rogers.

| Date. | Right Ascension. | | Declination. | | Correction to theory. | | | | |
|----------------|------------------|---------|--------------|---------|-----------------------|-------|-------|---------------------------------------|------------------------------------|
| | Observation. | Theory. | Observation. | Theory. | R. A. | Dec. | Long. | $\frac{\partial l}{\partial \lambda}$ | $\frac{\partial l}{\partial \rho}$ |
| | h m s | s | ° ' " | " | " | " | " | | |
| 1690, Dec. 23 | 3 43 16.2 | 14.7 | +19 35 19 | 7 | +22 | +12 | +24 | 1.04 | +0.27 |
| 1715, Mar. 4 | 11 22 40.2 | 38.7 | 4 54 31 | 48 | +22 | -17 | +28 | 1.06 | 0 |
| Mar. 5 | 11 22 31.5 | 29.1 | 4 55 21 | 49 | +36 | -28 | +44 | 1.06 | 0 |
| Mar. 10 | 11 21 43.4 | 41.0 | 5 0 48 | 56 | +36 | -8 | +36 | 1.06 | 0 |
| Apr. 29 | 11 15 1.6 | 1.5 | + 5 42 9 | 11 | + 1 | - 2 | + 2 | 1.04 | +0.36 |
| 1748, Oct. 21 | 21 4 37.93 | 35.42 | | ... | +37.6 | | +39.3 | 1.015 | +0.50 |
| 1750, Sept. 13 | 21 40 0.23 | 57.90 | | ... | +35.0 | | +35.7 | 1.05 | +0.22 |
| | ° ' " | " | | | | | | | |
| Oct. 14 | 324 15 24.6 | 47.6 | -15 1 40.4 | 47.0 | +37.0 | + 6.6 | +35.9 | 1.03 | +0.45 |

| Date. | Right Ascension. | | | | Declination. | | | | Correction to theory. | | | | |
|----------------|------------------|----|-------|---------|--------------|----|------|---------|-----------------------|--------|-------|---------------------------------------|-------------------------------------|
| | Observation. | | | Theory. | Observation. | | | Theory. | R. A. | Dec. | Long. | $\frac{\partial l}{\partial \lambda}$ | $\frac{\partial l}{\partial \beta}$ |
| | o | ' | " | " | o | ' | " | " | " | " | " | | |
| 1750, Dec. 3 | 324 | 34 | 53.5 | 15.4 | -14 | 53 | 20.2 | 32.4 | +38.1 | +12.2 | +38.8 | 0.98 | +0.47 |
| | h | m | s | s | | | | | | | | | |
| 1753, Dec. 3 | 22 | 23 | 21.60 | 19.34 | | | | | +33.8 | | +35.6 | 1.00 | +0.048 |
| | o | ' | " | " | | | | | | | | | |
| 1756, Sept. 25 | 348 | 0 | 54.5 | 25.0 | -6 | 1 | 49.4 | 46.0 | +29.5 | -3.4 | +25.7 | 1.05 | +0.013 |
| 1764, Jan. 15 | 12 | 37 | 39.0 | 56.4 | +4 | 43 | 47.2 | 56.2 | (-17.4) | (-9.0) | | | |
| 1768, Dec. 27 | 31 | 26 | 52.0 | 32.6 | 12 | 15 | 35.0 | 30.6 | +19.4 | +4.4 | +13.0 | 1.02 | +0.045 |
| Dec. 30 | 31 | 26 | 45.8 | 38.0 | 12 | 14 | 55.4 | 55.9 | +7.8 | -0.5 | | | |
| 1769, Jan. 15 | 31 | 22 | 7.7 | 55.8 | 12 | 14 | 26.0 | 29.7 | +11.9 | -3.7 | +12.5 | 1.01 | +0.049 |
| Jan. 16 | 31 | 22 | 23.4 | 11.1 | 12 | 14 | 36.3 | 37.0 | +12.3 | -0.7 | | | |
| Jan. 20 | 31 | 24 | 6.6 | 43.7 | 12 | 15 | 19.0 | 18.8 | +22.9 | +0.2 | | | |
| Jan. 21 | 31 | 24 | 33.8 | 14.1 | 12 | 15 | 31.8 | 30.8 | +19.7 | +1.0 | | | |
| Jan. 22 | 31 | 25 | 4.7 | 47.7 | 12 | 15 | 45.7 | 44.7 | +17.0 | +1.0 | | | |
| Jan. 23 | 31 | 25 | 28.5 | 24.2 | 12 | 16 | 7.5 | 59.6 | +4.3 | +7.9 | | | |

Where no declination has been observed the observed corrections in right ascension have been changed to corrections in longitude on the hypothesis that the theoretical latitude is correct. The approximate formula is

$$\delta l = \frac{\delta \alpha \cos \delta}{\sin E}, \text{ where}$$

$\cos E = \sin \epsilon \cos \alpha$, ϵ being the obliquity.

DISCUSSION OF THE MODERN OBSERVATIONS.

Reduction of the Published Results of Observations to a Uniform System.

We have now to discuss the great mass of observations made at the principal observatories of the world since the discovery of the planet by Herschel, in 1781. To make all the data of reduction rigorously homogeneous and uniform, it would be necessary to completely re-reduce the greater part of the observations made before 1850, using the modern values of the constants of reduction, and to compare each observation separately with the geocentric place deduced from the provisional theory. Such a reduction and comparison would be extremely desirable. Their execution would, however, involve an amount of labor far greater than it is now possible for the author to bestow upon the problem. We must, therefore, adopt the reductions which have been already made, applying such systematic corrections for reduction to a uniform system of star places as we have the means readily to determine. No reduced places are employed unless we can find data for some more or less accurate determination of these corrections, a rule which necessitates the rejection of a great mass of observations made at the minor observatories of the European continent, and published in the *Astronomischen Nachrichten*. We still have the following rich collection of materials at our disposal:

1. Observations at Greenwich, 1781 to 1872.
2. Paris, 1802 to 1827, and 1837 to 1869.
3. Königsberg, 1813 to 1835.
4. Vienna, 1822, and 1827 to 1839.
5. Speier, 1827-29.
6. Cambridge, 1828 to 1842.
7. Edinburgh, 1836 to 1844.
8. Berlin, 1838 to 1842.
9. Pulkowa, 1841 and 1842.
10. Washington, 1861 to 1872.
11. Leiden, 1863 to 1871.
12. Santiago, 1854 and 1855.

As to the general distribution of these observations in time, we may remark that during the first three or four years the planet was zealously observed at Greenwich. Observations then began gradually to fall off until 1798, in which year we find but one. From this time until 1814 only one or two observations were made at each opposition. They become a little more numerous, until 1829, when there is a sudden increase. Few interruptions have occurred since. With regard to the other observatories it may be said that from 1802 until 1830 there is a gradual increase in the number of observations, and that since the latter year the number of observations is entirely satisfactory.

A great number of the observations were reduced with the star places of the *Tabulæ Regiomontanæ*, and the entire Paris series are reduced with the star positions of Le Verrier, given in his "*Annales de l'Observatoire Imperial de Paris*," Tome II. As a preliminary to the discussion of the systematic corrections to the principal published reductions, I have prepared the following table, showing the corrections which must be applied to the places of the equatorial fundamental stars in the above catalogues to reduce them to the adopted standard, namely, Dr. Gould's coast survey list in right ascension, and Auwers' standard in declination.

In the table of right ascensions the first column after name of the star gives the annual variation of that co-ordinate for the epoch 1860.0, as derived from Le Verrier's tables of right ascensions just cited. Next we have the correction to this annual variation, expressed in units of the fourth place of decimals, to reduce it to that given in the "*Star Tables of the American Ephemeris*," the positions in which are founded on Dr. Gould's Catalogue. The fourth column gives the correction to the right ascensions of Le Verrier for 1860, in hundredths of a second of time. Subtracting from this column sixth-tenths of the preceding, we have the corresponding corrections for 1800. The last four columns give the corresponding numbers for the right ascensions of the *Tabulæ Regiomontanæ*.

The table of declinations shows, for different epochs, the corrections necessary to reduce the tabular positions to those given by Auwers in his paper on the declinations of the fundamental stars

I. RIGHT ASCENSIONS.

| Date. | Le Verrier's
ann. var.
1860. | Corrections to | | | Ann. var. of
Tab. Reg.
1860. | Corrections to | | |
|-------------------------|------------------------------------|----------------|----------------|----------------|------------------------------------|----------------|----------------|----------------|
| | | Ann. var. | R. A.
1860. | R. A.
1800. | | Ann. var. | R. A.
1860. | R. A.
1800. |
| α Andromedæ, | ^s
+3.0844 | + 14 | + 2 | - 6 | ^s
3.0840 | +18 | + 8 | - 3 |
| γ Pegasi, | 3.0801 | + 15 | + 3 | - 6 | 3.0824 | - 8 | - 9 | - 4 |
| α Arietis, | 3.3644 | + 12 | 0 | - 7 | 3.3636 | +20 | + 6 | - 6 |
| α Ceti, | 3.1266 | + 10 | + 5 | - 1 | 3.1267 | + 9 | + 7 | + 2 |
| α Tauri, | 3.4346 | - 1 | - 1 | 0 | 3.4335 | +10 | + 7 | + 1 |
| β Orionis, | 2.8797 | + 6 | + 2 | - 2 | 2.8800 | + 3 | + 5 | + 3 |
| β Tauri, | 3.7871 | - 4 | + 1 | + 3 | 3.7888 | -21 | - 4 | + 9 |
| α Orionis, | 3.2460 | + 5 | + 4 | + 1 | 3.2464 | + 1 | + 3 | + 2 |
| α Geminorum, | 3.8409 | - 2 | 0 | + 1 | 3.8386 | +21 | +18 | + 5 |
| α Canis Min. | 3.1462 | + 5 | + 9 | + 6 | 3.1455 | +12 | + 7 | 0 |
| β Geminorum, | 3.6828 | + 5 | + 3 | 0 | 3.6807 | +26 | +13 | - 3 |
| α Hydræ, | 2.9485 | + 12 | + 6 | - 1 | 2.9469 | +28 | +16 | - 1 |
| α Leonis, | 3.2030 | + 15 | + 6 | - 3 | 3.2014 | +31 | +13 | - 6 |
| β Leonis, | 3.0654 | + 11 | + 4 | - 3 | 3.0640 | +25 | +12 | - 3 |
| α Virginis, | 3.1495 | + 20 | + 5 | - 7 | 3.1497 | +18 | + 2 | - 9 |
| α Bootis, | 2.7325 | + 16 | + 3 | - 7 | 2.7327 | +14 | + 5 | - 3 |
| α^2 Libræ, | 3.3044 | + 6 | + 3 | - 1 | 3.3074 | -24 | - 6 | + 8 |
| α Coronæ, | 2.5378 | + 12 | + 1 | - 6 | 2.5373 | +17 | + 5 | - 5 |
| α Serpentis, | 2.9488 | + 8 | + 4 | - 1 | 2.9513 | -17 | - 7 | + 3 |
| α Scorpii, | 3.6654 | + 16 | + 4 | - 6 | 3.6672 | - 2 | - 4 | - 3 |
| α Herculis, | 2.7322 | + 7 | + 2 | - 2 | 2.7319 | + 9 | 0 | - 5 |
| α Ophiuchi, | 2.7808 | + 10 | 0 | - 6 | 2.7783 | +34 | +15 | - 5 |
| α Lyræ, | 2.0312 | + 2 | - 1 | - 2 | 2.0305 | + 9 | + 1 | - 4 |
| γ Aquilæ, | 2.8520 | + 9 | + 2 | - 3 | 2.8546 | -17 | - 8 | + 2 |
| α Aquilæ, | 2.9281 | + 4 | + 1 | - 1 | 2.9281 | + 4 | - 3 | - 5 |
| β Aquilæ, | 2.9466 | + 5 | + 3 | 0 | 2.9496 | -25 | -12 | + 3 |
| α^2 Capricornii, | 3.3338 | + 4 | + 2 | 0 | 3.3349 | - 7 | - 6 | - 2 |
| α Aquarii, | 3.0829 | + 13 | + 4 | - 4 | 3.0822 | +20 | + 4 | - 8 |
| α Piscis Aust. | 3.3311 | + 8 | + 4 | - 1 | 3.3326 | - 7 | -16 | -12 |
| α Pegasi, | 2.9828 | + 11 | 0 | - 7 | 2.9830 | + 9 | - 1 | - 6 |
| Sum, | | +254 | +81 | -72 | | +210 | +71 | -55 |
| Mean, | | +8.5 | +0.27 | -0.24 | | +7.0 | +0.24 | -0.18 |

| II. DECLINATIONS. | | | | | | |
|------------------------|-------------------------------------|-------|-------|-------|----------------------------|-------|
| | Corrections to Tabulæ Regiomontanæ. | | | | Corrections to Le Verrier. | |
| | 1780. | 1800. | 1820. | 1840. | 1820. | 1840. |
| | " | " | " | " | " | " |
| α Andromedæ, | +0.3 | +0.2 | +0.1 | 0.0 | +0.2 | 0.0 |
| γ Pegasi, | -0.2 | +0.1 | +0.4 | +0.8 | +0.1 | +0.3 |
| α Arietis, | -0.3 | 0.0 | +0.3 | +0.5 | +0.2 | +0.3 |
| α Ceti, | -1.0 | -0.1 | +0.7 | +1.6 | +0.1 | +0.7 |
| α Tauri, | 0.0 | +0.1 | +0.1 | +0.2 | +0.1 | 0.0 |
| β Orionis, | -0.5 | +0.1 | +0.6 | +1.2 | +0.3 | +0.7 |
| β Tauri, | -0.5 | 0.0 | +0.6 | +1.1 | +0.2 | +0.5 |
| α Orionis, | -1.2 | -0.6 | 0.0 | +0.6 | -0.2 | +0.3 |
| α Geminorum, | -0.8 | -0.5 | -0.1 | +0.3 | +0.5 | +1.0 |
| α Canis Min. | -0.2 | 0.0 | +0.3 | +0.6 | +0.2 | +0.5 |
| β Geminorum, | -0.2 | 0.0 | +0.3 | +0.6 | +0.2 | +0.5 |
| α Hydræ, | -0.2 | +0.3 | +0.7 | +1.2 | +0.3 | +0.7 |
| α Leonis, | +0.3 | +0.4 | +0.4 | +0.5 | +0.4 | +0.5 |
| β Leonis, | +0.2 | +0.2 | +0.2 | +0.3 | +0.2 | +0.3 |
| α Virginis, | +0.2 | +0.6 | +1.1 | +1.5 | +0.5 | +0.8 |
| α Bootis, | +0.4 | +0.4 | +0.3 | +0.2 | +0.4 | +0.4 |
| α^2 Libræ, | +0.5 | +0.5 | +0.6 | +0.6 | +0.5 | +0.6 |
| α Coronæ, | +0.7 | +0.6 | +0.4 | +0.3 | +0.4 | +0.4 |
| α Serpentis, | +0.2 | +0.6 | +1.1 | +1.5 | +0.5 | +0.9 |
| α Scorpii, | +0.1 | +0.6 | +1.0 | +1.4 | +0.4 | +0.7 |
| α Herculis, | +0.6 | +0.8 | +1.0 | +1.3 | +0.5 | +0.7 |
| α Ophiuchi, | +0.4 | +0.5 | +0.5 | +0.6 | +0.4 | +0.6 |
| α Lyræ, | +0.7 | +0.8 | +0.9 | +1.0 | +0.4 | +0.4 |
| γ Aquilæ, | 0.0 | +0.3 | +0.6 | +1.0 | +0.2 | +0.4 |
| α Aquilæ, | +0.1 | +0.4 | +0.7 | +1.0 | +0.1 | +0.4 |
| β Aquilæ, | +0.1 | +0.6 | +1.2 | +1.7 | +0.5 | +0.8 |
| α^2 Capricorni, | +0.2 | +0.9 | +1.6 | +2.3 | +0.6 | +1.1 |
| α Aquarii, | -0.5 | +0.1 | +0.8 | +1.5 | +0.5 | +1.0 |
| α Piscis Aust. | +1.0 | +2.4 | +3.9 | +5.3 | +1.7 | +2.3 |
| δ Pegasi, | +0.4 | +0.3 | +0.2 | +0.1 | +0.3 | +0.2 |
| Mean | +0.03 | +0.36 | +0.69 | +1.03 | +0.36 | +0.60 |

The correction to the reductions to apparent place given in the *Tabulæ Regiomontanae* on account of the correction to the constant of Nutation is;—

In right ascension:
 $-0''.46 \sin \Omega - 0''.18 \sin \Omega \sin \alpha \tan \delta - 0''.24 \sin \Omega \cos \alpha \tan \delta.$

In declination:
 $-0''.18 \sin \Omega \cos \alpha + 0.24 \cos \Omega \sin \alpha.$

The terms which contain $\tan \delta$ as a factor may be entirely neglected, as they are small, periodic, and contain $\tan \delta$ as a factor which is sometimes positive and sometimes negative. I shall also neglect the corrections in declination, as their sum is sensibly

$$0''.21 \sin (\alpha - \Omega)$$

the effect of which will generally be confounded with the accidental errors of observation.

The only correction we shall apply on account of nutation is, therefore,

$$\delta \alpha = -0''.030 \sin \Omega.$$

The values of this expression at the dates when it is zero, a maximum, or a minimum, are as follows:—

| y. | s. | y. | s. |
|--------|------|--------|------|
| 1778.5 | −.03 | 1820.3 | .00 |
| 1783.1 | .00 | 1825.0 | +.03 |
| 1787.7 | +.03 | 1829.6 | .00 |
| 1792.4 | .00 | 1834.3 | −.03 |
| 1797.0 | −.03 | 1838.9 | .00 |
| 1801.7 | .00 | 1843.6 | +.03 |
| 1806.3 | +.03 | 1848.2 | .00 |
| 1811.0 | .00 | 1852.9 | −.03 |
| 1815.6 | −.03 | 1857.5 | .00 |
| 1820.3 | .00 | | |

Having adopted this system of standard positions, we may adopt two ways of reducing the observations to it. One is to compare the positions of the stars adopted in the published reductions with the standard, and apply the mean difference to the reduced place of the planet. Another is to make a similar comparison of the standard catalogue with the positions of the fundamental stars which have been deduced from the observations by a system of reduction uniform with that employed in reducing the observations of the planet, and to regard the mean difference as a correction applicable to all the positions of the planet. If the standard catalogue and the observations are both free from systematic error, the results obtained in these two ways should be substantially identical. These are, however, conditions which we cannot expect to find fulfilled. In the following discussions I have sometimes used one, sometimes the other, and sometimes

combined both, the choice being determined by circumstances. We shall consider the different series of observations in succession.

Greenwich Observations from 1781 to 1830.

These observations are completely reduced by Airy and compared with Bouvard's Tables, in the work *Reduction of the Observations of Planets made at the Royal Observatory, Greenwich, from 1750 to 1830*. London, 1845. The concluded positions given in this work depend mainly on the star places of the *Tabulæ Regiomontanæ*, both in right ascension and declination. If we consider the first four oppositions—1781-1785—as forming a single group of which the mean epoch is 1783, we find that the general correction to the *Tabulæ Regiomontanæ* for this epoch is

$$\begin{aligned} \text{In right ascension,} & \quad -0^{\circ}.030; \\ \text{In declination,} & \quad +0^{\circ}.08. \end{aligned}$$

If, on the other hand, we consider only the particular stars compared with Uranus, the result will be a little different. The number of times each of the fundamental stars has been compared with Uranus, and the correction in right ascension corresponding to each star, are nearly as follows:—

| | | | |
|-------------------------|---------|---------------------------|---------------------------------|
| α Arietis, | $N = 2$ | Cor. = -0.09^{s} | $N \times C = -0.18^{\text{s}}$ |
| α Tauri, | 2 | -0.01 | -0.02 |
| γ Pegasi, | 2 | -0.03 | -0.06 |
| β Tauri, | 19 | $+0.13$ | $+2.47$ |
| α Orionis, | 33 | $+0.02$ | $+0.66$ |
| α Canis Minoris, | 33 | -0.02 | -0.66 |
| β Geminorum, | 34 | -0.07 | -2.38 |
| α Leonis, | 7 | -0.11 | -0.77 |
| β Leonis, | 2 | -0.07 | -0.14 |

The mean correction from these data comes out $-0^{\circ}.008$, differing by $0^{\circ}.022$ from the general mean correction. Our choice between the two corrections depends on whether we are to consider the relative positions of the *Tabulæ Regiomontanæ*, or those of the standard catalogue, as nearest the truth at the epoch 1783, and particularly upon whether we are to consider the large correction to the proper motion of β Tauri as real. In the absence of exact data for settling this question, the mean of the two results, or $-0^{\circ}.020$, has been adopted.

A similar anomaly is exhibited by the declinations. It is probable that the declinations of Uranus during this period mainly depend on stars in the first twelve hours in right ascension, for which the mean correction is about $-0^{\circ}.30$ instead of $+0^{\circ}.08$. I have adopted $-0^{\circ}.16$. Changing these corrections to longitude and latitude, we have, during the period 1781-1786:—

$$\begin{aligned} \text{Correction to observed longitude,} & \quad = -0^{\circ}.30; \\ \text{Correction to observed latitude,} & \quad -0^{\circ}.19. \end{aligned}$$

During the years 1788-1798 the above systematic difference in right ascension does not appear. The most probable correction seems to be

$$\begin{array}{l} \Delta\alpha = -0^{\circ}.025; \quad \Delta\delta = 0^{\circ}.00. \\ \text{Whence} \quad \Delta \text{ long.} = -0^{\circ}.34; \quad \Delta \text{ lat.} = -0^{\circ}.10. \end{array}$$

Between the years 1800 and 1823 the stars used for comparison are so widely scattered that I consider it safe to apply only the general mean correction for the epoch 1813, which is

$$\begin{array}{l} \Delta\alpha = -^{\circ}.005; \quad \Delta\delta = +0^{\circ}.66. \\ \text{Whence} \quad \Delta \text{ long.} = 0^{\circ}.00; \quad \Delta \text{ lat.} = +0^{\circ}.66. \end{array}$$

From 1825 to 1830 more than half the weight of the right ascension comes upon the stars α , β , and γ Aquilæ, the mean correction to which, during this interval, is $-0^{\circ}.035$. The general mean correction at this epoch is $+0^{\circ}.002$. I think the right ascensions of these three stars in the *Tabulæ Regiomontanæ* are really too great at this epoch by the entire difference of these results. We may, in fact, hereafter regard the positions of the standard catalogue as sufficiently accurate. The mean corrections to be applied will then be

$$\begin{array}{l} \Delta\alpha = -0^{\circ}.017; \quad \Delta\delta = +0^{\circ}.83. \\ \text{Whence} \quad \Delta \text{ long.} = -0^{\circ}.05; \quad \Delta \text{ lat.} = +0^{\circ}.86. \end{array}$$

From the year 1831 until the present time the Greenwich observations are regularly reduced in the several annual volumes of observations. But a reduction of the observations from 1831 to 1835, executed by Mr. Hugh Breen, is given in an appendix to the volume for the year 1864. The results here given differ from those published by Pond in the several annual volumes for the same interval. The right ascensions are altered only by applying the constant correction $-0^{\circ}.030$, which is found necessary to reduce Pond's right ascensions to those of the *Tabulæ Regiomontanæ*. This correction I have verified. The mean correction to reduce the right ascensions of the *Tabulæ Regiomontanæ* to our standard is at this time $+0^{\circ}.005$. On the other hand, when we compare the concluded right ascensions of stars within six hours of Uranus, as given by Pond in the Greenwich observations for 1834, with our standard, we find a mean correction of $-^{\circ}.034$ to reduce his positions to the standard, which implies a correction $-^{\circ}.004$ to Breen's reduction. The two results being $+^{\circ}.005$ and $-^{\circ}.004$, I have applied no correction whatever.

In the paper in question the declinations are completely re-reduced, using improved data of reduction, but, so far as I see, making no changes in Pond's method. The results differ strikingly from those of Pond, and suggest the desirableness of a complete re-examination of all Pond's determinations of declination. Having no catalogue of observed declinations of standard stars reduced in this same way, we cannot directly determine the systematic correction to the declinations. I therefore proceed as follows: A comparison of Pond's observed declinations of standard stars with Auwers' normal catalogue show that the former require the following corrections near the parallel of Uranus:

In 1831 — 1".42;
1834 — 2.10.

Then comparing Airy's reduced declinations of Uranus with Pond's, we find the following mean differences:

In 1831, Airy — Pond = — 3".18
1834, — 3.50.

To reduce Airy to Auwers we must there apply to the declinations

In 1831 + 1".76
1834 + 1.40.

I have regarded the correction + 1".60 as applicable throughout the period in question.

1836-72.

During this interval the corrections in right ascension have been derived by the following two sets of comparisons: (1) A comparison of the several collected six and seven year catalogues with Gould's standard, from which it appears that they require the following general corrections in right ascension:

| | |
|------------------------------|----------|
| Six year catalogue of 1840 | + 0".047 |
| Six year catalogue of 1845 | + 0.002 |
| Seven year catalogue of 1860 | + 0.003 |
| Seven year catalogue of 1864 | + 0.022 |

(2) A comparison of the corrections applied to the right ascensions of the individual years to reduce them to the standard of the catalogue, as given in the introduction to each catalogue. The sum of these two numbers gives the corrections for each year.

A slightly different method is to regard the above correction for each catalogue as applicable to all right ascensions which depend fundamentally upon that catalogue. I have sometimes combined both methods so as to derive what seemed to be the most probable result, and sometimes used but one.

The corrections to the declinations during the interval in question have been derived from Auwers' "*Tafeln zur Reduction der Declinationen verschiedener Sternverzeichnisse auf ein Fundamentalsystem*," *Astronomische Nachrichten*, No. 1536. These tables include the Greenwich seven year catalogue for 1860, when the correction corresponding to the declination of Uranus is about + 0".45. The corrections for the previous catalogues vary between 0".35 and 0".68. The correction corresponding to the interval 1861-67 has been derived by a direct comparison with Auwers' declinations, and the result is + 0".44, agreeing with the two preceding catalogues. But, on making a similar comparison with the annual catalogue for 1869, a considerable change was found, the correction being — 0".17, a change of more than half a second. I shall use this correction for and after the beginning of 1868, as the change is probably due to the introduction of a new constant of refraction in the reduction of the observations for 1868 and subsequent years.

Cambridge.

An extended series of planetary observations was commenced here by Professor Airy, in 1827. The series was continued by him and Professor Challis, his successor, until 1842. During the first three or four years the combined right ascensions depend on a few special stars, and mainly on α^2 Capricorni. Taking the mean correction to the adopted right ascensions of the stars actually compared as they are given in the introduction to each annual volume, giving to each star a weight proportional to the number of comparisons, the following corrections are deduced:

| | |
|---------|---------|
| 1828 | — 0°.10 |
| 1829-31 | — 0.16 |
| 1832-37 | — 0.19. |

In the introduction to the volume for 1838 it is stated that the adopted right ascensions are diminished by the average amount of 0°.083, which would still leave a correction of — 0°.107. Actual comparisons in two subsequent years give

| | |
|-------|---------------------------|
| 1840, | $\Delta\alpha = - 0°.087$ |
| 1842, | — 0.069. |

Although the positions deduced from each year's work were adopted for clock correction the year following, without any change of equinox, it seems that there was, effectively, a progressive change of about 0°.01 annually in the equinox as adopted.

No declinations were observed until 1830. On comparing the declinations deduced from several years' work with Auwers, it was evident that the correction increased with the polar distance of the star. The law of increase could be well enough represented by supposing the correction proportional to N. P. D. Thus the following corrections were deduced in three different years.

| | |
|-------|--|
| 1834, | $\delta \text{ dec.} = - \frac{1''.78 \times \text{N. P. D. in degrees}}{100}$ |
| 1840, | $- \frac{1''.00 \times \text{N. P. D. in degrees}}{100}$ |
| 1842, | $- \frac{1''.03 \times \text{N. P. D. in degrees}}{100}$. |

From which the correction for other years was deduced by interpolation. But, on applying these corrections, the results were found systematically different from those of other observatories, and on referring to Auwers' corrections to Airy's Cambridge Catalogue, it appeared that the mural circle required a large correction near the declination of Uranus during this period. The above results were therefore altered so as to conform as nearly as practicable to Auwers' law.

Edinburgh.

In reducing the observations of 1836 Henderson uses the right ascensions of the *Tabulæ Regiomontanæ*, to which the general correction is at this epoch +°.007.

But, if we take only the stars near Uranus, with which the latter was necessarily most frequently compared, the corrections will be negative. Comparing the concluded positions of the stars from α Serpentis through 0^h to β Orionis, we find the following mean corrections:

In right ascension, $-0^s.012$; in declination, $-0^s.09$.

In subsequent years it is stated that the adopted positions of clock stars used each year are derived from the right ascensions observed at Greenwich, Cambridge, and Edinburgh, during the year or the two years preceding, without any statement whether corrections were applied for difference of equinoxes. In some subsequent years the following corrections are deduced, sometimes from the adopted and sometimes from the concluded positions:

1837, $\Delta\alpha = 0^s.000$;
 1840, $\Delta\alpha = +0.015$; $\Delta \text{Dec.} = 0^s.00$;
 1844, $\Delta\alpha = +0.070$; $\Delta \text{Dec.} = +0.49$.

Paris.

All the positions of planets given by Le Verrier, in his "*Annales de l'Observatoire Imperial de Paris: Observations*" depend both in right ascension and N. P. D. on his adopted positions of fundamental stars, the corrections to which have already been given. As the corrections to the individual star places used by Le Verrier are not generally of a systematic character, the general mean correction is employed, which is:—

In right ascension $-0^s.024 + 0^s.085T$,
 In declination $+0^s.12 + 1^s.20T$,

T being the fraction of a century after 1800.

In 1854 a new and larger catalogue was introduced, and for this and the following years the correction in declination is derived from Auwers' tables.

A summary of the adopted corrections after 1830, as deduced from the preceding comparisons and discussions, is given in the following table:—

| Year. | Greenwich. | | Paris. | | Königsberg. | | Berlin. | | Cambridge. | | Edinburgh. | |
|-------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| | $\Delta\alpha$
s | $\Delta\delta$
" | $\Delta\alpha$
s | $\Delta\delta$
" | $\Delta\alpha$
s | $\Delta\delta$
" | $\Delta\alpha$
s | $\Delta\delta$
" | $\Delta\alpha$
s | $\Delta\delta$
" | $\Delta\alpha$
s | $\Delta\delta$
" |
| 1830 | | | | | .00 | | | +0.9 | -.16 | -1.7 | .00 | |
| 1831 | .00 | +1.8 | .00 | +0.5 | -.02 | +0.9 | | +0.9 | -.16 | -1.6 | | |
| 1832 | | +1.7 | | | -.02 | +0.9 | | | -.19 | -1.5 | | |
| 1833 | | +1.6 | | | | | -.04 | | | -1.4 | | |
| 1834 | | +1.4 | | | | | -.05 | | | -1.3 | | |
| 1835 | | +1.2 | | | | | -.05 | | | -1.2 | | |
| 1836 | -.04 | +1.0 | +0.01 | +0.6 | -.02 | +1.0 | -.04 | +1.0 | | -1.0 | -.01 | 0 |
| 1837 | -.03 | +0.9 | | | | | -.03 | | | -0.8 | .00 | 0 |
| 1838 | -.04 | +0.8 | | | | | -.02 | | -.11 | -0.5 | +0.01 | 0 |
| 1839 | | +0.7 | | | | +1.0 | -.01 | | -.10 | -0.3 | +0.01 | 0 |
| 1840 | | +0.6 | | | | | .00 | | -.09 | -0.2 | +0.02 | +0.1 |
| 1841 | | +0.5 | | | | | +0.01 | | -.08 | -0.2 | -.04 | +0.2 |

TABLE OF ADOPTED SYSTEMATIC CORRECTIONS.—Continued.

| Year. | Greenwich. | | Paris. | | Königsberg. | | Berlin. | | Cambridge. | | Edinburgh. | |
|---------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| | $\Delta\alpha$
s | $\Delta\delta$
" | $\Delta\alpha$
s | $\Delta\delta$
" | $\Delta\alpha$
s | $\Delta\delta$
" | $\Delta\alpha$
s | $\Delta\delta$
" | $\Delta\alpha$
s | $\Delta\delta$
" | $\Delta\alpha$
s | $\Delta\delta$
" |
| 1842 | -.02 | +0.4 | +.01 | +0.6 | | | +.03 | +1.1 | -.07 | -0.3 | -.02 | +0.3 |
| 1843 | +.07 | +0.4 | +.01 | +0.6 | | | +.04 | +1.1 | | | +.07 | +0.4 |
| 1844 | +.06 | +0.4 | +.01 | +0.6 | | | +.04 | +1.1 | | | +.07 | +0.5 |
| 1845 | +.08 | +0.4 | +.01 | +0.6 | | | +.04 | +1.1 | | | | |
| 1846 | +.04 | +0.4 | +.02 | +0.6 | | | +.03 | +1.1 | | | | |
| 1847 | +.05 | +0.4 | +.02 | +0.6 | | | +.02 | +1.1 | | | | |
| 1848 | +.05 | +0.4 | +.02 | +0.6 | | | +.01 | +1.2 | | | | |
| 1849-53 | .00 | +0.4 | +.02 | +0.6 | | | .00 | +1.2 | Santiago | | | |
| 1854-55 | .00 | +0.5 | +.02 | +0.2 | | | -.01 | +1.2 | .00 | +0.6 | | |
| 1856-60 | -.01 | +0.5 | +.02 | +0.2 | | | | | | | Washington | |
| 1861 | -.01 | +0.4 | +.03 | +0.2 | | | | | | | | |
| 1862-65 | .00 | +0.4 | +.03 | +0.2 | | | | | | | .00 | -0.5 |
| 1866 | .00 | +0.4 | +.03 | +0.2 | | | | | | | .00 | +1.1 |
| 1867 | .00 | +0.4 | +.03 | +0.2 | | | | | | | .00 | +1.1 |
| 1868 | .00 | -0.2 | +.03 | +0.2 | | | | | | | .00 | +1.2 |
| 1869 | .00 | -0.2 | +.03 | +0.2 | | | | | | | .00 | +0.6 |
| 1870 | +.01 | -0.2 | | | | | | | | | .00 | +0.4 |
| 1871 | +.01 | -0.2 | | | | | | | | | .00 | +0.4 |
| 1872 | +.02 | -0.2 | | | | | | | | | .00 | +0.4 |

Applying the preceding corrections to the positions of the planet as originally reduced and published, we have a series of observed positions as nearly homogeneous as it is possible to make them with the means now at our command. The next step in order will be the computation of the geocentric place of the planet from the provisional theory for the moment of every observation, to be compared with the results of the latter. The complete execution of this labor, *ab initio*, is, however, at present impracticable, and it is proposed to diminish it by making use of the published comparisons with the older tables. This can be done without danger of serious error, and with all the more ease that owing to the great distance of Uranus the errors of the solar tables are, for the most part, without appreciable effect upon the computed geocentric place of the planet. The method of making the comparison is different with different series of observations, and each series must therefore be described and discussed separately. The general plan, however, has been to replace observed and computed absolute positions by observed and computed corrections to the geocentric positions deduced from Bouvard's Tables. To carry out this plan it is necessary to have at our disposal an ephemeris both of the heliocentric and geocentric positions derived from these tables. The corrections to the latter given by the observations are then given by direct comparison. To obtain the corrections given by the provisional theory, the heliocentric longitudes, latitudes, and radii vectores given by that theory are interpolated to the dates of the heliocentric ephemeris from Bouvard's Tables, and compared with that ephemeris. The differences are then changed to differences of geocentric place by the usual differential formulæ, and thus the corrections given by theory are derived. The difference between the two sets of corrections is the difference

between the provisional theory and observation. A condensed summary of the results for each of the principal series of observations is here presented.

Greenwich, 1781-1830.

In Airy's reductions, already referred to, we have given for the moment of each individual observation a heliocentric place computed from Bouvard's Tables, and the geocentric longitudes and latitudes thence deduced. The observed right ascensions and declinations are then changed to longitudes and latitudes, and the apparent error of the tables thence deduced. The means of these errors are taken for groups of observations, and expressed in terms of the errors of heliocentric longitude, radius vector, and latitude. The mode in which these means have been treated is fully shown in the following table. The first column gives the mean date of each individual group of observations. The next three give the mean excesses of the co-ordinates interpolated from the heliocentric ephemeris, p. 100, and corrected for solar nutation, over those printed in the "Computations of tabular place, etc.," in the Greenwich reductions. In the fifth column these corrections are changed to corrections of geocentric longitude. In the next two columns we have the mean corrections to Bouvard's geocentric places given by observation. It is the negative of the mean error of tabular place printed in the "Reductions," corrected by the numbers already given to reduce the star places to a uniform system. Then we have the difference between these two sets of corrections, or, the mean correction to the geocentric place of the provisional theory as given by observation. Lastly, we have the differential coefficients for expressing the errors of geocentric in terms of the errors of heliocentric co-ordinates taken without change from the Greenwich volume.

| Mean Date. | From Provisional Theory. | | | | From Observations. | | | Correction to Prov. Theory. | | $\frac{\partial l}{\partial \lambda}$ | $\frac{\partial l}{\partial \rho}$ |
|---------------|---|------------|------|-------------|--------------------|-----------|-------------|-----------------------------|-----------|---------------------------------------|------------------------------------|
| | Correction to tabular position in Greenwich Reductions. | | | | Correction to | | | Geoc. long. | Hel. lat. | | |
| | Long. | Log. R. V. | Lat. | Geoc. long. | Geoc. long. | Hel. lat. | No. of Obs. | Geoc. long. | Hel. lat. | | |
| | " | | " | | " | " | | " | " | | |
| 1781, Oct. 10 | - 7.8 | | +1.1 | - 7.5 | - 6.4 | + 2.6 | 4 | +1.1 | + 1.5 | 1.01 | - 9 |
| Nov. 13 | - 7.5 | | +1.1 | - 7.5 | - 4.9 | - 1.6 | 4 | +2.6 | - 2.7 | 1.04 | - 6 |
| Dec. 27 | - 7.3 | - 159 | +1.0 | - 7.7 | - 5.6 | - 2.3 | 5 | +2.1 | - 3.3 | 1.05 | + 1 |
| 1782, Jan. 31 | - 7.4 | | +0.9 | - 8.0 | - 4.9 | - 1.1 | 4 | +3.1 | - 2.0 | 1.04 | + 6 |
| Mar. 4 | - 6.6 | | +0.9 | - 7.1 | - 4.6 | - 0.6 | 4 | +2.5 | - 1.5 | 1.01 | + 9 |
| Oct. 10 | - 5.6 | | +0.4 | - 5.3 | - 3.6 | - 4.0 | 4 | +1.7 | - 4.4 | 1.01 | - 9 |
| Nov. 26 | - 5.3 | | +0.3 | - 5.3 | - 6.0 | - 5.0 | 3 | -0.7 | - 5.3 | 1.04 | - 5 |
| 1783, Jan. 11 | - 5.0 | | +0.3 | - 5.4 | - 3.9 | - 3.7 | 3 | +1.4 | - 4.0 | 1.05 | + 3 |
| Feb. 24 | - 4.4 | | +0.2 | - 4.8 | - 2.2 | - 2.4 | 3 | +2.6 | - 2.6 | 1.02 | + 9 |
| Oct. 10 | - 3.5 | | -0.3 | - 3.2 | - 0.3 | - 1.0 | 2 | +2.9 | - 0.7 | 1.00 | -10 |
| Nov. 1 | - 3.2 | | -0.3 | - 3.0 | - 1.1 | - 2.3 | 2 | +1.9 | - 2.0 | 1.02 | - 9 |
| Dec. 15 | - 3.3 | - 121 | -0.4 | - 3.4 | - 2.2 | - 3.8 | 3 | +1.2 | - 3.4 | 1.05 | - 3 |
| 1784, Jan. 29 | - 3.1 | | -0.4 | - 3.4 | - 2.4 | - 2.4 | 3 | +1.0 | - 2.0 | 1.05 | + 5 |
| Mar. 12 | - 2.6 | | -0.5 | - 2.9 | - 0.3 | - 4.3 | 3 | +2.6 | - 3.8 | 1.01 | + 9 |
| Oct. 30 | - 1.8 | | -0.8 | - 1.5 | - 0.7 | + 1.4 | 2 | +0.8 | + 2.2 | 1.02 | - 9 |
| Dec. 14 | - 1.5 | - 128 | -0.9 | - 1.5 | - 2.4 | + 2.7 | 2 | -0.9 | + 3.6 | 1.05 | - 4 |

| Mean Date. | From Provisional Theory. | | | | From Observations. | | | Correction to Prov. Theory. | | $\frac{\partial l}{\partial \lambda}$ | $\frac{\partial l}{\partial \rho}$ |
|---------------|---|------------|------|-------------|--------------------|-----------|-------------|-----------------------------|-----------|---------------------------------------|------------------------------------|
| | Correction to tabular position in Greenwich observations. | | | | Correction to | | | Geoc. long. | Hel. lat. | | |
| | Long. | Log. R. V. | Lat. | Geoc. long. | Geoc. long. | Hel. lat. | No. of Obs. | | | | |
| | " | | " | | " | " | | " | " | | |
| 1785, Jan. 16 | - 1.2 | | -1.0 | - 1.3 | - 1.4 | - 1.4 | 2 | -0.1 | - 0.4 | 1.05 | + 2 |
| Feb. 13 | - 1.0 | | -1.1 | - 1.3 | - 1.2 | - 1.1 | 2 | +0.1 | 0.0 | 1.04 | + 7 |
| Mar. 20 | - 0.6 | | -1.1 | - 0.9 | - 2.1 | - 5.2 | 2 | -1.2 | - 4.1 | 1.01 | +10 |
| Nov. 8 | + 0.2 | | -1.5 | + 0.5 | + 1.2 | - 1.7 | 2 | +0.7 | - 0.2 | 1.02 | - 9 |
| 1788, Mar. 13 | + 2.1 | - 187 | -3.2 | + 1.8 | + 4.4 | - 4.9 | 3 | +2.6 | - 1.7 | 1.03 | + 9 |
| Oct. 26 | + 3.4 | | -3.6 | + 3.9 | + 3.0 | - 2.2 | 2 | -0.9 | + 1.4 | 1.00 | -10 |
| 1789, Jan. 18 | + 4.2 | - 224 | -3.7 | + 4.5 | +10.4 | -14.5 | 1 | (+5.9) | (-10.8) | 1.06 | - 1 |
| Apr. 8 | + 4.7 | | -3.8 | + 4.2 | + 9.4 | - 3.7 | 4 | +5.2 | + 0.1 | 1.01 | +10 |
| Oct. 31 | + 4.8 | | -4.2 | + 5.4 | + 4.7 | - 1.1 | 2 | -0.7 | + 3.1 | 1.00 | -10 |
| 1790, Jan. 24 | + 4.9 | | -4.4 | + 5.2 | + 3.8 | - 2.6 | 2 | -1.4 | + 1.8 | 1.06 | - 1 |
| Nov. 5 | + 5.1 | - 254 | -4.8 | + 5.8 | + 5.0 | - 2.3 | 3 | -0.8 | + 2.5 | 1.00 | -10 |
| 1791, Jan. 29 | + 5.1 | - 258 | -5.0 | + 5.4 | + 4.5 | - 3.8 | 3 | -0.9 | + 1.2 | 1.06 | 0 |
| Apr. 14 | + 5.0 | | -5.2 | + 4.5 | + 2.7 | - 4.8 | 1 | -1.8 | + 0.4 | 1.01 | +10 |
| Nov. 10 | + 5.0 | - 234 | -5.4 | + 5.6 | + 5.7 | - 4.0 | 2 | +0.1 | + 1.4 | 1.00 | -10 |
| 1792, Feb. 5 | + 5.2 | | -5.6 | + 5.5 | + 4.2 | - 4.0 | 1 | -1.3 | + 1.6 | 1.06 | 0 |
| Nov. 15 | + 5.6 | | -6.0 | + 6.2 | + 3.7 | - 2.5 | 3 | -2.5 | + 3.5 | 1.00 | -10 |
| 1793, Feb. 8 | + 5.4 | | -6.1 | + 5.7 | + 8.5 | - 5.0 | 2 | +2.8 | + 1.1 | 1.06 | 0 |
| Nov. 14 | + 5.6 | - 135 | -6.4 | + 5.9 | +10.4 | - 6.3 | 1 | +4.5 | + 0.1 | 0.99 | -10 |
| 1794, Feb. 15 | + 5.7 | - 100 | -6.5 | + 6.0 | + 7.3 | - 6.3 | 2 | +1.3 | + 0.2 | 1.06 | 0 |
| Nov. 19 | + 5.4 | - 67 | -6.7 | + 5.6 | + 4.1 | - 5.1 | 4 | -1.5 | + 1.6 | 1.00 | -10 |
| 1795, Feb. 20 | + 5.9 | - 46 | -6.9 | + 6.3 | + 6.0 | - 8.0 | 3 | -0.3 | - 1.1 | 1.06 | 0 |
| Nov. 29 | + 5.4 | - 13 | -7.0 | + 5.5 | + 3.9 | - 3.4 | 2 | -1.6 | + 3.6 | 1.00 | -10 |
| 1796, Feb. 24 | + 5.3 | - 8 | -7.2 | + 5.6 | + 4.5 | - 4.5 | 2 | -1.1 | + 2.7 | 1.06 | 0 |
| 1797, Feb. 27 | + 5.1 | + 62 | -7.5 | + 5.4 | + 4.7 | - 4.2 | 3 | -0.7 | + 3.3 | 1.06 | 0 |
| 1800, Mar. 14 | + 4.6 | + 224 | -8.4 | + 4.9 | + 4.9 | - 8.7 | 2 | 0.0 | - 0.3 | 1.06 | 0 |
| 1814, May 22 | - 1.2 | + 217 | -2.2 | - 1.3 | + 1.1 | + 0.2 | 2 | +2.4 | + 2.4 | 1.06 | 0 |
| 1815, May 25 | - 1.1 | + 214 | -1.5 | - 1.2 | + 1.4 | - 1.0 | 4 | +2.6 | + 0.5 | 1.06 | 0 |
| 1818, June 10 | - 0.5 | + 413 | -0.8 | - 0.5 | - 1.1 | + 4.8 | 2 | -0.6 | + 5.6 | 1.06 | 0 |
| 1819, June 14 | - 0.4 | + 483 | +1.5 | - 0.4 | - 1.4 | + 3.7 | 4 | -1.0 | + 2.2 | 1.06 | 0 |
| 1820, June 16 | - 0.1 | + 498 | +2.2 | - 0.1 | - 2.6 | + 4.0 | 2 | -2.5 | + 1.8 | 1.06 | 0 |
| 1823, July 1 | 0.0 | + 538 | +4.4 | 0.0 | + 0.2 | + 5.0 | 4 | +0.2 | + 0.6 | 1.05 | 0 |
| 1825, July 11 | - 2.4 | + 567 | +5.8 | - 2.5 | - 4.8 | + 7.8 | 2 | -2.3 | + 2.0 | 1.05 | 0 |
| 1826, July 16 | - 4.5 | + 582 | +6.6 | - 4.7 | - 3.6 | + 8.3 | 4 | +1.1 | + 1.7 | 1.05 | 0 |
| 1827, July 20 | - 6.6 | + 626 | +7.2 | - 6.9 | - 7.2 | + 9.6 | 6 | -0.3 | + 2.4 | 1.05 | 0 |
| 1828, July 23 | - 9.8 | + 710 | +7.9 | -10.3 | - 6.9 | + 6.9 | 3 | +3.4 | - 1.0 | 1.05 | 0 |
| 1829, Aug. 7 | -13.4 | + 835 | +8.5 | -13.7 | -16.3 | +10.0 | 14 | -2.6 | + 1.5 | 1.08 | + 2 |
| Oct. 4 | -14.2 | + 865 | +8.6 | -12.5 | -13.0 | + 9.9 | 8 | -0.5 | + 1.3 | 1.02 | + 8 |
| 1830, July 30 | -17.8 | + 970 | +9.0 | -18.8 | -21.4 | +11.1 | 3 | -2.6 | + 2.1 | | |
| Aug. 29 | -18.2 | + 982 | +9.1 | -17.9 | -20.0 | +10.2 | 5 | -2.1 | + 1.1 | | |
| Sept. 20 | -18.5 | + 992 | +9.1 | -17.2 | -19.1 | + 9.1 | 12 | -1.9 | 0.0 | | |
| Oct. 14 | -18.9 | +1009 | +9.1 | -16.8 | -17.3 | + 9.9 | 11 | -0.5 | + 0.8 | | |
| Nov. 13 | -19.3 | +1009 | +9.2 | -16.8 | -17.8 | + 9.3 | 8 | -1.0 | + 0.1 | | |

Paris, 1801-1827.

A complete reduction of this series is found in Le Verrier's *Annales de l'Observatoire Imperial de Paris, Observations*, tome I. No comparison with any ephemeris is given here, nor is there any complete ephemeris to compare them with. A complete geocentric ephemeris was therefore computed from the provisional theory for the principal groups of the Paris observations. The individual observations being compared with it, the resulting mean corrections are given in the following table:

| Mean date. | $\Delta\alpha$ | $\Delta\delta$ | <i>N.</i> | Mean date. | $\Delta\alpha$ | $\Delta\delta$ | <i>N.</i> |
|------------------|----------------|----------------|-----------|----------------|----------------|----------------|-----------|
| 1801, March 24, | -.02 | +1".2 | 2 | 1813, May 20, | +.19 | +1".8 | 6 |
| 1802, April 1, | +.08 | +0.6 | 13 | 1814, May 27, | +.21 | +0.8 | 4 |
| 1805, April 22, | +.10 | +2.2 | 13 | 1815, May 24, | -.02 | +2.2 | 5 |
| 1806, April 17, | -.01 | -1.6 | 5 | 1816, June 1, | -.01 | +0.8 | 7 |
| 1807, April 28, | +.17 | +0.4 | 16 | 1817, June 5, | -.08 | +1.6 | 5 |
| 1808, April 28, | +.02 | +1.4 | 6 | 1818, June 7, | +.12 | +2.2 | 9 |
| 1809, May 5, | +.20 | +0.1 | 9 | 1819, June 18, | -.07 | +1.8 | 7 |
| 1810, April 30, | +.22 | +2.6 | 16 | 1820, June 20, | -.20 | -2.4 | 8.5 |
| 1811, Febr'y 18, | +.21 | +2.2 | 3 | 1821, June 22, | +.05 | +1.0 | 5 |
| 1811, May 17, | +.14 | +2.6 | 8 | 1823, July 18, | +.02 | +1.8 | 5 |
| 1812, Febr'y 16, | +.28 | | 2 | 1824, July 13, | +.04 | 0.0 | 7 |
| 1812, May 10, | +.16 | +3.0 | 6 | 1827, July 25, | -.05 | +0.6 | 5 |
| 1813, Febr'y 25, | +.44 | | 3 | | | | |

Total number of observations in right ascension, 175.

The observations in this series exhibit numbers of discordances of that class which leave the astronomer in doubt whether the observation should be retained or rejected. This remark applies more especially to the declinations. If we determine the probable error of an observation in declination by the condition that it is that amount which the error falls short of as often as it exceeds, it is found to be about 2". Then, if the errors followed the commonly assumed law of probability, only about one in six of the errors should exceed 4", and one in twenty-three 6". But errors of these magnitudes are much more numerous, the deviations often amounting to six or eight seconds. I have rejected only a few in which the discordances approached 10".

Bessel's Königsberg Observations, 1814-1835.

I have made a complete re-reduction of the right ascensions of this important series, and of most of the declinations. In order to avoid the necessity of applying systematic corrections, Dr. Gould's right ascensions and Dr. Auwers' declinations were used throughout in these reductions. In this work a selection of the fundamental stars observed by Bessel was made for each observation of the planet, to be used for clock error. These were chosen so that the mean of their right ascensions and declinations should be as near as practicable to those of Uranus, a condition, however, which could not generally be fulfilled for the declinations, owing to the southern position of the planet. Bessel's instrumental cor-

rections were applied to his observed times of transit over the mean wire, and the resulting time was employed as that of transit. Each time, compared with the computed right ascension of the star gave a value of the clock correction, which was reduced to the time of transit of the planet by the known daily rate. If the instrumental errors were always accurately determined, the mean of these clock corrections would be used to obtain the right ascension of Uranus. But it was frequently found that the clock error varied systematically with the declination of the star, so that it was deemed advisable to add to the clock correction a term varying as the simple declination, which was deduced from all the stars, and used to reduce the correction to the parallel of Uranus.

It was intended to give the results of this reduction for each observation, but on comparing the results with those of Fleming in the *Astronomische Nachrichten*, Band 30, it appeared that the results were not materially better than his. It does not, therefore, seem necessary to give more than the mean results for each opposition.

From Bessel's declinations, with the old Cary circle, I was unable to obtain any satisfactory results, owing, apparently, to a want of knowledge of some peculiarity of the instrument. Fleming's reductions were therefore adopted. They are designated by the letter *F* in the following list.

Mean Corrections to the Provisional Ephemeris given by Bessel's Observations at Königsberg, 1814-1829.

| Mean date. | $\Delta\alpha$ | $\Delta\delta$ | <i>N.</i> | Mean date. | $\Delta\alpha$ | $\Delta\delta$ | <i>N.</i> |
|----------------|--------------------|----------------|-----------|----------------|--------------------|----------------|-----------|
| 1814, May 22, | + ^s .11 | +2'.5 <i>F</i> | 9 | 1822, June 24, | + ^s .10 | +1'.8 | 7 |
| 1815, May 25, | + .13 | +1.8 <i>F</i> | 11 | 1823, July 4, | - .05 | -1.6 <i>F</i> | 2 |
| 1816, May 27, | + .06 | +1.2 <i>F</i> | 11 | 1824, July 6, | + .01 | -1.0 <i>F</i> | 5 |
| 1817, June 6, | + .13 | +2.3 <i>F</i> | 8 | 1825, July 16, | + .01 | -2.4 <i>F</i> | 5 |
| 1818, June 8, | + .02 | +4.3 <i>F</i> | 13 | 1826, July 18, | - .01 | -3.0 <i>F</i> | 7 |
| 1820, June 21, | + .02 | +4.1 | 4 | 1828, July 25, | - .15 | -3.5 <i>F</i> | 7 |
| 1821, June 23, | + .12 | +1.5 | 5 | 1829, Aug. 1, | - .10 | -1.0 <i>F</i> | 9 |

Total numbers of observations, 103.

Results of Observations at various Observatories, from 1827 to 1829 inclusive.

During these three years we have, besides the observations already quoted, the following:—

1. Observations by Schwerd, at Speier, of which the originals are given in *Astronomische Beobachtungen angestellt auf der Sternwarte des Königl. Lyzeums in Speyer von F. M. Schwerd*, Speyer, 1829-30, and of which the reduced results are found in the *Astronomische Nachrichten*, Band 8, S. 264.

2. The series by Airy, at Cambridge, commenced in 1828, and found in the Cambridge Observations.

3. Littrow's Vienna Observations, found in the first series of *Annalen der K. K. Sternwarte in Wien*.

The mean corrections to the provisional ephemeris given by these series are shown in the following table. The observations have been divided in the usual

way into groups of about a month each, and the mean date and mean correction found for each group. The Paris and Königsberg results are repeated for the sake of clearness. The small figures show, as usual, the number of observations employed in forming the mean.

| | Date. | Observatory. | $\Delta\alpha$ | | $\Delta\delta$ |
|-------|---------------|--------------|---------------------|------------|--------------------|
| | | | Original. | Corrected. | |
| 1827, | July 22, | Speier, | -0.16 ₃ | -0.14 | |
| | July 25, | Paris, | -0.03 ₃ | -0.05 | +0.5 ₃ |
| | September 15, | Vienna, | -0.11 ₁₁ | -0.10 | 0.0 ₁₄ |
| | October 14, | Vienna, | -0.18 ₆ | -0.17 | -2.2 ₆ |
| 1828, | July 25, | Königsberg, | -0.15 ₇ | -0.15 | -3.5 ₇ |
| | July 29, | Vienna, | -0.24 ₂ | -0.20 | -1.4 ₂ |
| | August 14, | Vienna, | -0.13 ₁₀ | -0.09 | +1.1 ₁₀ |
| | August 27, | Speier, | -0.10 ₆ | -0.09 | |
| | September 18, | Vienna, | -0.03 ₉ | +0.01 | +1.0 ₉ |
| | September 25, | Cambridge, | -0.05 ₆ | -0.16 | |
| | October 17, | Vienna, | -0.13 ₁₄ | -0.09 | 0.0 ₁₄ |
| | October 17, | Cambridge, | -0.02 ₁₀ | -0.12 | |
| 1829, | August 1, | Königsberg, | -0.10 ₉ | -0.10 | -1.0 |
| | August 6, | Cambridge, | +0.11 ₈ | -0.08 | -1.1 ₁₇ |
| | August 28, | Speier, | -0.04 ₃ | -0.04 | |
| | September 23, | Cambridge, | +0.21 ₁₀ | +0.05 | |
| | November 6, | Cambridge, | +0.25 ₆ | +0.09 | |

Observations from 1830 to 1872.

Since the year 1830 heliocentric and geocentric ephemerides of Uranus computed from Bouvard's Tables are at our disposal. We make use of those in the Berlin Astronomisches Jahrbuch for the years 1830 to 1833, and of those in the Nautical Almanac from 1834 forward. The system of comparison is the same as that already explained. That is to say, we deduce separately:

(1) Mean corrections to the geocentric longitude and latitude of Uranus in the ephemeris as derived from observation.

(2) Mean corrections to the same, given by the provisional theory, as derived from a comparison of the heliocentric positions of that theory with the heliocentric positions in the ephemeris.

Then (1) - (2) is the correction to the provisional theory given by observation. The process of forming (1) and (2) is shown quite fully in the following pages. Each individual printed observation was first compared with the printed ephemeris, and a correction to the latter was thence deduced. When this correction was given with the observations themselves, it was of course not recomputed, unless in some doubtful cases. The observations were then divided into groups, usually of about a month each, and coinciding in time with the grouping of the Greenwich results. The mean of the dates and the mean of the corrections were then taken separately for each group and each observatory. The separate results are shown

in the proper columns of the following table, under the head "Mean dates," Mean cor. in R. A., and Mean cor. in Dec. These means are those given by the observations as printed, without the application of the systematic corrections on pages 120 and 121. In the columns "Corrected mean" these corrections are applied; this column would therefore exhibit no systematic differences between the results of the different observatories, unless the observations of Uranus were affected by errors different from those which affect the positions of the fundamental stars. A careful comparison of the differences in various parts of the table shows that this is unfortunately the case. A weight is next assigned to each individual result depending on the number of observations, the general sufficiency of the data of reduction, the mean discordance of the individual observations, and the quality of the instruments. The critical reader will notice a lack of homogeneity among the weights assigned, of which I shall speak presently. The mean of the separate group-results is then taken with regard to these weights, and also the mean of the mean dates, using for the latter the relative weights adopted for the several right ascensions. Thus, we have a mean result derived from all the observations for each month, or other group-period, which is written under the horizontal lines.

These corrections to right ascension and declination are next changed to corrections of longitude and latitude, using for this purpose the following table, which is computed from the formulæ of Gauss:

$$\cos E = \sin \epsilon \cos \alpha \sec b = \sin \epsilon \cos l \sec \delta$$

$$\Delta l = \frac{\sin E \cos \delta}{\cos b} \Delta \alpha + \frac{\cos E}{\cos b} \Delta \delta$$

$$\Delta b = -\cos E \cos \delta \Delta \alpha + \sin E \Delta \delta.$$

The differential coefficients in this table are expressed as a function of the right ascension of Uranus only, which may be done because, owing to the small inclination and great distance of the planet, its geocentric position on the celestial sphere is never more than about 2° from some point of the projection of its heliocentric orbit. The coefficients of $\Delta \alpha$ are multiplied by 15, that the right ascension may be expressed in time.

TO CONVERT ERRORS OF RIGHT ASCENSION AND DECLINATION OF URANUS INTO ERRORS OF LONGITUDE AND LATITUDE.

When the Right Ascension exceeds 12^h, enter with R. A. -12^h, and change the signs of the quantities

$$\frac{\partial b}{\partial \alpha} \text{ and } \frac{\partial l}{\partial \delta}$$

| R. A. | Logarithms of | | | | $\frac{\partial l}{\partial \alpha}$ | $\frac{\partial b}{\partial \alpha}$ | $\frac{\partial l}{\partial \delta}$ | $\frac{\partial b}{\partial \delta}$ |
|-------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
| | $\frac{\partial l}{\partial \alpha}$ | $\frac{\partial b}{\partial \alpha}$ | $\frac{\partial l}{\partial \delta}$ | $\frac{\partial b}{\partial \delta}$ | | | | |
| 0 ^h 0 ^m | 1.1386 | -0.7761 - 4 | +9.6000 - 4 | 9.9626 | +13.8+ | -5.97+ | +0.40- | +0.92+ |
| 10 | 1.1387 | -0.7757 - 14 | +9.5996 - 13 | 9.9626 | | 5.96 | 0.40 | |
| 20 | 1.1388 | -0.7743 - 23 | +9.5983 - 20 | 9.9628 | | 5.95 | 0.40 | |
| 30 | 1.1389 | -0.7720 - 33 | +9.5963 - 29 | 9.9632 | | 5.92 | 0.40 | |
| 40 | 1.1390 | -0.7687 - 44 | +9.5934 - 38 | 9.9637 | | 5.87 | 0.39 | |
| 50 | 1.1391 | -0.7643 - 55 | +9.5896 - 47 | 9.9645 | | 5.81 | 0.38 | |
| 1 0 | 1.1392 | -0.7588 - 63 | +9.5849 - 55 | 9.9653 | +13.8+ | -5.74+ | +0.38- | +0.92+ |
| 10 | 1.1393 | -0.7525 - 74 | +9.5794 - 64 | 9.9662 | | 5.66 | 0.38 | |
| 20 | 1.1394 | -0.7451 - 86 | +9.5730 - 74 | 9.9673 | | 5.56 | 0.37 | |
| 30 | 1.1394 | -0.7365 - 95 | +9.5656 - 83 | 9.9685 | | 5.45 | 0.37 | |
| 40 | 1.1395 | -0.7270 - 108 | +9.5573 - 94 | 9.9698 | | 5.33 | 0.36 | |
| 50 | 1.1395 | -0.7162 - 118 | +9.5479 - 104 | 9.9711 | | 5.20 | 0.35 | |

To CONVERT ERRORS OF RIGHT ASCENSION AND DECLINATION.—Continued.

| R. A. | Logarithms of | | | | | | | | | |
|-------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------|--------|
| | $\frac{\partial l}{\partial \alpha}$ | $\frac{\partial b}{\partial \alpha}$ | $\frac{\partial l}{\partial \delta}$ | $\frac{\partial b}{\partial \delta}$ | $\frac{\partial l}{\partial \alpha}$ | $\frac{\partial b}{\partial \alpha}$ | $\frac{\partial l}{\partial \delta}$ | $\frac{\partial b}{\partial \delta}$ | | |
| 2 ⁿ 0 ⁿ | 1.1395 | -0.7044 | -129 | +9.5375 | -115 | 9.9725 | +13.8+ | -5.06+ | +0.34- | +0.93+ |
| 10 | 1.1395 | -0.6915 | -142 | +9.5260 | -126 | 9.9740 | | 4.91 | 0.33 | |
| 20 | 1.1395 | -0.6773 | -155 | +9.5134 | -139 | 9.9756 | | 4.75 | 0.32 | |
| 30 | 1.1395 | -0.6618 | -168 | +9.4995 | -152 | 9.9772 | | 4.58 | 0.31 | |
| 40 | 1.1395 | -0.6450 | -184 | +0.4843 | -167 | 9.9788 | | 4.41 | 0.30 | |
| 50 | 1.1394 | -0.6266 | -197 | +9.4676 | -181 | 9.9804 | | 4.23 | 0.29 | |
| 3 0 | 1.1394 | -0.6069 | -215 | +9.4495 | -198 | 9.9821 | +13.8+ | -4.05+ | +0.28- | +0.96+ |
| 10 | 1.1394 | -0.5854 | -233 | +9.4297 | -216 | 9.9837 | | 3.86 | 0.27 | |
| 20 | 1.1393 | -0.5621 | -253 | +9.4081 | -237 | 9.9853 | | 3.66 | 0.26 | |
| 30 | 1.1393 | -0.5368 | -274 | +9.3844 | -258 | 9.9869 | | 3.45 | 0.24 | |
| 40 | 1.1392 | -0.5094 | -299 | +9.3586 | -284 | 9.9883 | | 3.24 | 0.23 | |
| 50 | 1.1391 | -0.4795 | -327 | +9.3302 | -312 | 9.9898 | | 3.02 | 0.21 | |
| 4 0 | 1.1390 | -0.4468 | -359 | +9.2990 | -346 | 9.9912 | +13.8+ | -2.80+ | +0.20- | +0.98+ |
| 10 | 1.1390 | -0.4109 | -399 | +9.2644 | -385 | 9.9925 | | 2.58 | 0.18 | |
| 20 | 1.1389 | -0.3710 | -443 | +9.2259 | -431 | 9.9938 | | 2.36 | 0.17 | |
| 30 | 1.1388 | -0.3267 | -498 | +9.1828 | -487 | 9.9949 | | 2.14 | 0.15 | |
| 40 | 1.1387 | -0.2769 | -571 | +9.1341 | -560 | 9.9959 | | 1.91 | 0.13 | |
| 50 | 1.1386 | -0.2198 | -659 | +9.0781 | -651 | 9.9969 | | 1.67 | 0.12 | |
| 5 0 | 1.1385 | -0.1539 | -786 | +9.0130 | -777 | 9.9977 | +13.8+ | -1.43+ | +0.10- | +0.99+ |
| 10 | 1.1384 | -0.0753 | -96 | +8.9353 | -956 | 9.9984 | | 1.19 | 0.08 | |
| 20 | 1.1383 | -9.979 | -125 | +8.8397 | -1240 | 9.9990 | | 0.95 | 0.07 | |
| 30 | 1.1382 | -9.854 | -175 | +8.7157 | -1757 | 9.9994 | | 0.71 | 0.05 | |
| 40 | 1.1381 | -9.679 | -301 | +8.540 | -300 | 9.9997 | | 0.48 | 0.03 | |
| 50 | 1.1380 | -9.378 | | +8.240 | | 9.9999 | | 0.24 | 0.02 | |
| 6 0 | 1.1379 | - OC | OC | OC | 0 | 9.9999 | +13.7+ | 0.00 | 0.00 | +1.00+ |
| 10 | 1.1378 | +9.378 | | -8.240 | | 9.9999 | | -0.24+ | -0.02+ | |
| 20 | 1.1377 | +9.679 | +301 | -8.540 | +300 | 9.9997 | | 0.48 | 0.03 | |
| 30 | 1.1376 | +9.854 | +175 | -8.7157 | +1757 | 9.9994 | | 0.71 | 0.05 | |
| 40 | 1.1375 | +9.978 | +124 | -8.8397 | +1240 | 9.9990 | | 0.95 | 0.07 | |
| 50 | 1.1374 | +0.0743 | +96 | -8.9353 | +956 | 9.9984 | | 1.18 | 0.08 | |
| 7 0 | 1.1373 | +0.1526 | +658 | -9.0130 | +651 | 9.9977 | +13.7+ | +1.42- | -0.10+ | +0.99+ |
| 10 | 1.1373 | +0.2184 | +569 | -9.0781 | +560 | 9.9969 | | 1.65 | 0.12 | |
| 20 | 1.1372 | +0.2753 | +497 | -9.1341 | +487 | 9.9959 | | 1.88 | 0.13 | |
| 30 | 1.1371 | +0.3250 | +442 | -9.1828 | +431 | 9.9949 | | 2.11 | 0.15 | |
| 40 | 1.1371 | +0.3692 | +397 | -9.2259 | +385 | 9.9938 | | 2.34 | 0.17 | |
| 50 | 1.1370 | +0.4089 | +359 | -9.2644 | +346 | 9.9925 | | 2.56 | 0.18 | |
| 8 0 | 1.1370 | +0.4448 | +325 | -9.2990 | +312 | 9.9912 | +13.7+ | +2.78- | -0.20+ | +0.98+ |
| 10 | 1.1370 | +0.4773 | +298 | -9.3302 | +284 | 9.9898 | | 3.00 | 0.21 | |
| 20 | 1.1370 | +0.5071 | +273 | -9.3586 | +258 | 9.9883 | | 3.22 | 0.23 | |
| 30 | 1.1370 | +0.5344 | +253 | -9.3844 | +237 | 9.9869 | | 3.43 | 0.24 | |
| 40 | 1.1370 | +0.5597 | +233 | -9.4081 | +216 | 9.9853 | | 3.63 | 0.26 | |
| 50 | 1.1370 | +0.5830 | +214 | -9.4297 | +198 | 9.9837 | | 3.83 | 0.27 | |
| 9 0 | 1.1370 | +0.6044 | +197 | -9.4495 | +181 | 9.9821 | +15.7+ | +4.02- | -0.28+ | +0.96+ |
| 10 | 1.1370 | +0.6241 | +184 | -9.4676 | +167 | 9.9804 | | 4.20 | 0.29 | |
| 20 | 1.1370 | +0.6425 | +169 | -9.4843 | +152 | 9.9788 | | 4.38 | 0.30 | |
| 30 | 1.1371 | +0.6594 | +155 | -9.4995 | +139 | 9.9772 | | 4.55 | 0.31 | |
| 40 | 1.1371 | +0.6749 | +143 | -9.5134 | +126 | 9.9756 | | 4.72 | 0.32 | |
| 50 | 1.1372 | +0.6892 | +131 | -9.5260 | +115 | 9.9740 | | 4.88 | 0.33 | |
| 10 0 | 1.1373 | +0.7023 | +119 | -9.5375 | +104 | 9.9725 | +13.7+ | +5.04- | -0.34+ | +0.93+ |
| 10 | 1.1374 | +0.7142 | +109 | -9.5479 | +94 | 9.9711 | | 5.18 | 0.35 | |
| 20 | 1.1375 | +0.7251 | +96 | -9.5573 | +83 | 9.9698 | | 5.31 | 0.36 | |
| 30 | 1.1376 | +0.7347 | +87 | -9.5656 | +74 | 9.9685 | | 5.43 | 0.37 | |
| 40 | 1.1377 | +0.7434 | +76 | -9.5730 | +64 | 9.9673 | | 5.54 | 0.37 | |
| 50 | 1.1378 | +0.7510 | +65 | -9.5794 | +55 | 9.9662 | | 5.64 | 0.38 | |
| 11 0 | 1.1379 | +0.7575 | +57 | -9.5849 | +47 | 9.9653 | +13.7+ | +5.72- | -0.38+ | +0.92+ |
| 10 | 1.1380 | +0.7632 | +46 | -9.5896 | +38 | 9.9645 | | 5.79 | 0.38 | |
| 20 | 1.1381 | +0.7678 | +35 | -9.5934 | +29 | 9.9637 | | 5.85 | 0.39 | |
| 30 | 1.1382 | +0.7713 | +24 | -9.5963 | +20 | 9.9632 | | 5.90 | 0.39 | |
| 40 | 1.1383 | +0.7737 | +17 | -9.5983 | +13 | 9.9628 | | 5.94 | 0.39 | |
| 50 | 1.1385 | +0.7754 | +7 | -9.5996 | +4 | 9.9626 | | 5.96 | 0.40 | |
| 12 0 | 1.1386 | +0.7761 | | -9.6000 | | 9.9626 | +13.8+ | +5.97- | -0.40+ | +0.92+ |

We thus have, for the interval occupied by each group of observations, a mean correction to the geocentric longitude and latitude of the planet given by observations, which are found in the ninth and tenth columns of the table, on the same horizontal line with the mean corrections in right ascension and declination from which they are derived. The next step is to obtain the corresponding corrections given by the provisional ephemeris.

This correction has been first obtained for every twentieth day of each of the forty-two oppositions included in the table. The heliocentric longitude, latitude, and radius vector were interpolated to the most convenient twenty-day intervals, and compared with the corresponding co-ordinates in the heliocentric ephemeris. This ephemeris was of course the one corresponding to that with which the observations were compared, namely, the Berliner Jahrbuch for the years 1830-33, and the Nautical Almanac for subsequent years. These comparisons are fully given at the end of this chapter, and the resulting corrections to the printed ephemeris are given in the proper columns of the table.

These corrections to the heliocentric co-ordinates were then changed to corrections of geocentric longitude and latitude by the following formulæ. Put

r' , the projection of the planet's radius vector on the ecliptic;

ρ' , the projection of the planet's distance from the earth on the same plane;

ρ , this distance itself;

λ, β , the planet's heliocentric longitude and latitude;

L , the sun's geocentric longitude;

R , its radius vector;

M , the modulus of the common logarithms;

$\delta l, \delta b$, the corrections to the geocentric longitude and latitude;

$\delta \rho$, the correction to the common logarithm of the radius vector.

Then

$$\begin{aligned} \delta l &= \frac{r'^2}{\rho'^2} \left\{ 1 + \frac{R}{r'} \cos(L - \lambda) \right\} \delta \lambda \\ &\quad - \frac{Rr'}{\rho'^2} \sin(L - \lambda) \frac{\delta \rho}{M \sin 1''} \\ &\quad + \frac{Rr'}{\rho'^2} \sin(L - \lambda) \tan \beta \delta \beta \\ \delta b &= \frac{r'}{\rho'} \left\{ 1 + \frac{r'R}{\rho'^2} \tan^2 \beta \cos(L - \lambda) \right\} \delta \beta \\ &\quad - \frac{r'^2 R}{\rho'^2} \tan \beta \sin(L - \lambda) \delta \lambda \\ &\quad + \frac{r'R^2}{\rho'^2} \left\{ 1 + \frac{r'}{R} \cos(L - \lambda) \right\} \sin \beta \frac{\delta \rho}{M \sin 1''}. \end{aligned}$$

The last term in δl and the last two terms of δb have been omitted in the computation, as they scarcely ever exceed a few hundredths of a second.

The values of δl and δb are printed in the last two columns of the table. The formula for δb might have contained the additional term

$$\delta b = \sin l \delta \omega$$

$\delta \omega$ being the correction to the obliquity of the ecliptic adopted in the ephemeris to reduce it to that employed in the provisional theory. This correction is, however, deferred until we come to form the equations of condition.

From the values of δl and δb thus obtained we are to find the mean values during each group of observations. If these quantities varied uniformly, the proper value would be that corresponding to the mean date of each group. But the second differences are so large that this value would generally be in error by one- or two-tenths of a second. Owing to the minuteness of this difference, it has been considered that when the mean date was near the middle of a twenty-day interval, the correction δl interpolated to that date without regard to second differences would furnish a sufficient approximation to the required mean value of δl during an interval of about 30 days. In other case the value of δl was interpolated to 5-day intervals through the period of each group of observations, and the mean value taken.

During the years 1850-1863 the sun's longitude employed in the ephemeris required a gradually increasing correction, amounting at the latter date to about 3". A small correction of which the maximum value is about 0".15 was applied to l to reduce it to the value it would have had if Hansen's tables had been employed.

The corrected mean values of l and b thus obtained are given in the last two columns of the following table, being inclosed in brackets and printed immediately above the values of Δl and Δb derived from observation.

I deem it proper to mention that the mechanical labor of constructing these tables of comparisons, in the manner just described, was in great part performed by Dr. C. L. F. Kampf, who was employed by the Smithsonian Institution to assist me in the work. Before using it I subjected the whole of the work to a careful revision, altering especially the relative weights of the corrected means in many cases. As the assigned weights now stand, each set of results which are combined into a single mean has its own unit of weight, which does not necessarily coincide with that of any other set. The use of a uniform scale of weights through this series of observations, and the assignment to every final mean of a weight equal to the sum of the weights of the quantities whose mean was taken, would have led to weights in many cases quite fictitious, owing to the obvious presence of systematic errors in the results. For this reason I have made no further use of the weights found in this table, and their lack of homogeneity therefore does no harm.

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS IN THE BERLINER JAHRBUCH AND THE NAUTICAL ALMANAC.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|---------------------------------------|-------------|-------------------------------|----------------|--------------------|------------------------------|----------------|--------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1830 | s | | s | " | " | " | " | " |
| Königsberg, | July 29 | -1.56 | 7 | -1.56 ₅ | +4.6 | 7 | +4.6 | | |
| Cambridge, | July 29 | -1.51 | 6 | -1.67 ₂ | | | | [-19.2] | [+ 9.5] |
| [20 ^h 40 ^m] | July 29 | | ... | -1.60 | | ... | | -20.8 | +10.3 |
| Königsberg, | Aug. 12 | -1.65 | 2 | -1.65 ₂ | +3.6 | 2 | +3.6 | | |
| Cambridge, | Aug. 25 | -1.36 | 4 | -1.52 ₂ | | | | [-18.5] | [+ 9.5] |
| [20 ^h 36 ^m] | Aug. 19 | | ... | -1.58 | | ... | | -20.8 | + 9.1 |
| Cambridge, | Sept. 19 | -1.46 | 7 | -1.62 ₃ | | ... | | [-17.2] | |
| [20 ^h 37 ^m] | | | | | | | | -21.2 | |
| Cambridge, | Oct. 17 | -1.34 | 8 | -1.50 ₄ | | ... | | [-16.7] | |
| [20 ^h 36 ^m] | | | | | | | | -19.5 | |
| Cambridge, | Nov. 14 | -1.36 | 8 | -1.52 ₄ | | ... | | [-16.6] | |
| [20 ^h 37 ^m] | | | | | | | | -19.8 | |
| | 1831 | | | | | | | | |
| Greenwich, | Aug. 3 | -1.72 | 11 | -1.72 ₄ | +2.5 | 11 | +4.3 | | |
| Cambridge, | Aug. 8 | -1.70 | 9 | -1.86 ₄ | | | | [-23.8] | [+10.0] |
| [20 ^h 58 ^m] | Aug. 6 | | ... | -1.79 | | ... | | -23.4 | +11.1 |
| Greenwich, | Sept. 7 | -1.67 | 5 | -1.67 ₃ | +3.5 | 5 | +5.3 | | |
| Cambridge, | Sept. 15 | -1.60 | 6 | -1.76 ₃ | | | | [-22.0] | [+10.0] |
| [20 ^h 52 ^m] | Sept. 11 | | ... | -1.72 | | ... | | -22.2 | +11.8 |
| Greenwich, | Nov. 4 | -1.43 | 7 | -1.48 ₄ | +3.1 | 7 | +4.9 | | |
| Cambridge, | Oct. 26 | -1.54 | 16 | -1.70 ₈ | | | | [-20.7] | [+ 9.8] |
| [20 ^h 50 ^m] | Oct. 31 | | ... | -1.63 | | ... | | -21.0 | +10.8 |
| | 1832 | | | | | | | | |
| Greenwich, | Aug. 9 | -2.02 | 3 | -2.02 ₂ | +1.7 | 2 | +3.4 ₂ | | |
| Königsberg, | Aug. 10 | -2.24 | 3 | -2.24 ₄ | | | | | |
| Cambridge, | Aug. 15 | -1.99 | 9 | -2.18 ₃ | | | | | |
| Vienna, | Aug. 3 | -2.33 | 3 | -2.33 ₁ | +0.2 | 3 | +1.1 | [-28.4] | [+10.3] |
| [21 ^h 17 ^m] | Aug. 12 | | ... | -2.19 | | ... | +2.5 | -29.2 | +11.9 |
| Cambridge, | Sept. 12 | -1.97 | 10 | -2.16 ₁ | | ... | | [-26.9] | |
| [21 ^h 12 ^m] | | | | | | | | -28.8 | |
| Cambridge, | Oct. 6 | -1.91 | 13 | -2.09 ₆ | | | | | |
| Vienna, | Oct. 12 | -1.90 | 15 | -1.90 ₄ | +1.9 | 15 | +2.8 | [-25.8] | [+10.2] |
| [21 ^h 9 ^m] | Oct. 7 | | ... | -2.01 | | ... | | -26.6 | +11.1 |
| Cambridge, | Nov. 16 | -1.85 | 7 | -2.04 ₇ | | | | | |
| Vienna, | Nov. 9 | -1.89 | 2 | -1.89 ₁ | +1.1 | ... | +2.0 | [-25.2] | [+10.1] |
| [21 ^h 10 ^m] | Nov. 15 | | ... | -2.02 | | ... | | -27.4 | +10.5 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—Continued

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|---------------------------------------|-------------|-------------------------------|----------------|--------------------|------------------------------|----------------|--------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1833 | s | | s | " | | " | " | " |
| Greenwich, | Aug. 22 | -2.57 | 7 | -2.57 ₂ | -1.4 | 7 | +0.2 ₆ | | |
| Königsberg, | Aug. 12 | -2.57 | 5 | -2.57 ₄ | -1.5 | 5 | -1.5 ₅ | | |
| Cambridge, | Aug. 15 | -2.40 | 11 | -2.59 ₅ | | | | [-33.8] | [+10.8] |
| [21 ^h 32 ^m] | Aug. 15 | | ... | -2.58 | | ... | -0.6 | -35.6 | +10.9 |
| Greenwich, | Sept. 18 | -2.33 | 5 | -2.33 ₂ | -0.8 | 4 | +0.8 ₄ | | |
| Cambridge, | Sept. 19 | -2.33 | 10 | -2.52 ₄ | +2.8 | 11 | +1.4 ₁₁ | | |
| Vienna, | Sept. 11 | -2.52 | 6 | -2.56 ₁ | +2.6 | 6 | +1.2 ₂ | [-32.2] | [+10.8] |
| [21 ^h 28 ^m] | Sept. 18 | | ... | -2.47 | | ... | +1.2 | -33.4 | +12.3 |
| Greenwich, | Oct. 11 | -2.25 | 4 | -2.25 ₁ | +0.1 | 4 | +1.7 ₄ | | |
| Cambridge, | Oct. 12 | -2.29 | 12 | -2.48 ₅ | +2.2 | 13 | +0.8 ₁₃ | | |
| Vienna, | Oct. 14 | -2.37 | 19 | -2.41 ₂ | | | | [-31.2] | [+10.6] |
| [21 ^h 26 ^m] | Oct. 12 | | ... | -2.43 | | ... | +1.0 | -33.0 | +11.9 |
| Cambridge, | Nov. 18 | -2.16 | 5 | -2.35 ₃ | + 1.3 | 4 | -0.1 | | |
| Vienna, | Nov. 14 | -2.24 | 4 | -2.28 ₁ | | | | [-30.3] | [+10.3] |
| [21 ^h 26 ^m] | Nov. 16 | | ... | -2.33 | | ... | -0.1 | -32.0 | +10.4 |
| | 1834 | | | | | | | | |
| Cambridge, | Aug. 15 | -2.58 | 11 | -2.77 ₅ | - 1.3 | 11 | -2.6 ₁₁ | | |
| Vienna, | Aug. 13 | -3.07 | 4 | -3.11 ₁ | - 4.4 | 4 | -3.5 ₁ | [-40.6] | [+11.1] |
| [21 ^h 49 ^m] | Aug. 14 | | ... | -2.83 | | ... | -2.7 | -39.7 | +11.3 |
| Greenwich, | Sept. 10 | -3.00 | 5 | -3.00 ₁ | - 4.4 | 5 | -3.0 ₅ | | |
| Cambridge, | Sept. 16 | -2.66 | 18 | -2.85 ₄ | - 1.2 | 18 | -2.5 ₁₈ | [-39.0] | [+11.1] |
| [21 ^h 45 ^m] | Sept. 13 | | ... | -2.88 | | ... | -2.6 | -40.5 | +11.2 |
| Greenwich, | Oct. 14 | -2.74 | 10 | -2.74 ₃ | - 3.1 | 10 | -1.7 ₁₀ | | |
| Cambridge, | Oct. 16 | -2.68 | 16 | -2.87 ₇ | - 0.8 | 16 | -2.1 ₁₆ | | |
| Vienna, | Oct. 20 | -2.75 | 4 | -2.80 ₁ | - 1.3 | 4 | -0.4 ₁ | [-37.5] | [+10.9] |
| [21 ^h 41 ^m] | Oct. 17 | | ... | -2.83 | | ... | -1.9 | -39.5 | +11.6 |
| Cambridge, | Nov. 16 | -2.64 | 9 | -2.83 ₁ | - 0.7 | 11 | -2.0 ₁₁ | | |
| Vienna, | Nov. 12 | -2.53 | 8 | -2.58 ₁ | - 2.8 | 8 | -1.9 ₄ | [-36.7] | [+10.7] |
| [21 ^h 41 ^m] | Nov. 14 | | ... | -2.77 | | ... | -2.0 | -38.7 | +11.3 |
| Cambridge, | Dec. 6 | -2.71 | 5 | -2.90 ₂ | - 1.2 | 4 | -2.5 ₄ | | |
| Vienna, | Dec. 7 | -2.55 | 1 | -2.59 ₀ | - 2.1 | 1 | -1.2 ₁ | [-36.2] | [+10.5] |
| [21 ^h 43 ^m] | Dec. 7 | | ... | -2.88 | | ... | -2.4 | -40.3 | +11.5 |
| | 1835 | | | | | | | | |
| Greenwich, | Aug. 17 | -3.25 | 4 | -3.25 ₁ | - 6.6 | 4 | -5.4 ₄ | | |
| Königsberg, | Aug. 20 | -3.31 | 11 | -3.31 ₈ | | | | | |
| Cambridge, | Aug. 14 | -3.28 | 20 | -3.47 ₉ | - 4.7 | 20 | -5.9 ₂₀ | | |
| Vienna, | Aug. 21 | -3.30 | 1 | -3.35 ₀ | - 8.8 | 1 | -7.9 ₁ | [-47.1] | [+11.3] |
| [22 ^h 4 ^m] | Aug. 17 | | ... | -3.39 | | ... | -5.9 | -48.6 | +11.8 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|--|-------------|-------------------------------|----------------|--------------------|------------------------------|----------------|---------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1835 | s | | s | " | | " | " | " |
| Cambridge,
Vienna, | Sept. 15 | -3.17 | 9 | -3.36 ₄ | - 4.6 | 9 | -5.8 ₉ | | |
| | Sept. 14 | -3.30 | 9 | -3.35 ₁ | - 6.7 | 9 | -5.8 ₂ | [-45.8] | [+11.2] |
| [22 ^h 1 ^m] | Sept. 15 | | ... | -3.36 | | ... | -5.8 | -48.1 | +11.6 |
| Greenwich,
Cambridge, | Oct. 10 | -3.27 | 4 | -3.27 ₂ | - 6.53 | 4 | -5.3 ₄ | | |
| | Oct. 17 | -3.11 | 8 | -3.30 ₄ | - 4.1 | 8 | -5.3 ₈ | [-44.4] | [+11.1] |
| [21 ^h 57 ^m] | Oct. 15 | | ... | -3.29 | | ... | -5.3 | -46.9 | +11.4 |
| Greenwich,
Cambridge, | Nov. 27 | -3.21 | 6 | -3.21 ₂ | - 5.6 | 7 | -4.4 ₇ | | |
| | Nov. 26 | -3.00 | 10 | -3.19 ₄ | - 4.5 | 9 | -5.7 ₉ | [-43.0] | [+10.7] |
| [21 ^h 57 ^m] | Nov. 26 | | ... | -3.20 | | ... | -5.1 | -45.6 | +11.2 |
| | 1836 | | | | | | | | |
| Greenwich,
Cambridge, | July 22 | -3.80 | 7 | -3.84 ₂ | -10.5 | 8 | -9.5 ₈ | | |
| | July 25 | -3.60 | 3 | -3.89 ₁ | - 8.9 | 2 | -9.9 ₂ | [-55.3] | [+11.5] |
| [22 ^h 24 ^m] | July 23 | | ... | -3.86 | | ... | -9.6 | -56.5 | +11.8 |
| Greenwich,
Königsberg, | Aug. 24 | -3.78 | 7 | -3.82 ₄ | - 9.8 | 8 | - 8.8 ₈ | | |
| | Aug. 30 | -3.63 | 5 | -3.63 ₄ | | | | | |
| Cambridge,
Edinburgh, | Aug. 16 | -3.78 | 12 | -3.97 ₃ | - 9.3 | 12 | -10.3 ₁₂ | | |
| | Aug. 19 | -4.09 | 9 | -4.10 ₃ | - 9.3 | 7 | - 9.3 ₇ | | |
| Vienna, | Aug. 20 | -3.77 | 1 | -3.81 ₀ | -12.5 | 1 | -11.5 ₀ | [-54.6] | [+11.6] |
| [22 ^h 20 ^m] | Aug. 22 | | ... | -3.87 | | ... | -9.6 | -56.6 | +11.5 |
| Greenwich,
Cambridge, | Sept. 13 | -3.77 | 7 | -3.81 ₃ | - 8.8 | 6 | -7.8 ₆ | | |
| | Sept. 16 | -3.70 | 10 | -3.89 ₄ | - 8.4 | 10 | -9.4 ₁₀ | | |
| Edinburgh,
Vienna, | Sept. 16 | -4.01 | 8 | -4.02 ₂ | - 8.6 | 8 | -8.6 ₈ | | |
| | Sept. 15 | -3.59 | 9 | -3.63 ₃ | -10.4 | 9 | -9.4 ₃ | [-53.4] | [+11.6] |
| [22 ^h 16 ^m] | Sept. 15 | | ... | -3.87 | | ... | -8.8 | -56.2 | +12.1 |
| Greenwich,
Cambridge, | Oct. 12 | -3.67 | 7 | -3.71 ₂ | - 8.6 | 8 | -7.6 ₈ | | |
| | Oct. 16 | -3.51 | 12 | -3.70 ₃ | - 8.5 | 11 | -9.5 ₁₁ | | |
| Edinburgh,
Vienna, | Oct. 15 | -4.05 | 8 | -4.06 ₂ | - 7.9 | 4 | -7.9 ₄ | | |
| | Oct. 11 | -3.57 | 10 | -3.61 ₁ | -10.2 | 10 | -9.2 ₃ | [-51.8] | [+11.3] |
| [22 ^h 13 ^m] | Oct. 15 | | ... | -3.77 | | ... | -8.6 | -54.8 | +11.7 |
| Greenwich,
Cambridge, | Nov. 13 | -3.59 | 11 | -3.63 ₃ | - 9.1 | 11 | -8.1 ₁₁ | | |
| | Nov. 13 | -3.37 | 8 | -3.56 ₄ | - 7.9 | 7 | -8.9 ₇ | | |
| Edinburgh,
Vienna, | Nov. 17 | -3.78 | 8 | -3.79 ₂ | | | | | |
| | Nov. 9 | -3.56 | 3 | -3.60 ₁ | -10.2 | 3 | -9.2 ₁ | [-50.6] | [+11.1] |
| [22 ^h 12 ^m] | Nov. 13 | | ... | -3.63 | | ... | -8.5 | -52.8 | +11.0 |
| Cambridge,
Edinburgh, | Dec. 12 | -3.40 | 7 | -3.59 ₃ | - 8.0 | 7 | -9.0 ₇ | | |
| | Dec. 12 | -3.29 | 5 | -3.30 ₁ | - 8.8 | 3 | -8.8 ₃ | [-50.0] | [+10.9] |
| [22 ^h 13 ^m] | Dec. 12 | | ... | -3.52 | | ... | -8.9 | -51.5 | +10.1 |
| | 1837 | | | | | | | | |
| Greenwich,
[22 ^h 40 ^m] | July 22 | -4.23 | 4 | -4.26 ₁ | -13.4 | 4 | -12.5 | [-62.9] | [+11.5] |
| | | | | | | | | -63.2 | +12.0 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|---------------------------------------|-------------|-------------------------------|----------------|---------------------|------------------------------|----------------|---------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1837 | s | | s | " | | " | " | " |
| Greenwich, | Aug. 18 | -4.30 | 10 | -4.33 ₂ | -13.4 | 10 | -12.5 ₁₀ | | |
| Cambridge, | Aug. 18 | -4.09 | 14 | -4.28 ₂ | -12.6 | 11 | -13.4 ₁₁ | | |
| Edinburgh, | Aug. 22 | -4.40 | 6 | -4.40 ₁ | -12.7 | 6 | -12.7 ₆ | | |
| Paris, | Aug. 14 | -4.34 | 9 | -4.33 ₂ | -12.0 | 10 | -11.4 ₁₀ | | |
| Vienna, | Aug. 25 | -4.29 | 4 | -4.33 ₀ | -12.6 | 4 | -11.6 ₁ | | |
| [22 ^h 36 ^m] | Aug 18 | | ... | -4.33 | | ... | -12.5 | [-62.9] | [+11.7] |
| | | | | | | | | -64.1 | +12.2 |
| Greenwich, | Sept. 17 | -4.23 | 14 | -4.26 ₁₃ | -13.4 | 14 | -12.5 ₁₄ | | |
| Königsberg, | Sept. 11 | -4.10 | 8 | -4.13 ₆ | | | | | |
| Cambridge, | Sept. 17 | -4.06 | 14 | -4.25 ₆ | -11.9 | 15 | -12.7 ₁₄ | | |
| Edinburgh, | Sept. 10 | -4.39 | 4 | -4.39 ₁ | -11.5 | 3 | -11.5 ₃ | | |
| Paris, | Sept. 18 | -4.20 | 12 | -4.19 ₆ | -12.3 | 13 | -11.7 ₁₃ | | |
| Vienna, | Sept. 13 | -4.12 | 5 | -4.15 ₁ | -13.9 | 5 | -12.9 ₁ | | |
| [22 ^h 32 ^m] | Sept. 16 | | ... | -4.21 | | ... | -12.3 | [-61.7] | [+11.6] |
| | | | | | | | | -62.3 | +11.5 |
| Greenwich, | Oct. 16 | -4.13 | 11 | -4.16 ₃ | -12.9 | 11 | -12.0 ₁₁ | | |
| Cambridge, | Oct. 17 | -4.00 | 10 | -4.19 ₄ | -11.6 | 11 | -12.4 ₁₁ | | |
| Paris, | Oct. 17 | -4.05 | 4 | -4.04 ₂ | -11.2 | 4 | -10.6 ₄ | | |
| Vienna, | Oct. 18 | -4.44 | 2 | -4.47 ₃ | -15.5 | 2 | -14.5 ₃ | | |
| [22 ^h 28 ^m] | Oct. 17 | | ... | -4.16 | | ... | -12.0 | [-59.8] | [+11.4] |
| | | | | | | | | -61.5 | +11.3 |
| Greenwich, | Nov. 8 | -4.10 | 5 | -4.13 ₂ | -12.4 | 5 | -11.5 ₃ | | |
| Cambridge, | Nov. 4 | -3.96 | 4 | -4.15 ₂ | -12.3 | 3 | -13.1 ₃ | | |
| Paris, | Nov. 6 | -4.07 | 3 | -4.06 ₂ | -11.3 | 3 | -10.7 ₃ | | |
| Vienna, | Nov. 2 | -4.09 | 2 | -4.12 ₃ | -12.9 | 2 | -11.9 ₃ | | |
| [22 ^h 27 ^m] | Nov. 6 | | ... | -4.11 | | ... | -11.7 | [-59.0] | [+11.2] |
| | | | | | | | | -60.7 | +11.3 |
| Greenwich, | Dec. 2 | -4.05 | 2 | -4.08 ₁ | -14.0 | 2 | -13.1 ₂ | | |
| Cambridge, | Nov. 30 | -3.87 | 6 | -4.06 ₃ | -11.6 | 7 | -12.4 ₃ | | |
| Paris, | Dec. 8 | -4.03 | 7 | -4.02 ₄ | -10.9 | 7 | -10.3 ₄ | | |
| Vienna, | Dec. 8 | -4.28 | 2 | -4.31 ₃ | -13.0 | 2 | -12.0 ₃ | | |
| [22 ^h 28 ^m] | Dec. 5 | | ... | -4.05 | | ... | -11.6 | [-57.7] | [+11.0] |
| | | | | | | | | -59.9 | +11.1 |
| | 1838 | | | | | | | | |
| Greenwich, | Aug. 20 | -4.72 | 11 | -4.76 ₃ | -15.9 | 11 | -15.1 ₁₁ | | |
| Königsberg, | Aug. 25 | -4.65 | 5 | -4.65 ₄ | -18.6 | 5 | -17.6 ₃ | | |
| Cambridge, | Aug. 19 | -4.67 | 11 | -4.78 ₅ | -15.8 | 12 | -16.3 ₁₂ | | |
| Edinburgh, | Aug. 25 | -4.96 | 3 | -4.95 ₁ | -15.0 | 4 | -15.0 ₄ | | |
| Paris, | Aug. 20 | -4.80 | 9 | -4.79 ₃ | -16.6 | 9 | -16.0 ₉ | | |
| [22 ^h 51 ^m] | Aug. 21 | | ... | -4.76 | | ... | -15.8 | [-70.9] | [+11.7] |
| | | | | | | | | -71.4 | +12.3 |
| Greenwich, | Sept. 12 | -4.62 | 8 | -4.66 ₂ | -16.2 | 8 | -15.4 ₃ | | |
| Königsberg, | Sept. 16 | -4.65 | 14 | -4.67 ₁₀ | -18.5 | 14 | -17.5 ₇ | | |
| Cambridge, | Sept. 16 | -4.61 | 11 | -4.72 ₅ | -14.9 | 12 | -15.4 ₁₂ | | |
| Edinburgh, | Sept. 15 | -4.74 | 9 | -4.73 ₄ | -15.8 | 7 | -15.8 ₇ | | |
| Paris, | Sept. 12 | -4.71 | 6 | -4.70 ₃ | -16.0 | 6 | -15.4 ₆ | | |
| Vienna, | Sept. 11 | -4.71 | 12 | -4.73 ₁ | -18.1 | 12 | -17.1 ₃ | | |
| Berlin, | Sept. 6 | -4.72 | 7 | -4.74 ₂ | -17.5 | 7 | -16.5 ₃ | | |
| [22 ^h 47 ^m] | Sept. 15 | | ... | -4.70 | | ... | -16.0 | [-70.4] | [+11.7] |
| | | | | | | | | -70.7 | +12.0 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R.A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|---------------------------------------|-------------|------------------------------|----------------|--------------------|------------------------------|----------------|---------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude | Latitude. |
| | 1838 | s | | s | " | | " | " | " |
| Greenwich, | Oct. 15 | -4.67 | 7 | -4.71 ₃ | -15.5 | 7 | -14.7 ₇ | | |
| Cambridge, | Oct. 17 | -4.49 | 7 | -4.60 ₃ | -15.3 | 6 | -15.8 ₆ | | |
| Edinburgh, | Oct. 16 | -4.56 | 12 | -4.55 ₃ | -15.0 | 8 | -15.0 ₈ | | |
| Paris, | Oct. 16 | -4.66 | 7 | -4.65 ₃ | -15.9 | 7 | -15.3 ₇ | | |
| Vienna, | Oct. 14 | -4.66 | 9 | -4.68 ₁ | -17.3 | 9 | -16.3 ₂ | | |
| [22 ^h 44 ^m] | Oct. 16 | | ... | -4.63 | | ... | -16.2 | [-68.6] | [+11.5] |
| | | | | | | | | -69.7 | +10.8 |
| Greenwich, | Nov. 9 | -4.52 | 6 | -4.56 ₂ | -16.0 | 6 | -15.2 ₆ | | |
| Cambridge, | Nov. 15 | -4.37 | 10 | -4.48 ₁ | -15.1 | 9 | -15.6 ₉ | | |
| Edinburgh, | Nov. 16 | -4.70 | 8 | -4.69 ₃ | -17.4 | 2 | -17.4 ₂ | | |
| Vienna, | Nov. 7 | -4.93 | 4 | -4.95 ₃ | -18.3 | 4 | -17.3 ₁ | | |
| [22 ^h 42 ^m] | Nov. 14 | | ... | -4.58 | | ... | -15.8 | [-66.9] | [+11.3] |
| | | | | | | | | -68.8 | +10.9 |
| Greenwich, | Dec. 9 | -4.60 | 2 | -4.64 ₁ | -15.9 | 2 | -15.1 ₂ | | |
| Cambridge, | Dec. 15 | -4.26 | 5 | -4.37 ₂ | -15.1 | 7 | -15.6 ₇ | | |
| Edinburgh, | Dec. 17 | -4.18 | 3 | -4.17 ₁ | -14.7 | 1 | -14.7 ₁ | | |
| Paris, | Dec. 5 | -4.40 | 7 | -4.39 ₁ | -15.1 | 7 | -14.5 ₇ | | |
| [22 ^h 43 ^m] | Dec. 10 | | ... | -4.38 | | ... | -15.0 | [-65.8] | [+11.1] |
| | | | | | | | | -65.7 | +10.4 |
| | 1839 | | | | | | | | |
| Greenwich, | Aug. 22 | -5.28 | 5 | -5.32 ₁ | -21.3 | 5 | -20.6 ₅ | | |
| Cambridge, | Aug. 24 | -5.11 | 8 | -5.22 ₄ | -20.7 | 7 | -21.0 ₇ | | |
| Paris, | Aug. 23 | -5.20 | 3 | -5.19 ₂ | -20.7 | 3 | -20.1 ₃ | | |
| Edinburgh, | Aug. 25 | -5.21 | 2 | -5.20 ₃ | | | | | |
| [23 ^h 6 ^m] | Aug. 23 | | ... | -5.22 | | ... | -20.7 | [-79.7] | [+11.7] |
| | | | | | | | | -79.7 | +11.0 |
| Greenwich, | Sept. 10 | -5.14 | 12 | -5.18 ₃ | -21.0 | 12 | -20.3 ₁₂ | | |
| Königsberg, | Sept. 12 | -5.11 | 12 | -5.13 ₃ | -21.7 | 12 | -20.7 ₆ | | |
| Berlin, | Sept. 10 | -5.10 | 14 | -5.11 ₄ | -21.1 | 14 | -20.1 ₇ | | |
| Cambridge, | Sept. 17 | -5.12 | 11 | -5.23 ₃ | -20.0 | 10 | -20.3 ₁₀ | | |
| Paris, | Sept. 14 | -5.23 | 10 | -5.22 ₃ | -20.7 | 10 | -20.1 ₁₀ | | |
| Edinburgh, | Sept. 16 | -5.16 | 17 | -5.15 ₃ | -20.5 | 5 | -20.5 ₅ | | |
| Vienna, | Sept. 17 | -5.32 | 9 | -5.33 ₁ | -20.8 | 9 | -19.8 ₂ | | |
| [23 ^h 3 ^m] | Sept. 14 | | ... | -5.17 | | ... | -20.3 | [-79.0] | [+11.6] |
| | | | | | | | | -78.8 | +11.0 |
| Greenwich, | Oct. 12 | -5.18 | 5 | -5.22 ₂ | -20.2 | 5 | -19.5 ₅ | | |
| Cambridge, | Oct. 15 | -5.07 | 8 | -5.18 ₁ | -19.2 | 5 | -19.5 ₅ | | |
| Edinburgh, | Oct. 15 | -5.14 | 10 | -5.13 ₁ | -19.9 | 8 | -19.9 ₈ | | |
| Vienna, | Oct. 10 | -5.17 | 13 | -5.18 ₁ | -20.5 | 13 | -19.5 ₄ | | |
| [22 ^h 59 ^m] | Oct. 14 | | ... | -5.18 | | ... | -19.6 | [-77.4] | [+11.5] |
| | | | | | | | | -78.7 | +11.8 |
| Greenwich, | Nov. 12 | -4.98 | 6 | -5.02 ₂ | -20.2 | 6 | -19.5 ₆ | | |
| Cambridge, | Nov. 19 | -4.85 | 6 | -4.96 ₂ | -19.1 | 5 | -19.4 ₅ | | |
| Paris, | Nov. 9 | -4.99 | 2 | -4.98 ₁ | -20.4 | 2 | -19.8 ₂ | | |
| Edinburgh, | Nov. 11 | -4.90 | 3 | -4.89 ₁ | -18.3 | 1 | -18.3 ₁ | | |
| [22 ^h 57 ^m] | Nov. 14 | | ... | -4.97 | | ... | -19.4 | [-75.3] | [+11.3] |
| | | | | | | | | -75.7 | +10.4 |
| Greenwich, | Dec. 6 | -4.84 | 2 | -4.88 ₁ | -19.0 | 2 | -18.3 ₂ | | |
| Cambridge, | Dec. 15 | -4.83 | 5 | -4.94 ₂ | -19.1 | 2 | -19.4 ₂ | | |
| Paris, | Dec. 3 | -4.90 | 3 | -4.89 ₂ | -17.8 | 3 | -17.2 ₃ | | |
| Edinburgh, | Dec. 28 | -4.96 | 4 | -4.95 ₁ | -18.5 | 3 | -18.5 ₃ | | |
| [22 ^h 57 ^m] | Dec. 12 | | ... | -4.92 | | ... | -18.2 | [-73.9] | [+11.0] |
| | | | | | | | | -74.6 | +11.2 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|---------------------------------------|-------------|-------------------------------|----------------|--------------------|------------------------------|----------------|---------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1840 | s | | s | " | | " | " | " |
| Greenwich, | Aug. 14 | -5.77 | 11 | -5.81 ₃ | -24.5 | 11 | -23.9 ₁₁ | | |
| Cambridge, | Aug. 14 | -5.64 | 1 | -5.73 ₄ | -25.0 | 1 | -25.2 ₁ | | |
| Edinburgh, | Aug. 31 | -5.65 | 1 | -5.64 ₃ | -23.8 | 1 | -23.7 ₁ | [-87.9] | [+11.4] |
| [23 ^h 20 ^m] | Aug. 16 | | ... | -5.78 | | ... | -23.9 | -88.8 | +11.9 |
| Greenwich, | Sept. 15 | -5.66 | 8 | -5.70 ₃ | -24.0 | 8 | -23.4 ₈ | | |
| Königsberg, | Sept. 13 | -5.74 | 6 | -5.76 ₄ | -25.1 | 6 | -24.1 ₃ | | |
| Cambridge, | Sept. 12 | -5.43 | 1 | -5.52 ₃ | | | | | |
| Edinburgh, | Sept. 16 | -5.70 | 13 | -5.69 ₄ | -24.6 | 9 | -24.5 ₉ | | |
| Paris, | Sept. 6 | -5.60 | 4 | -5.59 ₂ | -23.6 | 4 | -23.0 ₄ | | |
| Berlin, | Sept. 14 | -5.45 | 2 | -5.45 ₄ | -26.3 | 2 | -25.3 ₁ | | |
| Vienna, | Sept. 9 | -5.76 | 2 | -5.76 ₄ | -22.2 | 2 | -21.2 ₄ | [-87.6] | [+11.5] |
| [23 ^h 18 ^m] | Sept. 13 | | ... | -5.69 | | ... | -23.8 | -87.5 | +11.4 |
| Greenwich, | Oct. 10 | -5.61 | 9 | -5.65 ₃ | -24.5 | 9 | -23.9 ₉ | | |
| Cambridge, | Oct. 9 | -5.49 | 5 | -5.58 ₂ | -23.5 | 5 | -23.7 ₅ | | |
| Edinburgh, | Oct. 15 | -5.64 | 12 | -5.63 ₄ | -23.3 | 10 | -23.2 ₁₀ | | |
| Paris, | Oct. 11 | -5.60 | 4 | -5.59 ₂ | -23.3 | 4 | -22.7 ₄ | | |
| Berlin, | Oct. 27 | -5.50 | 1 | -5.50 ₄ | -22.8 | 1 | -21.8 ₁ | | |
| Vienna, | Oct. 19 | -5.76 | 5 | -5.76 ₄ | -22.5 | 4 | -21.5 ₁ | [-85.8] | [+11.4] |
| [23 ^h 14 ^m] | Oct. 12 | | ... | -5.62 | | ... | -23.3 | -86.3 | +11.2 |
| Greenwich, | Nov. 6 | -5.58 | 7 | -5.62 ₂ | -23.5 | 6 | -22.9 ₆ | | |
| Cambridge, | Nov. 3 | -5.34 | 2 | -5.43 ₁ | -24.2 | 1 | -24.4 ₁ | | |
| Edinburgh, | Nov. 17 | -5.41 | 8 | -5.40 ₃ | -22.9 | 1 | -22.8 ₁ | | |
| Paris, | Nov. 4 | -5.52 | 2 | -5.51 ₁ | -26.1 | 2 | -25.5 ₂ | | |
| Vienna, | Nov. 15 | -5.42 | 6 | -5.42 ₁ | -23.2 | 6 | -22.2 ₂ | [-83.7] | [+11.3] |
| [23 ^h 12 ^m] | Nov. 9 | | ... | -5.48 | | ... | -23.3 | -84.4 | +10.3 |
| Greenwich, | Dec. 3 | -5.36 | 8 | -5.40 ₂ | -23.5 | 9 | -22.9 ₉ | | |
| Cambridge, | Dec. 3 | -5.38 | 2 | -5.47 ₁ | -22.5 | 2 | -22.7 ₂ | | |
| Edinburgh, | Dec. 15 | -5.49 | 3 | -5.48 ₁ | -22.6 | 2 | -22.5 ₂ | | |
| Vienna, | Dec. 4 | -5.50 | 1 | | | | | [-82.2] | [+11.0] |
| [23 ^h 12 ^m] | Dec. 6 | | ... | -5.44 | | ... | -22.8 | -83.7 | +10.4 |
| | 1841 | | | | | | | | |
| Greenwich, | Aug. 20 | -6.16 | 5 | -6.20 ₂ | -28.6 | 5 | -28.1 ₅ | | |
| Paris, | Aug. 19 | -6.14 | 2 | -6.13 ₁ | -29.6 | 2 | -29.0 ₂ | [-96.9] | [+11.3] |
| [23 ^h 37 ^m] | Aug. 20 | | ... | -6.18 | | ... | -28.3 | -96.2 | +10.6 |
| Greenwich, | Sept. 11 | -6.16 | 10 | -6.20 ₁ | -29.0 | 10 | -28.5 ₁₀ | | |
| Königsberg, | Sept. 11 | -6.18 | 5 | -6.18 ₄ | -30.1 | 5 | -29.1 ₂ | | |
| Berlin, | Sept. 14 | -6.14 | 7 | -6.14 ₂ | -29.6 | 7 | -28.6 ₃ | | |
| Edinburgh, | Sept. 17 | -6.11 | 7 | -6.15 ₂ | -28.2 | 5 | -28.0 ₅ | | |
| Paris, | Sept. 14 | -6.16 | 5 | -6.15 ₃ | -29.0 | 3 | -28.4 ₃ | | |
| Vienna, | Sept. 17 | -6.37 | 11 | -6.37 ₁ | -30.3 | 11 | -29.3 ₃ | [-96.5] | [+11.4] |
| [23 ^h 33 ^m] | Sept. 13 | | ... | -6.18 | | ... | -28.5 | -96.3 | +10.4 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|---------------------------------------|-------------|-------------------------------|----------------|--------------------|------------------------------|----------------|---------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1841 | s | | s | " | " | " | " | " |
| Greenwich, | Oct. 17 | -6.09 | 7 | -6.13 ₂ | -28.2 | 7 | -27.7 ₇ | | |
| Berlin, | Oct. 20 | -5.87 | 2 | -5.87 ₁ | -28.2 | 2 | -27.2 ₁ | | |
| Edinburgh, | Oct. 13 | -6.00 | 4 | -6.04 ₁ | -27.2 | 4 | -27.0 ₄ | | |
| Paris, | Oct. 21 | -6.10 | 1 | -6.09 ₁ | -27.2 | 1 | -26.6 ₁ | | |
| Vienna, | Oct. 16 | -6.14 | 9 | -6.14 ₁ | -28.8 | 9 | -27.8 ₂ | | |
| [23 ^h 28 ^m] | Oct. 18 | | ... | -6.06 | | ... | -27.4 | [-94.6] | [+11.2] |
| | | | | | | | | -94.1 | +10.5 |
| Greenwich, | Nov. 17 | -5.95 | 5 | -5.99 ₂ | -28.0 | 5 | -27.5 ₅ | | |
| Berlin, | Nov. 16 | -6.15 | 2 | -6.15 ₁ | -28.4 | 2 | -27.4 ₁ | | |
| Edinburgh, | Nov. 2 | -5.89 | 3 | -5.93 ₁ | -26.5 | 4 | -26.3 ₄ | | |
| Vienna, | Nov. 11 | -5.98 | 4 | -5.98 ₁ | -30.0 | 4 | -29.0 ₁ | | |
| [23 ^h 26 ^m] | Nov. 13 | | ... | -6.01 | | ... | -27.2 | [-92.6] | [+11.0] |
| | | | | | | | | -93.3 | +10.4 |
| Greenwich, | Dec. 14 | -5.86 | 8 | -5.90 ₂ | -27.4 | 8 | -26.9 ₃ | | |
| Berlin, | Dec. 15 | -5.60 | 3 | -5.60 ₁ | -25.5 | 3 | -24.5 ₂ | | |
| Edinburgh, | Dec. 20 | -5.68 | 4 | -5.73 ₁ | | | | | |
| Paris, | Dec. 15 | -5.78 | 4 | -5.77 ₃ | -26.7 | 4 | -26.1 ₄ | | |
| [23 ^h 26 ^m] | Dec. 16 | | ... | -5.78 | | ... | -26.3 | [-90.5] | [+10.7] |
| | | | | | | | | -89.9 | + 9.8 |
| | 1842 | | | | | | | | |
| Greenwich, | Aug. 19 | -6.62 | 5 | -6.64 ₁ | -32.1 | 6 | -31.7 ₆ | | |
| Cambridge, | Aug. 24 | -6.48 | 2 | -6.55 ₁ | -32.7 | 2 | -33.0 ₂ | | |
| [23 ^h 51 ^m] | Aug. 21 | | ... | -6.61 | | ... | -32.0 | [-105.6] | [+11.1] |
| | | | | | | | | -103.6 | +10.0 |
| Greenwich, | Sept. 12 | -6.55 | 4 | -6.57 ₁ | -32.2 | 4 | -31.8 ₄ | | |
| Königsberg, | Sept. 18 | -6.69 | 10 | -6.69 ₂ | -33.4 | 10 | -32.4 ₅ | | |
| Berlin, | Sept. 11 | -6.70 | 3 | -6.69 ₁ | -31.7 | 3 | -30.7 ₁ | | |
| Paris, | Sept. 14 | -6.65 | 5 | -6.64 ₃ | -32.1 | 5 | -31.5 ₃ | | |
| Cambridge, | Sept. 14 | -6.60 | 13 | -6.67 ₆ | -31.9 | 14 | -32.2 ₄ | | |
| Edinburgh, | Sept. 23 | -6.57 | 4 | -6.59 ₁ | -30.9 | 4 | -30.6 ₂ | | |
| Pulkowa, | Sept. 17 | -6.63 | 6 | -6.64 ₂ | | | | | |
| Vienna, | Sept. 19 | -6.81 | 2 | -6.80 ₃ | -33.5 | 2 | -32.5 ₄ | | |
| [23 ^h 47 ^m] | Sept. 16 | | ... | -6.66 | | ... | -32.0 | [-105.4] | [+11.2] |
| | | | | | | | | -104.3 | +10.3 |
| Greenwich, | Oct. 20 | -6.66 | 12 | -6.68 ₃ | -31.8 | 12 | -31.4 ₁₂ | | |
| Berlin, | Oct. 23 | -6.54 | 4 | -6.53 ₁ | -30.7 | 4 | -29.7 ₂ | | |
| Paris, | Oct. 17 | -6.64 | 12 | -6.63 ₆ | -31.5 | 9 | -30.9 ₉ | | |
| Cambridge, | Oct. 17 | -6.60 | 11 | -6.67 ₅ | -31.4 | 10 | -31.7 ₁₀ | | |
| Edinburgh, | Oct. 15 | -6.57 | 12 | -6.59 ₃ | -31.1 | 8 | -30.8 ₅ | | |
| Vienna, | Oct. 10 | -6.74 | 4 | -6.73 ₃ | -34.6 | 5 | -33.6 ₁ | | |
| [23 ^h 44 ^m] | Oct. 17 | | ... | -6.64 | | ... | -31.2 | [-103.6] | [+11.0] |
| | | | | | | | | -103.7 | +10.9 |
| Greenwich, | Nov. 23 | -6.36 | 7 | -6.38 ₃ | -31.0 | 7 | -30.6 ₇ | | |
| Berlin, | Nov. 8 | -6.42 | 2 | -6.41 ₃ | -30.2 | 2 | -29.2 ₁ | | |
| Cambridge, | Nov. 17 | -6.40 | 7 | -6.47 ₁ | -30.7 | 7 | -31.0 ₇ | | |
| Edinburgh, | Nov. 16 | -6.39 | 3 | -6.32 ₁ | | | | | |
| Vienna, | Nov. 20 | -6.39 | 3 | -6.38 ₂ | -30.0 | 2 | -29.0 ₁ | | |
| [23 ^h 42 ^m] | Nov. 19 | | ... | -6.41 | | ... | -30.6 | [-100.8] | [+10.8] |
| | | | | | | | | -100.2 | +10.0 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|---------------------------------------|-------------|-------------------------------|----------------|---------------------|------------------------------|----------------|---------------------|---------------------|------------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1842 | s | | s | " | | " | " | " |
| Greenwich, | Dec. 16 | -6.31 | 8 | -6.33 ₃ | -30.2 | 8 | -29.8 ₈ | | |
| Berlin, | Dec. 10 | -6.16 | 4 | -6.15 ₁ | -31.4 | 4 | -30.4 ₃ | | |
| Paris, | Dec. 14 | -6.23 | 5 | -6.22 ₃ | -30.9 | 5 | -30.3 ₃ | | |
| Cambridge, | Dec. 16 | -6.22 | 9 | -6.29 ₄ | -30.1 | 8 | -30.4 ₈ | | |
| Edinburgh, | Dec. 13 | -6.25 | 2 | -6.18 ₄ | | | | | |
| Pulkowa, | Dec. 13 | -6.28 | 3 | -6.28 ₂ | | | | | |
| [23 ^h 41 ^m] | Dec. 15 | | ... | -6.27 | | ... | -30.2 | [- 98.9] | [+ 10.5] |
| | | | | | | | | - 98.2 | + 9.6 |
| | 1843 | | | | | | | [- 97.5] | [+ 10.3] |
| Edinburgh, | Jan. 9 | -6.32 | 7 | -6.25 | -31.4 | 4 | -31.0 | - 98.2 | + 8.6 |
| Greenwich, | Aug. 20 | -7.11 | 7 | -7.04 ₃ | -35.1 | 7 | -34.7 ₇ | | |
| Paris, | Aug. 21 | -7.20 | 3 | -7.19 ₃ | -37.5 | 5 | -37.1 ₃ | | |
| [0 ^h 6 ^m] | Aug. 20 | | ... | -7.10 | | ... | -35.7 | [- 114.3] | [+ 10.7] |
| | | | | | | | | - 111.9 | + 9.6 |
| Greenwich, | Sept. 17 | -7.23 | 9 | -7.16 ₃ | -35.4 | 9 | -35.0 ₉ | | |
| Paris, | Sept. 15 | -7.17 | 11 | -7.16 ₆ | -36.8 | 14 | -36.2 ₁₄ | | |
| Edinburgh, | Sept. 18 | -7.26 | 7 | -7.19 ₂ | -36.8 | 5 | -36.4 ₃ | | |
| Pulkowa, | Sept. 22 | -7.18 | 10 | -7.18 ₁₀ | | | | | |
| [0 ^h 3 ^m] | Sept. 19 | | ... | -7.17 | | ... | -35.8 | [- 114.1] | [+ 10.6] |
| | | | | | | | | - 112.9 | + 10.0 |
| Greenwich, | Oct. 18 | -7.10 | 10 | -7.03 ₃ | -35.3 | 10 | -34.9 ₁₀ | | |
| Paris, | Oct. 17 | -7.07 | 6 | -7.06 ₃ | -37.1 | 5 | -36.5 ₅ | | |
| Edinburgh, | Oct. 17 | -7.05 | 12 | -6.98 ₃ | -35.1 | 5 | -34.7 ₅ | | |
| [23 ^h 59 ^m] | Oct. 17 | | ... | -7.02 | | ... | -35.2 | [- 112.6] | [+ 10.5] |
| | | | | | | | | - 110.6 | + 9.6 |
| Greenwich, | Nov. 20 | -7.01 | 9 | -6.94 ₃ | -34.3 | 9 | -33.9 ₉ | | |
| Königsberg, | Nov. 11 | -6.84 | 5 | -6.83 ₄ | -34.4 | 5 | -33.4 ₂ | | |
| Paris, | Nov. 15 | -7.03 | 3 | -7.02 ₂ | -36.3 | 3 | -35.7 ₃ | | |
| Edinburgh, | Nov. 15 | -6.93 | 8 | -6.86 ₂ | -35.7 | 6 | -35.3 ₆ | | |
| [23 ^h 56 ^m] | Nov. 15 | | ... | -6.90 | | ... | -34.5 | [- 110.1] | [+ 10.4] |
| | | | | | | | | - 108.6 | + 9.5 |
| | 1844 | | | | | | | | |
| Greenwich, | Jan. 3 | -6.70 | 3 | -6.64 ₁ | -34.1 | 3 | -33.7 ₃ | | |
| Edinburgh, | Jan. 10 | -6.67 | 6 | -6.60 ₂ | -35.5 | 2 | -34.9 ₂ | | |
| Paris, | Jan. 3 | -6.72 | 2 | -6.71 ₁ | -35.5 | 2 | -34.9 ₂ | | |
| [23 ^h 56 ^m] | Jan. 7 | | ... | -6.64 | | ... | -34.2 | [- 106.1] | [+ 9.9] |
| | | | | | | | | - 105.0 | + 8.2 |
| Greenwich, | Aug. 19 | -7.73 | 10 | -7.67 ₃ | -39.6 | 10 | -39.2 ₁₀ | | |
| Paris, | Aug. 17 | -7.70 | 3 | -7.69 ₂ | -40.0 | 3 | -39.4 ₃ | | |
| [0 ^h 22 ^m] | Aug. 18 | | ... | -7.68 | | ... | -39.2 | [- 122.3] | [+ 10.2] |
| | | | | | | | | - 121.2 | + 9.6 |
| Greenwich, | Sept. 25 | -7.74 | 11 | -7.68 ₄ | -39.9 | 11 | -39.5 ₁₁ | | |
| Edinburgh, | Sept. 19 | -7.67 | 11 | -7.60 ₃ | -40.3 | 10 | -39.8 ₁₀ | | |
| Königsberg, | Sept. 17 | -7.65 | 10 | -7.64 ₇ | -40.7 | 10 | -39.7 ₅ | | |
| Paris, | Sept. 10 | -7.63 | 10 | -7.62 ₃ | -39.4 | 15 | -38.8 ₁₅ | | |
| [0 ^h 18 ^m] | Sept. 17 | | ... | -7.63 | | ... | -39.3 | [- 122.6] | [+ 10.3] |
| | | | | | | | | - 120.6 | + 9.3 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|---------------------------------------|-------------|-------------------------------|----------------|---------------------|------------------------------|----------------|---------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1844 | s | | s | " | | " | " | " |
| Greenwich, | Oct. 17 | -7.70 | 9 | -7.64 ₃ | -39.2 | 9 | -38.8 ₉ | | |
| Edinburgh, | Oct. 13 | -7.58 | 10 | -7.51 ₂ | -40.5 | 1 | -40.0 ₁ | [-121.5] | [+10.2] |
| [0 ^h 15 ^m] | Oct. 15 | | ... | -7.59 | | .. | -39.0 | -120.0 | + 9.4 |
| Greenwich, | Nov. 26 | -7.44 | 9 | -7.38 ₃ | -38.7 | 9 | -38.3 ₉ | | |
| Edinburgh, | Nov. 19 | -7.41 | 7 | -7.35 ₂ | | | | [-118.2] | [+ 9.9] |
| [0 ^h 12 ^m] | Nov. 23 | | ... | -7.37 | | .. | -38.3 | -116.6 | + 8.8 |
| Edinburgh, | Dec. 18 | -7.24 | 5 | -7.18 ₁ | | | | | |
| Paris, | Dec. 22 | -7.10 | 2 | -7.09 ₁ | -39.3 | 3 | -38.7 ₃ | [-115.7] | [+ 9.6] |
| [0 ^h 10 ^m] | Dec. 20 | | ... | -7.14 | | .. | -38.7 | -113.7 | + 7.1 |
| | 1845 | | | | | | | | |
| Greenwich, | Jan. 13 | -7.18 | 1 | -7.10 ₁ | -38.4 | 1 | -38.0 | | |
| Edinburgh, | Jan. 14 | -7.16 | 8 | -7.08 ₁ | | | | [-114.0] | [+ 9.5] |
| [0 ^h 11 ^m] | Jan. 14 | | ... | -7.08 | | .. | -38.0 | -112.5 | + 7.3 |
| Greenwich, | Aug. 25 | -8.25 | 5 | -8.17 ₂ | -42.7 | 6 | -42.3 | [-131.0] | [+ 9.6] |
| | | | | | | | | -129.1 | + 9.2 |
| Greenwich, | Sept. 18 | -8.21 | 10 | -8.13 ₄ | -42.6 | 10 | -42.2 ₁₀ | | |
| Königsberg, | Sept. 30 | -8.16 | 5 | -8.15 ₄ | -43.4 | 5 | -42.4 ₂ | | |
| Paris, | Sept. 14 | -8.24 | 5 | -8.23 ₃ | -43.9 | 3 | -43.3 ₃ | [-131.6] | [+ 9.9] |
| [0 ^h 33 ^m] | Sept. 22 | | ... | -8.17 | | .. | -42.5 | -129.2 | + 9.1 |
| Greenwich, | Oct. 17 | -8.22 | 10 | -8.14 ₄ | -42.2 | 10 | -41.8 ₁₀ | | |
| Paris, | Oct. 20 | -8.01 | 6 | -8.00 ₃ | -43.7 | 6 | -43.1 ₆ | [-130.2] | [+ 9.8] |
| [0 ^h 29 ^m] | Oct. 18 | | ... | -8.08 | | .. | -42.3 | -128.0 | + 8.9 |
| Greenwich, | Nov. 9 | -8.06 | 6 | -7.98 | -43.3 | 7 | -42.9 | [-128.4] | [+ 9.6] |
| [0 ^h 27 ^m] | | | ... | | | .. | | -126.9 | [+ 7.9] |
| Greenwich, | Dec. 14 | -7.85 | 8 | -7.77 | -49.2 | 8 | -39.8 | [-124.8] | [+ 9.3] |
| [0 ^h 25 ^m] | | | ... | | | .. | | -122.8 | + 9.6 |
| | 1846 | | | | | | | | |
| Greenwich, | Sept. 8 | -8.80 | 8 | -8.76 ₃ | -46.2 | 8 | -45.8 ₈ | | |
| Paris, | Sept. 12 | -8.73 | 19 | -8.72 ₁₀ | -46.9 | 14 | -46.3 ₁₄ | [-139.9] | [+ 9.2] |
| [0 ^h 50 ^m] | Sept. 11 | | ... | -8.73 | | .. | -46.1 | -138.2 | + 8.3 |
| Greenwich, | Oct. 8 | -8.74 | 7 | -8.70 ₃ | -46.8 | 7 | -46.4 ₇ | | |
| Königsberg, | Oct. 4 | -8.61 | 11 | -8.61 ₈ | -44.8 | 11 | -43.8 ₆ | | |
| Paris, | Oct. 14 | -8.69 | 13 | -8.68 ₇ | -46.7 | 13 | -46.1 ₁₃ | [-139.7] | [+ 9.2] |
| [0 ^h 46 ^m] | Oct. 9 | | ... | -8.65 | | .. | -45.8 | -137.1 | + 8.3 |
| Greenwich, | Nov. 10 | -8.62 | 6 | -8.58 ₂ | -46.8 | 6 | -46.4 ₆ | | |
| Paris, | Nov. 16 | -8.48 | 9 | -8.47 ₅ | -46.4 | 9 | -45.8 ₉ | [-136.8] | [+ 9.0] |
| [0 ^h 41 ^m] | Nov. 14 | | ... | -8.50 | | .. | -46.0 | -135.1 | + 7.6 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|---|-------------|-------------------------------|----------------|---------------------|------------------------------|----------------|---------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1846 | s | | s | " | | " | " | " |
| Greenwich,
Paris,
[0 ^h 39 ^m] | Dec. 15 | —8.31 | 9 | —8.27 ₃ | —46.0 | 10 | —45.6 ₁₀ | | |
| | Dec. 16 | —8.25 | 6 | —8.24 ₃ | —46.1 | 4 | —45.5 ₄ | [—134.5] | [+8.9] |
| | Dec. 16 | | ... | —8.26 | | .. | —45.6 | —131.7 | +6.5 |
| | 1847 | | | | | | | | |
| Greenwich,
Paris,
[0 ^h 40 ^m] | Jan. 13 | —8.25 | 4 | —8.20 ₂ | —43.1 | 4 | —42.7 ₄ | | |
| | Jan. 11 | —8.14 | 6 | —8.12 ₃ | —45.4 | 6 | —44.8 ₆ | [—130.9] | [+8.6] |
| | Jan. 12 | | ... | —8.15 | | .. | —44.0 | —129.6 | +7.4 |
| Greenwich,
[1 ^h 6 ^m] | Sept. 3 | —9.21 | 8 | —9.16 | —49.4 | 8 | —49.0 | [—148.4] | [+8.7] |
| | | | ... | | | .. | | —144.9 | +6.8 |
| Greenwich,
Paris,
[1 ^h 2 ^m] | Oct. 1 | —9.25 | 6 | —9.20 ₃ | —49.0 | 6 | —48.6 ₆ | | |
| | Oct. 12 | —9.23 | 18 | —9.21 ₁₀ | —49.2 | 16 | —48.6 ₁₆ | [—148.0] | [+8.7] |
| | Oct. 10 | | ... | —9.21 | | .. | —48.6 | —145.5 | +7.8 |
| Greenwich,
Paris,
[0 ^h 57 ^m] | Nov. 3 | —9.10 | 9 | —9.05 ₃ | —49.2 | 9 | —48.8 ₉ | | |
| | Nov. 12 | —9.08 | 10 | —9.06 ₅ | —49.3 | 9 | —48.7 ₉ | [—146.2] | [+8.5] |
| | Nov. 8 | | ... | —9.06 | | .. | —48.8 | —142.6 | +7.2 |
| Greenwich,
Paris,
[0 ^h 54 ^m] | Dec. 3 | —8.96 | 7 | —8.91 ₂ | —49.0 | 7 | —48.6 ₇ | | |
| | Dec. 12 | —8.84 | 9 | —8.82 ₅ | —48.5 | 7 | —47.9 ₇ | [—143.0] | [+8.3] |
| | Dec. 9 | | ... | —8.85 | | .. | —48.2 | —140.6 | +6.8 |
| | 1848 | | | | | | | | |
| Greenwich,
Paris,
[0 ^h 54 ^m] | Jan. 10 | —8.83 | 2 | —8.78 ₂ | —47.1 | 2 | —46.7 ₂ | | |
| | Jan. 11 | —8.53 | 5 | —8.51 ₅ | —47.6 | 5 | —47.0 ₅ | [—139.4] | [+8.1] |
| | Jan. 11 | | ... | —8.57 | | .. | —46.9 | —136.2 | +6.0 |
| Greenwich,
Paris,
[1 ^h 19 ^m] | Sept. 8 | —9.84 | 7 | —9.79 ₇ | —51.1 | 7 | —50.7 ₇ | | |
| | Sept. 22 | —9.82 | 11 | —9.80 ₁₁ | —51.5 | 9 | —50.9 ₉ | [—156.4] | [+8.0] |
| | Sept. 17 | | ... | —9.80 | | .. | —50.8 | —154.1 | +7.5 |
| Greenwich,
Paris,
[1 ^h 15 ^m] | Oct. 19 | —9.82 | 7 | —9.77 ₇ | —52.2 | 7 | —51.8 ₇ | | |
| | Oct. 16 | —9.79 | 11 | —9.77 ₁₁ | —52.2 | 10 | —51.6 ₁₀ | [—156.4] | [+8.0] |
| | Oct. 17 | | ... | —9.77 | | .. | —51.7 | —154.2 | +6.9 |
| Greenwich,
Paris,
[1 ^h 11 ^m] | Nov. 13 | —9.68 | 7 | —9.63 ₇ | —52.3 | 7 | —51.9 ₇ | | |
| | Nov. 13 | —9.55 | 4 | —9.53 ₄ | —52.2 | 3 | —51.6 ₃ | [—154.6] | [+8.0] |
| | Nov. 13 | | ... | —9.59 | | .. | —51.8 | —151.8 | +6.3 |
| Greenwich,
Paris,
[1 ^h 9 ^m] | Dec. 14 | —9.41 | 5 | —9.36 ₅ | —51.2 | 5 | —50.8 ₅ | | |
| | Dec. 9 | —9.43 | 5 | —9.41 ₅ | —51.3 | 6 | —50.7 ₆ | [—150.2] | [+7.7] |
| | Dec. 12 | | ... | —9.38 | | .. | —50.7 | —148.6 | +6.2 |
| | 1849 | | | | | | | | |
| Greenwich,
Paris,
[1 ^h 9 ^m] | Jan. 6 | —9.37 | 1 | —9.37 ₁ | —51.2 | 1 | —50.8 ₁ | | |
| | Jan. 18 | —9.11 | 1 | —9.09 ₁ | —50.0 | 1 | —49.4 ₁ | [—147.7] | [+7.5] |
| | Jan. 12 | | ... | —9.23 | | .. | —50.1 | —146.2 | +6.0 |

MEAN CORRECTIONS TO THE EPIHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|---|-------------|-------------------------------|----------------|---------------------|------------------------------|----------------|--------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1849 | s | | s | " | | " | " | " |
| Greenwich,
[1 ^h 34 ^m] | Sept. 16 | -10.27 | 3 | -10.27 | -53.3 | 3 | -52.9 | [+165.1] | [+7.3] |
| | | | | | | | | +160.8 | +6.2 |
| Greenwich,
Paris,
[1 ^h 30 ^m] | Oct. 28 | -10.24 | 6 | -10.24 ₆ | -53.9 | 6 | -53.5 ₆ | | |
| | Oct. 21 | -10.26 | 5 | -10.24 ₅ | -54.8 | 5 | -54.2 ₅ | [-164.4] | [+7.4] |
| | Oct. 25 | | ... | -10.24 | | .. | -53.8 | -161.0 | +5.8 |
| Greenwich,
Paris,
[1 ^h 26 ^m] | Nov. 16 | -10.12 | 3 | -10.12 | -54.4 | 3 | -54.0 ₃ | | |
| | Nov. 17 | -10.22 | 5 | -10.20 ₅ | -54.1 | 4 | -53.5 ₄ | [-162.5] | [+7.3] |
| | Nov. 17 | | ... | -10.17 | | .. | -53.7 | -160.1 | +6.0 |
| Greenwich,
Paris,
[1 ^h 24 ^m] | Dec. 13 | -9.95 | 6 | -9.95 ₆ | -53.1 | 9 | -52.7 ₉ | | |
| | Dec. 16 | -9.88 | 2 | -9.86 ₂ | -54.4 | 2 | -53.8 ₂ | [-160.4] | [+7.1] |
| | Dec. 14 | | ... | -9.93 | | .. | -52.9 | -156.6 | +5.7 |
| | 1850 | | | | | | | | |
| Greenwich,
Paris,
[1 ^h 24 ^m] | Jan. 7 | -9.71 | 1 | -9.71 ₁ | -53.0 | 1 | -52.6 ₁ | | |
| | Jan. 5 | -9.77 | 2 | -9.75 ₂ | -53.0 | 3 | -52.4 ₃ | [-156.8] | [+6.9] |
| | Jan. 6 | | ... | -9.74 | | .. | -52.5 | -153.8 | +6.2 |
| Greenwich,
[1 ^h 52 ^m] | Sept. 6 | -10.83 | 9 | -10.83 | -53.9 | 9 | -53.5 | [-171.7] | [+6.5] |
| | | | | | | | | -168.1 | +5.9 |
| Greenwich,
Paris,
[1 ^h 47 ^m] | Oct. 11 | -10.92 | 8 | -10.92 | -55.3 | 7 | -54.9 ₇ | | |
| | Oct. 17 | -19.87 | 7 | -10.85 ₇ | -56.3 | 5 | -55.7 | [-173.3] | [+6.4] |
| | Oct. 16 | | ... | -10.87 | | .. | -55.3 | -169.6 | +5.3 |
| Greenwich,
Paris,
[1 ^h 43 ^m] | Nov. 6 | -10.79 | 5 | -10.79 ₅ | -55.7 | 6 | -55.3 ₆ | | |
| | Nov. 10 | -10.89 | 3 | -10.87 ₃ | -56.1 | 4 | -55.5 ₄ | [-172.4] | [+6.4] |
| | Nov. 8 | | ... | -10.82 | | .. | -55.4 | -169.1 | +5.6 |
| Greenwich,
Königsberg,
[1 ^h 41 ^m] | Dec. 7 | -10.56 | 8 | -10.56 ₈ | -55.0 | 9 | -54.6 ₉ | | |
| | Dec. 15 | -10.50 | 2 | -10.48 ₂ | -56.0 | 2 | | [-169.0] | [+6.3] |
| | Dec. 9 | | ... | -10.54 | | .. | -54.6 | -165.0 | +5.2 |
| | 1851 | | | | | | | | |
| Greenwich,
[1 ^h 41 ^m] | Jan. 19 | -10.21 | 4 | -10.21 | -54.8 | 4 | -54.4 | [-163.5] | [+6.0] |
| | | | | | | | | -160.4 | +3.5 |
| Greenwich,
Königsberg,
Paris,
[2 ^h 7 ^m] | Sept. 11 | -11.42 | 2 | -11.42 ₂ | -55.7 | 2 | -55.3 ₂ | | |
| | Sept. 19 | -11.41 | 2 | -11.41 ₁ | | .. | | | |
| | Sept. 14 | -11.59 | 1 | -11.57 ₁ | -57.5 | 1 | -56.9 ₁ | [-180.0] | [+5.7] |
| | Sept. 14 | | ... | -11.46 | | .. | -55.8 | -176.9 | +4.4 |
| Königsberg,
Paris,
[2 ^h 2 ^m] | Oct. 22 | -11.75 | 3 | -11.75 ₂ | -58.4 | 3 | -57.4 | | |
| | Oct. 22 | -11.46 | 3 | -11.44 ₃ | | .. | | [-181.8] | [+5.8] |
| | Oct. 23 | | ... | -11.56 | | .. | -57.4 | -179.1 | +4.3 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|---|-------------|-------------------------------|----------------|---------------------|------------------------------|----------------|--------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1851 | s | | s | " | | " | " | " |
| Greenwich,
Paris, | Nov. 2 | -11.50 | 5 | -11.50 ₅ | -57.5 | 5 | -57.1 ₅ | | |
| | Nov. 2 | -11.48 | 2 | -11.46 ₅ | -58.1 | 1 | -57.5 ₁ | [-181.4] | [+5.7] |
| [2 ^h 0 ^m] | Nov. 2 | | ... | -11.49 | | .. | -57.2 | -178.1 | +4.5 |
| Greenwich,
Paris, | Nov. 22 | -11.43 | 5 | -11.43 | -58.3 | 5 | -57.9 ₅ | | |
| | Nov. 14 | -11.39 | 1 | -11.37 ₁ | | .. | | [-179.9] | [+5.6] |
| [1 ^h 58 ^m] | Nov. 21 | | ... | -11.42 | | .. | -57.9 ₅ | -177.6 | +3.8 |
| Greenwich,
Paris, | Dec. 22 | -11.15 | 3 | -11.15 ₃ | -57.2 | 3 | -56.8 ₃ | | |
| | Dec. 25 | -11.00 | 3 | -10.98 ₃ | -57.2 | 3 | -56.6 ₃ | [-175.7] | [+5.3] |
| [1 ^h 54 ^m] | Dec. 24 | | ... | -11.06 | | .. | -56.7 | -172.3 | +3.8 |
| | 1852 | | | | | | | | |
| Greenwich,
Paris, | Jan. 11 | -10.85 | 7 | -10.85 ₇ | -57.2 | 7 | -56.8 ₇ | | |
| | Jan. 14 | -10.89 | 5 | -10.87 ₅ | -56.5 | 5 | -55.9 ₅ | [-173.1] | [+5.2] |
| [1 ^h 54 ^m] | Jan. 12 | | ... | -10.86 | | .. | -56.4 | -169.4 | +3.1 |
| Greenwich,
[2 ^h 22 ^m] | Sept. 12 | -12.06 | 7 | -12.06 ₇ | -56.5 | 7 | -56.1 | [-187.4] | [+4.9] |
| | | | ... | | | .. | | -184.5 | +3.9 |
| Greenwich,
Paris, | Oct. 17 | -12.23 | 8 | -12.23 ₈ | -58.6 | 8 | -58.2 ₈ | | |
| | Oct. 23 | -12.06 | 3 | -12.04 ₃ | -59.6 | 3 | -59.0 ₃ | [-189.6] | [+4.7] |
| [2 ^h 18 ^m] | Oct. 19 | | ... | -12.13 | | .. | -58.4 | -187.2 | +3.1 |
| Greenwich,
Paris, | Nov. 13 | -12.00 | 5 | -12.00 ₅ | -59.7 | 5 | -59.3 ₅ | | |
| | Nov. 15 | -12.08 | 5 | -12.06 ₅ | -60.2 | 4 | -59.6 ₄ | [-188.9] | [+4.7] |
| [2 ^h 14 ^m] | Nov. 14 | | ... | -12.03 | | .. | -59.4 | -185.6 | +2.4 |
| Greenwich,
Königsberg,
Paris, | Dec. 19 | -11.72 | 7 | -11.72 ₇ | -58.7 | 7 | -58.3 ₇ | | |
| | Dec. 16 | -11.67 | 3 | | -59.3 | 3 | -58.3 ₂ | | |
| | Dec. 19 | -11.77 | 5 | -11.75 ₅ | -59.0 | 5 | -58.4 ₅ | [-184.8] | [+4.6] |
| [2 ^h 10 ^m] | Dec. 19 | | ... | -11.73 | | .. | -58.3 | -181.3 | +2.8 |
| | 1853 | | | | | | | | |
| Greenwich,
Paris, | Jan. 12 | -11.42 | 7 | -11.42 ₇ | -57.5 | 7 | -57.1 | | |
| | Jan. 15 | -11.46 | 4 | -11.44 ₄ | | .. | | [-181.0] | [+4.4] |
| [2 ^h 9 ^m] | Jan. 13 | | ... | -11.43 | | .. | -57.1 | -176.8 | +2.6 |
| Greenwich,
Paris, | Sept. 17 | -12.69 | 4 | -12.69 ₄ | -56.7 | 4 | -56.3 ₄ | | |
| | Sept. 16 | -12.58 | 2 | -12.56 ₂ | -57.0 | 2 | -56.4 ₂ | [-194.6] | [+4.0] |
| [2 ^h 39 ^m] | Sept. 17 | | ... | -12.65 | | .. | -56.3 | -191.6 | +2.5 |
| Greenwich,
Paris, | Oct. 15 | -12.69 | 7 | -12.69 ₇ | -58.8 | 7 | -58.4 ₇ | | |
| | Oct. 27 | -12.66 | 6 | -12.64 ₆ | -59.0 | 6 | -58.4 ₆ | [-196.7] | [+3.8] |
| [2 ^h 34 ^m] | Oct. 22 | | ... | -12.66 | | .. | -58.4 | -192.7 | +1.7 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R.A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|--|-------------|------------------------------|----------------|----------------------|------------------------------|----------------|---------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1853 | s | | s | " | | " | " | |
| Greenwich,
Paris,
[2 ^h 31 ^m] | Nov. 12 | -12.65 | 14 | -12.65 ₁₄ | -59.4 | 14 | -59.0 ₁₄ | | |
| | Nov. 15 | -12.67 | 10 | -12.65 ₁₀ | -59.2 | 8 | -58.6 ₈ | [-196.0] | [+3.7] |
| | Nov. 14 | | ... | -12.65 | | .. | -58.9 | -192.9 | +1.9 |
| Paris,
[2 ^h 27 ^m] | Dec. 12 | -12.28 | 1 | -12.26 | -59.0 | 1 | -58.4 | [-193.4] | [+3.6] |
| | | | ... | | | .. | | -187.9 | +1.5 |
| | 1854 | | | | | | | | |
| Greenwich,
Paris,
[2 ^h 25 ^m] | Jan. 13 | -12.02 | 4 | -12.02 ₄ | -59.7 | 4 | -59.2 ₄ | | |
| | Jan. 20 | -11.87 | 2 | -11.85 ₂ | -58.1 | 1 | -57.9 ₁ | [-188.1] | [+3.5] |
| | Jan. 15 | | ... | -11.96 | | .. | -58.9 | -183.9 | +0.1 |
| Paris,
[2 ^h 26 ^m] | Feb. 8 | -11.89 | 1 | -11.87 | -57.8 | 1 | -57.6 | [-184.5] | [+3.5] |
| | | | ... | | | .. | | -182.1 | +0.7 |
| Greenwich,
[2 ^h 53 ^m] | Sept. 21 | -13.12 | 6 | -13.12 | -56.0 | 6 | -55.5 | [-201.3] | [+3.0] |
| | | | ... | | | .. | | -196.9 | +1.7 |
| Greenwich,
Paris,
[2 ^h 50 ^m] | Oct. 26 | -13.24 | 7 | -13.24 ₇ | -58.0 | 7 | -57.5 ₂ | | |
| | Oct. 29 | -13.12 | 1 | -13.10 ₁ | -57.2 | 3 | -57.0 ₁ | [-203.7] | [+2.8] |
| | Oct. 27 | | ... | -13.22 | | .. | -57.3 | -199.0 | +1.2 |
| Greenwich,
Paris,
Santiago,
[2 ^h 48 ^m] | Nov. 12 | -13.22 | 5 | -13.22 ₅ | -58.2 | 5 | -57.5 ₅ | | |
| | Nov. 16 | -13.21 | 6 | -13.19 ₆ | -58.7 | 1 | -58.5 ₁ | | |
| | Nov. 17 | -13.20 | 4 | -13.20 ₃ | -58.7 | 3 | -58.1 ₂ | [-203.0] | [+2.7] |
| | Nov. 15 | | ... | -13.20 | | .. | -57.8 | -199.1 | +1.2 |
| Greenwich
Paris,
Santiago,
[2 ^h 44 ^m] | Dec. 14 | -12.93 | 7 | -12.93 ₇ | -58.6 | 7 | -58.1 ₂ | | |
| | Dec. 9 | -13.14 | 1 | -13.12 ₁ | -56.9 | 4 | -56.7 ₁ | | |
| | Dec. 15 | -12.85 | 10 | -12.85 ₇ | -59.7 | 9 | -59.1 ₁ | [-200.1] | [+2.6] |
| | Dec. 14 | | ... | -12.90 | | .. | -58.0 | -195.3 | +0.7 |
| | 1855 | | | | | | | | |
| Greenwich,
[2 ^h 42 ^m] | Jan. 18 | -12.55 | 2 | -12.55 ₂ | -56.9 | 2 | -56.4 | [-194.6] | [+2.5] |
| | | | ... | | | .. | | -190.1 | +1.1 |
| Greenwich,
Paris,
[3 ^h 9 ^m] | Oct. 13 | -13.71 | 6 | -13.71 ₆ | -55.7 | 6 | -55.2 ₃ | | |
| | Oct. 25 | -13.93 | 3 | -13.91 ₃ | -55.8 | 2 | -55.6 ₁ | [-209.2] | [+1.8] |
| | Oct. 17 | | ... | -13.78 | | .. | -55.3 | -204.9 | +0.1 |
| Greenwich,
Paris,
Santiago,
[3 ^h 5 ^m] | Nov. 11 | -13.85 | 4 | -13.85 ₁ | -57.6 | 5 | -57.1 ₅ | | |
| | Nov. 17 | -13.62 | 4 | -13.60 ₁ | -56.6 | 4 | -56.4 ₄ | | |
| | Nov. 16 | -13.82 | 5 | -13.82 ₁ | -57.2 | 5 | -56.6 ₄ | [-209.4] | [+1.7] |
| | Nov. 15 | | ... | -13.76 | | .. | -56.7 | -205.3 | -0.2 |
| Greenwich,
Paris,
Santiago,
[3 ^h 0 ^m] | Dec. 18 | -13.48 | 10 | -13.48 ₁₀ | -57.3 | 10 | -56.8 ₃ | | |
| | Dec. 16 | -13.52 | 2 | -13.50 ₂ | -56.7 | 3 | -56.5 ₁ | | |
| | Dec. 18 | -13.53 | 6 | -13.53 ₁ | -58.1 | 6 | -57.5 ₂ | [-206.2] | [+1.6] |
| | Dec. 18 | | ... | -13.50 | | .. | -57.0 | -202.1 | -0.1 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|--|-------------|-------------------------------|----------------|----------------------|------------------------------|---------------|---------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1856 | s | | s | " | | " | " | " |
| Greenwich,
Paris, | Jan. 22 | -12.97 | 6 | -12.97 ₆ | -56.1 | 6 | -55.6 ₃ | | |
| | Feb. 10 | -12.87 | 2 | -12.85 ₂ | -54.8 | 2 | -54.6 ₁ | [-199.5] | [+1.4] |
| [2 ^h 58 ^m] | Jan. 27 | | ... | -12.94 | | .. | -55.4 | -194.7 | -0.5 |
| Greenwich,
Paris, | Oct. 16 | -14.36 | 4 | -14.36 ₄ | -53.8 | 4 | -53.3 | | |
| | Oct. 16 | -14.38 | 2 | -14.36 ₃ | | | | [-213.8] | [+0.7] |
| [3 ^h 27 ^m] | Oct. 16 | | ... | -14.36 | | .. | -53.3 | -211.0 | -1.4 |
| Greenwich,
Paris, | Nov. 18 | -14.23 | 8 | -14.23 ₃ | -54.8 | 8 | -54.3 ₃ | | |
| | Nov. 17 | -14.32 | 6 | -14.30 ₆ | -54.0 | 7 | -53.8 ₇ | [-214.7] | [+0.7] |
| [3 ^h 22 ^m] | Nov. 18 | | ... | -14.26 | | .. | -54.1 | -210.2 | -0.9 |
| Greenwich,
Paris, | Dec. 18 | -13.92 | 5 | -13.93 ₅ | -55.9 | 5 | -55.4 ₅ | | |
| | Dec. 19 | -14.11 | 7 | -14.09 ₇ | -55.0 | 6 | -54.8 ₆ | [-211.6] | [+0.6] |
| [3 ^h 17 ^m] | Dec. 19 | | ... | -14.03 | | .. | -55.1 | -207.7 | -1.2 |
| | 1857 | | | | | | | | |
| Greenwich,
Paris, | Jan. 21 | -13.59 | 10 | -13.60 ₁₀ | -53.4 | 8 | -52.9 ₃ | | |
| | Jan. 12 | -13.87 | 3 | -13.85 ₃ | -55.3 | 3 | -55.1 ₁ | [-206.4] | [+0.5] |
| [3 ^h 15 ^m] | Jan. 19 | | ... | -13.66 | | .. | -53.5 | -202.3 | -0.4 |
| Greenwich,
[3 ^h 46 ^m] | Oct. 8 | -14.68 | 4 | -14.69 | -49.1 | 4 | -48.6 | [-216.8] | [-0.3] |
| | | | ... | | | .. | | -213.0 | -1.9 |
| Greenwich,
Paris, | Nov. 6 | -14.80 | 4 | -14.81 ₄ | -51.3 | 5 | -50.8 ₅ | | |
| | Nov. 10 | -14.83 | 9 | -14.81 ₉ | -51.5 | 8 | -51.3 ₃ | | |
| Königsberg, | Nov. 19 | -14.77 | 3 | -14.75 ₄ | -52.7 | 3 | -51.5 ₂ | [-218.9] | [-0.4] |
| [3 ^h 41 ^m] | Nov. 11 | | ... | -14.80 | | .. | -51.2 | -215.5 | -2.4 |
| Greenwich,
Paris, | Dec. 11 | -14.55 | 6 | -14.56 ₆ | -53.2 | 5 | -52.7 ₃ | | |
| | Dec. 16 | -14.48 | 7 | -14.46 ₇ | -52.8 | 6 | -52.6 ₆ | | |
| Königsberg, | Dec. 8 | -14.56 | 1 | -14.54 ₂ | -53.7 | 1 | -52.9 ₃ | [-217.1] | [-0.5] |
| [3 ^h 36 ^m] | Dec. 13 | | ... | -14.51 | | .. | -52.6 | -212.2 | -3.0 |
| | 1858 | | | | | | | | |
| Greenwich,
Paris, | Jan. 17 | -14.13 | 13 | -14.14 ₁₃ | -52.8 | 13 | -52.3 ₁₃ | | |
| | Jan. 16 | -14.19 | 8 | -14.17 ₈ | -52.8 | 9 | -52.6 ₉ | [-211.8] | [-0.6] |
| [3 ^h 32 ^m .2] | Jan. 17 | | ... | -14.15 | | .. | -52.4 | -207.5 | -2.8 |
| Greenwich,
Paris, | Feb. 11 | -13.81 | 10 | -13.82 ₁₀ | -52.1 | 10 | -51.6 ₁₀ | | |
| | Feb. 12 | -13.91 | 1 | -13.89 ₁ | -52.7 | 1 | -52.5 ₁ | [-207.0] | [-0.6] |
| [3 ^h 31 ^m .9] | Feb. 11 | | ... | -13.83 | | .. | -51.7 | -203.1 | -3.1 |
| Greenwich,
[4 ^h 3 ^m .3] | Oct. 17 | -15.06 | 5 | -15.06 | -46.4 | 5 | -45.9 | [-220.6] | [-1.4] |
| | | | ... | | | .. | | -216.2 | -4.0 |
| Greenwich,
Paris, | Nov. 14 | -15.16 | 10 | -15.17 ₁₀ | -48.2 | 11 | -47.7 ₁₁ | | |
| | Nov. 20 | -15.19 | 9 | -15.17 ₉ | -47.8 | 10 | -47.6 ₁₀ | [-222.1] | [-1.5] |
| [3 ^h 58 ^m .5] | Nov. 17 | | ... | -15.17 | | .. | -47.7 | -218.6 | -3.8 |

MEAN CORRECTIONS TO THE EPIHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|---|-------------|-------------------------------|----------------|----------------------|------------------------------|----------------|--------------------|---------------------|------------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Latitude. | Longitude. |
| | 1858 | s | | s | " | | " | " | " |
| Greenwich,
Paris,
[3 ^h 53 ^m .6] | Dec. 16 | -14.96 | 6 | -14.97 ₆ | -49.2 | 6 | -48.7 ₂ | | |
| | Dec. 16 | -14.78 | 5 | -14.76 ₅ | -49.5 | 3 | -49.3 ₁ | [-220.7] | [-1.6] |
| | Dec. 16 | | ... | -14.87 | | .. | -48.9 | -215.1 | -4.1 |
| | 1859 | | | | | | | | |
| Greenwich,
Paris,
[3 ^h 50 ^m .0] | Jan. 19 | -14.56 | 8 | -14.57 ₈ | -48.9 | 8 | -48.4 ₂ | | |
| | Jan. 10 | -14.72 | 4 | -14.70 ₄ | -48.2 | 4 | -48.0 ₁ | [-216.1] | [-1.5] |
| | Jan. 16 | | ... | -14.61 | | .. | -48.3 | -211.5 | -3.1 |
| Greenwich,
Paris,
[3 ^h 49 ^m .3] | Feb. 18 | -14.18 | 5 | -14.19 ₅ | -48.1 | 5 | -47.6 ₅ | | |
| | Feb. 11 | -14.37 | 6 | -14.35 ₆ | -48.3 | 4 | -48.1 ₄ | [-210.9] | [-1.6] |
| | Feb. 14 | .. | ... | -14.28 | | .. | -47.8 | -207.1 | -3.4 |
| Greenwich,
[4 ^h 20 ^m .8] | Oct. 25 | -15.43 | 6 | -15.44 | -42.6 | 6 | -42.1 | [-223.2] | [-2.2] |
| | | | ... | | | .. | | -219.6 | -5.5 |
| Greenwich,
Paris,
[4 ^h 17 ^m .2] | Nov. 18 | -15.56 | 6 | -15.57 ₆ | -43.3 | 5 | -42.8 ₅ | | |
| | Nov. 17 | -15.61 | 8 | -15.59 ₈ | -43.0 | 9 | -42.8 ₉ | [-224.7] | [-2.2] |
| | Nov. 17 | | ... | -15.58 | | .. | -42.8 | -221.9 | -4.6 |
| Greenwich,
Paris,
[4 ^h 12 ^m .5] | Dec. 16 | -15.43 | 9 | -15.44 ₉ | -45.2 | 8 | -44.7 ₈ | | |
| | Dec. 11 | -15.46 | 5 | -15.44 ₅ | -44.5 | 5 | -44.3 ₅ | [-223.9] | [-2.3] |
| | Dec. 14 | | ... | -15.44 | | .. | -44.5 | -220.7 | -4.9 |
| | 1860 | | | | | | | | |
| Greenwich,
Paris,
[4 ^h 8 ^m .0] | Jan. 16 | -15.08 | 8 | -15.09 ₈ | -45.8 | 8 | -45.3 ₈ | | |
| | Jan. 15 | -15.08 | 8 | -15.06 ₈ | -44.3 | 5 | -44.1 ₅ | [-219.6] | [-2.3] |
| | Jan. 16 | | ... | -15.08 | | .. | -44.8 | -216.1 | -4.5 |
| Greenwich,
Paris,
[4 ^h 7 ^m .0] | Feb. 17 | -14.64 | 12 | -14.65 ₁₂ | -45.0 | 12 | -44.5 | | |
| | Feb. 6 | -14.79 | 2 | -14.77 ₂ | | | | [-213.9] | [-2.3] |
| | Feb. 15 | | ... | -14.67 | | .. | -44.5 | -210.4 | -5.0 |
| Greenwich,
[4 ^h 40 ^m .9] | Oct. 13 | -15.52 | 2 | -15.52 | -35.2 | 3 | -34.7 | [-223.0] | [-2.9] |
| | | | ... | | | .. | | -218.3 | -5.4 |
| Greenwich,
Paris,
[4 ^h 35 ^m .8] | Nov. 15 | -15.88 | 9 | -15.89 ₉ | -38.3 | 9 | -37.8 ₂ | | |
| | Nov. 22 | -15.79 | 5 | -15.77 ₅ | -37.3 | 5 | -37.1 ₁ | [-226.6] | [-3.1] |
| | Nov. 18 | | ... | -15.85 | | .. | -37.6 | -223.5 | -5.8 |
| Greenwich,
Paris,
[4 ^h 31 ^m .6] | Dec. 12 | -15.77 | 4 | -15.78 ₄ | -41.5 | 5 | -41.0 ₅ | | |
| | Dec. 12 | -15.79 | 3 | -15.77 ₃ | -38.5 | 3 | -38.3 ₃ | [-226.2] | [-3.2] |
| | Dec. 12 | | ... | -15.78 | | .. | -40.0 | -223.2 | -6.8 |
| | 1861 | | | | | | | | |
| Greenwich,
Paris,
[4 ^h 26 ^m .8] | Jan. 13 | -15.45 | 10 | -15.46 ₁₀ | -40.5 | 10 | -40.1 ₃ | | |
| | Jan. 13 | -15.52 | 6 | -15.49 ₆ | -40.2 | 3 | -40.0 ₁ | [-222.4] | [-3.2] |
| | Jan. 13 | | ... | -15.47 | | .. | -40.1 | -219.3 | -5.8 |

| MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—Continued. | | | | | | | | | |
|---|------------------------------------|-------------------------------|----------------|----------------------|------------------------------|----------------|---------------------|---------------------|-----------|
| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1861 | s | | s | " | | " | " | " |
| Greenwich,
Paris, | Feb. 10 | -15.12 | 2 | -15.13 ₂ | -40.7 | 3 | -40.3 | | |
| | Feb. 5 | -15.23 | 1 | -15.20 ₁ | | | | [-217.9] | [-3.1] |
| [4 ^h 21 ^m .0] | Feb. 8 | | ... | -15.15 | | .. | -40.3 | -215.0 | -6.0 |
| Greenwich,
[4 ^h 56 ^m .4] | Nov. 10 | -15.99 | 9 | -16.00 ₉ | -30.9 | 8 | -30.5 | [-226.2] | [-4.0] |
| | | | ... | | | .. | | -223.4 | -6.1 |
| Greenwich,
Paris,
Washington, | Dec. 9 | -16.01 | 4 | -16.02 ₁ | -33.4 | 4 | -33.0 ₄ | | |
| | Dec. 10 | -16.04 | 11 | -16.01 ₁₁ | -32.8 | 13 | -32.6 ₁₃ | | |
| | Dec. 18 | -16.10 | 6 | -16.07 ₆ | | | | [-227.3] | [-4.0] |
| [4 ^h 50 ^m .8] | Dec. 12 | | ... | -16.03 | | .. | -32.7 | -224.4 | -6.4 |
| | 1862 | | | | | | | | |
| Greenwich,
Paris, | Jan. 19 | -15.67 | 8 | -15.67 ₈ | -35.1 | 9 | -34.7 ₉ | | |
| | Jan. 20 | -15.72 | 5 | -15.69 ₅ | -34.1 | 5 | -33.9 ₅ | [-223.2] | [-4.0] |
| [4 ^h 44 ^m .9] | Jan. 19 | | ... | -15.68 | | .. | -34.4 | -220.2 | -6.3 |
| Greenwich,
Washington,
Paris, | Feb. 22 | -15.18 | 7 | -15.18 ₇ | -34.3 | 7 | -33.9 ₇ | | |
| | Feb. 19 | -15.31 | 3 | -15.31 ₃ | | | | | |
| [4 ^h 43 ^m .1] | Feb. 15 | -15.47 | 5 | -15.44 ₅ | -34.6 | 6 | -34.4 ₆ | [-217.5] | [-4.0] |
| | Feb. 19 | | ... | -15.30 | | .. | -34.1 | -215.1 | -6.0 |
| Greenwich,
[5 ^h 16 ^m .2] | Nov. 9 | -16.13 | 9 | -16.13 | -24.5 | 9 | -24.1 | [-226.2] | [-4.7] |
| | | | ... | | | .. | | -223.6 | -7.1 |
| Greenwich,
Paris,
Leyden, | Dec. 12 | -16.27 | 7 | -16.27 ₇ | -27.2 | 7 | -26.8 ₇ | | |
| | Dec. 11 | -16.27 | 5 | -16.24 ₅ | -26.5 | 5 | -26.3 ₅ | | |
| | Dec. 8 | -16.16 | 4 | -16.13 ₃ | -25.2 | 4 | -25.2 ₄ | [-227.6] | [-4.7] |
| [5 ^h 10 ^m .7] | Dec. 19 | | ... | -16.23 | | .. | -26.3 | -225.4 | -7.2 |
| | 1863 | | | | | | | | |
| Greenwich,
Paris,
Washington,
Leyden, | Jan. 17 | -15.96 | 10 | -15.96 ₁₀ | -28.3 | 10 | -27.9 ₁₀ | | |
| | Jan. 20 | -15.94 | 2 | -15.91 ₂ | -27.4 | 1 | -27.2 ₁ | | |
| | Jan. 16 | -15.92 | 4 | -15.92 ₄ | | | | | |
| | Jan. 16 | -16.03 | 6 | -16.00 ₄ | -27.9 | 6 | -27.9 ₄ | [-224.3] | [-4.8] |
| [5 ^h 4 ^m .6] | Jan. 17 | | ... | -15.96 | | .. | -27.9 | -222.2 | -6.8 |
| Greenwich,
Paris,
Washington,
Leyden, | Feb. 15 | -15.55 | 10 | -15.55 ₁₀ | -29.2 | 10 | -28.8 ₁₀ | | |
| | Feb. 15 | -15.47 | 17 | -15.44 ₁₇ | -28.6 | 17 | -28.4 ₁₇ | | |
| | Feb. 9 | -15.64 | 2 | -15.64 ₂ | | | | | |
| | Feb. 15 | -15.57 | 6 | -15.54 ₄ | -28.5 | 7 | -28.5 ₇ | [-219.3] | [-4.8] |
| [5 ^h 2 ^m .2] | Feb. 15 | | ... | -15.50 | | .. | -28.6 | -216.0 | -7.2 |
| Greenwich,
Paris, | Mar. 3 | -15.36 | 2 | -15.36 ₂ | -29.0 | 3 | -28.6 ₃ | | |
| | Mar. 5 | -15.30 | 4 | -15.27 ₄ | -28.0 | 4 | -27.8 ₄ | [-215.9] | [-4.8] |
| | [5 ^h 2 ^m .2] | Mar. 4 | | ... | -15.30 | | .. | -28.2 | -213.3 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R.A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|---|-------------|------------------------------|----------------|----------------------|------------------------------|----------------|--------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1863 | s | | s | " | | " | " | " |
| Paris,
Leyden,
[5 ^h 34 ^m] | Nov. 29 | -16.33 | 3 | -16.30 ₃ | -18.4 | 3 | -18.2 ₃ | | |
| | Nov. 14 | -16.23 | 6 | -16.20 ₄ | -18.1 | 6 | -18.1 ₄ | [-225.9] | [-5.6] |
| | Nov. 21 | | ... | -16.24 | | .. | -18.2 | -224.1 | -8.3 |
| Greenwich,
Leyden,
[5 ^h 29 ^m] | Dec. 19 | -16.32 | 3 | -16.32 ₃ | -20.0 | 3 | -19.6 ₃ | | |
| | Dec. 14 | -16.51 | 2 | -16.48 ₁ | -17.1 | 1 | -17.1 ₁ | [-226.8] | [-5.7] |
| | Dec. 18 | | ... | -16.36 | | .. | -19.0 | -225.9 | -7.0 |
| | 1864 | | | | | | | | |
| Washington,
Leyden,
[5 ^h 25 ^m] | Jan. 15 | -16.01 | 4 | -16.01 ₁₁ | | 11 | | | |
| | Jan. 10 | -16.07 | 4 | -16.04 ₂ | -20.0 | 4 | -20.0 | [-224.6] | [-5.8] |
| | Jan. 14 | | ... | -16.02 | | .. | | -221.4 | -6.7 |
| Washington,
[5 ^h 21 ^m] | Feb. 15 | -15.74 | 4 | -15.74 | -21.5 | 9 | | [-219.6] | [-5.7] |
| | | | ... | | | .. | | -217.8 | |
| Greenwich,
[5 ^h 21 ^m] | May 3 | -15.57 | 2 | -15.57 | -22.2 | 2 | -21.8 ₂ | [-216.1] | [-5.6] |
| | | | ... | | | .. | | -215.6 | -7.3 |
| Paris,
Washington,
Leyden,
[5 ^h 49 ^m] | Dec. 20 | -16.32 | 4 | -16.29 ₄ | -13.2 | 4 | -12.8 ₄ | | |
| | Dec. 10 | -16.22 | 2 | -16.22 ₂ | -11.4 | 4 | | | |
| | Dec. 18 | -16.19 | 5 | -16.16 ₄ | -12.4 | 5 | -12.4 ₄ | [-225.1] | [-6.5] |
| | Dec. 17 | | ... | -16.22 | | .. | -12.6 | -223.1 | -8.4 |
| | 1865 | | | | | | | | |
| Greenwich,
Paris,
Washington,
Leyden,
[5 ^h 45 ^m] | Jan. 5 | -16.28 | 2 | -16.28 ₂ | -13.7 | 2 | -13.3 ₂ | | |
| | Jan. 15 | -16.22 | 1 | -16.19 ₂ | -15.1 | 2 | -14.9 ₂ | | |
| | Jan. 17 | -16.11 | 9 | -16.11 ₉ | -14.4 | 9 | | | |
| | Jan. 16 | -16.03 | 3 | -16.00 ₂ | -14.7 | 3 | -14.7 ₂ | [-223.4] | [-6.5] |
| | Jan. 15 | | ... | -16.13 | | .. | -14.3 | -220.0 | -8.8 |
| Washington,
Leyden,
[5 ^h 41 ^m] | Feb. 14 | -15.77 | 9 | -15.77 ₉ | -15.4 | 6 | | | |
| | Feb. 13 | -15.85 | 2 | -15.82 ₂ | -15.6 | 2 | -15.6 | [-218.9] | [-6.4] |
| | Feb. 14 | | ... | -15.78 | | .. | | -217.4 | -8.6 |
| Greenwich,
Washington,
[6 ^h 17 ^m] | Oct. 10 | -15.59 | 1 | -15.59 ₁ | - 2.4 | 1 | - 2.0 ₁ | | |
| | Oct. 18 | -15.59 | 5 | -15.59 ₅ | | | | [-215.3] | [-7.0] |
| | Oct. 17 | | ... | -15.59 | | .. | - 2.0 | -214.0 | -8.1 |
| Greenwich,
Leyden,
Washington,
[6 ^h 11 ^m] | Dec. 6 | -16.13 | 2 | -16.13 ₂ | - 5.2 | 2 | - 4.8 ₂ | | |
| | Dec. 7 | -16.26 | 2 | -16.23 ₂ | - 4.8 | 2 | - 4.6 ₂ | | |
| | Dec. 18 | -16.17 | 8 | -16.17 ₈ | - 3.8 | 5 | | [-222.2] | [-7.3] |
| | Dec. 14 | | ... | -16.17 | | .. | - 4.7 | -222.0 | -8.8 |

MEAN CORRECTIONS TO THE EPIHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|---------------------------------------|-------------|-------------------------------|----------------|----------------------|------------------------------|----------------|--------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1866 | s | | s | " | | " | " | " |
| Greenwich, | Jan. 10 | -16.20 | 8 | -16.20 ₈ | - 7.2 | 9 | - 6.8 ₄ | | |
| Paris, | Jan. 12 | -16.04 | 1 | -16.01 ₁ | - 6.3 | 1 | - 6.1 ₁ | | |
| Washington, | Jan. 17 | -16.06 | 9 | -16.06 ₉ | - 7.6 | 9 | - 6.5 ₄ | | |
| Leyden, | Jan. 17 | -16.02 | 6 | -15.99 ₄ | - 7.1 | 6 | - 7.1 ₂ | | |
| [6 ^h 5 ^m] | Jan. 14 | | ... | -16.09 | | .. | - 6.6 | [-221.6] | [- 7.3] |
| | | | | | | | | -221.0 | - 9.0 |
| Greenwich, | Feb. 14 | -15.81 | 10 | -15.81 ₁₀ | - 9.1 | 11 | - 8.7 ₄ | | |
| Washington, | Feb. 14 | -15.81 | 6 | -15.81 ₆ | - 8.4 | 8 | - 7.3 ₄ | | |
| Leyden, | Feb. 14 | -15.75 | 8 | -15.72 ₆ | - 8.6 | 8 | - 8.6 ₄ | | |
| [6 ^h 0 ^m] | Feb. 14 | | ... | -15.79 | | .. | - 8.2 | [-217.3] | [- 7.3] |
| | | | | | | | | -216.9 | - 8.2 |
| Greenwich, | Mar. 12 | -15.47 | 2 | -15.47 ₂ | - 8.1 | 2 | - 7.7 ₁ | | |
| Washington, | Mar. 9 | -15.49 | 6 | -15.49 ₆ | - 9.7 | 6 | - 8.6 ₂ | | |
| Leyden, | Mar. 3 | -15.59 | 2 | -15.56 ₂ | - 9.0 | 2 | - 9.0 ₁ | | |
| [6 ^h 0 ^m] | Mar. 9 | | ... | -15.50 | | .. | - 8.5 | [-212.9] | [- 7.1] |
| | | | | | | | | -212.9 | - 8.5 |
| Washington, | Oct. 13 | -15.36 | 5 | -15.36 | + 4.4 | 5 | + 5.5 | [-209.6] | [- 7.5] |
| [6 ^h 37 ^m] | | | | | | .. | | -211.2 | - 7.9 |
| Greenwich, | Nov. 9 | -15.65 | 2 | -15.65 ₂ | + 3.5 | 2 | + 3.9 ₁ | | |
| Washington, | Nov. 15 | -15.72 | 10 | -15.72 ₁₀ | + 3.8 | 10 | + 4.9 ₂ | | |
| [6 ^h 35 ^m] | Nov. 14 | | ... | -15.71 | | .. | + 4.6 | [-215.1] | [- 7.8] |
| | | | | | | | | -215.9 | - 8.3 |
| Greenwich, | Dec. 11 | -16.01 | 10 | -16.01 ₁₀ | + 2.8 | 10 | + 3.2 ₆ | | |
| Washington, | Dec. 12 | -16.00 | 7 | -16.00 ₇ | + 2.0 | 7 | + 3.1 ₄ | | |
| Leyden, | Dec. 10 | -15.87 | 2 | -15.84 ₂ | + 1.6 | 2 | + 1.6 ₂ | | |
| [6 ^h 31 ^m] | Dec. 11 | | ... | -15.99 | | .. | + 2.9 | [-218.2] | [- 8.0] |
| | | | | | | | | -219.7 | - 9.7 |
| | 1867 | | | | | | | | |
| Greenwich, | Jan. 21 | -15.89 | 5 | -15.89 ₅ | - 1.4 | 5 | - 1.0 ₃ | | |
| Washington, | Jan. 21 | -15.93 | 12 | -15.93 ₁₂ | - 1.2 | 12 | - 0.1 ₅ | | |
| Leyden, | Jan. 9 | -15.82 | 4 | -15.79 ₃ | - 0.3 | 3 | - 0.3 ₁ | | |
| [6 ^h 24 ^m] | Jan. 19 | | ... | -15.90 | | .. | - 0.4 | [-217.8] | [- 8.0] |
| | | | | | | | | -218.3 | - 9.3 |
| Paris, | Feb. 13 | -15.72 | 3 | -15.69 ₃ | - 2.4 | 4 | - 2.0 ₂ | | |
| Washington, | Feb. 15 | -15.65 | 9 | -15.65 ₉ | - 2.0 | 10 | - 0.9 ₄ | | |
| Leyden, | Feb. 18 | -15.54 | 3 | -15.51 ₂ | - 0.5 | 3 | - 0.5 ₂ | | |
| [6 ^h 20 ^m] | Feb. 15 | | ... | -15.64 | | .. | - 1.1 | [-214.5] | [- 7.9] |
| | | | | | | | | -214.7 | - 8.6 |
| Greenwich, | Mar. 2 | -15.45 | 7 | -15.45 ₇ | - 2.9 | 8 | - 2.5 ₄ | | |
| Washington, | Mar. 11 | -15.32 | 4 | -15.32 ₄ | - 2.5 | 2 | - 1.4 ₂ | | |
| [6 ^h 19 ^m] | Mar. 5 | | ... | -15.40 | | .. | - 2.1 | [-211.1] | [- 7.8] |
| | | | | | | | | -211.3 | - 9.0 |
| Greenwich, | Dec. 11 | -15.61 | 5 | -15.61 ₅ | + 8.5 | 5 | + 8.9 ₃ | | |
| Washington, | Dec. 18 | -15.70 | 3 | -15.70 ₃ | + 6.9 | 3 | + 8.0 ₂ | | |
| [6 ^h 51 ^m] | Dec. 13 | | ... | -15.64 | | .. | + 8.5 | [-213.1] | [- 8.6] |
| | | | | | | | | -215.3 | -10.4 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|--|-------------|-------------------------------|----------------|----------------------|------------------------------|----------------|---------------------|---------------------|-----------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1868 | s | | s | " | | " | " | " |
| Greenwich,
[6 ^h 45 ^m] | Jan. 13 | -15.67 | 4 | -15.67 ₄ | + 6.9 | 4 | + 6.7 ₄ | [-213.4] | [- 8.7] |
| | | | | | | | | -215.5 | -10.1 |
| Greenwich,
Leyden,
Washington,
Paris, | Feb. 15 | -15.43 | 8 | -15.43 ₈ | + 4.8 | 8 | + 4.6 ₄ | | |
| | Feb. 5 | -15.43 | 3 | -15.40 ₂ | + 5.9 | 3 | + 5.9 ₂ | | |
| | Feb. 13 | -15.50 | 7 | -15.50 ₈ | + 4.9 | 6 | + 6.0 ₂ | | |
| | Feb. 14 | -15.41 | 7 | -15.38 ₄ | | | | [-210.3] | [- 8.6] |
| [6 ^h 41 ^m] | | | | -15.44 | | | + 5.3 | -212.3 | - 9.7 |
| Leyden,
Washington, | Mar. 11 | -15.02 | 2 | -14.99 ₁ | + 4.6 | 2 | + 4.6 ₂ | | |
| | Mar. 19 | -14.91 | 1 | -14.91 ₁ | + 3.4 | 1 | + 4.5 ₁ | [-205.0] | [- 8.5] |
| [6 ^h 39 ^m] | Mar. 15 | | | -14.95 | | | + 4.6 | -205.5 | - 9.2 |
| Washington,
[7 ^h 16 ^m] | Oct. 18 | -14.57 | 6 | -14.57 | +12.3 | 5 | +13.3 | [-198.3] | [- 8.7] |
| | | | | | | | | -201.5 | -12.9 |
| Washington,
[7 ^h 16 ^m] | Nov. 7 | -14.83 | 5 | -14.83 | +16.2 | 5 | +17.3 | [-202.0] | [- 9.1] |
| | | | | | | | | -205.6 | - 9.3 |
| Greenwich,
Washington, | Dec. 25 | -15.24 | 5 | -15.24 ₅ | +15.1 | 5 | +14.9 ₅ | | |
| | Dec. 12 | -15.16 | 6 | -15.16 ₆ | +14.6 | 4 | +15.6 ₄ | [-206.5] | [- 9.2] |
| [7 ^h 10 ^m] | Dec. 18 | | | -15.20 | | | +15.2 | -210.3 | -10.0 |
| | 1869 | | | | | | | | |
| Greenwich,
Washington, | Jan. 19 | -15.28 | 1 | -15.28 ₁ | +12.6 | 1 | +12.4 ₁ | | |
| | Jan. 18 | -15.31 | 8 | -15.31 ₈ | +12.9 | 8 | +13.4 ₈ | [-207.1] | [- 9.3] |
| [7 ^h 5 ^m] | Jan. 18 | | | -15.31 | | | +13.3 | -211.4 | -10.3 |
| Greenwich,
Washington,
Paris, | Feb. 14 | -15.04 | 10 | -15.04 ₁₀ | +11.1 | 10 | +10.9 ₁₀ | | |
| | Feb. 14 | -15.13 | 13 | -15.13 ₁₃ | +11.2 | 13 | +11.7 ₁₃ | | |
| | Feb. 10 | -15.09 | 10 | -15.06 ₇ | +11.7 | 10 | +12.2 ₁₀ | [-204.7] | [- 9.2] |
| [7 ^h 1 ^m] | Feb. 13 | | | -15.08 | | | +11.7 | -208.1 | -10.2 |
| Greenwich,
Washington, | Mar. 3 | -14.81 | 2 | -14.81 ₂ | +10.8 | 2 | +10.6 ₂ | | |
| | Mar. 7 | -14.87 | 2 | -14.87 ₂ | +11.4 | 2 | +11.9 ₂ | [-201.5] | [- 9.2] |
| [6 ^h 59 ^m] | Mar. 5 | | | -14.84 | | | +11.3 | -204.8 | - 9.3 |
| Greenwich,
[7 ^h 32 ^m] | Dec. 8 | -14.50 | 2 | -14.50 ₂ | +21.1 | 2 | +20.9 | [-197.1] | [- 9.8] |
| | | | | | | | | -202.1 | -10.3 |
| | 1870 | | | | | | | | |
| Greenwich,
[7 ^h 25 ^m] | Jan. 19 | -14.73 | 10 | -14.72 ₅ | +19.0 | 10 | +18.8 | [-199.3] | [- 9.9] |
| | | | | | | | | -204.5 | -10.8 |
| Greenwich,
Washington, | Feb. 16 | -14.58 | 7 | -14.57 ₅ | +17.2 | 7 | +17.0 | | |
| | Feb. 16 | -14.49 | 1 | -14.49 ₁ | +16.7 | 2 | +17.1 | [-197.2] | [- 9.9] |
| [7 ^h 21 ^m] | Feb. 16 | | | -14.55 | | | +17.0 | -201.9 | -10.9 |

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—Continued.

| Observatory.
[R. A. of
Uranus.] | Mean dates. | Observed corrections in R. A. | | | Observed corrections in Dec. | | | Corr. to Geocentric | |
|--|-------------|-------------------------------|----------------|---------------------|------------------------------|----------------|--------------------|----------------------|--------------------|
| | | Mean. | No. of
obs. | Corrected
mean. | Mean. | No. of
obs. | Corrected
mean. | Longitude. | Latitude. |
| | 1870 | | | | | | | | |
| Greenwich,
Washington,
[7 ^h 18 ^m] | Mar. 12 | -14.29 ^s | 5 | -14.28 ₈ | +16.2 | 5 | +16.0 | | |
| | Mar. 11 | -14.30 | 6 | -14.30 ₆ | +15.9 | 6 | +16.3 | | |
| | Mar. 12 | | ... | -14.29 | | .. | +16.2 | [-193.6]
[-198.1] | [- 9.7]
[-10.1] |
| | 1871 | | | | | | | | |
| Greenwich,
[7 ^h 47 ^m] | Jan. 9 | -13.99 | 5 | -13.98 | +24.8 | 5 | +24.6 | [-190.0]
[-196.1] | [-10.2]
[-10.7] |
| | | | ... | | | .. | | | |
| Greenwich,
[7 ^h 41 ^m] | Feb. 15 | -13.92 | 2 | -13.91 | +21.8 | 2 | +21.6 | [-188.8]
[-194.4] | [-10.1]
[-11.5] |
| | | | ... | | | .. | | | |
| Greenwich,
[7 ^h 38 ^m] | Mar. 14 | -13.82 | 14 | -13.81 | +20.7 | 14 | +20.5 | [-184.9]
[-192.8] | [-10.0]
[-11.5] |
| | | | ... | | | .. | | | |
| Greenwich,
[8 ^h 11 ^m] | Dec. 21 | -13.16 | 3 | -13.15 | +29.2 | 3 | +29.0 | [-177.9]
[-186.5] | [-10.4]
[-11.4] |
| | | | ... | | | .. | | | |
| | 1872 | | | | | | | | |
| Greenwich,
Washington,
[8 ^h 6 ^m .1] | Jan. 6 | -13.32 | 6 | -13.31 | +28.3 | 6 | +28.1 ₂ | | |
| | Jan. 21 | -13.31 | 3 | -13.31 | +7.8 | 3 | +28.2 ₁ | | |
| | Jan. 18 | | ... | -13.31 | | .. | +28.1 | [-179.8]
[-188.3] | [-10.6]
[-11.3] |
| Greenwich,
Washington,
[8 ^h 1 ^m .0] | Feb. 15 | -13.23 | 7 | -13.22 | +26.2 | 6 | +26.0 ₂ | | |
| | Feb. 21 | -13.24 | 3 | -13.24 | +26.4 | 3 | +26.8 ₁ | | |
| | Feb. 17 | | ... | -13.23 | | .. | +26.3 | [-179.3]
[-186.6] | [-10.5]
[-11.3] |
| Greenwich,
Washington,
[7 ^h 57 ^m .9] | Mar. 15 | -13.04 | 14 | -13.03 ₄ | +25.4 | 14 | +25.2 ₂ | | |
| | Mar. 15 | -13.07 | 10 | -13.07 ₆ | +24.5 | 8 | +24.9 ₁ | | |
| | Mar. 15 | | ... | -13.05 | | .. | +25.1 | [-176.4]
[-183.7] | [-10.4]
[-11.1] |
| Washington,
Greenwich,
[7 ^h 57 ^m .3] | April 8 | -12.79 | 3 | -12.79 ₃ | +24.9 | 3 | +25.3 ₃ | | |
| | April 8 | -12.77 | 1 | -12.76 ₁ | +25.0 | 1 | +24.8 ₁ | | |
| | April 8 | | ... | -12.78 | | .. | +25.2 | [-172.4]
[-180.2] | [-10.2]
[-10.8] |

CORRECTIONS TO BE APPLIED TO THE POSITIONS OF URANUS IN THE BERLIN JAHRBUCH AND THE NAUTICAL ALMANAC TO REDUCE THEM TO THE POSITIONS FROM THE PROVISIONAL THEORY.

| Date. | Heliocentric. | | | Geocentric. | |
|---------------|----------------------|---------------|--------------------|-----------------|-----------------|
| | $\delta\lambda$
" | $M\delta\rho$ | $\delta\beta$
" | δl
" | δb
" |
| 1830, July 24 | -18.0 | +1001 | + 9.0 | -19.3 | + 9.5 |
| Aug. 13 | 18.3 | 1007 | 9.0 | 18.8 | 9.5 |
| Sept. 2 | 18.4 | 1014 | 9.1 | 17.9 | 9.5 |
| Sept. 22 | 18.5 | 1020 | 9.1 | 17.1 | 9.4 |
| Oct. 12 | 18.7 | 1027 | 9.1 | 16.7 | 9.3 |
| Nov. 1 | 19.0 | 1031 | 9.2 | 16.5 | 9.2 |
| Nov. 21 | -19.3 | +1038 | + 9.2 | -16.7 | + 9.1 |
| 1831, July 19 | -22.5 | +1112 | + 9.5 | -24.5 | +10.0 |
| Aug. 8 | 22.6 | 1119 | 9.5 | 23.7 | 10.0 |
| Aug. 28 | 22.7 | 1123 | 9.6 | 22.7 | 10.1 |
| Sept. 17 | 22.8 | 1128 | 9.6 | 21.7 | 10.0 |
| Oct. 7 | 22.9 | 1133 | 9.6 | 20.9 | 9.9 |
| Oct. 27 | 23.2 | 1138 | 9.7 | 20.6 | 9.8 |
| Nov. 16 | 23.5 | 1142 | 9.7 | 20.6 | 9.6 |
| Dec. 6 | -23.8 | +1146 | + 9.7 | -21.0 | 9.5 |
| 1832, Aug. 2 | -26.9 | +1198 | + 9.8 | -28.7 | +10.3 |
| Aug. 22 | 27.2 | 1201 | 9.9 | 27.9 | 10.4 |
| Sept. 11 | 27.4 | 1202 | 9.9 | 26.9 | 10.3 |
| Oct. 1 | 27.6 | 1205 | 9.9 | 26.1 | 10.2 |
| Oct. 21 | 27.8 | 1208 | 10.0 | 25.4 | 10.2 |
| Nov. 10 | 28.0 | 1209 | 10.1 | 25.1 | 10.1 |
| Nov. 30 | -28.3 | +1209 | +10.2 | -25.2 | +10.0 |
| 1833, July 28 | -31.9 | +1233 | +10.2 | -34.5 | +10.7 |
| Aug. 17 | 32.2 | 1236 | 10.3 | 33.7 | 10.8 |
| Sept. 6 | 32.4 | 1240 | 10.3 | 32.7 | 10.8 |
| Sept. 26 | 32.6 | 1242 | 10.3 | 31.8 | 10.7 |
| Oct. 16 | 32.9 | 1245 | 10.3 | 31.0 | 10.5 |
| Nov. 5 | 33.2 | 1247 | 10.4 | 30.4 | 10.4 |
| Nov. 25 | -33.5 | +1250 | +10.4 | -30.2 | +10.3 |
| 1834, July 23 | -38.0 | +1264 | +10.5 | -41.2 | +11.0 |
| Aug. 12 | 38.4 | 1266 | 10.5 | 40.7 | 11.1 |
| Sept. 1 | 38.4 | 1268 | 10.6 | 39.6 | 11.1 |
| Sept. 21 | 38.7 | 1270 | 10.6 | 38.6 | 11.1 |
| Oct. 11 | 39.1 | 1275 | 10.6 | 37.7 | 10.9 |
| Oct. 31 | 39.4 | 1277 | 10.6 | 36.9 | 10.8 |
| Nov. 20 | 39.7 | 1282 | 10.7 | 36.6 | 10.7 |
| Dec. 10 | -40.1 | +1282 | +10.7 | -36.1 | +10.5 |
| 1835, July 18 | -43.9 | +1309 | +10.8 | -47.7 | +11.3 |
| Aug. 7 | 44.5 | 1311 | 10.7 | 47.6 | 11.3 |
| Aug. 27 | 44.5 | 1313 | 10.7 | 46.6 | 11.3 |
| Sept. 16 | 45.1 | 1316 | 10.7 | 45.8 | 11.2 |
| Oct. 6 | 45.4 | 1318 | 10.8 | 44.8 | 11.2 |
| Oct. 26 | 45.9 | 1321 | 10.7 | 44.0 | 10.9 |
| Nov. 15 | 46.5 | 1324 | 10.8 | 43.6 | 10.8 |
| Dec. 5 | -46.6 | +1327 | +10.7 | -42.5 | +10.6 |
| 1836, July 12 | -51.0 | +1361 | +11.0 | -55.3 | +11.4 |
| Aug. 1 | 51.3 | 1364 | 11.0 | 55.2 | 11.5 |
| Aug. 21 | 51.5 | 1365 | 11.1 | 54.6 | 11.7 |
| Sept. 10 | -51.9 | +1368 | +11.0 | -53.7 | +11.6 |

CORRECTIONS TO BE APPLIED TO THE POSITIONS OF URANUS—*Continued.*

| Date. | Heliocentric. | | | Geocentric. | |
|----------------|----------------------|---------------|--------------------|-----------------|-----------------|
| | $\delta\lambda$
" | $M\delta\rho$ | $\delta\beta$
" | δl
" | δb
" |
| 1836, Sept. 30 | -52.2 | +1370 | +11.0 | -52.4 | +11.5 |
| Oct. 20 | 52.9 | 1372 | 11.0 | 51.6 | 11.3 |
| Nov. 9 | 53.1 | 1375 | 11.0 | 50.5 | 11.1 |
| Nov. 29 | 53.9 | 1377 | 11.0 | 50.4 | 11.0 |
| Dec. 19 | -53.8 | +1380 | +11.1 | -49.8 | +10.9 |
| 1837, July 7 | -58.3 | +1387 | +11.0 | -62.9 | +11.4 |
| July 27 | 58.4 | 1387 | 11.0 | 62.9 | 11.5 |
| Aug. 16 | 59.2 | 1388 | 11.1 | 63.1 | 11.7 |
| Sept. 5 | 59.2 | 1389 | 11.1 | 62.0 | 11.7 |
| Sept. 25 | 60.0 | 1391 | 11.0 | 61.3 | 11.5 |
| Oct. 15 | 60.0 | 1393 | 11.0 | 59.6 | 11.4 |
| Nov. 4 | 61.0 | 1394 | 11.0 | 59.1 | 11.2 |
| Nov. 24 | 61.3 | 1394 | 11.0 | 58.2 | 11.0 |
| Dec. 14 | -62.1 | +1393 | +11.0 | -57.1 | +10.9 |
| 1838, Aug. 11 | -66.4 | +1395 | +11.1 | -71.1 | +11.6 |
| Aug. 31 | 67.2 | 1396 | 11.1 | 71.0 | 11.7 |
| Sept. 20 | 67.7 | 1395 | 11.1 | 70.2 | 11.7 |
| Oct. 10 | 68.1 | 1392 | 11.1 | 68.9 | 11.6 |
| Oct. 30 | 68.5 | 1390 | 11.1 | 67.6 | 11.4 |
| Nov. 19 | 68.9 | 1388 | 11.1 | 66.4 | 11.2 |
| Dec. 9 | 69.4 | 1388 | 11.1 | 65.8 | 11.1 |
| Dec. 29 | -69.7 | +1389 | +11.2 | -65.4 | +11.0 |
| 1839, Aug. 6 | -74.7 | +1382 | +11.1 | -80.0 | +11.6 |
| Aug. 26 | 75.1 | 1381 | 11.1 | 79.7 | 11.7 |
| Sept. 15 | 75.5 | 1380 | 11.0 | 79.0 | 11.6 |
| Oct. 5 | 76.0 | 1379 | 11.1 | 77.9 | 11.6 |
| Oct. 25 | 76.4 | 1379 | 11.1 | 76.5 | 11.5 |
| Nov. 14 | 76.8 | 1379 | 11.1 | 75.2 | 11.3 |
| Dec. 4 | 77.2 | 1378 | 11.1 | 74.0 | 11.1 |
| Dec. 24 | 77.6 | 1377 | 11.1 | 73.5 | 10.9 |
| 1840, Jan. 13 | -78.0 | +1376 | +11.0 | -73.1 | +10.7 |
| June 24 | -81.9 | +1376 | +11.0 | -86.0 | +11.1 |
| Aug. 20 | 82.7 | 1374 | 11.0 | 87.9 | 11.5 |
| Sept. 9 | 83.4 | 1376 | 11.0 | 87.8 | 11.6 |
| Sept. 29 | 83.5 | 1377 | 11.0 | 86.4 | 11.5 |
| Oct. 19 | 84.2 | 1376 | 11.0 | 85.5 | 11.4 |
| Nov. 8 | 84.4 | 1376 | 11.0 | 83.5 | 11.3 |
| Nov. 13 | 85.3 | 1377 | 11.0 | 82.7 | 11.1 |
| Dec. 13 | 85.5 | 1378 | 11.0 | 81.5 | 10.9 |
| 1841, Jan. 7 | -86.1 | +1377 | +10.9 | -81.3 | +10.7 |
| June 16 | -89.8 | +1378 | +10.8 | -93.2 | +10.8 |
| Aug. 15 | 91.1 | 1384 | 10.8 | 97.0 | 11.3 |
| Sept. 4 | 91.5 | 1385 | 10.8 | 96.9 | 11.4 |
| Sept. 24 | 91.9 | 1388 | 10.7 | 96.1 | 11.3 |
| Oct. 14 | 92.3 | 1390 | 10.7 | 94.8 | 11.2 |
| Nov. 3 | 92.9 | 1392 | 10.7 | 93.4 | 11.1 |
| Nov. 23 | 93.4 | 1393 | 10.8 | 91.8 | 10.9 |
| Dec. 13 | 93.8 | 1395 | 10.7 | 90.5 | 10.7 |
| 1842, Jan. 2 | 94.3 | 1398 | 10.8 | 89.7 | 10.6 |
| Feb. 11 | -95.0 | +1404 | +10.7 | -89.6 | +10.3 |

CORRECTIONS TO BE APPLIED TO THE POSITIONS OF URANUS—*Continued.*

| Date. | | Heliocentric. | | | Geocentric. | |
|-------|----------|----------------------|---------------|--------------------|-----------------|-----------------|
| | | $\delta\lambda$
" | $M\delta\rho$ | $\delta\beta$
" | δl
" | δb
" |
| 1842, | June 11 | — 96.7 | +1409 | +10.6 | — 98.5 | +10.5 |
| | Aug. 10 | 99.1 | 1408 | 10.6 | 105.3 | 10.0 |
| | Aug. 30 | 99.6 | 1409 | 10.7 | 105.8 | 10.2 |
| | Sept. 19 | 99.9 | 1409 | 10.6 | 105.2 | 10.2 |
| | Oct. 9 | 100.5 | 1410 | 10.5 | 104.2 | 10.0 |
| | Oct. 29 | 100.9 | 1412 | 10.5 | 102.6 | 10.9 |
| | Nov. 18 | 101.3 | 1413 | 10.5 | 100.8 | 10.8 |
| | Dec. 8 | 101.8 | 1414 | 10.5 | 99.2 | 10.6 |
| | Dec. 28 | 102.2 | 1415 | 10.5 | 98.0 | 10.4 |
| 1843, | Jan. 17 | —102.5 | +1417 | +10.4 | — 97.2 | +10.2 |
| | Aug. 5 | —107.2 | +1422 | +10.3 | —113.7 | +10.7 |
| | Aug. 25 | 107.7 | 1420 | 10.2 | 114.4 | 10.7 |
| | Sept. 14 | 108.0 | 1420 | 10.1 | 114.3 | 10.6 |
| | Oct. 4 | 108.5 | 1420 | 10.1 | 113.4 | 10.6 |
| | Oct. 24 | 109.0 | 1419 | 10.1 | 112.1 | 10.5 |
| | Nov. 13 | 109.4 | 1418 | 10.1 | 110.2 | 10.4 |
| | Dec. 3 | 110.0 | 1418 | 10.1 | 108.6 | 10.3 |
| | Dec. 23 | 110.4 | 1417 | 10.1 | 106.9 | 10.1 |
| 1844, | Jan. 12 | —110.8 | +1418 | +10.0 | —105.9 | + 9.8 |
| | July 30 | —115.1 | +1406 | + 9.7 | —121.2 | +10.0 |
| | Aug. 19 | 115.4 | 1403 | 9.8 | 122.3 | 10.2 |
| | Sept. 8 | 115.8 | 1400 | 9.8 | 122.7 | 10.3 |
| | Sept. 28 | 116.3 | 1397 | 9.8 | 122.4 | 10.3 |
| | Oct. 18 | 116.9 | 1396 | 9.7 | 121.3 | 10.2 |
| | Nov. 7 | 117.4 | 1392 | 9.6 | 119.7 | 10.0 |
| | Nov. 27 | 117.8 | 1389 | 9.6 | 117.6 | 9.8 |
| | Dec. 17 | 118.3 | 1384 | 9.6 | 115.7 | 9.7 |
| 1845, | Jan. 6 | 118.6 | 1381 | 9.5 | 114.3 | 9.4 |
| | Jan. 26 | —119.1 | +1378 | + 9.6 | —113.3 | + 9.4 |
| | Aug. 14 | —123.5 | +1340 | + 9.1 | —130.4 | + 9.4 |
| | Sept. 3 | 123.9 | 1335 | 9.4 | 131.4 | 9.8 |
| | Sept. 23 | 124.4 | 1330 | 9.4 | 131.7 | 9.9 |
| | Oct. 13 | 124.9 | 1327 | 9.3 | 130.6 | 9.8 |
| | Nov. 2 | 125.3 | 1322 | 9.3 | 129.0 | 9.7 |
| | Nov. 22 | 125.7 | 1318 | 9.2 | 127.0 | 9.5 |
| | Dec. 12 | 126.2 | 1313 | 9.2 | 125.0 | 9.3 |
| 1846, | Jan. 1 | 126.7 | 1309 | 9.2 | 123.2 | 9.2 |
| | Jan. 21 | —127.1 | +1303 | + 9.2 | —121.9 | — 9.0 |
| | Aug. 29 | —132.0 | +1253 | + 8.8 | —139.5 | + 9.2 |
| | Sept. 18 | 132.3 | 1249 | 8.8 | 139.9 | 9.2 |
| | Oct. 8 | 132.8 | 1246 | 8.8 | 139.8 | 9.3 |
| | Oct. 28 | 133.2 | 1242 | 8.7 | 138.4 | 9.1 |
| | Nov. 17 | 133.5 | 1238 | 8.7 | 136.4 | 9.0 |
| | Dec. 7 | 134.1 | 1234 | 8.7 | 134.4 | 8.9 |
| | Dec. 27 | 134.6 | 1233 | 8.6 | 132.3 | 8.7 |
| 1847, | Jan. 16 | —134.9 | +1230 | + 8.6 | —130.5 | + 8.5 |
| | Aug. 24 | —139.5 | +1197 | + 8.3 | —146.7 | + 8.6 |
| | Sept. 13 | 140.0 | 1194 | 8.3 | 148.0 | 8.7 |
| | Oct. 3 | 140.5 | 1191 | 8.3 | 148.3 | 8.7 |
| | Oct. 23 | —140.9 | +1188 | + 8.2 | —147.4 | + 8.6 |

CORRECTIONS TO BE APPLIED TO THE POSITIONS OF URANUS—*Continued.*

| Date. | Heliocentric. | | | Geocentric. | |
|---------------|----------------------|---------------|--------------------|-----------------|-----------------|
| | $\delta\lambda$
" | $M\delta\rho$ | $\delta\beta$
" | δl
" | δb
" |
| 1847, Nov. 12 | -141.4 | +1185 | +8.2 | -145.8 | +8.5 |
| Dec. 2 | 141.9 | 1183 | 8.1 | 143.7 | 8.3 |
| Dec. 22 | 142.4 | 1182 | 8.1 | 141.4 | 8.2 |
| 1848, Jan. 11 | -142.8 | +1182 | +8.1 | -139.3 | +8.1 |
| Sept. 7 | -147.9 | +1165 | +7.6 | -156.0 | +7.9 |
| Sept. 27 | 148.3 | 1166 | 7.6 | 156.7 | 8.0 |
| Oct. 17 | 148.7 | 1164 | 7.6 | 156.4 | 8.0 |
| Nov. 6 | 149.3 | 1164 | 7.6 | 155.3 | 8.0 |
| Nov. 26 | 149.8 | 1163 | 7.6 | 153.2 | 7.9 |
| Dec. 16 | 150.2 | 1162 | 7.6 | 150.7 | 7.8 |
| 1849, Jan. 5 | 150.6 | 1161 | 7.5 | 148.3 | 7.5 |
| Jan. 25 | -151.3 | +1161 | +7.5 | -146.6 | +7.4 |
| Sept. 2 | -155.8 | +1149 | +7.0 | -164.6 | +7.3 |
| Sept. 22 | 156.2 | 1150 | 7.0 | 165.1 | 7.3 |
| Oct. 12 | 156.5 | 1149 | 7.0 | 165.1 | 7.4 |
| Nov. 1 | 156.8 | 1148 | 7.0 | 164.0 | 7.4 |
| Nov. 21 | 157.4 | 1149 | 7.0 | 162.4 | 7.3 |
| Dec. 11 | 157.9 | 1148 | 6.9 | 160.0 | 7.1 |
| Dec. 31 | 158.3 | 1147 | 6.8 | 157.5 | 6.9 |
| 1850, Jan. 20 | -158.6 | +1147 | +6.8 | -155.0 | +6.8 |
| Aug. 28 | -163.3 | +1135 | +6.2 | -170.6 | +6.4 |
| Sept. 17 | 163.7 | 1133 | 6.2 | 172.6 | 6.5 |
| Oct. 7 | 164.2 | 1131 | 6.2 | 173.3 | 6.5 |
| Oct. 27 | 164.7 | 1129 | 6.1 | 173.1 | 6.4 |
| Nov. 16 | 165.2 | 1127 | 6.1 | 171.8 | 6.4 |
| Dec. 6 | 165.6 | 1126 | 6.1 | 169.4 | 6.3 |
| Dec. 26 | 166.0 | 1127 | 6.0 | 166.7 | 6.2 |
| 1851, Jan. 15 | 166.3 | 1127 | 6.1 | 163.8 | 6.1 |
| Feb. 4 | -166.7 | +1124 | +6.0 | -161.8 | +5.9 |
| Sept. 12 | -171.5 | +1109 | +5.5 | -180.0 | +5.7 |
| Oct. 2 | 171.9 | 1105 | 5.5 | 181.4 | 5.8 |
| Oct. 22 | 172.4 | 1103 | 5.5 | 181.9 | 5.8 |
| Nov. 11 | 172.9 | 1103 | 5.4 | 180.8 | 5.7 |
| Dec. 1 | 173.3 | 1103 | 5.3 | 178.7 | 5.5 |
| Dec. 21 | 173.8 | 1102 | 5.2 | 176.1 | 5.3 |
| 1852, Jan. 10 | 174.3 | 1101 | 5.2 | 173.4 | 5.3 |
| Jan. 30 | -174.7 | +1098 | +5.1 | -170.2 | +5.1 |
| Sept. 6 | -179.0 | +1074 | +4.7 | -186.8 | +4.9 |
| Sept. 26 | 179.4 | 1073 | 4.6 | 188.9 | 4.8 |
| Oct. 16 | 179.8 | 1071 | 4.5 | 189.7 | 4.7 |
| Nov. 5 | 180.3 | 1068 | 4.5 | 189.6 | 4.7 |
| Nov. 25 | 180.7 | 1067 | 4.4 | 187.8 | 4.6 |
| Dec. 15 | 181.2 | 1065 | 4.4 | 185.3 | 4.6 |
| 1853, Jan. 4 | 181.5 | 1064 | 4.4 | 182.3 | 4.5 |
| Jan. 24 | 181.8 | 1062 | 4.4 | 179.6 | 4.4 |
| Feb. 13 | -182.1 | +1060 | +4.3 | -176.9 | +4.2 |
| Sept. 1 | -185.8 | +1055 | +3.9 | -192.4 | +4.0 |
| Sept. 21 | 186.2 | 1055 | 3.8 | 195.2 | 3.9 |
| Oct. 11 | -186.5 | +1055 | +3.7 | -196.7 | +3.9 |

CORRECTIONS TO BE APPLIED TO THE POSITIONS OF URANUS—*Continued.*

| Date. | | Heliocentric. | | | Geocentric. | |
|-------|----------|----------------------|---------------|--------------------|-----------------|-----------------|
| | | $\delta\lambda$
" | $M\delta\rho$ | $\delta\beta$
" | δl
" | δb
" |
| 1853, | Oct. 31 | -186.8 | +1054 | +3.6 | -196.8 | +3.8 |
| | Nov. 20 | 187.1 | 1054 | 3.5 | 195.6 | 3.7 |
| | Dec. 10 | 187.5 | 1053 | 3.5 | 193.5 | 3.6 |
| | Dec. 30 | 187.9 | 1053 | 3.5 | 190.7 | 3.6 |
| 1854, | Jan. 19 | 188.1 | 1053 | 3.5 | 187.4 | 3.5 |
| | Feb. 8 | -188.5 | +1052 | +3.5 | -184.5 | +3.5 |
| | Sept. 16 | -192.2 | +1034 | +2.9 | -200.5 | +3.0 |
| | Oct. 6 | 192.6 | 1032 | 2.8 | 202.9 | 2.9 |
| | Oct. 26 | 193.0 | 1032 | 2.7 | 203.7 | 2.8 |
| | Nov. 15 | 193.3 | 1032 | 2.6 | 203.0 | 2.7 |
| | Dec. 5 | 193.7 | 1029 | 2.6 | 201.3 | 2.7 |
| | Dec. 25 | 194.0 | 1025 | 2.5 | 198.6 | 2.6 |
| 1855, | Jan. 14 | 194.2 | 1021 | 2.5 | 195.2 | 2.5 |
| | Feb. 3 | -194.5 | +1018 | +2.4 | -192.0 | +2.4 |
| | Oct. 1 | -198.3 | +1010 | +1.8 | -207.9 | +1.9 |
| | Oct. 21 | 198.7 | 1009 | 1.7 | 209.6 | 1.8 |
| | Nov. 10 | 199.0 | 1008 | 1.6 | 209.7 | 1.7 |
| | Nov. 30 | 199.3 | 1007 | 1.5 | 208.4 | 1.6 |
| | Dec. 20 | 199.6 | 1006 | 1.5 | 206.0 | 1.6 |
| 1856, | Jan. 9 | 199.8 | 1005 | 1.5 | 202.6 | 1.5 |
| | Jan. 29 | 200.1 | 1003 | 1.4 | 199.2 | 1.4 |
| | Feb. 18 | -200.3 | +1001 | +1.3 | -196.1 | +1.3 |
| | Oct. 15 | -203.3 | + 962 | +0.7 | -213.9 | +0.7 |
| | Nov. 4 | 203.7 | 957 | 0.7 | 214.9 | 0.7 |
| | Nov. 24 | 203.9 | 953 | 0.7 | 214.5 | 0.7 |
| | Dec. 14 | 204.3 | 949 | 0.6 | 212.4 | 0.6 |
| 1857, | Jan. 3 | 204.4 | 945 | 0.6 | 209.3 | 0.6 |
| | Jan. 23 | 204.5 | 941 | 0.5 | 205.6 | 0.5 |
| | Feb. 12 | -204.7 | + 934 | +0.4 | -202.2 | +0.4 |
| | Sept. 20 | -206.9 | + 879 | -0.2 | -214.0 | -0.2 |
| | Oct. 10 | 207.2 | 874 | 0.3 | 217.1 | 0.3 |
| | Oct. 30 | 207.5 | 868 | 0.3 | 218.8 | 0.3 |
| | Nov. 19 | 207.8 | 861 | 0.4 | 219.0 | 0.4 |
| | Dec. 9 | 208.0 | 853 | 0.4 | 217.6 | 0.4 |
| | Dec. 29 | 208.1 | 846 | 0.5 | 215.0 | 0.5 |
| 1858, | Jan. 18 | 208.2 | 839 | 0.6 | 211.5 | 0.6 |
| | Feb. 7 | 208.3 | 830 | 0.6 | 207.7 | 0.6 |
| | Feb. 27 | -208.4 | + 823 | -0.6 | -204.2 | -0.6 |
| | Oct. 5 | -210.4 | + 741 | -1.3 | -219.1 | -1.3 |
| | Oct. 25 | 210.6 | 732 | 1.4 | 221.4 | 1.5 |
| | Nov. 14 | 210.8 | 723 | 1.4 | 222.2 | 1.5 |
| | Dec. 4 | 211.0 | 713 | 1.5 | 221.9 | 1.6 |
| | Dec. 24 | 211.2 | 704 | 1.5 | 219.7 | 1.6 |
| 1859, | Jan. 13 | 211.3 | 695 | 1.5 | 216.6 | 1.5 |
| | Feb. 2 | 211.4 | 686 | 1.6 | 213.0 | 1.6 |
| | Feb. 22 | -211.6 | + 677 | -1.5 | -209.4 | -1.5 |
| | Oct. 20 | -213.1 | + 576 | -2.1 | -222.8 | -2.2 |
| | Nov. 9 | 213.2 | 569 | 2.1 | 224.5 | 2.2 |
| | Nov. 29 | -213.4 | + 561 | -2.1 | -224.8 | -2.2 |

CORRECTIONS TO BE APPLIED TO THE POSITIONS OF URANUS—*Continued.*

| Date. | Heliocentric. | | | Geocentric. | |
|---------------|----------------------|---------------|--------------------|-----------------|-----------------|
| | $\delta\lambda$
" | $M\delta\rho$ | $\delta\beta$
" | δl
" | δb
" |
| 1859, Dec. 19 | -213.5 | +554 | -2.2 | -223.5 | -2.3 |
| 1860, Jan. 8 | 213.5 | 546 | 2.2 | 220.9 | 2.3 |
| Jan. 28 | 213.6 | 539 | 2.2 | 217.4 | 2.3 |
| Feb. 17 | -213.7 | +533 | -2.3 | -213.5 | -2.3 |
| Sept. 24 | -214.7 | +456 | -2.8 | -219.7 | -2.9 |
| Oct. 14 | 214.8 | 451 | 2.8 | 223.1 | 2.9 |
| Nov. 3 | 215.0 | 444 | 2.9 | 225.6 | 3.0 |
| Nov. 23 | 215.1 | 437 | 2.9 | 226.8 | 3.1 |
| Dec. 18 | 215.2 | 431 | 3.0 | 226.3 | 3.2 |
| 1861, Jan. 2 | 215.3 | 424 | 3.1 | 223.8 | 3.2 |
| Jan. 22 | 215.3 | 417 | 3.1 | 221.0 | 3.2 |
| Feb. 12 | -215.3 | +411 | -3.1 | -217.3 | -3.1 |
| Oct. 29 | -215.6 | +339 | -3.7 | -225.0 | -3.9 |
| Nov. 13 | 215.6 | 333 | 3.8 | 226.9 | 4.0 |
| Dec. 8 | 215.7 | 329 | 3.8 | 227.4 | 4.0 |
| Dec. 28 | 215.8 | 324 | 3.8 | 226.3 | 4.0 |
| 1862, Jan. 17 | 215.8 | 320 | 3.9 | 223.5 | 4.0 |
| Feb. 6 | 215.8 | 315 | 4.0 | 220.0 | 4.1 |
| Feb. 26 | -215.9 | +310 | -4.0 | -216.0 | -4.0 |
| Oct. 24 | -216.0 | +246 | -4.4 | -224.2 | -4.6 |
| Nov. 13 | 216.0 | 241 | 4.5 | 226.6 | 4.7 |
| Dec. 3 | 216.0 | 237 | 4.5 | 227.7 | 4.7 |
| Dec. 23 | 215.9 | 232 | 4.6 | 227.2 | 4.8 |
| 1863, Jan. 12 | 215.8 | 226 | 4.7 | 225.0 | 4.9 |
| Feb. 1 | 215.8 | 219 | 4.7 | 221.8 | 4.8 |
| Feb. 21 | 215.7 | 213 | 4.7 | 218.1 | 4.8 |
| Mar. 13 | -215.7 | +208 | -4.8 | -214.1 | -4.8 |
| Nov. 8 | -215.3 | +139 | -5.3 | -224.6 | -5.5 |
| Nov. 28 | 215.2 | 133 | 5.4 | 226.5 | 5.7 |
| Dec. 18 | 215.1 | 126 | 5.4 | 226.8 | 5.7 |
| 1864, Jan. 7 | 215.0 | 120 | 5.5 | 225.4 | 5.8 |
| Jan. 27 | 214.9 | 114 | 5.5 | 222.7 | 5.7 |
| Feb. 16 | 214.8 | 108 | 5.6 | 219.3 | 5.7 |
| March 7 | -214.8 | +103 | -5.6 | -215.3 | -5.6 |
| Oct. 13 | -213.8 | + 21 | -6.0 | -217.8 | -6.1 |
| Nov. 2 | 213.7 | 16 | 6.0 | 221.3 | 6.2 |
| Nov. 22 | 213.7 | 9 | 6.1 | 224.0 | 6.4 |
| Dec. 12 | 213.6 | + 1 | 6.2 | 225.1 | 6.5 |
| 1865, Jan. 1 | 213.4 | - 6 | 6.2 | 224.7 | 6.5 |
| Jan. 21 | 213.2 | 14 | 6.2 | 222.7 | 6.5 |
| Feb. 10 | 213.1 | 20 | 6.2 | 219.6 | 6.4 |
| March 2 | -213.0 | - 27 | -6.3 | -215.7 | -6.4 |
| Oct. 8 | -211.6 | -103 | -6.8 | -213.5 | -6.9 |
| Oct. 28 | 211.4 | 110 | 6.8 | 217.2 | 7.0 |
| Nov. 17 | 211.2 | 117 | 6.9 | 220.0 | 7.2 |
| Dec. 7 | 211.1 | 124 | 6.9 | 222.0 | 7.3 |
| Dec. 27 | 210.9 | 132 | 6.9 | 222.4 | 7.3 |
| 1866, Jan. 16 | 210.7 | 139 | 7.0 | 221.5 | 7.4 |
| Feb. 5 | 210.6 | 145 | 7.0 | 218.8 | 7.3 |
| Feb. 25 | 210.4 | 151 | 7.0 | 215.4 | 7.2 |
| Mar. 17 | -210.1 | -158 | -7.1 | -211.2 | -7.1 |

CORRECTIONS TO BE APPLIED TO THE POSITIONS OF URANUS—*Continued.*

| Date. | Heliocentric. | | | Geocentric. | |
|---------------|----------------------|---------------|--------------------|-----------------|-----------------|
| | $\delta\lambda$
" | $M\delta\rho$ | $\delta\beta$
" | δl
" | δb
" |
| 1866, Oct. 3 | —208.2 | — 225 | — 7.4 | —207.3 | — 7.4 |
| Oct. 23 | 208.1 | 230 | 7.4 | 211.5 | 7.6 |
| Nov. 12 | 207.9 | 235 | 7.5 | 214.9 | 7.8 |
| Dec. 2 | 207.8 | 241 | 7.6 | 217.6 | 8.0 |
| Dec. 22 | 207.6 | 246 | 7.6 | 218.8 | 8.0 |
| 1867, Jan. 11 | 207.4 | 251 | 7.6 | 218.5 | 8.0 |
| Jan. 31 | 207.2 | 256 | 7.7 | 216.7 | 8.0 |
| Feb. 20 | 207.0 | 262 | 7.7 | 213.8 | 7.9 |
| Mar. 12 | —206.7 | — 268 | — 7.7 | —209.8 | — 7.8 |
| Nov. 27 | —203.2 | — 351 | — 8.2 | —211.4 | — 8.6 |
| Dec. 17 | 203.0 | 358 | 8.2 | 213.5 | 8.6 |
| 1868, Jan. 6 | 202.8 | 365 | 8.2 | 213.7 | 8.7 |
| Jan. 26 | 202.4 | 371 | 8.3 | 212.7 | 8.7 |
| Feb. 15 | 202.2 | 376 | 8.3 | 210.3 | 8.6 |
| Mar. 6 | 201.8 | 382 | 8.4 | 207.0 | 8.6 |
| Mar. 26 | —201.4 | — 387 | — 8.4 | —202.9 | — 8.4 |
| Oct. 12 | —198.0 | — 458 | — 8.6 | —197.2 | — 8.6 |
| Nov. 1 | 197.8 | 465 | 8.7 | 201.1 | 8.9 |
| Nov. 21 | 197.5 | 473 | 8.8 | 203.8 | 9.1 |
| Dec. 11 | 197.2 | 481 | 8.8 | 206.2 | 9.2 |
| Dec. 31 | 196.8 | 490 | 8.8 | 207.5 | 9.3 |
| 1869, Jan. 20 | 196.4 | 498 | 8.8 | 207.1 | 9.3 |
| Feb. 9 | 196.0 | 507 | 8.8 | 205.4 | 9.2 |
| Mar. 1 | 195.6 | 515 | 8.9 | 202.4 | 9.2 |
| Mar. 21 | —195.3 | — 524 | — 8.9 | —198.4 | — 9.0 |
| Dec. 6 | —190.0 | | | —197.0 | |
| Dec. 26 | 189.5 | | | 198.9 | |
| 1870, Jan. 15 | 189.0 | — 665 | — 9.4 | 199.5 | — 9.9 |
| Feb. 14 | 188.6 | 676 | 9.4 | 198.7 | 9.9 |
| Feb. 24 | 188.0 | 686 | 9.4 | 196.2 | 9.8 |
| Mar. 16 | 187.5 | 697 | 9.4 | 193.0 | 9.6 |
| April 5 | —187.2 | — 708 | — 9.4 | —189.4 | — 9.4 |
| Dec. 1 | —181.5 | — 846 | — 9.6 | —186.5 | —10.0 |
| Dec. 21 | 181.1 | 859 | 9.6 | 189.0 | 10.1 |
| 1871, Jan. 11 | 180.5 | 872 | 9.7 | 190.3 | 10.2 |
| Jan. 30 | 179.8 | 883 | 9.8 | 190.1 | 10.3 |
| Feb. 19 | 179.1 | 894 | 9.7 | 188.3 | 10.1 |
| Mar. 11 | 178.4 | 906 | 9.7 | 185.6 | 10.0 |
| Mar. 31 | —177.0 | — 919 | — 9.8 | —181.2 | — 9.9 |
| Dec. 16 | —171.6 | —1083 | —10.0 | —177.4 | —10.4 |
| 1872, Jan. 5 | 171.0 | 1095 | 10.1 | 179.3 | 10.6 |
| Jan. 25 | 170.5 | 1107 | 10.0 | 180.2 | 10.6 |
| Feb. 14 | 169.8 | 1120 | 10.0 | 179.6 | 10.5 |
| Mar. 5 | 169.3 | 1133 | 10.1 | 177.8 | 10.5 |
| Mar. 25 | 168.7 | 1145 | 10.1 | 174.9 | 10.3 |
| April 14 | 168.1 | 1156 | 10.1 | 171.3 | 10.1 |
| May 4 | —167.6 | —1168 | —10.1 | —166.3 | —10.0 |

CHAPTER VII.

FORMATION AND SOLUTION OF THE EQUATIONS OF CONDITION RESULTING FROM THE PRECEDING COMPARISONS.

IN the preceding chapter we have obtained from observations a series of corrections to the geocentric positions of Uranus resulting from the provisional theory. The further operations are as follows:—

1. To reduce all the corrections in right ascension and declination to corrections in geocentric longitude and latitude. Most of the corrections are already so expressed, so that this reduction is necessary in only a few cases.

2. To find the mean value of the correction in geocentric longitude during each opposition, and to express this mean value in terms of the correction to the heliocentric co-ordinates.

3. To express these corrections to the heliocentric co-ordinates in terms of corrections to the elements of Uranus and the mass of Neptune.

4. To solve the equations of condition thus formed.

The first of these processes is too simple to make it necessary to present any details of it. With regard to the second I have sought, not the simple correction to the geocentric longitude, but this correction multiplied by such a factor as it was supposed would make the probable error of the correction $0''.5$. The equations for expressing the error of geocentric longitude in terms of errors of heliocentric longitude and radius vector have been given on page 129. The first observation of Flamstead, p. 107, gives the equation

$$+ 2'' = 1.04\delta\lambda + .027\delta\rho$$

$\delta\lambda$ being the correction to the heliocentric longitude, and $\delta\rho$ that to the Neperian logarithm of the radius vector. From the discordance of Flamstead's clock errors it may be estimated that the probable error of the first member of this equation is $10''$. Therefore we divide the equation by 20, which gives

$$\frac{1}{20}\delta l = 1''.1 = .052\delta\lambda + .001\delta\rho.$$

In the opposition of 1715 we have four observations. The best were those of March 4 and 10, of which we may estimate the probable error at $10''$, and the worst that of March 5, of which the probable error may be estimated at $20''$, while that of April 29 is intermediate in certainty. The separate observations give the equations

| | | |
|-----------|--|------------------------------|
| March 4, | $\delta l = + 28'' = 1.06\delta\lambda;$ | Weight, 4 |
| March 5, | $\delta l = + 44 = 1.06\delta\lambda;$ | Weight, 1 |
| March 10, | $\delta l = + 36 = 1.06\delta\lambda;$ | Weight, 4 |
| April 29, | $\delta l = + 2 = 1.04\delta\lambda + .04\delta\rho;$ | Weight, 2. |
| Mean | $\delta l = + 27.6 = 1.056\delta\lambda + .003\delta\rho;$ | probable error = $\pm 6''$. |

Applying the correction $-1''.1$ for equinox, and dividing by 12, the equation of condition becomes

$$\frac{1}{12} \delta l = + 2''.2 = 0.088 \delta \lambda.$$

In this way the following equations were obtained. It is deemed unnecessary to give the details of the process, as it is one which every one can go over for himself from the data already given, and can reproduce all the results, except so far as they depend on the relative weights assigned to the different groups of observations during one and the same opposition.

| No. | Date. | Equations. | Number of observations in R. A. |
|-----|---------|--|---------------------------------|
| 1 | 1691.0; | $\frac{1}{20} \delta l = + 1.1 = .052 \delta \lambda + .001 \delta \rho$ | 1 |
| 2 | 1715.2 | $\frac{1}{12} = + 2.2 = .088$ | 4 |
| 3 | 1748.8 | $\frac{1}{3} = + 12.8 = .338 + .017$ | 1 |
| 4 | 1750.8 | $\frac{1}{3} = + 11.8 = .345 + .010$ | 3 |
| 5 | 1753.9 | $\frac{1}{3} = + 11.6 = .333 + .016$ | 1 |
| 6 | 1756.7 | $\frac{1}{6} = + 5.0 = .210 + .003$ | 1 |
| 7 | 1769.0 | $\frac{2}{5} = + 4.8 = .203 + .010$ | 8 |
| 8 | 1782.0 | $\frac{4}{3} = + 3.0 = 1.370$ | 21 |
| 9 | 1783.0 | $1 = + 1.25 = 1.030 - .002$ | 13 |
| 10 | 1784.0 | $1 = + 1.92 = 1.026 - .008$ | 13 |
| 11 | 1785.0 | $1 = - 0.26 = 1.034 + .006$ | 10 |
| 12 | 1788.0 | $\frac{2}{3} = + 1.23 = 0.684 + .006$ | 5 |
| 13 | 1789.0 | $\frac{1}{2} = + 1.58 = 0.504 + .008$ | 6 |
| 14 | 1790.0 | $\frac{1}{2} = - 0.52 = 0.514 - .013$ | 4 |
| 15 | 1791.0 | $\frac{2}{3} = - 0.56 = 0.684 - .010$ | 7 |
| 16 | 1792.0 | $\frac{1}{3} = - 0.12 = 0.340 - .011$ | 3 |
| 17 | 1793.0 | $\frac{1}{2} = - 0.19 = 0.512 - .015$ | 5 |
| 18 | 1794.0 | $\frac{1}{3} = + 0.79 = 0.344 - .006$ | 3 |
| 19 | 1795.0 | $\frac{2}{3} = - 0.66 = 0.683 - .019$ | 7 |
| 20 | 1796.0 | $\frac{1}{2} = - 0.68 = 0.514 - .015$ | 4 |
| 21 | 1797.1 | $\frac{1}{2} = - 0.35 = 0.528$ | 3 |
| 22 | 1800.2 | $\frac{1}{3} = 0.00 = 0.352$ | 2 |
| 23 | 1801.2 | $\frac{1}{3} = - 0.26 = 0.352$ | 2 |
| 24 | 1802.3 | $1 = + 0.85 = 1.05$ | 13 |
| 25 | 1805.3 | $1 = + 0.69 = 1.05$ | 13 |
| 26 | 1806.3 | $\frac{1}{2} = + 0.20 = 0.52$ | 5 |
| 27 | 1807.3 | $1 = + 2.34 = 1.045$ | 16 |
| 28 | 1808.3 | $\frac{1}{2} = - 0.04 = 0.52$ | 6 |
| 29 | 1809.3 | $\frac{2}{3} = + 1.80 = 0.70$ | 9 |
| 30 | 1810.3 | $1 = + 2.39 = 1.05$ | 16 |
| 31 | 1811.3 | $1 = + 1.49 = 1.04 - 0.01$ | 11 |
| 32 | 1812.4 | $\frac{1}{2} = + 0.8 = 0.53$ | 8 |
| 33 | 1813.4 | $\frac{1}{2} = + 1.2 = 0.53$ | 9 |
| 34 | 1814.4 | $1 = + 1.7 = 1.05$ | 15 |

| No. | Date. | Equations. | Number of observations in R. A. |
|-----|---------|---|---------------------------------|
| 35 | 1815.4 | $\frac{3}{2} \mathcal{U} = + 2.1 = 1.58 \delta \lambda$ | 20 |
| 36 | 1818.4* | $\frac{3}{2} = + 0.3 = 1.58$ | 24 |
| 37 | 1819.4 | 1 = - 0.8 = 1.05 | 11 |
| 38 | 1820.5 | 1 = - 1.3 = 1.05 | 14 |
| 39 | 1821.5 | $\frac{2}{3} = + 0.9 = 0.70$ | 10 |
| 40 | 1822.5 | $\frac{2}{3} = + 0.9 = 0.70$ | 7 |
| 41 | 1823.5 | $\frac{2}{3} = 0.0 = 0.70$ | 11 |
| 42 | 1824.5 | 1 = + 0.5 = 1.05 | 12 |
| 43 | 1825.5 | $\frac{2}{3} = - 0.4 = 0.70$ | 7 |
| 44 | 1826.5 | 1 = + 0.3 = 1.05 | 11 |
| 45 | 1827.7 | 2 = - 2.7 = 2.07 + 0.04 $\delta \rho$ | 37 |
| 46 | 1828.7 | $2\frac{1}{2} = - 2.7 = 2.57 + 0.07$ | 67 |
| 47 | 1829.7 | $2\frac{1}{2} = - 2.4 = 2.59 + 0.04$ | 61 |
| 48 | 1830.7 | $2\frac{1}{2} = - 4.9 = 2.56 + 0.07$ | 73 |
| 49 | 1831.7 | 2 = 0.0 = 2.06 + 0.05 | 54 |
| 50 | 1832.7 | 2 = - 2.2 = 2.07 + 0.05 | 65 |
| 51 | 1833.8 | $2\frac{1}{2} = - 4.1 = 2.58 + 0.08$ | 88 |
| 52 | 1834.8 | $2\frac{1}{2} = - 3.9 = 2.57 + 0.08$ | 91 |
| 53 | 1835.8 | $2\frac{1}{2} = - 5.1 = 2.59 + 0.05$ | 82 |
| 54 | 1836.8 | 3 = - 7.0 = 3.11 + 0.08 | 157 |
| 55 | 1837.8 | 3 = - 3.6 = 3.11 + 0.04 | 162 |
| 56 | 1838.8 | 3 = - 1.6 = 3.11 + 0.06 | 193 |
| 57 | 1839.8 | 3 = - 1.2 = 3.11 + 0.06 | 170 |
| 58 | 1840.8 | 3 = - 1.5 = 3.11 + 0.06 | 124 |
| 59 | 1841.8 | 3 = + 0.8 = 3.10 + 0.06 | 108 |
| 60 | 1842.8 | 3 = + 1.6 = 3.10 + 0.06 | 169 |
| 61 | 1843.8 | 3 = + 4.7 = 3.12 + 0.04 | 111 |
| 62 | 1844.9 | 3 = + 5.1 = 3.11 + 0.03 | 106 |
| 63 | 1845.9 | 2 = + 4.2 = 2.08 + 0.02 | 55 |
| 64 | 1846.9 | 3 = + 6.3 = 3.10 + 0.04 | 98 |
| 65 | 1847.9 | 2 = + 5.8 = 2.07 + 0.03 | 74 |
| 66 | 1848.9 | 2 = + 4.4 = 2.08 + 0.02 | 59 |
| 67 | 1849.9 | 2 = + 6.6 = 2.08 + 0.04 | 33 |
| 68 | 1850.9 | 2 = + 7.2 = 2.08 + 0.01 | 46 |
| 69 | 1851.9 | 2 = + 6.3 = 2.07 + 0.04 | 42 |
| 70 | 1852.9 | 2 = + 6.6 = 2.07 + 0.03 | 54 |
| 71 | 1853.9 | 2 = + 7.9 = 2.09 + 0.01 | 49 |
| 72 | 1854.9 | 2 = + 8.9 = 2.09 + 0.02 | 49 |
| 74 | 1855.9 | 2 = + 8.5 = 2.08 + 0.04 | 48 |
| 75 | 1856.9 | 2 = + 7.9 = 2.08 + 0.04 | 45 |
| 76 | 1858.0 | $2\frac{1}{2} = + 10.3 = 2.61 + 0.09$ | 66 |

* The results for 1816 and 1817 were omitted in this list through oversight.

| No. | Date. | Equations. | Number of observations in R. A. |
|-----|--------|---|---------------------------------|
| 77 | 1859.0 | $2\frac{1}{2} \delta l = + 10.6 = 2.60 \delta \lambda + 0.03 \delta \rho$ | 58 |
| 78 | 1860.0 | $2\frac{1}{2} = + 8.2 = 2.60 + 0.05$ | 64 |
| 79 | 1861.0 | $2 = + 6.3 = 2.09 + 0.03$ | 41 |
| 80 | 1862.0 | $2\frac{1}{2} = + 7.0 = 2.60 + 0.05$ | 60 |
| 81 | 1863.0 | $2\frac{1}{2} = + 6.6 = 2.59 + 0.08$ | 88 |
| 82 | 1864.0 | $2 = + 4.3 = 2.09 + 0.03$ | 35 |
| 83 | 1865.0 | $1\frac{1}{2} = + 3.9 = 1.57 + 0.04$ | 37 |
| 84 | 1866.0 | $2\frac{1}{2} = + 1.1 = 2.60 + 0.06$ | 76 |
| 85 | 1867.0 | $2\frac{1}{2} = - 1.8 = 2.60 + 0.02$ | 83 |
| 86 | 1868.0 | $2 = - 3.8 = 2.09 + 0.04$ | 40 |
| 87 | 1869.0 | $2\frac{1}{2} = - 9.1 = 2.61 + 0.02$ | 68 |
| 88 | 1870.0 | $2 = - 9.6 = 2.084 + 0.054$ | 31 |
| 89 | 1871.0 | $1\frac{1}{2} = - 10.6 = 1.560 + 0.040$ | 21 |
| 90 | 1872.1 | $2\frac{1}{2} = - 19.1 = 2.600 + 0.070$ | 50 |

Total number of observations in R. A., 3763.

We have next to express the values of $\delta \lambda$ and $\delta \rho$ in terms of the corrections to the elements. Differentiating the expressions

$$\lambda = l + (2e - \frac{1}{4}e^3) \sin(l - \pi) + \frac{1}{4}e^2 \sin(2l - 2\pi) + \frac{1}{8}e^3 \sin e^3 \sin(3l - 3\pi) + \text{etc.}$$

$$\rho = v + \frac{1}{4}e^2 + (-e + \frac{3}{8}e^3) \cos(l - \pi) - \frac{3}{4}e^2 \cos(2l - 2\pi) - \text{etc.},$$

with respect to $l, e,$ and $\pi,$ and reducing the coefficients to numbers, we find

$$\frac{\partial \lambda}{\partial l} = 1 + 0.0939 \cos g + 0.0055 \cos 2g$$

$$\frac{\partial \lambda}{\partial e} = 1.999 \sin g + 0.117 \sin 2g + 0.007 \sin 3g$$

$$\frac{\partial \lambda}{\partial \pi} = - 2.000 \cos g - 0.117 \cos 2g - 0.007 \cos 3g$$

We have here put l for the mean longitude, or

$$l = nt + \epsilon$$

whence

$$\frac{\partial \lambda}{\partial \epsilon} = \frac{\partial \lambda}{\partial l}$$

$$\frac{\partial \lambda}{\partial n} = t \frac{\partial \lambda}{\partial l}$$

Also, from the expression for ρ

$$\frac{\partial \rho}{\partial l} = - \frac{\partial \rho}{\partial \pi} = (e - \frac{3}{8}e^3) \sin g + \frac{3}{4}e^2 \sin 2g + \dots$$

$$\frac{\partial \rho}{\partial e} = \frac{1}{2}e - (1 - \frac{3}{8}e^2) \cos g - \frac{3}{2}e \cos 2g - \text{etc.}$$

$$\frac{\partial \rho}{\partial n} = t \frac{\partial \rho}{\partial l} - \frac{2}{3n}$$

The values of these coefficients which depend only on g are shown in the following table:

| g | $\frac{\partial \lambda}{\partial \varepsilon}$ | $\frac{\partial \lambda}{\partial e}$ | $\frac{\partial \lambda}{e d \pi}$ | $\frac{\partial \rho}{\partial \varepsilon}$ | $\frac{\partial \rho}{\partial e}$ | $\frac{\partial \rho}{e d \pi}$ |
|-----|---|---------------------------------------|------------------------------------|--|------------------------------------|---------------------------------|
| 0° | +1.099 | 0 | -2.124 | 0 | -1.00 | 0 |
| 1 | 1.099 | +0.039 | 2.124 | +0.001 | 1.00 | -0.02 |
| 2 | 1.099 | 0.079 | 2.123 | 0.002 | 1.00 | 0.03 |
| 3 | 1.099 | 0.118 | 2.121 | 0.002 | 1.00 | 0.05 |
| 4 | 1.099 | 0.158 | 2.118 | 0.003 | 1.00 | 0.07 |
| 5 | +1.099 | +0.196 | -2.114 | +0.004 | -1.00 | -0.09 |
| 6 | 1.099 | 0.235 | 2.110 | 0.005 | 0.99 | 0.10 |
| 7 | 1.099 | 0.274 | 2.105 | 0.006 | 0.99 | 0.12 |
| 8 | 1.098 | 0.313 | 2.099 | 0.006 | 0.99 | 0.14 |
| 9 | 1.098 | 0.352 | 2.092 | 0.007 | 0.99 | 0.16 |
| 10 | +1.098 | +0.391 | -2.085 | +0.008 | -0.98 | -0.17 |
| 11 | 1.097 | 0.430 | 2.077 | 0.009 | 0.98 | 0.19 |
| 12 | 1.097 | 0.468 | 2.069 | 0.010 | 0.98 | 0.21 |
| 13 | 1.097 | 0.505 | 2.059 | 0.011 | 0.97 | 0.22 |
| 14 | 1.096 | 0.544 | 2.049 | 0.011 | 0.97 | 0.24 |
| 15 | +1.096 | +0.581 | -2.038 | +0.012 | -0.97 | -0.26 |
| 16 | 1.095 | 0.618 | 2.027 | 0.013 | 0.96 | 0.28 |
| 17 | 1.095 | 0.655 | 2.014 | 0.014 | 0.96 | 0.29 |
| 18 | 1.094 | 0.693 | 2.001 | 0.015 | 0.95 | 0.31 |
| 19 | 1.094 | 0.729 | 1.987 | 0.015 | 0.95 | 0.33 |
| 20 | +1.093 | +0.765 | -1.973 | +0.016 | -0.94 | -0.34 |
| 21 | 1.092 | 0.801 | 1.957 | 0.017 | 0.93 | 0.36 |
| 22 | 1.091 | 0.836 | 1.941 | 0.018 | 0.93 | 0.37 |
| 23 | 1.090 | 0.872 | 1.925 | 0.018 | 0.92 | 0.39 |
| 24 | 1.090 | 0.907 | 1.907 | 0.019 | 0.91 | 0.41 |
| 25 | +1.089 | +0.942 | -1.890 | +0.020 | -0.91 | -0.42 |
| 26 | 1.088 | 0.976 | 1.872 | 0.021 | 0.90 | 0.44 |
| 27 | 1.087 | 1.010 | 1.852 | 0.021 | 0.89 | 0.45 |
| 28 | 1.086 | 1.043 | 1.832 | 0.022 | 0.88 | 0.47 |
| 29 | 1.085 | 1.076 | 1.811 | 0.023 | 0.87 | 0.48 |
| 30 | +1.084 | +1.108 | -1.790 | +0.023 | -0.87 | -0.50 |
| 31 | 1.084 | 1.140 | 1.769 | 0.024 | 0.86 | 0.51 |
| 32 | 1.083 | 1.172 | 1.747 | 0.025 | 0.85 | 0.53 |
| 33 | 1.082 | 1.203 | 1.724 | 0.026 | 0.84 | 0.54 |
| 34 | 1.080 | 1.233 | 1.700 | 0.026 | 0.83 | 0.56 |
| 35 | +1.079 | +1.264 | -1.676 | +0.027 | -0.82 | -0.57 |
| 36 | 1.078 | 1.294 | 1.651 | 0.028 | 0.81 | 0.59 |
| 37 | 1.077 | 1.323 | 1.626 | 0.028 | 0.80 | 0.60 |
| 38 | 1.076 | 1.351 | 1.601 | 0.029 | 0.79 | 0.62 |
| 39 | 1.074 | 1.379 | 1.574 | 0.030 | 0.78 | 0.63 |
| 40 | +1.073 | +1.407 | -1.548 | +0.030 | -0.77 | -0.64 |
| 41 | 1.072 | 1.434 | 1.521 | 0.031 | 0.76 | 0.66 |
| 42 | 1.070 | 1.460 | 1.494 | 0.031 | 0.74 | 0.67 |
| 43 | 1.069 | 1.486 | 1.466 | 0.032 | 0.73 | 0.68 |
| 44 | 1.068 | 1.511 | 1.438 | 0.033 | 0.72 | 0.70 |
| 45 | +1.067 | +1.536 | -1.409 | +0.033 | -0.71 | -0.71 |
| 46 | 1.065 | 1.561 | 1.380 | 0.034 | 0.70 | 0.71 |
| 47 | 1.064 | 1.584 | 1.351 | 0.034 | 0.68 | 0.72 |
| 48 | 1.063 | 1.606 | 1.320 | 0.035 | 0.67 | 0.74 |
| 49 | 1.061 | 1.629 | 1.290 | 0.036 | 0.66 | 0.75 |
| 50 | +1.060 | +1.651 | -1.260 | +0.036 | -0.64 | -0.76 |
| 51 | 1.058 | 1.672 | 1.229 | 0.037 | 0.63 | 0.77 |
| 52 | 1.057 | 1.692 | 1.197 | 0.037 | 0.62 | 0.78 |
| 53 | 1.055 | 1.712 | 1.165 | 0.037 | 0.60 | 0.79 |
| 54 | 1.054 | 1.731 | 1.133 | 0.038 | 0.59 | 0.80 |
| 55 | +1.052 | +1.750 | -1.100 | +0.038 | -0.57 | -0.81 |
| 56 | 1.051 | 1.768 | 1.067 | 0.039 | 0.56 | 0.82 |
| 57 | 1.049 | 1.785 | 1.034 | 0.039 | 0.54 | 0.83 |
| 58 | 1.047 | 1.802 | 1.002 | 0.040 | 0.53 | 0.84 |
| 59 | +1.046 | +1.817 | -0.968 | +0.040 | -0.51 | -0.85 |

| g | $\frac{\partial \lambda}{\partial \varepsilon}$ | $\frac{\partial \lambda}{\partial e}$ | $\frac{\partial \lambda}{e d \pi}$ | $\frac{\partial \rho}{\partial \varepsilon}$ | $\frac{\partial \rho}{\partial e}$ | $\frac{\partial \rho}{e d \pi}$ |
|-----|---|---------------------------------------|------------------------------------|--|------------------------------------|---------------------------------|
| 60° | +1.044 | +1.833 | -0.935 | +0.041 | -0.50 | -0.86 |
| 61 | 1.043 | 1.848 | 0.901 | 0.041 | 0.49 | 0.87 |
| 62 | 1.041 | 1.862 | 0.867 | 0.041 | 0.47 | 0.87 |
| 63 | 1.040 | 1.875 | 0.832 | 0.042 | 0.45 | 0.88 |
| 64 | 1.038 | 1.888 | 0.798 | 0.042 | 0.44 | 0.89 |
| 65 | +1.036 | +1.901 | -0.763 | +0.043 | -0.42 | -0.90 |
| 66 | 1.034 | 1.912 | 0.728 | 0.043 | 0.41 | 0.91 |
| 67 | 1.033 | 1.922 | 0.693 | 0.043 | 0.39 | 0.91 |
| 68 | 1.031 | 1.932 | 0.659 | 0.044 | 0.37 | 0.92 |
| 69 | 1.030 | 1.942 | 0.624 | 0.044 | 0.36 | 0.93 |
| 70 | +1.028 | +1.950 | -0.588 | +0.044 | -0.34 | -0.94 |
| 71 | 1.027 | 1.959 | 0.553 | 0.044 | 0.33 | 0.95 |
| 72 | 1.025 | 1.967 | 0.518 | 0.045 | 0.31 | 0.95 |
| 73 | 1.023 | 1.974 | 0.483 | 0.045 | 0.29 | 0.96 |
| 74 | 1.021 | 1.980 | 0.447 | 0.045 | 0.28 | 0.96 |
| 75 | +1.019 | +1.985 | -0.412 | +0.045 | -0.26 | -0.97 |
| 76 | 1.018 | 1.990 | 0.376 | 0.046 | 0.24 | 0.97 |
| 77 | 1.016 | 1.994 | 0.341 | 0.046 | 0.22 | 0.97 |
| 78 | 1.014 | 1.997 | 0.305 | 0.046 | 0.21 | 0.98 |
| 79 | 1.013 | 2.000 | 0.269 | 0.046 | 0.19 | 0.98 |
| 80 | +1.011 | +2.003 | -0.233 | +0.046 | -0.17 | -0.98 |
| 81 | 1.010 | 2.005 | 0.198 | 0.046 | 0.16 | 0.99 |
| 82 | 1.008 | 2.007 | 0.163 | 0.046 | 0.14 | 0.99 |
| 83 | 1.006 | 2.007 | 0.128 | 0.047 | 0.12 | 0.99 |
| 84 | 1.005 | 2.006 | 0.093 | 0.047 | 0.11 | 0.99 |
| 85 | +1.003 | +2.005 | -0.057 | +0.047 | -0.09 | -1.00 |
| 86 | 1.002 | 2.004 | -0.022 | 0.047 | 0.07 | 1.00 |
| 87 | 1.000 | 2.002 | +0.013 | 0.047 | 0.05 | 1.00 |
| 88 | 0.998 | 2.000 | 0.048 | 0.047 | 0.03 | 1.00 |
| 89 | 0.997 | 1.997 | 0.082 | 0.047 | -0.02 | 1.00 |
| 90 | +0.995 | +1.993 | +0.117 | +0.047 | 0.00 | -1.00 |
| 91 | 0.993 | 1.989 | 0.152 | 0.047 | +0.02 | 1.00 |
| 92 | 0.992 | 1.984 | 0.186 | 0.047 | 0.03 | 1.00 |
| 93 | 0.990 | 1.978 | 0.220 | 0.047 | 0.05 | 1.00 |
| 94 | 0.988 | 1.972 | 0.254 | 0.047 | 0.07 | 1.00 |
| 95 | +0.987 | +1.965 | +0.287 | +0.047 | +0.09 | -1.00 |
| 96 | 0.985 | 1.958 | 0.321 | 0.047 | 0.11 | 0.99 |
| 97 | 0.984 | 1.950 | 0.354 | 0.047 | 0.12 | 0.99 |
| 98 | 0.982 | 1.943 | 0.387 | 0.046 | 0.14 | 0.99 |
| 99 | 0.980 | 1.933 | 0.421 | 0.046 | 0.16 | 0.99 |
| 100 | +0.979 | +1.924 | +0.453 | +0.046 | +0.17 | -0.98 |
| 101 | 0.977 | 1.914 | 0.487 | 0.046 | 0.19 | 0.98 |
| 102 | 0.976 | 1.903 | 0.519 | 0.046 | 0.21 | 0.98 |
| 103 | 0.974 | 1.893 | 0.551 | 0.046 | 0.22 | 0.97 |
| 104 | 0.972 | 1.882 | 0.582 | 0.046 | 0.24 | 0.97 |
| 105 | +0.971 | +1.869 | +0.614 | +0.045 | +0.26 | -0.97 |
| 106 | 0.969 | 1.857 | 0.645 | 0.045 | 0.28 | 0.96 |
| 107 | 0.968 | 1.844 | 0.677 | 0.045 | 0.29 | 0.96 |
| 108 | 0.967 | 1.829 | 0.707 | 0.045 | 0.31 | 0.95 |
| 109 | 0.965 | 1.815 | 0.737 | 0.044 | 0.33 | 0.95 |
| 110 | +0.964 | +1.800 | +0.768 | +0.044 | +0.34 | -0.94 |
| 111 | 0.962 | 1.786 | 0.798 | 0.044 | 0.36 | 0.93 |
| 112 | 0.961 | 1.770 | 0.827 | 0.044 | 0.37 | 0.92 |
| 113 | 0.959 | 1.754 | 0.855 | 0.043 | 0.39 | 0.91 |
| 114 | 0.958 | 1.738 | 0.884 | 0.043 | 0.41 | 0.91 |
| 115 | 0.956 | +1.721 | +0.913 | +0.043 | +0.42 | -0.90 |
| 116 | 0.955 | 1.704 | 0.942 | 0.042 | 0.44 | 0.89 |
| 117 | 0.954 | 1.686 | 0.970 | 0.042 | 0.45 | 0.88 |
| 118 | 0.953 | 1.668 | 0.997 | 0.041 | 0.47 | 0.87 |
| 119 | +0.951 | +1.650 | +1.025 | +0.041 | +0.49 | -0.87 |

| g | $\frac{\partial \lambda}{\partial \varepsilon}$ | $\frac{\partial \lambda}{\partial e}$ | $\frac{\partial \lambda}{\partial i \pi}$ | $\frac{\partial \rho}{\partial \varepsilon}$ | $\frac{\partial \rho}{\partial e}$ | $\frac{\partial \rho}{\partial i \pi}$ |
|------|---|---------------------------------------|---|--|------------------------------------|--|
| 120° | +0.950 | +1.631 | +1.051 | +0.041 | +0.50 | -0.86 |
| 121 | 0.949 | 1.611 | 1.077 | 0.040 | 0.51 | 0.85 |
| 122 | 0.948 | 1.592 | 1.104 | 0.040 | 0.53 | 0.84 |
| 123 | 0.947 | 1.571 | 1.129 | 0.039 | 0.54 | 0.83 |
| 124 | 0.945 | 1.551 | 1.154 | 0.039 | 0.56 | 0.82 |
| 125 | +0.944 | +1.530 | +1.180 | +0.038 | +0.57 | -0.81 |
| 126 | 0.943 | 1.509 | 1.205 | 0.038 | 0.59 | 0.80 |
| 127 | 0.942 | 1.488 | 1.229 | 0.037 | 0.60 | 0.79 |
| 128 | 0.941 | 1.466 | 1.253 | 0.037 | 0.62 | 0.78 |
| 129 | 0.940 | 1.443 | 1.277 | 0.037 | 0.63 | 0.77 |
| 130 | +0.939 | +1.421 | +1.300 | +0.036 | +0.64 | -0.76 |
| 131 | 0.937 | 1.397 | 1.322 | 0.036 | 0.66 | 0.75 |
| 132 | 0.936 | 1.374 | 1.344 | 0.035 | 0.67 | 0.74 |
| 133 | 0.936 | 1.350 | 1.367 | 0.034 | 0.68 | 0.72 |
| 134 | 0.935 | 1.327 | 1.388 | 0.034 | 0.70 | 0.71 |
| 135 | +0.934 | +1.302 | +1.409 | +0.033 | +0.71 | -0.71 |
| 136 | 0.933 | 1.277 | 1.430 | 0.033 | 0.71 | 0.70 |
| 137 | 0.932 | 1.253 | 1.451 | 0.032 | 0.72 | 0.68 |
| 138 | 0.931 | 1.228 | 1.470 | 0.031 | 0.74 | 0.67 |
| 139 | 0.930 | 1.202 | 1.489 | 0.031 | 0.75 | 0.66 |
| 140 | +0.929 | +1.177 | +1.508 | +0.030 | +0.76 | -0.64 |
| 141 | 0.928 | 1.151 | 1.527 | 0.030 | 0.77 | 0.63 |
| 142 | 0.927 | 1.124 | 1.545 | 0.029 | 0.78 | 0.62 |
| 143 | 0.927 | 1.099 | 1.562 | 0.028 | 0.79 | 0.60 |
| 144 | 0.926 | 1.072 | 1.580 | 0.028 | 0.80 | 0.59 |
| 145 | +0.925 | +1.044 | +1.596 | +0.027 | +0.81 | -0.57 |
| 146 | 0.924 | 1.016 | 1.613 | 0.026 | 0.82 | 0.56 |
| 147 | 0.923 | 0.989 | 1.629 | 0.026 | 0.83 | 0.54 |
| 148 | 0.922 | 0.962 | 1.644 | 0.025 | 0.84 | 0.53 |
| 149 | 0.922 | 0.934 | 1.659 | 0.024 | 0.85 | 0.51 |
| 150 | +0.921 | +0.906 | +1.674 | +0.023 | +0.86 | -0.50 |
| 151 | 0.921 | 0.878 | 1.688 | 0.023 | 0.87 | 0.49 |
| 152 | 0.920 | 0.849 | 1.702 | 0.022 | 0.87 | 0.47 |
| 153 | 0.919 | 0.820 | 1.714 | 0.021 | 0.88 | 0.45 |
| 154 | 0.919 | 0.792 | 1.727 | 0.021 | 0.89 | 0.44 |
| 155 | +0.919 | +0.762 | +1.740 | +0.020 | +0.90 | -0.42 |
| 156 | 0.918 | 0.733 | 1.751 | 0.019 | 0.91 | 0.41 |
| 157 | 0.918 | 0.704 | 1.763 | 0.018 | 0.91 | 0.39 |
| 158 | 0.917 | 0.674 | 1.773 | 0.018 | 0.92 | 0.37 |
| 159 | 0.916 | 0.645 | 1.783 | 0.017 | 0.93 | 0.36 |
| 160 | +0.916 | +0.615 | +1.793 | +0.016 | +0.94 | -0.34 |
| 161 | 0.915 | 0.585 | 1.803 | 0.015 | 0.95 | 0.33 |
| 162 | 0.915 | 0.555 | 1.811 | 0.015 | 0.95 | 0.31 |
| 163 | 0.915 | 0.525 | 1.820 | 0.014 | 0.96 | 0.29 |
| 164 | 0.915 | 0.494 | 1.829 | 0.013 | 0.96 | 0.28 |
| 165 | +0.914 | +0.465 | +1.836 | +0.012 | +0.97 | -0.26 |
| 166 | 0.914 | 0.435 | 1.843 | 0.011 | 0.97 | 0.24 |
| 167 | 0.914 | 0.403 | 1.849 | 0.011 | 0.97 | 0.22 |
| 168 | 0.913 | 0.373 | 1.855 | 0.010 | 0.98 | 0.21 |
| 169 | 0.913 | 0.343 | 1.860 | 0.009 | 0.98 | 0.19 |
| 170 | +0.913 | +0.311 | +1.865 | +0.008 | +0.98 | -0.17 |
| 171 | 0.912 | 0.280 | 1.870 | 0.007 | 0.99 | 0.16 |
| 172 | 0.912 | 0.249 | 1.875 | 0.006 | 0.99 | 0.14 |
| 173 | 0.912 | 0.219 | 1.879 | 0.006 | 0.99 | 0.12 |
| 174 | 0.912 | 0.187 | 1.882 | 0.005 | 0.99 | 0.11 |
| 175 | +0.911 | +0.156 | +1.884 | +0.004 | +1.00 | -0.09 |
| 176 | 0.911 | 0.126 | 1.886 | 0.003 | 1.00 | 0.07 |
| 177 | 0.911 | 0.094 | 1.888 | 0.002 | 1.00 | 0.05 |
| 178 | 0.911 | 0.063 | 1.889 | 0.002 | 1.00 | 0.03 |
| 179 | +0.911 | +0.031 | +1.890 | +0.001 | +1.00 | -0.02 |

In the equations of condition ten years has been adopted for the unit of time, in order to make the general value of the coefficients as nearly equal as possible, and the time has been counted from the epoch 1830.0, in order to have the positive and negative values of t in the equations more nearly balanced. To distinguish these values of $\delta\epsilon$ and δn they are marked with an accent. This unit of time gives 0.8914 for the value of $\frac{2}{3n}$ in arc, whence

$$\frac{\partial \rho}{\partial n'} = t \frac{\partial \rho}{\partial t} = 0.891.$$

The equations of condition are now formed by putting in the preceding equations for heliocentric longitude and radius vector

$$\begin{aligned} \delta\lambda &= \frac{\partial\lambda}{\partial\epsilon} \delta\epsilon + \frac{\partial\lambda}{\partial v} \delta v + \frac{\partial\lambda}{\partial e} \delta e + \frac{\partial\lambda}{\partial\pi} e\delta\pi + \frac{\partial\lambda}{\partial u'} \delta u' \\ \delta\rho &= \frac{\partial\rho}{\partial\epsilon} \delta\epsilon + \frac{\partial\rho}{\partial n} \delta n + \frac{\partial\rho}{\partial e} \delta e + \frac{\partial\rho}{\partial\pi} e\delta\pi. \end{aligned}$$

For the coefficients $\frac{\partial\lambda}{\partial u'}$ have been taken one-hundredth the perturbations of longitude produced by Neptune, as given in the heliocentric ephemeris at the end of Chapter V. The corrected mass of Neptune will then be

$$\frac{1}{17000} \left(1 + \frac{\delta u'}{100}\right)$$

Finally, I remark that all the preceding comparisons are made with the heliocentric ephemeris as printed, without the correction indicated in the column adjoining it, but in the following equations this correction is for the first time introduced.

Equations of condition given by the Corrections in Longitude.

| | | | | | | |
|----|------------------------|--------------------|------------------|--------------------|-------------------|----------|
| 1 | 0.05 $\delta\epsilon'$ | - 0.70 $\delta n'$ | -0.10 δe | +0.03 $e\delta\pi$ | +0.12 $\delta u'$ | = + 1".1 |
| 2 | 0.10 | - 1.11 | +0.01 | -0.19 | +0.12 | = + 2.2 |
| 3 | 0.31 | - 2.55 | +0.33 | +0.51 | -0.22 | = +12.8 |
| 4 | 0.32 | - 2.52 | +0.27 | +0.58 | -0.18 | = +11.8 |
| 5 | 0.30 | - 2.32 | +0.12 | +0.59 | -0.09 | = +11.6 |
| 6 | 0.19 | - 1.40 | -0.01 | +0.39 | 0.00 | = + 5.0 |
| 7 | 0.19 | - 1.18 | -0.29 | +0.23 | +0.22 | = + 4.8 |
| 8 | 1.41 | - 6.75 | -2.69 | -0.75 | +2.47 | = + 2.8 |
| 9 | 1.06 | - 5.01 | -1.98 | -0.72 | +1.85 | = + 1.1 |
| 10 | 1.07 | - 4.91 | -1.94 | -0.86 | +1.85 | = + 1.7 |
| 11 | 1.09 | - 4.87 | -1.86 | -1.04 | +1.85 | = - 0.5 |
| 12 | 0.73 | - 3.06 | -1.04 | -0.97 | +1.16 | = + 1.0 |
| 13 | 0.54 | - 2.20 | -0.71 | -0.78 | +0.82 | = + 1.4 |
| 14 | 0.55 | - 2.19 | -0.68 | -0.81 | +0.81 | = - 0.6 |
| 15 | 0.73 | - 2.86 | -0.82 | -1.16 | +1.03 | = - 0.9 |
| 16 | 0.37 | - 1.39 | -0.36 | -0.60 | +0.49 | = - 0.2 |
| 17 | 0.56 | - 2.05 | -0.47 | -0.95 | +0.70 | = - 0.4 |
| 18 | 0.37 | - 1.33 | -0.26 | -0.65 | +0.44 | = + 0.6 |
| 19 | 0.75 | - 2.59 | -0.38 | -1.34 | +0.82 | = - 1.0 |

| | | | | | | | |
|------|---------------------------------|---|---------------------------------|----------------------------------|-----------------------------------|----------------------------------|---------|
| 20 | 0.56 ^s _{ε'} | — | 1.89 ^s _{n'} | —0.22 ^s _{δe} | —1.04 ^s _{εδπ} | +0.58 ^s _{μ'} | = — 0.9 |
| 21 | 0.58 | — | 1.92 | —0.13 | —1.12 | +0.55 | = — 0.5 |
| 22 | 0.38 | — | 1.15 | +0.09 | —0.74 | +0.24 | = — 0.2 |
| 23 | 0.38 | — | 1.11 | +0.15 | —0.73 | +0.21 | = — 0.3 |
| 24 | 1.15 | — | 3.18 | +0.65 | —2.13 | +0.50 | = + 0.6 |
| 25 | 1.13 | — | 2.81 | +1.11 | —1.91 | +0.19 | = + 0.5 |
| 26 | 0.56 | — | 1.33 | +0.62 | —0.89 | +0.05 | = + 0.1 |
| 27 | 1.12 | — | 2.54 | +1.38 | —1.68 | 0.00 | = + 2.1 |
| 28 | 0.56 | — | 1.21 | +0.75 | —0.79 | —0.04 | = — 0.1 |
| 29 | 0.70 | — | 1.55 | +1.09 | —0.97 | —0.10 | = + 1.6 |
| 30 | 1.11 | — | 2.19 | +1.73 | —1.32 | —0.23 | = + 2.2 |
| 31 | 1.09 | — | 2.04 | +1.81 | —1.14 | —0.29 | = + 1.3 |
| 32 | 0.56 | — | 0.98 | +0.96 | —0.51 | —0.18 | = + 0.7 |
| 33 | 0.55 | — | 0.91 | +1.00 | —0.43 | —0.21 | = + 1.1 |
| 34 | 1.08 | — | 1.69 | +2.03 | —0.71 | —0.46 | = + 1.5 |
| 35 | 1.63 | — | 2.37 | +3.11 | —0.82 | —0.74 | = + 2.0 |
| 35' | 1.07 | — | 1.45 | +2.08 | —0.38 | —0.52 | = + 0.4 |
| 35'' | 1.06 | — | 1.33 | +2.10 | —0.22 | —0.55 | = + 0.8 |
| 36 | 1.58 | — | 1.83 | +3.18 | —0.09 | —0.85 | = + 0.3 |
| 37 | 1.05 | — | 1.11 | +2.10 | +0.08 | —0.59 | = — 0.8 |
| 38 | 1.04 | — | 0.99 | +2.07 | +0.26 | —0.59 | = — 1.3 |
| 39 | 0.69 | — | 0.58 | +1.36 | +0.27 | —0.39 | = + 0.8 |
| 40 | 0.69 | — | 0.51 | +1.33 | +0.38 | —0.39 | = + 0.8 |
| 41 | 0.68 | — | 0.44 | +1.29 | +0.47 | —0.39 | = 0.0 |
| 42 | 1.01 | — | 0.56 | +1.88 | +0.84 | —0.59 | = + 0.3 |
| 43 | 0.67 | — | 0.30 | +1.20 | +0.64 | —0.38 | = — 0.5 |
| 44 | 1.00 | — | 0.35 | +1.72 | +1.09 | —0.57 | = + 0.2 |
| 45 | 1.95 | — | 0.50 | +3.21 | +2.39 | —1.12 | = — 2.5 |
| 46 | 2.42 | — | 0.37 | +3.77 | +3.21 | —1.36 | = — 2.6 |
| 47 | 2.42 | — | 0.12 | +3.53 | +3.52 | —1.37 | = — 2.4 |
| 48 | 2.39 | + | 0.12 | +3.22 | +3.69 | —1.33 | = — 4.9 |
| 49 | 1.91 | + | 0.28 | +2.37 | +3.14 | —1.07 | = — 0.1 |
| 50 | 1.91 | + | 0.48 | +2.15 | +3.30 | —1.08 | = — 2.4 |
| 51 | 2.37 | + | 0.83 | +2.34 | +4.32 | —1.34 | = — 4.5 |
| 52 | 2.36 | + | 1.06 | +2.02 | +4.44 | —1.36 | = — 4.3 |
| 53 | 2.37 | + | 1.32 | +1.68 | +4.62 | —1.37 | = — 5.5 |
| 54 | 2.85 | + | 1.86 | +1.66 | +5.64 | —1.68 | = — 7.5 |
| 55 | 2.84 | + | 2.17 | +1.22 | +5.74 | —1.71 | = — 4.1 |
| 56 | 2.84 | + | 2.44 | +0.81 | +5.84 | —1.74 | = — 2.1 |
| 57 | 2.83 | + | 2.71 | +0.40 | +5.88 | —1.78 | = — 1.7 |
| 58 | 2.83 | + | 3.00 | —0.06 | +5.88 | —1.82 | = — 2.0 |
| 59 | 2.82 | + | 3.28 | —0.44 | +5.84 | —1.86 | = + 0.3 |
| 60 | 2.83 | + | 3.57 | —0.84 | +5.81 | —1.91 | = + 1.2 |
| 61 | 2.85 | + | 3.90 | —1.27 | +5.78 | —1.98 | = + 4.4 |
| 62 | 2.85 | + | 4.21 | —1.74 | +5.64 | —2.02 | = + 4.8 |
| 63 | 1.91 | + | 3.02 | —1.42 | +3.69 | —1.39 | = + 4.0 |
| 64 | 2.85 | + | 4.78 | —2.50 | +5.32 | —2.14 | = + 6.0 |
| 65 | 1.91 | + | 3.39 | —1.92 | +3.46 | —1.47 | = + 5.6 |
| 66 | 1.92 | + | 3.61 | —2.18 | +3.32 | —1.52 | = + 4.3 |
| 67 | 1.93 | + | 3.82 | —2.40 | +3.17 | —1.56 | = + 6.6 |
| 68 | 1.94 | + | 4.05 | —2.65 | +2.98 | —1.60 | = + 7.3 |
| 69 | 1.94 | + | 4.20 | —2.83 | +2.80 | —1.64 | = + 6.5 |

| | | | | | | |
|----|------------------------|--------------------|------------------|--------------------|------------------|----------|
| 70 | 1.95 $\delta\epsilon'$ | + 4.44 $\delta n'$ | -3.04 δe | +2.59 $e\delta\pi$ | -1.68 δu | = + 6".8 |
| 71 | 1.98 | + 4.72 | -3.23 | +2.37 | -1.73 | = + 8.1 |
| 72 | 1.99 | + 4.93 | -3.44 | +2.15 | -1.76 | = + 9.1 |
| 73 | 1.99 | + 5.13 | -3.58 | +1.91 | -1.78 | = + 8.7 |
| 74 | 2.00 | + 5.35 | -3.73 | +1.64 | -1.80 | = + 8.1 |
| 75 | 2.53 | + 7.02 | -4.83 | +1.73 | -2.30 | = +10.6 |
| 76 | 2.54 | + 7.34 | -4.96 | +1.33 | -2.29 | = +10.9 |
| 77 | 2.56 | + 7.64 | -5.07 | +0.97 | -2.31 | = + 8.2 |
| 78 | 2.07 | + 6.38 | -4.14 | +0.47 | -1.86 | = + 6.3 |
| 79 | 2.59 | + 8.25 | -5.20 | +0.21 | -2.32 | = + 7.0 |
| 80 | 2.60 | + 8.51 | -5.21 | -0.15 | -2.29 | = + 6.6 |
| 81 | 2.11 | + 7.14 | -4.19 | -0.51 | -1.83 | = + 4.3 |
| 82 | 1.60 | + 5.56 | -3.13 | -0.58 | -1.36 | = + 3.9 |
| 83 | 2.67 | + 9.59 | -5.12 | -1.37 | -2.20 | = + 1.1 |
| 84 | 2.68 | + 9.92 | -5.00 | -1.80 | -2.14 | = - 1.8 |
| 85 | 2.17 | + 8.22 | -3.92 | -1.75 | -1.67 | = - 3.8 |
| 86 | 2.73 | +10.61 | -4.97 | -2.59 | -2.01 | = - 9.0 |
| 87 | 2.20 | + 8.78 | -3.63 | -2.33 | -1.52 | = - 9.5 |
| 88 | 1.65 | + 6.74 | -2.58 | -1.96 | -1.08 | = -10.5 |
| 89 | 2.77 | +11.61 | -4.04 | -3.61 | -1.69 | = -19.9 |

The following are the approximate normals to which these equations give rise. Inaccuracies being detected in several of the equations of condition after these normals were formed, they do not accurately correspond to those equations as written.

| | | | | | |
|--------------------------|----------------------|--------------------|----------------------|---------------------|-----------|
| 283.64 $\delta\epsilon'$ | + 414.36 $\delta n'$ | -151.63 δe | +247.23 $e\delta\pi$ | -176.03 $\delta u'$ | = +123".5 |
| 414.36 | +1619.44 | -689.11 | +260.26 | -436.02 | = +103.2 |
| -151.63 | - 689.11 | +557.82 | + 38.45 | +122.88 | = -399.8 |
| 247.23 | + 260.26 | + 38.45 | +618.45 | -194.60 | = +267.3 |
| -176.03 | - 436.02 | +122.88 | -194.60 | +163.13 | = -128.1 |

The values of the unknown quantities deduced from these normals were substituted in the equations of condition, and a farther approximation was made by solving the equations given by the residuals. The following are the first approximations given by the normals, and the finally concluded corrections

| | Preliminary. | Final. |
|----------------------------|-------------------|-------------------|
| $\delta u'$, | -15.00 | -13.44 |
| $e\delta\pi$, | - 0.52 | - 0.36 |
| δe , | - 4.25 | - 4.04 |
| $10\delta n = \delta n'$, | - 4.73 | - 4.33 |
| $\delta\epsilon'$, | - 3.78 | - 3.44 |
| Mass of Neptune | $\frac{1}{20000}$ | $\frac{1}{19640}$ |

The final values of the corrections being substituted in the equations leave the following system of residuals, or outstanding excesses of the observed longitudes over theory. Column $f\delta l$ gives the residual of the equation itself; the probable error of which has always been judged to be 0".5, while in column δl this residual is divided by f to obtain the residual correction of the longitude itself. The values of the factors f are found with the original equations on pages 159 and 160.

| No. of Eq. | Year. | $f\delta Z$
" | δZ
" | No. of Eq. | Year. | $f\delta Z$
" | δZ
" |
|------------|--------|------------------|-----------------|------------|--------|------------------|-----------------|
| 1 | 1691.0 | -0.5 | -10. | 35" | 1817.4 | -0.3 | - 0.3 |
| 2 | 1715.2 | -0.8 | -10. | 36 | 1818.4 | -0.7 | - 0.5 |
| 3 | 1748.8 | +1.5 | + 4.5 | 37 | 1819.4 | -1.4 | - 1.4 |
| 4 | 1750.8 | +0.8 | + 2.4 | 38 | 1820.5 | -1.4 | - 1.4 |
| 5 | 1753.9 | +2.2 | + 6.6 | 39 | 1821.5 | +0.9 | + 1.4 |
| 6 | 1756.7 | -0.2 | - 1.0 | 40 | 1822.5 | +1.4 | + 2.1 |
| 7 | 1769.0 | -0.2 | - 1.0 | 41 | 1823.5 | +0.6 | + 0.9 |
| 8 | 1782.0 | +0.1 | + 0.2 | 42 | 1824.5 | +1.2 | + 1.2 |
| 9 | 1783.0 | -0.5 | - 0.5 | 43 | 1825.5 | +0.7 | + 1.0 |
| 10 | 1784.0 | +0.6 | + 0.6 | 44 | 1826.5 | +2.0 | + 2.0 |
| 11 | 1785.0 | -1.0 | - 1.0 | 45 | 1827.7 | +1.0 | + 0.5 |
| 12 | 1788.0 | +1.2 | + 1.8 | 46 | 1828.7 | +2.5 | + 1.0 |
| 13 | 1789.0 | +1.7 | + 3.4 | 47 | 1829.7 | +2.8 | + 1.1 |
| 14 | 1790.0 | -0.5 | - 1.0 | 48 | 1830.7 | +0.4 | + 0.2 |
| 15 | 1791.0 | -0.7 | - 1.0 | 49 | 1831.7 | +4.1 | + 2.0 |
| 16 | 1792.0 | 0.0 | 0.0 | 50 | 1832.7 | +1.8 | + 0.9 |
| 17 | 1793.0 | -0.1 | - 0.2 | 51 | 1833.8 | +0.5 | + 0.2 |
| 18 | 1794.0 | +0.5 | + 1.5 | 52 | 1834.8 | +0.1 | 0.0 |
| 19 | 1795.0 | -0.6 | - 0.9 | 53 | 1835.8 | -1.4 | - 0.6 |
| 20 | 1796.0 | -0.5 | - 1.0 | 54 | 1836.8 | -3.4 | - 1.1 |
| 21 | 1797.1 | -0.6 | - 1.2 | 55 | 1837.8 | -0.6 | - 0.2 |
| 22 | 1800.2 | -1.4 | - 4.2 | 56 | 1838.8 | +0.5 | + 0.2 |
| 23 | 1801.2 | -1.4 | - 4.2 | 57 | 1839.8 | -0.4 | - 0.1 |
| 24 | 1802.3 | -0.6 | - 0.6 | 58 | 1840.8 | -1.7 | - 0.6 |
| 25 | 1805.3 | -1.4 | - 1.4 | 59 | 1841.8 | -0.3 | - 0.1 |
| 26 | 1806.3 | -1.0 | - 2.0 | 60 | 1842.8 | -0.5 | - 0.2 |
| 27 | 1807.3 | -0.1 | - 0.1 | 61 | 1843.8 | +1.6 | + 0.5 |
| 28 | 1808.3 | -1.0 | - 2.0 | 62 | 1844.9 | +0.7 | + 0.2 |
| 29 | 1809.3 | +0.1 | + 0.2 | 63 | 1845.9 | +0.6 | + 0.3 |
| 30 | 1810.3 | -0.1 | - 0.1 | 64 | 1846.9 | -0.4 | - 0.1 |
| 31 | 1811.3 | -0.9 | - 0.9 | 65 | 1847.9 | +0.6 | + 0.3 |
| 32 | 1812.4 | -0.3 | - 0.6 | 66 | 1848.9 | -1.6 | - 0.8 |
| 33 | 1813.4 | +0.1 | + 0.2 | 67 | 1849.9 | +0.2 | + 0.1 |
| 34 | 1814.4 | -0.4 | - 0.4 | 68 | 1850.9 | +0.3 | + 0.2 |
| 35 | 1815.4 | -0.3 | - 0.2 | 69 | 1851.9 | -1.2 | - 0.6 |
| 35' | 1816.4 | -0.8 | - 0.8 | 70 | 1852.9 | -1.2 | - 0.6 |
| | | | | 71 | 1853.9 | -0.2 | - 0.1 |

| No. of Eq. | Year. | $f\delta l$ | δl | No. of Eq. | Year. | $f\delta l$ | δl |
|------------|--------|-------------|------------|------------|--------|-------------|------------|
| 72 | 1854.9 | +0.5 | + 0.2 | 81 | 1864.0 | +0.7 | + 0.4 |
| 73 | 1855.9 | 0.0 | 0 0 | 82 | 1865.0 | +2.2 | + 1.5 |
| 74 | 1856.9 | -0.5 | - 0.2 | 83 | 1866.0 | +1.0 | + 0.4 |
| 75 | 1858.0 | -0.2 | - 0.1 | 84 | 1867.0 | +0.5 | + 0.2 |
| 76 | 1859.0 | +0.9 | + 0.4 | 85 | 1868.0 | +0.4 | + 0.2 |
| 77 | 1860.0 | -1.1 | - 0.4 | 86 | 1869.0 | -1.9 | - 0.8 |
| 78 | 1861.0 | -0.6 | - 0.3 | 87 | 1870.0 | -0.1 | 0.0 |
| 79 | 1862.0 | -0.6 | - 0.2 | 88 | 1871.0 | -1.2 | - 0.8 |
| 80 | 1863.0 | +0.2 | + 0.1 | 89 | 1872.1 | +0.4 | + 0.2 |

A simple glance at the course of the residuals shows (1) that their probable value is considerably greater than the probable error attributed to the equations of condition, being more nearly 0".7 than 0".5, and yet larger in the later years; (2) that during certain periods they are of a systematic character. During the years 1748 to 1753 the observations show a decided positive correction to the theory of a magnitude greater than we can consider probable, amounting to about one-third of a second of time in the mean of Bradley's two observations of 1748 and 1753. About 1800 the correction becomes negative, and so continues for 20 years with an average value of about 1". In 1821 it suddenly becomes positive, and so continues until 1833. From this year forward the residuals are not systematic in character.

In order to show clearly the general course of the outstanding corrections, they have been divided into groups, generally including about five years each. The mean outstanding correction for each group, taken with respect to the weights indicated by the factors f , is as follows. In the column ϵ is shown what the probable error of the residual should be if the weights assigned to the several equations were strictly correct, and no systematic errors were present either in theory or observation.

| Year. | δl | ϵ | Year. | δl | ϵ |
|--------|------------|------------|--------|------------|------------|
| 1715.2 | -10. | $\pm 6.$ | 1824.8 | + 1.50 | ± 0.16 |
| 1751.1 | + 3.7 | ± 0.8 | 1829.7 | + 0.91 | ± 0.10 |
| 1769.0 | - 1.0 | ± 2.5 | 1835.2 | - 0.27 | ± 0.09 |
| 1783.3 | - 0.18 | ± 0.23 | 1839.8 | - 0.17 | ± 0.07 |
| 1790.0 | + 0.62 | ± 0.40 | 1844.8 | + 0.14 | ± 0.03 |
| 1795.0 | - 0.55 | ± 0.41 | 1849.9 | - 0.21 | ± 0.11 |
| 1802.0 | - 1.25 | ± 0.45 | 1854.9 | - 0.14 | ± 0.11 |
| 1806.5 | - 1.00 | ± 0.31 | 1860.0 | - 0.13 | ± 0.09 |
| 1810.5 | - 0.37 | ± 0.32 | 1865.0 | + 0.36 | ± 0.10 |
| 1814.5 | - 0.37 | ± 0.23 | 1870.0 | - 0.21 | ± 0.11 |
| 1819.5 | - 0.37 | ± 0.18 | | | |

A simple glance at the residuals δl shows that they are much greater than the purely accidental residuals resulting from the theory of least squares. We may divide the possible causes of these systematic errors into three classes.

1. *Systematic Errors of Observation.*—These may result from deviation of the line of collimation of the instrument from a true great circle, or from any peculiarity of the observer which leads to his registering the transit of Uranus earlier or later than that of a fixed star. If we compare the corrections derived from the work of different observatories as given in the last chapter, we shall find frequent cases not only of systematic differences between the results of different observatories, but between those of the same observatory in two successive years. An instance which particularly attracted my attention on first preparing the comparisons of theory and observation is that of the Greenwich observations for 1831, which, as compared with observations at the same observatory during the years preceding and following, seem to be affected with some constant error in R. A. of about 2". I find that this discrepancy can be attributed only to the original observations.

2. *Errors in the Theory compared.*—These may arise from errors in the preceding theoretical computations, from the omission of the terms of the second order produced by Neptune, from the adoption of an erroneous mass of Saturn, or from the attraction of an unknown planet. With regard to the probability of these different sources of error it may be remarked that errors of computation seem possible only in the terms of the second order, that the mass of Saturn is taken from the exhaustive discussion of the Saturnian system by Bessel, in which an error sufficient to influence the theory of Uranus seems highly improbable, and that a trans-Neptunian planet large enough to produce a sensible deviation of the orbit of Uranus from an ellipse in the course of a century would be too large to have escaped detection. The choice of the elliptic elements of Uranus and Neptune is such that the terms of the second order, due to the action of Neptune, can scarcely become sensible within a century of the epoch.

3. *Errors in the various Reductions by which Theory and Observation are compared.*—In the method adopted for comparing theory and observation a number of small uncertainties incident to the imperfections of the older data of reduction necessarily creep in. In the early observations the imperfections arise principally from the uncertainty of the instrumental corrections, and the errors in the adopted positions of the fundamental stars, and indeed in nearly all the data of reduction. In the late years they arise principally from the great magnitude of the correction to Bouvard's tables, and the consequent rapid change of the corrections to the geocentric ephemerides, which make the determination of the corrections Δl and δl from theory and observation somewhat uncertain. Errors from this source will necessarily be in part of a systematic character, and, in view of their possibility, I regret not having been able to completely re-reduce all the observations before 1840, and to compare all since directly with ephemerides computed from the provisional theory. In order, however, to test the question whether they are sensible, I have prepared an ephemeris from the provisional theory for the three recent oppositions of 1861-2, 1862-3, and 1872, and compared it directly with

the observations. The mean corrections in geocentric longitude for groups of observations are given in columns (2), column (1) showing the correction given by the work of the last chapter.

| Opposition 1861-2 | | 1862-3 | | 1872 | |
|-------------------|--------------------|--------|--------------------|-------|--------------------|
| (1) | (2) | (1) | (2) | (1) | (2) |
| +2".8 | +2".4 ₁ | +2".6 | +2".6 ₁ | -8".6 | -9".0 ₁ |
| 2.9 | 2.4 ₃ | 2.3 | 2.3 ₂ | -8.5 | -8.1 ₂ |
| 3.0 | 3.1 ₂ | 2.1 | 2.0 ₃ | -7.3 | -7.5 ₃ |
| 2.4 | 2.4 ₂ | 3.3 | 2.9 ₄ | -7.3 | -7.7 ₅ |
| ... | ... | 2.6 | 2.7 ₁ | -7.8 | -7.6 ₁ |
| Mean | +2.79 | +2.65 | +2.50 | -7.65 | -7.82 |

A systematic difference of 0".16 would seem to be indicated, and on account of it a correction of 0".10 was applied to the comparisons of the last few years in forming the equations of condition.

In view of the possibility of systematic errors from this source it may be considered that too great relative weight has been assigned to the results of the later observations. If the residuals arise from errors of comparison and of theory, their probable magnitude is nearly as great at one epoch as at another. It may therefore be interesting to inquire what result we should get if, instead of assigning such different weights to the comparisons at different epochs, we sought only for the best general agreement with observations during the period the planet has been observed. The preceding system of mean residuals will enable us to discuss this question quite easily. In the first solution we shall reject the results from Flamstead's observations, owing to their assured uncertainty, and those from Le Monnier's of 1769, owing to the possible maladjustment of his quadrant. The equations from the remaining residuals will be the following:

| | | | | | | |
|-------------------------|-------------------|------------------|---------------------|---------------------|---------|-------------------|
| 1.0 $\delta^2\epsilon'$ | -7.6 δ^2n' | +0.8 δ^2e | +1.7 $e\delta^2\pi$ | -0.5 $\delta^2\mu'$ | = +3".7 | Wt. $\frac{1}{2}$ |
| 1.1 | -5.0 | -2.0 | -0.8 | +1.8 | = -0.18 | 2 |
| 1.1 | -4.4 | -1.4 | -1.6 | +1.6 | = +0.62 | 1 |
| 1.1 | -3.9 | -0.6 | -2.0 | +1.2 | = -0.55 | 1 |
| 1.2 | -3.2 | +0.6 | -2.1 | +0.5 | = -1.25 | 1 |
| 1.1 | -2.6 | +1.3 | -1.8 | 0.0 | = -1.00 | 2 |
| 1.1 | -2.2 | +1.7 | -1.3 | -0.2 | = -0.37 | 2 |
| 1.1 | -1.7 | +2.0 | -0.7 | -0.5 | = -0.37 | 2 |
| 1.0 | -1.1 | +2.1 | +0.1 | -0.6 | = -0.37 | 2 |
| 1.0 | -0.5 | +1.8 | +0.9 | -0.6 | = +1.50 | 2 |
| 1.0 | 0.0 | +1.4 | +1.4 | -0.5 | = +0.91 | 3 |
| 0.9 | +0.5 | +0.8 | +1.8 | -0.5 | = -0.27 | 3 |
| 0.9 | +0.9 | +0.1 | +2.0 | -0.6 | = -0.17 | 3 |
| 1.0 | +1.4 | -0.6 | +1.9 | -0.7 | = +0.14 | 3 |
| 1.0 | +1.9 | -1.2 | +1.6 | -0.8 | = -0.21 | 3 |
| 1.0 | +2.5 | -1.7 | +1.1 | -0.9 | = -0.14 | 3 |
| 1.0 | +3.1 | -2.0 | +0.4 | -0.9 | = -0.13 | 3 |
| 1.1 | +3.7 | -2.1 | -0.4 | -0.9 | = +0.36 | 3 |
| 1.1 | +4.4 | -1.8 | -1.2 | -0.8 | = -0.21 | 3 |

Giving these nineteen equations equal weights, we have the second of the following solutions, and the second of the series of residuals the first corresponding to the primitive solution. Solving them again and assigning the weights attached to the respective equations, which I judge to be those to which they are entitled when a liberal allowance is made for systematic errors of observation and of comparison of theory with observations, adding also the equations given by the observations of Flamsteed and Le Monnier, which are as follows:

| | | | |
|------|---|-------------------|-------|
| 1690 | $0.0\delta^2\varepsilon' - 0.7\delta^2n' - 0.1\delta^2e + 0.0e\delta^2\pi + 0.1\delta^2\mu' = -0''.5$ | $f, \frac{1}{20}$ | Wt, 1 |
| 1715 | $0.1 \quad -1.1 \quad 0.0 \quad -0.2 \quad +0.1 = -0.5$ | $\frac{1}{12}$ | 1 |
| 1769 | $0.2 \quad -1.2 \quad -0.3 \quad +0.2 \quad +0.2 = +2.2$ | $\frac{1}{6}$ | 1 |

we have the second solution, and the third series of residuals.

| | (1) | (2) | (3) |
|-------------------------|-------------------|-------------------|-------------------|
| $\delta^2\varepsilon_1$ | 0 | -0''.39 | -0.21 |
| δ^2n_1 | 0 | -0.38 | -0.19 |
| δ^2e | 0 | -0.33 | -0.15 |
| $e\delta^2\pi$ | 0 | +0.25 | +0.19 |
| $\delta^2\mu'$ | 0 | -1.02 | -0.49 |
| m' | $\frac{1}{19640}$ | $\frac{1}{19370}$ | $\frac{1}{19750}$ |

RESIDUALS.

| Year. | RESIDUALS. | | |
|--------|--------------|--------------|--------------|
| | $\Delta_1 l$ | $\Delta_2 l$ | $\Delta_3 l$ |
| 1691.0 | -10. | -14. | -12. |
| 1715.2 | -10. | - 9. | - 7. |
| 1751.1 | + 3.7 | + 0.5 | + 2.0 |
| 1769.0 | - 1.0 | - 2.6 | - 1.9 |
| 1783.3 | - 0.18 | - 0.30 | - 0.17 |
| 1790.0 | + 0.62 | + 0.93 | + 0.89 |
| 1795.0 | - 0.55 | - 0.09 | - 0.18 |
| 1802.0 | - 1.25 | - 0.78 | - 0.87 |
| 1806.5 | - 1.00 | - 0.69 | - 0.73 |
| 1810.5 | - 0.37 | - 0.10 | - 0.05 |
| 1814.5 | - 0.37 | - 0.26 | - 0.28 |
| 1819.5 | - 0.37 | - 0.34 | - 0.37 |
| 1824.8 | + 1.50 | + 1.46 | + 1.41 |
| 1829.7 | + 0.91 | + 0.90 | + 0.81 |
| 1835.2 | - 0.27 | - 0.43 | - 0.46 |
| 1839.8 | - 0.17 | - 0.56 | - 0.48 |
| 1844.8 | + 0.14 | - 0.31 | - 0.18 |
| 1849.9 | - 0.21 | - 0.70 | - 0.52 |
| 1854.9 | - 0.14 | - 0.54 | - 0.36 |
| 1860.0 | - 0.13 | - 0.23 | - 0.14 |
| 1865.0 | + 0.36 | + 0.70 | + 0.62 |
| 1870.0 | - 0.21 | + 0.80 | + 0.44 |

It will be seen that the effect of these changes of weights is, that the older observations are a little better, and the later a little worse represented. I conceive that our choice must lie between the first and third solutions, the first being the more probable if we conceive the outstanding residuals to be due to errors of observation only, and the third if we suppose them equally due to errors of computation. On the whole, I consider the mean of the two to be about the most probable, and this will give the mass of Neptune very near the round number

$$\frac{1}{19700}$$

which will be adopted as the definitive value. The definitive corrections to Elements III (p. 99) will then be

| | |
|--------------------------|---------|
| $\delta\epsilon'$ (1830) | — 3".56 |
| $\delta\epsilon$ (1850) | — 12.45 |
| $10\delta n$ | — 4.44 |
| δe | — 4.12 |
| $e\delta\pi$ | — 0.25 |
| $\delta\mu$ | — 0.137 |

Corrections to the Inclination and Node.

These corrections have been derived entirely from the modern observations, the ancient ones being too uncertain to add anything to the weight of the result. The mode in which the correction to the latitude of the provisional ephemeris has been concluded from the observations has been sufficiently explained: it is only necessary to add that the immediate results from the data of the preceding chapter require two corrections, namely:

(1) A correction to the theoretical latitude for the change in the adopted mass of Neptune. The value of this correction, as derived from the data of Chapter V, is with sufficient approximation

$$\delta\beta = 0''.25 T \cos g.$$

(2) A correction to the observed latitude on account of the difference between the obliquity of the ecliptic adopted in the various ephemerides compared, and that of Hansen's *Tables du Soleil*, which having been adopted in the theory should be used throughout.

Applying the correction (2) — (1) to all the observed latitudes, we have the following corrections to the latitude of the provisional ephemeris derived from all the observations of each opposition since 1781. The third column gives the number of observations in declination. These numbers may, however, in some cases be inaccurate. The fourth and fifth columns give the sine and cosine of the argument of latitude, to be used in forming the equations of condition.

| Year. | $\delta\beta$ | No. of obs. | $\sin u$ | $\cos u$ | Year. | $\delta\beta$ | No. of obs. | $\sin u$ | $\cos u$ |
|--------|---------------|-------------|----------|----------|--------|---------------|-------------|----------|----------|
| | " | | | | | " | | | |
| 1782.0 | -0.4 | 21 | +0.31 | +0.95 | 1830.7 | +0.2 | 48 | -0.82 | -0.57 |
| 1783.0 | -3.5 | 13 | 0.38 | 0.92 | 1831.7 | +1.0 | 23 | 0.87 | 0.50 |
| 1784.0 | -2.4 | 13 | 0.45 | 0.89 | 1832.7 | +0.9 | 20 | -0.90 | -0.44 |
| 1785.0 | -0.1 | 10 | 0.52 | 0.85 | 1833.7 | +0.6 | 54 | 0.93 | 0.37 |
| 1788.0 | -1.0 | 5 | +0.71 | +0.71 | 1834.7 | +0.3 | 92 | 0.95 | 0.31 |
| 1789.0 | +0.9 | 6 | 0.77 | 0.64 | 1835.7 | +0.2 | 71 | -9.97 | -0.24 |
| 1790.0 | +2.6 | 4 | 0.81 | 0.58 | 1836.7 | +0.1 | 135 | 0.98 | 0.17 |
| 1791.0 | +1.9 | 7 | 0.86 | 0.51 | 1837.8 | 0.0 | 154 | 0.99 | 0.10 |
| 1792.0 | +1.8 | 3 | +0.90 | +0.44 | 1838.8 | -0.2 | 182 | -1.00 | -0.03 |
| 1793.0 | +2.9 | 5 | 0.93 | 0.37 | 1839.8 | -0.4 | 142 | 1.00 | +0.03 |
| 1794.0 | +0.5 | 3 | 0.95 | 0.30 | 1840.8 | -0.3 | 106 | 0.99 | 0.10 |
| 1795.0 | +0.7 | 7 | +0.97 | +0.22 | 1841.8 | -0.7 | 101 | -0.98 | +0.17 |
| 1796.0 | +3.6 | 4 | 0.99 | 0.14 | 1842.8 | -0.7 | 145 | 0.97 | 0.24 |
| 1797.0 | +3.6 | 3 | 1.00 | +0.05 | 1843.8 | -0.9 | 88 | 0.95 | 0.31 |
| 1800.2 | 0.0 | 2 | +0.98 | -0.21 | 1844.9 | -1.0 | 87 | -0.93 | +0.37 |
| 1801.2 | +1.2 | 2 | 0.97 | 0.26 | 1845.9 | -0.6 | 55 | 0.90 | 0.44 |
| 1802.3 | +1.4 | 13 | 0.94 | 0.34 | 1846.9 | -1.1 | 92 | 0.87 | 0.50 |
| 1805.3 | +1.1 | 3 | +0.83 | -0.56 | 1847.9 | -1.1 | 69 | -0.83 | +0.56 |
| 1806.3 | -1.4 | 5 | 0.78 | 0.63 | 1848.9 | -0.9 | 56 | 0.79 | 0.62 |
| 1807.3 | +1.6 | 16 | 0.72 | 0.69 | 1849.9 | -1.0 | 36 | 0.74 | 0.67 |
| 1808.3 | +1.8 | 6 | +0.67 | -0.74 | 1850.9 | -0.7 | 46 | -0.69 | +0.72 |
| 1809.3 | +1.3 | 9 | 0.60 | 0.80 | 1851.9 | -1.2 | 35 | 0.64 | 0.77 |
| 1810.3 | +3.7 | 16 | 0.53 | 0.85 | 1852.9 | -1.3 | 49 | 0.59 | 0.81 |
| 1811.3 | +3.4 | 11 | +0.45 | -0.89 | 1853.9 | -1.8 | 48 | -0.53 | +0.85 |
| 1812.4 | +3.6 | 6 | 0.38 | 0.93 | 1854.9 | -1.0 | 47 | 0.47 | 0.88 |
| 1813.4 | +2.4 | 6 | 0.29 | 0.96 | 1855.9 | -1.3 | 49 | 0.41 | 0.91 |
| 1814.4 | +2.5 | 13 | +0.22 | -0.98 | 1856.9 | -1.1 | 41 | -0.34 | +0.94 |
| 1815.4 | +2.2 | 16 | 0.14 | 0.99 | 1858.0 | -1.7 | 65 | 0.26 | 0.97 |
| 1816.4 | +1.3 | 18 | 0.06 | 1.00 | 1859.0 | -1.7 | 56 | 0.19 | 0.98 |
| 1817.5 | +2.4 | 13 | -0.01 | -1.00 | 1860.0 | -1.9 | 58 | -0.12 | +0.99 |
| 1818.5 | +4.2 | 22 | 0.09 | 1.00 | 1861.0 | -2.3 | 41 | 0.05 | 1.00 |
| 1819.5 | +1.7 | 7 | 0.17 | 0.99 | 1862.0 | -1.6 | 52 | +0.02 | 1.00 |
| 1820.5 | +1.0 | 9 | -0.24 | -0.97 | 1863.0 | -1.7 | 83 | +0.10 | +0.99 |
| 1821.5 | +1.3 | 10 | 0.31 | 0.95 | 1864.0 | -2.2 | 39 | 0.18 | 0.99 |
| 1822.5 | +1.8 | 7 | 0.38 | 0.92 | 1865.0 | -1.7 | 37 | 0.25 | 0.97 |
| 1823.5 | +0.6 | 7 | -0.45 | -0.89 | 1866.0 | -0.9 | 72 | +0.33 | +0.95 |
| 1824.5 | -0.2 | 11 | 0.51 | 0.86 | 1867.0 | -0.6 | 83 | 0.40 | 0.92 |
| 1825.5 | +1.5 | 5 | 0.57 | 0.82 | 1668.1 | -0.9 | 32 | 0.47 | 0.88 |
| 1826.5 | +1.2 | 7 | -0.63 | -0.78 | 1869.1 | -0.8 | 65 | +0.54 | +0.84 |
| 1827.6 | +2.0 | 5 | 0.68 | 0.73 | 1870.1 | -0.4 | 32 | 0.60 | 0.80 |
| 1828.6 | -1.5 | 7 | 0.73 | 0.68 | 1871.1 | -0.7 | 21 | +0.66 | 0.75 |
| 1829.7 | +0.9 | 9 | -0.78 | -0.63 | 1872.1 | -0.4 | 47 | +0.72 | +0.69 |

It will be remembered that the observed declinations have, as far as possible, been reduced to Auwers' standard. We have no positive proof that this standard is correct. If it be affected by a constant error, the result will be that the orbit of the planet on the celestial sphere, as deduced from observation, instead of being a great circle, as we know the real orbit to be, will be a small one, and the comparison of a uniform series of observations extending through an entire revolution of the planet, after making the best correction to the position of the orbit, will leave a constant residual. Now, we can best determine this residual by including it as an unknown quantity in our equations.

Again, the error of the standard is not necessarily constant, but may contain a term proportional to the time, arising from erroneous proper motions of the standard stars. Therefore, instead of supposing the residual constant, we shall suppose it of the form $a + bt$. Each observed correction to the theoretical latitude will then give the equation,

$$\sin u\delta\phi - \cos u\phi\delta\theta + a + bT = \delta\beta.$$

To facilitate the solution of these equations they have been divided into groups, each group usually comprehending three oppositions, and combined into a single equation multiplied by such a factor as would make its probable error half a second. The factor by which the correction of the latitude is multiplied in the equation is the same with the coefficient of a . The year 1840.0 is taken as the epoch for b . Thus we have the following:

| EQUATIONS OF LATITUDE. | | | | | |
|------------------------|-------------|------------------|-------------------------|----------|-----------------|
| Dates of oppositions. | No. of opp. | Equation. | | | |
| 1782.0-85.0 | 4 | 0.4 $\delta\phi$ | -0.9 $\phi\delta\theta$ | +1.0 a | -0.6 b = -1.6 |
| 1788.0-91.0 | 4 | 0.8 | -0.6 | +1.0 | -0.5 b = +1.1 |
| 1792.0-94.0 | 3 | 0.5 | -0.2 | +0.5 | -0.2 = +0.9 |
| 1795.0-97.1 | 3 | 0.3 | 0.0 | +0.5 | -0.2 = +0.7 |
| 1800.2-02.3 | 3 | 0.0 | +0.3 | +1.0 | -0.4 = +1.0 |
| 1805.3-07.3 | 3 | 0.8 | +0.6 | +1.0 | -0.3 = +0.7 |
| 1808.3-10.3 | 3 | 0.6 | +0.8 | +1.0 | -0.3 = +2.6 |
| 1811.3-13.4 | 3 | 0.4 | +0.9 | +1.0 | -0.3 = +3.1 |
| 1814.4-16.4 | 3 | 0.2 | +1.5 | +1.5 | -0.4 = +3.0 |
| 1817.4-19.5 | 3 | -0.1 | +1.5 | +1.5 | -0.3 = +4.8 |
| 1820.5-22.5 | 3 | -0.5 | +1.4 | +1.5 | -0.3 = +2.0 |
| 1823.5-25.5 | 3 | -0.5 | +0.9 | +1.0 | -0.2 = +0.6 |
| 1826.5-28.6 | 3 | -0.7 | +0.7 | +1.0 | -0.1 = +0.6 |
| 1829.6-31.7 | 3 | -1.2 | +0.8 | +1.5 | -0.1 = +1.1 |
| 1832.7-34.7 | 3 | -2.8 | +1.1 | +3.0 | -0.2 = +1.8 |
| 1835.7-37.8 | 3 | -2.9 | +0.5 | +3.0 | -0.1 = +0.3 |
| 1838.8-40.8 | 3 | -3.0 | -0.1 | +3.0 | 0.0 = -0.9 |
| 1841.8-43.8 | 3 | -2.9 | -0.7 | +3.0 | +0.1 = -2.3 |
| 1844.8-46.8 | 3 | -2.7 | -1.3 | +3.0 | +0.2 = -2.7 |
| 1847.8-49.9 | 3 | -2.4 | -1.9 | +3.0 | +0.3 = -3.0 |
| 1850.9-52.9 | 3 | -1.9 | -2.3 | +3.0 | +0.4 = -3.2 |
| 1853.9-55.9 | 3 | -1.4 | -2.6 | +3.0 | +0.4 = -4.1 |
| 1856.9-59.0 | 3 | -0.8 | -2.9 | +3.0 | +0.5 = -4.5 |
| 1860.0-62.0 | 3 | -0.2 | -3.0 | +3.0 | +0.6 = -5.8 |
| 1863.0-65.0 | 3 | +0.5 | -3.0 | +3.0 | +0.7 = -5.6 |
| 1866.0-68.0 | 3 | +1.2 | -2.8 | +3.0 | +0.8 = -2.4 |
| 1869.1-70.0 | 2 | +1.1 | -1.6 | +2.0 | +0.6 = -1.2 |
| 1871.1-72.1 | 2 | +1.4 | -1.4 | +2.0 | +0.6 = -1.1 |

Treating these equations by the method of least squares, we find the normal equations

$$\begin{aligned} & 63.60\delta\phi + 6.45\phi\delta\theta - 52.10a - 1.08b = + 28.33 \\ & 6.45\delta\phi + 69.59\phi\delta\theta - 52.60a - 14.25b = + 111.02 \\ & -52.10\delta\phi - 52.60\phi\delta\theta + 133.50a + 8.95b = - 76.55 \\ & - 1.08\delta\phi - 14.25\phi\delta\theta + 8.95a + 4.49b = - 23.69 \end{aligned}$$

The solution of these equations gives

$$\begin{aligned}\delta\phi &= +0''.28 + 0''.75 a = +0''.54 \\ \phi\delta\theta &= +1.57 + 0.686a + 0.205b = +1''.75 \\ a &= +0.35 \\ b &= -0.28\end{aligned}$$

These values of a and b indicate that at the epoch 1840 Auwers' equatorial declinations are too great, or his north polar distances are too small by $0''.35$, and that this error is diminishing at the rate of $0''.28$ per century. If the older measures in declination had been comparable in precision with those made at the present time, and if the possible periodic error in the reduced right ascensions had been carefully eliminated, I should regard this determination as entitled to considerable weight. In view of the great uncertainty of the declinations previous to 1820, it can be regarded as little more than a rough attempt at a determination. For this reason the first two normal equations have been solved, leaving a and b indeterminate, so as to show the values of $\delta\phi$ and $\phi\delta\theta$ in terms of these quantities. It will be seen that had we neglected a and b entirely, the value of $\delta\phi$ would have been smaller by $0''.26$, and that of $\phi\delta\theta$ smaller by $0''.18$ than those actually concluded. As the observations with the Washington Transit Circle, and those with the Pulkowa Vertical Circle, both indicate an increase of Auwers' polar distances, I shall take for the definitive corrections to the inclination and node those which follow from the above values of a and b , or,

$$\begin{aligned}\delta\phi &= +0''.54 \\ \phi\delta\theta &= +1.75.\end{aligned}$$

The following table shows the residuals of the equations, and the mean outstanding corrections to the latitude, (1) when the concluded values of $\delta\phi$ and $\phi\delta\theta$ and a and b are all used, and (2) when a and b are supposed zero, and the values of $\delta\phi$ and $\phi\delta\theta$, corresponding to this supposition, are used:

| Year. | Residuals. | | $\delta\beta$ | |
|-------|------------|----------|---------------|----------|
| | (1)
" | (2)
" | (1)
" | (2)
" |
| 1783 | -0.8 | -0.3 | -0.8 | -0.3 |
| 1789 | +1.2 | +1.8 | +1.2 | +1.8 |
| 1793 | +0.7 | +1.1 | +1.4 | +2.2 |
| 1796 | +0.3 | +0.6 | +0.6 | +1.2 |
| 1801 | -0.5 | +0.3 | -0.5 | +0.3 |
| 1806 | -1.2 | -0.5 | -1.2 | -0.5 |
| 1809 | +0.5 | +1.2 | +0.5 | +1.2 |
| 1812 | +0.9 | +1.6 | +0.9 | +1.6 |
| 1815 | -0.3 | +0.6 | -0.2 | +0.4 |
| 1818 | +1.6 | +2.5 | +1.1 | +1.7 |
| 1821 | -0.7 | -0.1 | -0.5 | -0.1 |
| 1824 | -1.3 | -0.7 | -1.3 | -0.7 |
| 1827 | -0.6 | -0.3 | -0.6 | -0.3 |

| Year. | Residuals. | | δ, β | |
|-------|------------|----------|-----------------|----------|
| | (1)
" | (2)
" | (1)
" | (2)
" |
| 1830 | -0.2 | +0.2 | -0.1 | +0.1 |
| 1833 | +0.3 | +0.9 | +0.10 | +0.30 |
| 1836 | 0.0 | +0.3 | 0.00 | +0.10 |
| 1839 | -0.1 | +0.1 | -0.03 | +0.03 |
| 1842 | -0.5 | -0.4 | -0.17 | -0.13 |
| 1845 | +0.1 | 0.0 | +0.03 | 0.00 |
| 1848 | +0.6 | +0.7 | +0.20 | +0.23 |
| 1851 | +0.9 | +0.9 | +0.30 | +0.30 |
| 1854 | +0.3 | +0.3 | +0.10 | +0.10 |
| 1858 | +0.1 | +0.3 | +0.03 | +0.10 |
| 1861 | -1.4 | -1.0 | -0.47 | -0.33 |
| 1864 | -1.5 | -1.0 | -0.50 | -0.33 |
| 1867 | +1.1 | +1.7 | +0.37 | +0.57 |
| 1869 | +0.5 | +1.0 | +0.17 | +0.33 |
| 1871 | 0.0 | +0.7 | 0.00 | +0.23 |

The sum of the squares of the residuals is in the first case $17''.94$, and in the second $25''.41$, so that the introduction of a and b makes a decided improvement in the representation of the observations.

I have not attempted a rigorous investigation of the probable error of any of these results for the reason that the values of the probable error deducible by the method of least squares would, in a case like the present, be entirely untrustworthy. It is, however, very desirable that we should be able to form some judgment of the uncertainty of the mass of Neptune. From the last system of equations of condition the value of μ' comes out with the weight 3.13, or nearly that assigned to the mean result of each five years of modern observations. Regarding these results as independent, their mean error would be about $0''.5$, so that the probable error of μ' would be 0.5, and that of μ would be .005, or about $\frac{1}{200}$ the entire mass of Neptune. A probable error derived from the original equations would have been much smaller, and when, in the last equations, we allow for the systematic character of the residuals, it will be larger. If we suppose the theory to be perfect, I conceive we may fairly estimate the probable error of the mass of Neptune to be $\frac{1}{100}$ of its entire amount, and its possible error two or three times greater. If there is any error or imperfection in the theory, the error may be much larger.

CHAPTER VIII.

COMPLETION AND ARRANGEMENT OF THE THEORY TO FIT IT FOR
PERMANENT USE.

IN the preceding discussions the terms of the second order due to the action of Neptune have been neglected, the elements of Uranus and Neptune being so chosen that these terms can scarcely become sensible within a century of the epoch. But this very choice will make them larger in the course of centuries than if mean elements had been chosen. They will be most sensible in the case of the great inequality of 4300 years between Uranus and Neptune, an inequality which will make centuries of observation necessary to an accurate determination of the mean elements of the two orbits. The uncertainty arising from the great inequality is probably of the same order of magnitude with the omitted terms of the second order, and, such being the case, the theory would really be made but little more accurate by the addition of those terms. I conceive, however, that the theory will be made much more satisfactory by the computation of at least the largest of the terms in question, if only to arrive at a certain determination of their order of magnitude, and of their effect on the planet during the period in which it has been observed.

The term in question, being of very long period, may be most advantageously treated by the method of variation of elements, more especially as it has in the theory been already treated as such a perturbation. The largest of the perturbations in question are those of the mean longitude which are multiplied by the square of the integrating factor ν , which is nearly 51, but which also contain the eccentricities as factors, and those of the eccentricity and perihelion which are independent of the eccentricities, but are multiplied by only the first power of ν . These terms will probably comprise nearly or quite nine-tenths of those arising from the term of long period.

Let us begin with the perturbations of mean longitude. These are given by the integration of the equation

$$\frac{d^2l}{dt^2} = -3m'\omega n^2 \{ek_1 \sin(2l' - l - \pi) + ek_2 \sin(2l' - l - \pi')\}$$

k_1 and k_2 being functions of the ratio of the mean distances, or α . If we integrate this equation, supposing all the quantities in the second member except l' and l to be constant, and these two to be of the form $nt + \varepsilon$, n and ε being constants, we shall reproduce the principal term of long period already found. But in the second approximation we must suppose all the elements variable. It is not, however, necessary to take into account the variations of a , n , and k , because these are

of a lower order of magnitude. The perturbations to be added will be those of $l, l', e, e', \pi, \text{ and } \pi'$.

The point from which the longitudes are counted being arbitrary, we shall take the position of the perihelion of Uranus for 1850.0 as the origin, and put, as before, g for the mean longitude of Uranus counted from this point, and let l' represent the mean longitude of Neptune counted from the same point. The terms of $\frac{d^2l}{dt^2}$ within the brackets will thus become

$$ek_1 \sin (2l' - g - \delta\pi) + e'k_2 \sin (2l' - g - (\pi' - \pi))$$

or, if we put

$$\begin{aligned} 2l' - g &= N \\ e' \sin (\pi' - \pi) &= h' \\ e' \cos (\pi' - \pi) &= k' \end{aligned}$$

and notice, that to terms of the first order we have, $\sin \delta\pi = \delta\pi$, $\cos \delta\pi = 1$, we shall have

$$\frac{d^2l}{dt^2} = -3m'\alpha n^2 \{ (ek_1 + k'h_2) \sin N - (ek_1\delta\pi + h'k_2) \cos N \}$$

differentiating the quantities, of which the perturbations are to be considered with respect to the sign δ , we find for the terms of the second order.

$$\begin{aligned} \delta \frac{d^2l}{dt^2} &= -3m'\alpha n^2 \{ (k_1\delta e + k_2\delta k' + h'k_2\delta N) \sin N \\ &\quad + ((ek_1 + k'h_2) \delta N - ek_1\delta\pi - k_2\delta h') \cos N \}. \end{aligned}$$

We have now to substitute in this expression the numerical values of the quantities within parentheses. Those of the perturbations of Uranus have already been given in Chapter III, but it is necessary to diminish them by the factor 0.145* for the altered mass of Neptune. Those of Neptune are taken from my investigation of the orbit of that planet (p. 38). The mass of Uranus there adopted is $\frac{1}{210000}$, while the investigation of Dr. Von Asten,† from the observations of Struve and others, shows it to be $\frac{1}{220000}$. The perturbations are therefore diminished by $\frac{1}{2}$. In accordance with the system adopted throughout both investigations, constants are added to all the perturbations to make them vanish at the epoch 1850.0. A term is also added to make $\frac{dl}{dt}$ also vanish at the epoch; this corresponds to the constant which ought to be added to $\delta\alpha$. The numerical values thus obtained, are :

* This factor was adopted before the mass of Neptune to be employed had been finally decided upon. Hence the difference between it and that in the preceding chapter.

† Mémoires de l'Académie de St. Pétersbourg, tome xviii, vii série.

$$\begin{aligned}
\delta N &= + 7260'' \sin N - 6658'' + 4''.26t \\
e\delta\pi &= - 414 \sin N + 380 \\
\delta e &= - 414 \cos N - 165 \\
\delta h' &= + 120 \sin N - 110 \\
\delta k' &= + 120 \cos N + 48 \\
k_1 &= - 1.234 \\
k_2 &= + 0.452 \\
h' &= + 0.00695 \\
k' &= - 0.00486
\end{aligned}$$

Substituting these values in the expression for $\delta \frac{d^2 l}{dt^2}$ and integrating twice, we find, putting b for the coefficient of the time in N , of which the value, taking the century as the unit, is $+0.1472$, and putting T for the time in centuries,

$$\begin{aligned}
\delta l = \frac{3}{2} m' a v^2 \left\{ (411'' + \frac{102''.8}{b} + 2''.6T) \sin N + (1837'' + \frac{5''.3}{b} - 51''.4T) \cos N \right. \\
\left. - 109''.3 \sin 2N - 5''.5 \cos 2N \right\} \\
- 16''.5 m' a n^2 t^2 + cT + c',
\end{aligned}$$

c and c' being the arbitrary constants of integration, which are to be chosen so that both δl and its first differential coefficient shall vanish at the epoch. Reducing to numbers, we find

$$\begin{aligned}
\delta l &= (140''.70 + 0''.32T) \sin N \\
&+ (232.60 - 6.37T) \cos N \\
&- 13.60 \sin 2N \\
&- 0.70 \cos 2N \\
&- 0.03T^2 \\
&+ 34.27T \\
&- 46.76,
\end{aligned}$$

the last two terms being arbitrary.

When we carry the perturbations of the eccentricity and perihelion to quantities of the second order, we are troubled by the introduction of large terms depending on the square of the disturbing force, which disappear from the rigorous expressions for the co-ordinates. These may be avoided by substituting for the eccentricity and perihelion the quantities h and k determined by the condition

$$\begin{aligned}
h &= e \sin \pi \\
k &= e \cos \pi
\end{aligned}$$

If, as before, we count the longitudes from the perihelion of Uranus at the epoch 1850, we should substitute $\delta\pi$ for π in these expressions. The values of h and k will then be given by the integration of the equations

$$\begin{aligned}
\frac{dh}{dt} &= m' a n k_1 \cos N \\
\frac{dk}{dt} &= - m' a n k_1 \sin N.
\end{aligned}$$

Differentiating with respect to δ , we find for the terms of the second order

$$\begin{aligned} \delta \frac{dh}{dt} &= -m' \alpha n k_1 \sin N \delta N \\ \delta \frac{dk}{dt} &= -m' \alpha n k_1 \cos N \delta N. \end{aligned}$$

Substituting for δN its numerical value just given and integrating, we find

$$\begin{aligned} \delta h &= m' \alpha n k_1 \{ -2895'' \sin N - (6658'' - 4''.26t) \cos N + 1815'' \sin 2N \} \\ &\quad - 3630'' m' \alpha k_1 n t \quad + \text{constant}; \\ \delta k &= m' \alpha n k_1 \{ -2895'' \cos N + (6658'' - 4''.26t) \sin N + 1815'' \cos 2N \} \\ &\quad + \text{constant}; \end{aligned}$$

the constants being so chosen that δh and δk shall vanish at the epoch.

Reducing the values of δh and δk to numbers, they become

$$\begin{aligned} \delta h &= 5''.82 \sin N + (13''.40 - 0''.86T) \cos N - 3''.65 \sin 2N + 1''.08T - 2''.67, \\ \delta k &= 5''.82 \cos N - (13''.40 - 0''.86T) \sin N - 3''.65 \cos 2N \quad + 12''.12, \end{aligned}$$

the last two terms being arbitrary constants.

Computing the values of these terms of δl , δh , and δk , for intervals of 50 years, from 1600 to 2000, we find them to be as follows:

| Year | δl | δh | δk |
|------|------------|------------|------------|
| 1600 | -1''.34 | +0''.10 | -0''.02 |
| 1650 | -0''.71 | +0''.05 | -0''.02 |
| 1700 | -0''.31 | +0''.02 | -0''.01 |
| 1750 | -0''.10 | 0''.00 | -0''.01 |
| 1800 | -0''.01 | 0''.00 | 0''.00 |
| 1850 | 0''.00 | 0''.00 | 0''.00 |
| 1900 | 0''.00 | 0''.00 | 0''.00 |
| 1950 | +0''.04 | 0''.00 | -0''.01 |
| 2000 | +0''.18 | -0''.01 | -0''.02 |

We see that although the ultimate effect of these terms is very considerable, their effect, during the period that Uranus has been observed, is insignificant.

Concluded Elements and Perturbations of Uranus.

The corrections found in the last chapter being applied to the final provisional elements (p. 99) give the following elements for 1850, affected by the great inequality produced by Neptune:

Elements IV of Uranus.

Epoch, 1850, Jan. 0, Greenwich mean noon.

| | |
|--------------|----------------|
| π , | 168° 15' 6''.7 |
| ϵ , | 28 25 17.05 |
| θ , | 73 14 8.0 |
| ϕ , | 0 46 20.54 |
| e , | .0469236 |

| | |
|----------------|-----------|
| e (in sec.), | 9678".69 |
| n , | 15425.752 |
| $\log a$, | 1.2829072 |
| $\log a_1$, | 1.2831044 |

$\log a_1$ includes, as before, the constant term in the perturbations of the logarithm of the radius vector which, with the corrected mass of Neptune, is $+.0001972$.

To find the corresponding elements at any other epoch, the following secular and long-period perturbations are to be applied. Those produced by Neptune are derived from the expressions in Chapter III by correcting them for the new mass of Neptune, and for the change in the value of the small divisor $2n' - n$ produced by the correction of the elements of Uranus. The logarithms of the factors for correction are,

| | |
|--|---------|
| Correction of mass of Neptune | 9.93598 |
| Correction of divisor | 0.00051 |
| Log. factor for δl | 9.93496 |
| Log. factor for $\delta e, \delta \pi, \delta \nu$ | 9.93547 |

Including the perturbations of the second order just found, we have, by putting

$$N = 2l' - g,$$

$$= 113^\circ 30' 46".0 + 8^\circ 26' 51".9T,$$

$$\begin{aligned} \delta l = & (-2850".41 + 0".32T) \sin N + (387".67 - 6".37T) \cos N \\ & + 112.72 \sin 2N \quad - 47.28 \cos 2N \\ & - 7.72 \sin 3N \quad + 4.33 \cos 3N \\ & + 0.55 \sin 4N \quad - 0.46 \cos 4N \\ & - 0.03T^2 - 83".78T + 2811".41. \end{aligned}$$

$$\begin{aligned} \delta h = & -412".18 \sin N + (14".03 - 0".86T) \cos N \\ & + 29.20 \sin 2N - 6.09 \cos 2N \\ & - 3.11 \sin 3N + 1.19 \cos 3N \\ & + 0.28 \sin 4N - 0.13 \cos 4N \\ & + 14.76T \quad + 398.33 \end{aligned}$$

$$\begin{aligned} \delta k = & -411".53 \cos N - (13".65 - 0".86T) \sin N \\ & + 29.33 \cos 2N + 6.17 \sin 2N \\ & - 3.12 \cos 3N - 1.21 \sin 3N \\ & + 0.29 \cos 4N + 0.13 \sin 4N \\ & - 5.453T \quad - 124.72 \end{aligned}$$

$\delta \nu$ (in units of the 7th place of decimals).

$$\begin{aligned} & = 1963 \cos N + 103 \sin N \\ & - 171 \cos 2N - 67 \sin 2N \\ & + 15 \cos 3N + 6 \sin 3N \\ & \quad + 511.0. \end{aligned}$$

The perturbations of $e\delta\pi$ and δe are here replaced by those of h and k , defined by the equations

$$\begin{aligned} h & = e \sin (\pi - \pi_0) \\ k & = e \cos (\pi - \pi_0) \end{aligned}$$

π_0 representing the perihelion of 1850 = $168^\circ 15' 6''.7$. We then have for the eccentricity and longitude of perihelion at any epoch

$$\begin{aligned} e \sin (\pi - \pi_0) &= \delta h \\ e \cos (\pi - \pi_0) &= e_0 + \delta k. \end{aligned}$$

In the above terms multiplied by the time we have included the secular variations produced by Jupiter and Saturn. If the perturbations of the elements due to each particular planet are required, we have

Action of Jupiter,

$$\delta h = + 5''.73T; \delta k = - 0''.608T.$$

Action of Saturn,

$$\delta h = + 5''.56T; \delta k = - 4.589T.$$

Subtracting these from the above expressions all the remaining terms will be due to the action of Neptune. The values of δl and δv are due entirely to the action of Neptune.

For the sake of rigor, we may suppose the perturbations produced by each planet to be multiplied by a factor representing the number by which the adopted mass of the planet must be multiplied to obtain the true mass.

It will add to the homogeneousness of the theory to express the perturbations of long period, which are multiplied by the product of the masses of Jupiter and Saturn, as perturbations of the elements. These terms, as found on page 88, are

$$\begin{aligned} (v.c.0) &= - 0''.55 \sin N_6 - 0''.03 \cos N_6 \\ &\quad + 40.65 \sin N_7 - 10.50 \cos N_7 \\ (v.s.1) &= + 2.64 \sin N_6 + 4.64 \cos N_6 \\ &\quad + 7.35 \sin N_7 + 4.41 \cos N_7 \\ (v.c.1) &= - 4.23 \sin N_6 - 3.87 \cos N_6 \\ &\quad + 8.06 \sin N_7 - 8.38 \cos N_7 \end{aligned}$$

These terms, together with the arbitrary corrections of the elements which have been applied to make them very small at the epoch, may be replaced by the following corrections to the elements:

$$\begin{aligned} \delta l &= - 0''.55 \sin N_6 - 0''.03 \cos N_6 \\ &\quad + 40.65 \sin N_7 - 10.50 \cos N_7 \\ &\quad + 27''.27 - 11''.72T. \\ \delta h &= + 2.09 \sin N_6 + 1.94 \cos N_6 \\ &\quad - 2.13 \sin N_7 + 3.71 \cos N_7 \\ &\quad + 1''.28. \\ \delta k &= + 1.32 \sin N_6 + 2.32 \cos N_6 \\ &\quad + 3.68 \sin N_7 + 2.21 \cos N_7 \end{aligned}$$

$$\delta v = 27 \sin N_7 + 104 \cos N_7 + 76 \text{ (in units of the 7th decimal).}$$

The amount of the perturbations of the elements for every half century, from the year 1000 to 2200, is given in the following table. Column (1) gives the perturbations by Neptune, Saturn, and Jupiter, computed from the expressions

on page 182; column (2) those just given depending on the product of the masses of Jupiter and Saturn.

| Year. | δl | | δh | | δk | | δx | | δz | $\delta \gamma$ |
|-------|------------|---------|------------|-------|------------|-------|------------|------|------------|-----------------|
| | (1)
" | (2) | (1)
" | (2) | (1)
" | (2) | (1) | (2) | | |
| 1000 | +2050.31 | +160.69 | + 42.66 | +1.94 | -389.51 | +6.08 | +1955 | -140 | -2.06 | +1.20 |
| 1050 | 1841.17 | 149.08 | 27.79 | 3.60 | 375.65 | 6.68 | 1882 | 156 | 2.10 | 1.37 |
| 1100 | 1638.76 | 136.39 | 14.03 | 4.99 | 360.31 | 6.82 | 1802 | 169 | 2.12 | 1.54 |
| 1150 | 1444.87 | 122.87 | + 1.45 | 6.01 | 343.49 | 6.47 | 1717 | 178 | 2.08 | 1.66 |
| 1200 | 1260.24 | 108.85 | - 9.90 | 6.60 | 325.27 | 5.64 | 1626 | 183 | 2.02 | 1.77 |
| 1250 | +1085.76 | + 94.67 | -19.84 | +6.75 | -305.69 | +4.42 | +1529 | +183 | -1.94 | +1.86 |
| 1300 | 921.76 | 80.67 | 28.34 | 6.49 | 284.90 | 2.86 | 1426 | 180 | 1.84 | 1.91 |
| 1350 | 769.32 | 67.12 | 35.27 | 5.90 | 252.90 | +1.16 | 1318 | 173 | 1.70 | 1.92 |
| 1400 | 629.00 | 54.35 | 40.56 | 5.08 | 239.78 | -0.57 | 1205 | 161 | 1.54 | 1.91 |
| 1450 | 501.39 | 42.62 | 44.11 | 4.16 | 215.68 | 2.17 | 1086 | 147 | 1.37 | 1.85 |
| 1500 | + 387.18 | + 32.11 | - 45.83 | +3.24 | -190.66 | -3.52 | + 963 | +129 | -1.19 | +1.74 |
| 1550 | 286.65 | 22.78 | 45.63 | 2.42 | 164.84 | 4.52 | 835 | 110 | 0.99 | 1.60 |
| 1600 | 200.65 | 15.38 | 43.46 | 1.80 | 138.31 | 5.12 | 704 | 90 | 0.80 | 1.42 |
| 1650 | 129.43 | 9.34 | 39.20 | 1.41 | 111.22 | 5.30 | 568 | 68 | 0.61 | 1.20 |
| 1700 | 73.46 | 4.86 | 32.79 | 1.23 | 83.70 | 5.09 | 430 | 48 | 0.44 | 0.95 |
| 1750 | + 33.06 | + 1.90 | - 24.16 | +1.23 | - 55.90 | -4.56 | + 288 | + 29 | -0.27 | +0.66 |
| 1800 | 8.51 | 0.33 | - 13.26 | 1.36 | - 27.93 | 3.81 | + 145 | + 11 | -0.12 | +0.34 |
| 1850 | 0.00 | 0.00 | 0.00 | 1.52 | 0.00 | 2.96 | 0 | - 5 | 0.00 | 0.00 |
| 1900 | 7.64 | 0.71 | + 15.65 | 1.63 | + 27.74 | 2.10 | - 145 | 18 | +0.10 | -0.36 |
| 1950 | 31.47 | 2.23 | 33.70 | 1.59 | 55.09 | 1.34 | 290 | 26 | 0.16 | 0.73 |
| 2000 | + 71.43 | + 4.25 | + 54.20 | +1.34 | + 81.81 | -0.73 | - 433 | - 32 | +0.20 | -1.10 |
| 2050 | 127.33 | 6.52 | 77.12 | 0.85 | 107.77 | 0.34 | 574 | 30 | 0.20 | 1.47 |
| 2100 | 198.86 | 8.75 | 102.46 | +0.13 | 132.67 | 0.14 | 711 | 27 | 0.17 | 1.83 |
| 2150 | 285.72 | 10.59 | 130.15 | -0.77 | 156.33 | 0.11 | 843 | 20 | 0.11 | 2.17 |
| 2200 | 387.11 | + 11.77 | +160.15 | -1.76 | +178.48 | -0.19 | 968 | - 9 | +0.02 | -2.48 |

Mean Elements of Uranus.

If, instead of the elements of Uranus affected by the great inequality, we wish the absolute mean elements, these are to be obtained by adding to the elements already given the constants applied to the perturbations δl , δh , δk , and δx to make the perturbations vanish at the epoch 1850.0, and also the corrections (p. 113) which we have subtracted from the elements and added to the perturbations to reduce the latter to a small quantity during the period for which the tables are likely to be used. We thus find the following mean elements:

| | |
|---|-------------------------------|
| Elements V of Uranus. Epoch, 1850, Jan. 0, Greenwich mean noon. | |
| Longitude of the perihelion | 170° 38' 48".7 + 8698". μ |
| Mean longitude at epoch, | 29 12 43.73 + 2811.4 μ |
| Longitude of the node, | 73 14 37.6 + 29.6 μ |
| Inclination of the orbit, | 0 46 20.92 + 0.38 μ |
| Eccentricity, | .0463592 - 5236 μ |
| Eccentricity in seconds, | 9562".27 - 108".0 μ |
| Mean motion, | 15424.797 - 0".838 μ |
| Log mean distance (uncorrected), | 1.2829251 + 179 μ |
| The same corrected, | 1.2831223 + 179 μ |
| True mass of Neptune, | $\frac{1 + \mu}{19700}$ |

Supposing the mass of Neptune to be uncertain by one-fiftieth of its entire amount, which is quite possible, it will be seen the longitude of the mean perihelion is from this cause uncertain by more than two minutes, the mean longitude of Uranus itself by nearly a minute, and the mean motion by nearly two seconds in a century.

It will be seen that the logarithm of the mean distance just given does not accurately correspond to that of elements IV plus the constant term of $\delta n \times 0.4343$, as it should. This difference arises from the rejection of the terms of the second order in δn , which can not affect the geocentric longitude of the planet by a tenth of a second for a number of centuries.

It is to be remarked that these mean elements are those to be used in the general theory of the secular variation of the planetary orbits.

Concluded Theory of Uranus.

The elliptic longitude and radius vector of Uranus, affected by the secular and long period perturbations of the elements, will be given by the following equations. Put

$$\begin{aligned} l_0 &= n_0 t + \varepsilon_0, \\ l &= l_0 + \delta l, \\ g &= l - \pi_0, \\ h &= \delta h, \\ k &= e_0 + \delta k, \\ e^2 &= h^2 + k^2, \end{aligned}$$

the zeros indicating elements IV, and δh , δk , and δl being the perturbations of these three elements just given. Then

Elliptic longitude in orbit = l

$$\begin{aligned} &+ \left\{ 2 - \frac{1}{4}e^2 + \frac{5}{96}e^4 \right\} \left\{ k \sin g - h \cos g \right\} \\ &+ \left\{ \frac{5}{4} - \frac{11}{24}e^2 \right\} \left\{ (k^2 - h^2) \sin 2g - 2hk \cos 2g \right\} \\ &+ \left\{ \frac{13}{12} - \frac{43}{64}e^2 \right\} \left\{ (k^3 - 3k^2h) \sin 3g - (3hk^2 - h^3) \cos 3g \right\} \\ &+ \frac{103}{96} \left\{ (k^4 - 6k^2h^2 + h^4) \sin 4g - (4k^3h - 4lh^3) \cos 4g \right\} \\ &+ \frac{1097}{960} \left\{ (k^5 - 10k^3h^2 + 5kh^4) \sin 5g - (5k^4h - 10k^2h^3 + k^5) \cos 5g \right\} \end{aligned}$$

Neperian logarithm of $r = n + \frac{1}{4}e^2 + \frac{1}{32}e^4$

$$\begin{aligned} &- \left\{ 1 - \frac{3}{8}e^2 \right\} \left\{ k \cos g + h \sin g \right\} \\ &- \left\{ \frac{3}{4} - \frac{11}{24}e^2 \right\} \left\{ (k^2 - h^2) \cos 2g + 2hk \sin 2g \right\} \end{aligned}$$

$$\begin{aligned}
& - \frac{17}{24} \left\{ (k^3 - 3kh^2) \cos 3g + (3k^2h - h^3) \sin 3g \right\} \\
& - \frac{71}{96} \left\{ (k^4 - 6k^2h^2 + h^4) \cos 4g + (4k^3h - 4kh^3) \sin 4g \right\}
\end{aligned}$$

In computing these expressions it will be sufficient for several centuries before or after 1850 to develop h , δk , and δl to their first dimensions: it will, however, be more convenient to correct the mean anomaly g for the perturbation δl before obtaining the equation of the centre. Developing the perturbations of h and k to terms of the first order, we have for the effects of the perturbations of those elements:

$$(v.s.1) = \left(2 - \frac{3}{4}e_0^2\right) \delta k$$

$$(v.c.1) = -\left(2 - \frac{1}{4}e_0^2\right) \delta h$$

$$(v.s.2) = \left(\frac{5}{2}e_0 - \frac{11}{6}e_0^3\right) \delta k$$

$$(v.c.2) = -\left(\frac{5}{2}e_0 - \frac{11}{24}e_0^3\right) \delta h$$

$$(v.s.3) = \frac{13}{4}e_0^2 \delta k$$

$$(v.c.3) = -\frac{13}{4}e_0^2 \delta h$$

$$(v.s.4) = \frac{103}{24}e_0^3 \delta k$$

$$(v.c.4) = -\frac{103}{24}e_0^3 \delta h$$

$$(\rho.c.0) = \delta \nu + \frac{1}{2}e_0 \delta k$$

$$(\rho.s.1) = -\left(1 - \frac{3}{8}e_0^2\right) \delta h$$

$$(\rho.c.1) = -\left(1 - \frac{9}{8}e_0^2\right) \delta k$$

$$(\rho.s.2) = -\frac{3}{2}e_0 \delta h$$

$$(\rho.c.2) = -\frac{3}{2}e_0 \delta k$$

$$(\rho.s.3) = -\frac{17}{8}e_0^2 \delta h$$

$$(\rho.c.3) = -\frac{17}{8}e_0^2 \delta k$$

These coefficients for ρ must, of course, be multiplied by the modulus 0.434294 to reduce the perturbations to those of the common logarithm of the radius vector.

Among the elliptic terms may be included the effect of the following minute constants introduced by the perturbations.

$$\begin{aligned}(v.s.2) &= -0''.144 \\(v.c.2) &= +0.130 \\0.4343 (\rho.c.0) &= +1972 \text{ in units of the 7th place} \\0.4343 (\rho.s.1) &= +63 \quad \text{of decimals.} \\0.4343 (\rho.c.1) &= +73 \\0.4343 (\rho.s.2) &= +5 \\0.4343 (\rho.c.2) &= +4\end{aligned}$$

This term ($\rho.c.0$) is that added as a correction to the logarithm of the mean distance.

To the coefficients ($v.s.1$), ($v.c.1$), etc., are still to be added the following periodic terms:—

1. The periodic terms due to the action of Jupiter, given in Chapter V, omitting the terms multiplied by T , which are included in the perturbations of the elements.

2. The periodic terms produced by Saturn, including those terms multiplied both by T and by $\sin A_2$ or $\cos A_2$, but omitting those multiplied by T only for the same reason as in the case of Jupiter.

3. The periodic terms produced by Neptune, multiplied by the factor 0.86294 on account of the correction to the mass of that planet, and omitting the terms multiplied by δl , δe , and $e^\delta g$.

4. The periodic terms multiplied by the product of the masses of Jupiter and Saturn, given on page 88, omitting the terms multiplied by the sine and cosine of N_6 and N_7 , because they are replaced by the terms of δl , δh , and δk , given on page 183, and tabulated in the columns headed (2) on page 184. The result will be the same whether we employ the terms of ($v.c.0$), ($v.s.1$), etc., given at the bottom of page 88 and the top of page 89, omitting the numbers in the columns 2 on page 184 from the expressions on page 186, or whether we include the latter and omit the former.

The true anomaly of Uranus will then be:

$$g_0 + \delta l + (\text{equation of centre from elements IV, using for mean anomaly } g_0 + \delta l) \\ + \Sigma (v.s.i) \sin ig + \Sigma (v.c.i) \cos ig.$$

The logarithm of the radius vector will be:

$$\log r \text{ in elliptic orbit from elements IV.} \\ + \Sigma (\rho.s.i) \sin ig + \Sigma (\rho.c.i) \cos ig$$

care being taken to multiply the coefficients by the modulus where that has not already been done. All the terms in Chapter V are so multiplied.

To pass from the true anomaly to the true longitude we must investigate the secular motion of the planes of the orbit and of the ecliptic. The effect of this motion on ϕ , θ , and τ will be found by successive approximations from the formulæ

(34), correcting the data for the new mass of Neptune. We shall also use the same motion of the ecliptic adopted on p. 95. We have thus:

$$\frac{d\rho}{dt} = -4''.53$$

$$\frac{d\rho'}{dt} = +5.43 + 0''.38T.$$

$$\frac{dq}{dt} = -5.17$$

$$\frac{dq'}{dt} = -46.78 + 0.12T.$$

As a first approximation we have

$$\begin{aligned} \theta = \tau &= 73^\circ 14' 8'' - 3169''.2T \\ \phi &= 0 46 20.54 + 2.48T \end{aligned}$$

Substituting these values in (34) and integrating we find

$$\begin{aligned} \phi &= \phi_0 + 2''.47T + 0''.13T^2 \\ \theta &= \theta_0 - 3168.42T + 3.00T^2 \\ \tau &= \tau_0 - 3168.76T + 3.00T^2 \end{aligned}$$

For tabulating we shall use, instead of θ and τ , the distance of the perihelion from the ascending node, or $\pi - \tau$, and the value of θ corrected for Struve's precession. Since the mean motion has been derived without making any distinction between τ and θ , it will be necessary to correct the motion of mean anomaly by the difference of those quantities. We thus obtain for the values of the three principal arguments:—

$$\begin{aligned} g &= 220^\circ 10' 10''.35 + 1542574''.86T + \delta l \\ \omega &= 95 0 58.70 + 3168.76T - 3.00T^2 \\ \theta &= 73 14 8.00 + 1856.82T + 4.12T^2 \end{aligned}$$

If we represent all the inequalities of the true longitude by Δl , so that we shall have for the true anomaly

$$f = g + \Delta l,$$

the argument of latitude will be

$$u = f + \omega.$$

The reduction to the ecliptic will then be

$$R = -(9''.37 + 0''.016T) \sin 2u,$$

the true longitude on the ecliptic referred to the mean equinox of date,

$$\lambda = u + \theta + R,$$

and the sine of the elliptic latitude,

$$\sin \beta_0 = \sin \phi_0 \sin u.$$

The perturbations of the latitude will be

$$(b.c.0) + (b.c.1) \cos g + (b.s.1) \sin g + \text{etc.}$$

The periodic terms of (*b.c.0*), (*b.s.1*), (*b.c.1*), etc., are given in Chapter V, on pages 86 and 87, and are to be taken without any farther modification than the multiplication of those due to the action of Neptune by the factor 0.863. The constant, secular, and long period terms are

$$\begin{aligned}
 b.c.0 &= +0''.26 - 0''.12T - 0.011\delta\eta + 0.046\delta\kappa \\
 (b.s.1) &= -0.22T - 0.05T^2 + 0.975\delta\eta + 0.221\delta\kappa \\
 (b.c.1) &= +2.47T + 0.12T^2 + 0.221\delta\eta - 0.975\delta\kappa \\
 (b.s.2) &= -0.06 - 0.01T + 0.046\delta\eta + 0.011\delta\kappa \\
 (b.c.2) &= -0.01 + 0.12T + 0.011\delta\eta - 0.046\delta\kappa
 \end{aligned}$$

The values of $\delta\eta$ and $\delta\kappa$ to be used in these expressions are those the expressions for which are given on page 97, and which are tabulated in the last two columns of the table on page 184.

The following tables are based on the elements and theory laid down in this chapter.

CHAPTER IX.

GENERAL TABLES OF URANUS.

Enumeration of the Quantities contained in the several Tables.

THE first six tables are designed to give the values of the three arguments of the elliptic motion, g , ω , and θ , and of the nine arguments of the tables of perturbations. The argument ω is, however, diminished by $3'$, the sums of the constants added to the perturbations of (*v.c.*0) to make these quantities positive, and θ by $10''$, the constant added to the reduction to ecliptic. The expressions for the arguments of perturbations are as follows, the mean longitude of each planet, counted from the perihelion of Uranus, being represented by the initial letter of the planet. All these arguments are expressed in units, of which 600 make an entire circumference, so that each unit is $36'$. The time t is counted in Julian years from the fundamental epoch,

1850, January 0, Greenwich mean noon.

| | | | |
|----------|----------------|---|---------------------|
| Arg. 1 = | $J - U$ | = | 219.190 + 43.44028t |
| 2 = | $S - U$ | = | 577.349 + 13.22717t |
| 3 = | $U - N$ | = | 88.884 + 3.50035t |
| 4 = | $J - 2S$ | = | 497.6 + 9.8445t |
| 5 = | $3S - U - J$ | = | 79.8 + 3.3825t |
| 6 = | $4S - 2U - J$ | = | 57.1 + 16.610t |
| 7 = | $2J - 3S - 3U$ | = | 238.7 + 18.633t |
| 8 = | $2J - 4S - 2U$ | = | 261.3 + 5.4058t |
| 9 = | $7S - 2J - 3U$ | = | 136.9 + 19.992t |

Table I gives the corrections which must be applied to the values of the arguments at any time during the nineteenth century to reduce them to the corresponding time in any preceding or following century between the Christian era and the year 2300. Since ω and θ each contains a term proportional to the square of the time, the correction for these quantities is not constant during each century, but is of the form

$$\omega + \omega' T$$

ω and ω' being constant during each century, and T being the fraction of the century counted from its beginning.

Table II gives the value of g , $\omega - 3'$, $\theta - 10''$, and the above nine arguments for Greenwich mean noon of Jan. 0 of each leap year from 1752 to 1948, and for January - 1 of the years 1800 and 1900, corresponding to December 30 of the years 1799 and 1899. The corrections for the perturbations of long period are not

applied in this table. The numbers at the bottom of this table, in the line $\Delta_{120}^{(1)}$, show the variation of the corresponding quantity in 120 days, for the epoch 1850.0. In the line "Factor T " is given the change of this variation in a century, while $\Delta_{120}^{(2)}$ is the second difference for intervals of 120 days. By means of these numbers, when the arguments are computed for any date, their values for other dates at intervals of 120 days may be found by successive addition.

Table III gives the motion of the several arguments between the epochs of the preceding table and the zero day of each month in the course of a four-year cycle. The variable motions, ω and θ , correspond to the epoch 1850, and rigorously they each require a correction for any other four-year cycle than that between 1848 and 1852. But, owing to the small inclination of the orbit of Uranus it is not necessary that either ω or θ should be exact, if only their sum is exact. The column θ' of this table, therefore, gives the correction which must be applied to the motion of θ at the end of a century (1950) in order that, being applied to θ alone, $\omega + \theta$ may be exact. This correction is, in fact, that for the secular variation of the precession.

Tables IV and V give the motion of the arguments for days and hours. The motion for hours is, however, not necessary in the case of any argument but g , as all the others can be readily enough interpolated to fractions of a day.

Table VI gives the corrections to the arguments on account of the terms of long period from 1000 to 2200. The terms in question are, in the case of Jupiter, the great inequality produced by the action of Saturn, in the case of Neptune the great inequality produced by Uranus, and, in the case of Uranus, the inequalities in the mean longitude tabulated in the preceding chapter. The numerical expressions are

$$\begin{aligned}\delta J &= 0.535 \sin (116^\circ 21' + 40^\circ 45' 20'' T) \\ \delta U &= \delta l \\ \delta N &= -0.75 \delta l.\end{aligned}$$

The corrections to the several arguments are

$$\begin{aligned}\delta g &= \delta l \\ \delta \text{arg. 1} &= \delta J - \delta l \\ \delta \text{arg. 2} &= -\delta l \\ \delta \text{arg. 3} &= \delta l - \delta N = 1.75 \delta l\end{aligned}$$

No correction to the mean longitude of Saturn is applied, all its inequalities being taken account of in the terms of the second order.

The corrections, expressed in seconds, have been reduced to units of the argument by dividing them by 2160".

Outside the limits of the table these corrections must be computed from their formulæ.

Table VII gives the equation of the centre, and the elliptic part of the logarithm of the radius vector. No constant is applied to the former, but the latter is diminished by .0003400, the sum of the constants added to (*p.c.0*) in Tables VIII, IX, X and XVII.

The formulæ for the Tables are

$$\begin{aligned} \text{Equation of centre} = & 19352''.06 \sin \\ & + 567.24 \sin \\ & + 23.05 \sin 3g \\ & + 1.07 \sin 4g \\ & + 0.05 \sin 5g \end{aligned}$$

$$\begin{aligned} \text{Elliptic log. } r = & 1.2833435 \\ & - .0003400 \\ & - .0203618 \cos g \\ & - .0007165 \cos 2g \\ & - .0000318 \cos 3g \\ & - .0000016 \cos 4g. \end{aligned}$$

Table VIII gives the coefficients (*v.c.0*), (*v.c.1*), etc., for the perturbations of the longitude and logarithm of radius vector produced by the action of Jupiter. They are computed from the periodic terms of the formulæ on page 83, with the addition of the following constants to make all the numbers of the table positive:

$$\begin{aligned} \text{Constant of } (v.c.0) &= 55''. \\ (v.s.1) &= 6. \\ (v.c.1) &= 4. \\ (v.s.2) &= 0.20 \\ (v.c.2) &= 0.20 \\ (\rho.c.0) &= 1200 \\ (\rho.s.1) &= 150 \\ (\rho.c.1) &= 100 \\ (\rho.s.2) &= 10 \\ (\rho.c.2) &= 10 \end{aligned}$$

Table IX gives the periodic part of the coefficients due to the action of Saturn, taken without change from the expressions on page 84, together with the secular variations, the latter including only the terms of (*v.s.1*), (*v.c.1*), (*v.s.2*), and (*v.c.2*), which are multiplied by *T* and by $\sin A_2$ or $\cos A_2$. The coefficients of *T* are given in the columns Sec. Var. and each number is increased by the constant 1''.50 to make it positive. The term $-0''.06T \sin A_2$ in (*v.c.0*) is omitted entirely, as it will not amount to a tenth of a second until after the year 2000. The constant terms added to the quantities of these tables to make all the numbers positive, are:

$$\begin{aligned} \text{Constant of } (v.c.0) &= 30. \\ (v.s.1) &= 150. + 1.50T \\ (v.c.1) &= 150. + 1.50T \\ (v.s.2) &= 130. + 1.50T \\ (v.c.2) &= 130. + 1.50T \\ (v.s.3) &= 8. \\ (v.c.3) &= 6. \\ (v.s.4) &= 1. \\ (v.c.4) &= 1. \end{aligned}$$

| | |
|------------------------------|------|
| Constant of ($\rho.c.0$) = | 800 |
| ($\rho.s.1$) = | 1500 |
| ($\rho.c.1$) = | 1500 |
| ($\rho.s.2$) = | 400 |
| ($\rho.c.2$) = | 400 |
| ($\rho.s.3$) = | 100 |
| ($\rho.c.3$) = | 100 |

Table X gives the coefficients produced by the action of Neptune, computed from the periodic terms on pages 85 and 86 without any other change than the multiplication of all the numbers by the factor 0.863 to reduce them to the new mass of Neptune. The constants added to the several quantities, are

| | |
|------------------------------|--------|
| " | |
| Constant of ($v.c.0$) = | 92.85 |
| ($v.s.1$) = | 20.00 |
| ($v.c.1$) = | 31.00 |
| ($v.s.2$) = | 5.00 |
| ($v.c.2$) = | 5.00 |
| ($v.s.3$) = | 1.00 |
| ($v.c.3$) = | 1.00 |
| ($v.s.4$) = | — 1.00 |
| ($v.c.4$) = | — 1.00 |
| Constant of ($\rho.c.0$) = | 400 |
| ($\rho.s.1$) = | 200 |
| ($\rho.c.1$) = | 200 |
| ($\rho.s.2$) = | 40 |
| ($\rho.c.2$) = | 40 |

Tables XI to XVI give the terms of the second order and of short period which contain the products of the masses of Jupiter and Saturn, which, with the constants added to the numbers of the several tables, are as follows:

| | | | |
|---------------|--|------------|---------------|
| ($v.c.0$) = | + 0".08 sin A_5 + 0".51 cos A_5 ; | Table XII; | const = 0".60 |
| | + 0.04 sin A_6 + 0.01 cos A_6 ; | XIII; | = 0.05 |
| | — 0.01 sin A_7 + 0.05 cos A_7 ; | XIV; | = 0.05 |
| | — 0.35 sin A_8 — 1.30 cos A_8 ; | XV; | = 1.35 |
| | — 0.05 sin A_9 + 0.03 cos A_9 ; | XVI; | = 0.10 |
| | | | — — — |
| | Sum of constants added to these tables | | 2.15 |

| | | | |
|---------------|---------------------------------------|-----------|---------------|
| ($v.s.1$) = | + 0".26 sin A_1 + 0".27 cos A_1 ; | Table XI; | const = 0".40 |
| | — 0.04 sin A_5 — 0.17 cos A_5 ; | XII; | = 0.20 |
| | + 0.08 sin A_6 + 0.03 cos A_6 ; | XIII; | = 0.10 |
| | — 0.02 sin A_7 + 0.08 cos A_7 ; | XIV; | = 0.10 |
| | + 0.30 sin A_8 — 0.58 cos A_8 ; | XV; | 0.75 |
| | — 0.04 sin A_9 ; | XVI; | 0.10 |

| | | |
|--|-----------|------------------|
| $(v.c.1) = + 0''.06 \sin A_4 - 0''.27 \cos A_4;$ | Table XI; | const = $0''.40$ |
| $+ 0.18 \sin A_5 + 0.01 \cos A_5;$ | XII; | 0.20 |
| $- 0.03 \sin A_6 + 0.08 \cos A_6;$ | XIII; | 0.10 |
| $- 0.02 \sin A_7 + 0.09 \cos A_7;$ | XIV; | 0.10 |
| $- 0.44 \sin A_8 - 0.61 \cos A_8;$ | XV; | 0.75 |
| $- 0.10 \sin A_9 + 0.03 \cos A_9;$ | XVI; | 0.10 |

Sum of constants added to $(v.s.1)$ and $(v.c.1)$ in these tables 1.65

The term of $(\rho.c.0)$

$$11 \sin A_3 - 3 \cos A_3$$

is omitted from the tables entirely.

Tables XVII *a* and XVII *b* give the constant, secular, and long-period terms of $(v.s.1)$ $(v.c.1)$, computed from the formulæ p. 186, with the following additions:

1. The constant terms introduced by the perturbations, given on p. 187.
2. The negatives of the constants added to the tables VII to XVI inclusive to make the numbers of those tables positive. The values of these terms are

| | Pert. Const. | Tables VIII to XVI. | (1) — (2) |
|--------------|--------------|-----------------------|-------------------------|
| $(v.s.1)$ | 0 | $177''.65 + 1''.50 T$ | $- 177''.65 - 1''.50 T$ |
| $(v.c.1)$ | 0 | $186.65 + 1.50 T$ | $- 186.65 - 1.50 T$ |
| $(v.s.2)$ | $- 0''.14$ | $135.20 + 1.50 T$ | $- 135.34 - 1.50 T$ |
| $(v.c.2)$ | $+ 0.13$ | $135.20 + 1.50 T$ | $- 135.07 - 1.50 T$ |
| $(v.s.3)$ | 0 | 9.00 | $- 9.00$ |
| $(v.c.3)$ | 0 | 7.00 | $- 7.00$ |
| $(\rho.c.0)$ | [+1972] | -1000 | +1000 |
| $(\rho.s.1)$ | + 63 | 1850 | -1787 |
| $(\rho.c.1)$ | + 73 | 1800 | -1727 |
| $(\rho.s.2)$ | + 5 | 450 | - 445 |
| $(\rho.c.2)$ | + 4 | 450 | - 446 |

The perturbation constant of $(\rho.c.0)$, being added to $\log a$ in forming the elliptic radius vector, is not included in this table.

Table XVIII gives the reduction to the ecliptic

$$- 9''.37 \sin 2u.$$

The constant $10''$ is added to make the numbers always positive, which constant has been already subtracted from θ .

Table XIX gives the principal term of the latitude

$$46' 20''.54 \times \sin u.$$

Table XX gives the coefficients $(b.s.1)$ and $(b.c.1)$ for the perturbations of the latitude produced by Jupiter. They are given by the formulæ

$$(b.s.1) = 0''.65 \cos (J - U + 40^\circ)$$

$$(b.c.1) = 0.65 \sin (J - U + 40^\circ)$$

The constant $0''.70$ is added to make all the numbers of the table positive.

Table XXI gives the corresponding coefficients for the action of Saturn, computed from the expressions on p. 87 with the addition of the following constants.

$$\begin{aligned} \text{Const. of } (b.c.0) &= 0''.10 \\ (b.s.1) &= 3.30 \\ (b.c.1) &= 3.10 \\ (b.s.2) &= 0.20 \\ (b.c.2) &= 0.20 \end{aligned}$$

Table XXII gives the coefficients for the action of Neptune from the formulæ on p. 87, all the numbers being multiplied by the factor 0.863 to reduce them to the adopted mass of Neptune. The following constants are added:

$$\begin{aligned} \text{To } (b.c.0) &. . . . 0''.06 \\ (b.s.1) &1.00 \\ (b.c.1) &1.20 \\ (b.s.2) &0.20 \\ (b.c.2) &0.20 \end{aligned}$$

Table XXIII gives the secular and long-period terms for various epochs computed from the formulæ of p. 189. The sums of the several constants added in the three preceding tables are here subtracted again so that these expressions become

$$\begin{aligned} b.c.0 - \Sigma c &= 0''.10 - 0''.12T - .011\delta\eta + .046\delta x \\ (b.s.1) - \Sigma c &= -5.00 - 0.22T - 0''.05T^2 + .975\delta\eta + .221\delta x \\ (b.c.1) - \Sigma c &= -5.00 + 2.47T + 0.12T^2 + .221\delta\eta - .975\delta x \\ (b.s.2) - \Sigma c &= -0.46 - 0.01T + .046\delta\eta + .011\delta x \\ (b.c.2) - \Sigma c &= -0.41 + 0.12T + .011\delta\eta - .046\delta x \end{aligned}$$

Precepts for the use of the Tables.

Express the date for which the position of Uranus is required in years, months, days, and hours of Greenwich mean time, according to the Julian Calendar if the date is earlier than 1500, according to the Gregorian Calendar if it is later than 1600, and according to either calendar between these epochs.

Enter Table I with the beginning of the century, and take out the values of g , ω , ω' , θ , θ' , and arguments 1 to 9. Multiply ω' and θ' by the fraction of a century corresponding to the date, and write the products with their proper algebraic signs under ω and θ . If the calendar is the Julian, the century marked J must be taken, and if the Gregorian, that marked G . Between the dates 1752 and 1951 it is not necessary to enter Table I at all.

If Table I was not entered, enter Table II with the year, or the first preceding year found therein. If Table I was entered, enter Table II between the year 1800 and 1896 as if the number of the century were changed to 18. Take out the values of g , ω , θ , and the arguments, and write them under the corresponding quantities from Table I.

Enter Table III with the excess of the actual year over that with which Table II was entered, and with the month. Write the corresponding values of g , ω , θ , and the arguments under the previous values. Multiply θ' by the fraction of a

century after 1850, corresponding to the date with which Table II was entered, and write the product under θ , or add it to it in writing θ . If Table II was entered with a date before 1850, this product is negative.

Enter Table IV with the day of the month and write down the corresponding values of g , ω , etc., under the former values.

If the date does not correspond to Greenwich mean noon, the motion of g for the hours must be computed from Table V, and the other quantities must be interpolated to the fraction of a day in entering Table IV.

Enter Table VI with the year, find by interpolation the values of g , and arguments 1, 2, and 3, corresponding to the date, and write them under the former values.

Add up all the partial values of g , ω , θ , and the arguments, attending to the algebraic signs of the products. Subtract from the arguments as many times 600 as possible, and the results will be the final values of those quantities.

Enter Table VII with g as the argument, the seconds being first reduced to fractions of a minute, and interpolate the quantities E and $\log r$. When g exceeds 180° the former quantity is to receive the negative sign; the latter is always positive.

Enter Tables VIII to XVI inclusive with their respective arguments, and take out the values of the quantities ($v.c.0$), ($v.s.1$), ($v.c.1$), etc., ($\rho.c.0$), ($\rho.s.1$), etc., so far as they are found in the tables, writing the quantities having the same designation under each other. In Table IX the quantities Sec. Var. must be multiplied by the centuries and fraction of a century of the actual date after 1850, and the product must be included with the corresponding quantities, ($v.s.1$), ($v.c.1$), etc. Before 1850 this product will always be negative; afterward always positive. All the quantities taken from these tables are positive except ($v.s.4$) and ($v.c.4$) in Table IX, which are negative.

Add up all the partial values of ($v.c.0$), ($v.s.1$), etc., thus obtained from Tables VIII to XVI, and from their sum take the corresponding quantities obtained from Table XVII by interpolating to the date. The required quantities are all given in Table XVII *b*; Table XVII *a* being only an expansion of a part of XVII *b* for the present century. The final values of ($v.s.1$), ($v.c.1$), ($v.s.2$), etc., ($\rho.s.1$), ($\rho.c.1$), etc., thus obtained are to be multiplied by the sines and cosines of the corresponding multiples of g , in doing which four place logarithms are sufficient if the computation is carefully made. The products are then all added together, and to g , ω , E , and ($v.c.0$); in the case of v , and to $\log r$, ($\rho.c.0$) in the case of ρ . That is, we are to form the expressions:

$$u = g + \omega + E + (v.c.0) + (v.s.1) \sin g + (v.c.1) \cos g \\ + (v.s.2) \sin 2g + (v.c.2) \cos 2g \\ + \quad \text{etc.} \quad + \quad \text{etc.}$$

$$\log r = \log r \text{ (from Table VII)} + (\rho.c.0) \\ + (\rho.s.1) \sin g \quad + (\rho.c.1) \cos g \\ + (\rho.s.2) \sin 2g \quad + (\rho.c.2) \cos 2g \\ + (\rho.s.3) \sin 3g \quad + (\rho.c.3) \cos 3g.$$

u will then be the true argument of latitude, and $\log r$ the logarithm of the radius vector with seven places of decimals.

Under u write θ ; enter Table XVIII with the argument u and take out the reduction to the ecliptic. Add it to u and θ , and the sum of the three quantities will be the heliocentric longitude of Uranus referred to the mean equinox and ecliptic of the date. Applying nutation the longitude will be reduced to the true equinox.

Enter Table XIX with u as the argument, or, when u exceeds 180° , with $u - 180^\circ$, and take out the principal term of the latitude, which will be positive when u is less than 180° , and negative when it is greater.

Enter Tables XX, XXI, XXII, and XXIII with their respective arguments, the argument for the last being the date, and add up the various quantities having the same designation, noticing that in the first three tables all the quantities are positive, while in the last they are all negative except (*b.c.0*). Then form the expression,

$$(b.c.0) + (b.s.1) \sin g + (b.c.1) \cos g + (b.s.2) \sin 2g + (b.c.2) \cos 2g,$$

and add it to the principal term of the latitude, with regard to the algebraic signs. The sum will be the heliocentric latitude of Uranus above the ecliptic of the date.

When an ephemeris of Uranus is to be computed for a series of years, some modifications may be introduced, which will save the computer labor. In the first place an equidistant series of dates being selected for computation, it will be sufficient to compute g , ω , θ , and the arguments for every sixth, eighth, or tenth date, and to fill in the arguments for the intermediate dates by adding the nearly constant differences corresponding to the adopted intervals. The agreement of the numbers thus obtained for the last date with those found by the original computation will prove the whole process. This interval may be as great as 120 days without detracting from the accuracy with which the places for the immediate dates can be interpolated, and the differences for this interval may be deduced from the numbers at the bottom of Table II. If these numbers are used without change the values of ω and θ for the last date may not always come out right. But these errors, if less than a second, will be of no importance if the one quantity comes out as much too great as the other is too small, and they may be avoided entirely by making a small change in the constant difference to be added.

Tables XI to XVI, inclusive, need be entered only for every third or fourth date, and the sums of the quantities can be then interpolated to every date, and added up with the corresponding quantities from the other tables.

Again, it will be found convenient to compute the sum of the small terms (*v.s.3*) $\sin 3g + (v.c.3) \cos 3g + (v.s.4) \sin 4g + (v.c.4) \cos 4g$, as well as the corresponding terms of the radius vector, and all the terms of the latitude, not for the dates adopted, but for every fourth entire degree of g . Having a series of values computed in this way, the sum can be interpolated to the value of g corresponding to the date. To facilitate the formation of the smaller products for entire degrees of g , a table of products of numbers by the sine and cosine of every degree is appended to these tables, by which the products in question can be formed at sight

whenever the coefficient to be multiplied is less than $32''$. The values of these coefficients, (*v.s.3*), (*v.c.3*), etc., corresponding to the entire degrees of g , may be either formed by interpolation at sight from those corresponding to the dates of computation, or the values of the arguments 2 and 3 corresponding to the required degrees of g may be computed, and the values of (*v.s.3*), etc., corresponding to these values of the arguments may be taken from Tables IX and X, while Table XVII must be entered with the corresponding dates.

If the heliocentric ephemeris is computed for ten years at a time, the last of these modifications in the mode of computation will greatly facilitate the computation of the smaller terms. We first find the date, and the values of arguments 1, 2, and 3, to one place of decimals, for some entire degree of g preceding that which corresponds to the first date, and then find the dates and the values of the arguments corresponding to successive values of g , differing by 2° or 4° , until we pass the last date of computation. We then take out the values of (*v.s.3*), (*v.c.3*), (*v.s.4*), (*v.c.4*), (*p.s.3*), (*p.c.3*), (*b.c.0*), (*b.s.1*), (*b.c.1*), (*b.s.2*), and (*b.c.2*), with these values of the dates and arguments, form their products by the sines and cosines of the corresponding multiples of g by means of the supplementary tables, and add the proper products together so as to form three small tables with g as the argument. These terms are then interpolated to the values of g corresponding to the original dates of computation.

As a first example of the use of the Tables we will compute the heliocentric co-ordinates of Uranus for Greenwich mean noon of the date 1753, Dec. 3. In computing the arguments we shall make use of Table I, though it is not necessary to do so. The computation of the arguments is as follows:

| | g | | | ω | | | θ | | | Arg. 1 |
|-------------------|---------|---------|-------|----------|-----|-------|----------|-----|-------|---------|
| | ° | ' | " | ° | ' | " | ° | ' | " | |
| Table I, 1700 | 291 | 31 | 7. | 359 | 7 | 5.33 | 359 | 29 | 11.47 | 456.092 |
| Product by 0.5392 | | | | +3.24 | | | -4.45 | | | |
| Table II, 1852 | 228 | 44 | 0.73 | 94 | 59 | 2.03 | 73 | 14 | 35.11 | 306.010 |
| III, Y. 1, Dec. | 8 | 12 | 43.38 | 0 | 1 | 0.73 | 0 | 0 | 35.59 | 83.252 |
| IV, 3 days | 0 | 2 | 6.70 | | | 0.26 | | | 0.15 | 0.357 |
| VI, 1753.92 | 0 | 0 | 32.30 | | | | | | | 0.507 |
| 1753, Dec. 3 | 168 | 30 | 30.48 | 94 | 7 | 11.59 | 72 | 44 | 17.87 | 246.218 |
| | Arg. 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | | |
| Table I, 1700 | 477.319 | 249.975 | 216 | 262 | 139 | 537 | 59.4 | 401 | | |
| II, 1852 | 3.785 | 95.880 | 517 | 87 | 90 | 276 | 272.1 | 177 | | |
| III, Y. 1, Dec. | 25.349 | 6.709 | 19 | 6 | 32 | 36 | 10.3 | 38 | | |
| IV, 3 days | 0.109 | .029 | 0 | 0 | 0 | 0 | 0.0 | 0 | | |
| VI, 1753.9 | -0.014 | +0.24 | | | | | | | | |
| 1753, Dec. 3 | 506.548 | 352.617 | 152 | 355 | 261 | 249 | 341.8 | 16 | | |

| | (v.c.0) | (v.s.1) | (v.c.1) | (v.s.2) | (v.c.2) | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) |
|------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | " | " | " | " | " | " |
| Table VIII | 83.58 | 9.97 | 6.61 | 0.37 | 0.31 | | | | |
| IX | 21.32 | 22.24 | 223.49 | 60.92 | 238.89 | 1.42 | 11.07 | 0.32 | 1.64 |
| X | 71.08 | 19.02 | 18.30 | 6.64 | 5.38 | 0.70 | 1.19 | -0.95 | -1.04 |
| XI | .. | .13 | .47 | | | | | | |
| XII | .13 | .36 | .10 | | | | | | |
| XIII | .06 | .11 | .01 | | | | | | |
| XIV | .01 | .04 | .03 | | | | | | |
| XV | 2.38 | 1.40 | 1.12 | | | | | | |
| XVI | .12 | .09 | .11 | | | | | | |

| | | | | | | | | | |
|------------|--------|---------|---------|---------|---------|-------|-------|-------|-------|
| Σ | 178.68 | 53.36 | 250.24 | 67.93 | 244.58 | 2.12 | 12.26 | | |
| Table XVII | | -292.55 | -140.91 | -140.73 | -131.03 | -9.41 | -6.85 | | |
| | | -239.19 | +109.35 | -72.80 | +113.55 | -7.29 | +5.41 | -0.63 | +0.60 |

| | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | (p.s.3) | (p.c.3) |
|------------|---------|---------|---------|---------|---------|---------|---------|
| Table VIII | 251 | 188 | 32 | | | | |
| IX | 1086 | 741 | 360 | 621 | 597 | 154 | 111 |
| X | 326 | 259 | 222 | 21 | 42 | | |
| | 1663 | 1188 | 614 | 642 | 639 | 154 | 111 |
| XVII | 1104 | -1321 | -500 | -412 | -360 | -98 | -94 |
| | 2767 | -133 | +114 | +230 | +279 | +56 | +17 |

| | | | |
|----------------------|----------------------|----------------------|----------------------|
| log (v.s.1) - 2.3787 | log (v.c.1) + 2.0388 | log (v.s.2) - 1.8621 | log (v.c.2) + 2.0552 |
| log sin + 9.2994 | log cos g - 9.9912 | log sin 2g - 9.5916 | log cos 2g + 9.9641 |
| log (p.s.1) - 2.124 | log (p.c.1) + 2.057 | log (p.s.2) + 2.362 | log (p.c.2) + 2.446 |

| | o | ' | " | | (b.s.1) | (b.c.1) | (b.s.2) | (b.c.2) |
|--------------------|-----|-------|--------|----------------|-----------|--------------------------|---------|---------|
| g | 168 | 30 | 30.48 | Table XX | ".06 | 0".62 | | |
| ω | 94 | 7 | 11.59 | XXI | 3.91 | 5.88 | 0.41 | 0.17 |
| E | 1 | 0 | 46.21 | XXII | .84 | 1.36 | 0.18 | 0.17 |
| (v.c.0) | | 2 | 58.68 | | 4.81 | 7.86 | 0.59 | 0.34 |
| (v.s.1) sin g | | | -47.66 | XXIII | -4.28 | -6.88 | -0.43 | -0.51 |
| (v.c.1) cos g | -1 | | 47.15 | | +0.53 | +0.98 | +0.16 | -0.17 |
| (v.s.2) sin 2g | | | 28.43 | | | | | |
| (v.c.2) cos 2g | 1 | | 44.55 | | | | | |
| (v.s.3) sin 3g | | | -4.13 | | | | | |
| (v.c.3) cos 3g | | | -4.46 | log r | | | | |
| (v.s.4) sin 4g | | | 0.47 | Table VII | 1.3023222 | β ₀ Table XIX | -0 46' | 3."73 |
| (v.c.4) cos 4g | | | 0.41 | (p.c.0) | 2767 | (b.c.0) Table XXI | | 0.16 |
| u | 263 | 40 | 57.42 | (p.s.1) sin g | -26 | XXII | | 0.09 |
| θ | 72 | 44 | 17.87 | (p.c.1) cos g | -112 | XXIII | | 0.20 |
| R | | | 7.95 | (p.s.2) sin 2g | -90 | (b.s.1) sin g | | +0.11 |
| Long. mean Eq. 336 | 25 | 23.24 | | (p.c.2) cos 2g | +257 | (b.c.1) cos g | | -0.96 |
| Nutation | | +6.51 | | (p.s.3) sin 3g | +31 | (b.s.2) sin 2g | | -0.06 |
| Long. true Eq. 336 | 25 | 29.75 | | (p.c.3) cos 3g | -14 | (b.c.2) cos 2g | | -0.16 |
| | | | | log r | 1.326035 | Latitude | -0 46' | 4.35 |

As a second example we will take the computation of an ephemeris for the years 1876 and 1877. We take as the extreme dates 1875, December 15, and 1878, April 3, between which are seven intervals of 120 days each, which we adopt as those of computation. We first form the arguments for the extreme dates as follows:

| I. For 1875, Dec. 15 = 1875.96. | | | | | | | | | | |
|---------------------------------|----------|---------|-------|----------|-----|-------|----------|-----|-------|---------|
| | <i>g</i> | | | ω | | | θ | | | Arg. 1 |
| | ° | ' | " | ° | ' | " | ° | ' | " | |
| Table II, 1872 | 314 | 25 | 55.70 | 95 | 9 | 35.64 | 73 | 20 | 46.67 | 574.815 |
| III, 3 Y. Dec. | 16 | 46 | 33.76 | 0 | 2 | 4.06 | 0 | 1 | 12.72 | 170.072 |
| IV, 15 days | 0 | 10 | 33.50 | | | 1.30 | | | 0.76 | 1.784 |
| VI, 1875.96 | | | 2.20 | | | | | | | + 0.426 |
| For 1875, Dec. 15 | 331 | 23 | 5.16 | 95 | 11 | 41.00 | 73 | 22 | 0.15 | 147.097 |
| | Arg. 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | | |
| Table II, 1872 | 268.328 | 165.888 | 114 | 154 | 423 | 49 | 380.3 | 577 | | |
| III, 3 Y. Dec. | 51.785 | 13.705 | 39 | 13 | 65 | 73 | 21.1 | 78 | | |
| IV, 15 days | 0.543 | 0.144 | 0 | 0 | 1 | 1 | 0.2 | 1 | | |
| VI, 1875.96 | — 1 | + 2 | | | | | | | | |
| For 1875, Dec. 15 | 320.655 | 179.739 | 153 | 167 | 489 | 123 | 401.6 | 56 | | |

| II. For 1878, April 3 = 1878.26. | | | | | | | | | | |
|----------------------------------|----------|---------|-------|----------|-----|-------|----------|-----|-------|---------|
| | <i>g</i> | | | ω | | | θ | | | Arg. 1 |
| | ° | ' | " | ° | ' | " | ° | ' | " | |
| Table II, 1876 | 331 | 34 | 18.70 | 95 | 11 | 42.33 | 73 | 22 | 1.03 | 148.576 |
| III, 2 Y. April | 9 | 37 | 53.62 | 0 | 1 | 11.23 | 0 | 0 | 41.75 | 97.643 |
| IV, 3d | | 2 | 6.70 | | | 0.26 | | | 0.15 | .357 |
| VI, 1878.26 | | | 2.62 | | | | | | | .421 |
| For 1878, April 3 | 341 | 14 | 21.64 | 95 | 12 | 53.82 | 73 | 22 | 42.93 | 246.997 |
| | Arg. 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | | |
| Table II, 1876 | 321.237 | 179.889 | 153 | 168 | 489 | 123 | 401.9 | 57 | | |
| III, 2 Y. April | 29.732 | 7.868 | 22 | 7 | 37 | 42 | 12.1 | 45 | | |
| IV, 3d | .109 | .029 | 0 | 0 | 0 | 0 | 0 | 0 | | |
| VI, 1878.26 | — .001 | + .002 | | | | | | | | |
| For 1878, April 3 | 351.077 | 187.788 | 175 | 175 | 526 | 165 | 414.0 | 102 | | |

We now fill in the values of *g*, the arguments 1—9, and the times with which Table XVII is to be entered, for the intermediate dates, by adding the nearly constant differences deduced from the numbers at the bottom of Table II. The seconds of *g* are first reduced to fractions of a minute, with which to enter Table VII. In making the subsequent computation we have used none of the devices previously described except in the case of the small longitude terms, as follows:

| | ° | ° | ° | ° | ° | ° | ° |
|---------------------------|-------|-------|-------|-------|-------|-------|-------|
| g | 330 | 332 | 334 | 336 | 338 | 340 | 342 |
| $3g$ | 270 | 276 | 282 | 288 | 294 | 300 | 306 |
| $4g$ | 240 | 248 | 256 | 264 | 272 | 280 | 288 |
| | " | " | " | " | " | " | " |
| (<i>v.s.3</i>) | -1.14 | -1.12 | -1.13 | -1.16 | -1.23 | -1.33 | -1.48 |
| (<i>v.c.3</i>) | -2.46 | -2.56 | -2.66 | -2.76 | -2.88 | -3.01 | -3.16 |
| (<i>v.s.4</i>) | +0.59 | +0.60 | +0.59 | +0.55 | +0.50 | +0.43 | +0.36 |
| (<i>v.c.4</i>) | -0.10 | -0.20 | -0.30 | -0.40 | -0.47 | -0.57 | -0.64 |
| (<i>v.s.3</i>) sin $3g$ | +1.14 | +1.12 | +1.11 | +1.11 | +1.12 | +1.15 | +1.20 |
| (<i>v.c.3</i>) cos $3g$ | .00 | -0.26 | -0.55 | -0.85 | -1.17 | -1.50 | -1.86 |
| (<i>v.s.4</i>) sin $4g$ | -.51 | -.56 | -.57 | -.55 | -.50 | -.42 | -.34 |
| (<i>v.c.4</i>) cos $4g$ | +.05 | +.07 | +.07 | +.04 | -.02 | -.10 | -.19 |
| Sum | +0.68 | +0.37 | +0.06 | -0.25 | -0.57 | -0.87 | -1.19 |

It will be seen that we have here computed twice as many numbers as are necessary to interpolate with all attainable accuracy.

The rest of the computation is fully given on the four following pages. First we have the values of g and the nine arguments for the intermediate dates, filled in by successive addition of the nearly constant difference. The arguments thus obtained for the last date may be compared with those just computed on the preceding page.

The numerals in the first columns of the sections of computation following indicate the arguments with which tables are entered to obtain the separate values of the quantities (*v.c.0*), (*v.s.1*), (*v.c.1*), etc. The negative terms in Table XVII being taken from the sum of all the periodic terms from Tables VIII to XVI with argument 1 to 9, we have the final values of (*v.c.0*), (*v.s.1*), etc.

The final computation of the products (*v.s.i*) sin ig , etc., and the addition of the separate terms which make up the three co-ordinates, are shown on page 205. The expressions *c.0*, *s.1*, etc., are employed for brevity, instead of (*v.c.0*), (*v.s.1*) sin g , etc.

The longitude finally given by the tables is referred to the mean equinox, and must therefore be corrected for nutation before being used to compute the geocentric place.

| Date, { | 1875, Dec. 15
1875.955. | 1876, Apr. 13
1876.283. | Aug. 11
1876.612. | Dec. 9
1876.940. | 1877, Apr. 8
1877.269. | Aug. 6
1877.597. | Dec. 4
1877.925. | 1878, Apr. 3
1878.254. |
|-----------|----------------------------|----------------------------|----------------------|---------------------|---------------------------|---------------------|---------------------|---------------------------|
| | ° / | ° / | ° / | ° / | ° / | ° / | ° / | ° / |
| | 331 23.086 | 332 47.554 | 334 12.021 | 335 36.489 | 337 0.957 | 338 25.425 | 339 49.893 | 341 14.361 |
| Arg. 1 | 147.097 | 161.368 | 175.639 | 189.910 | 204.182 | 218.453 | 232.724 | 246.995 |
| 2 | 320.655 | 325.001 | 329.346 | 333.692 | 338.038 | 342.383 | 346.729 | 351.074 |
| 3 | 179.739 | 180.889 | 182.038 | 183.188 | 184.338 | 185.488 | 186.638 | 187.788 |
| 4 | 153 | 156. | 159. | 163. | 166. | 169. | 172. | 175. |
| 5 | 167. | | | | | | | 175. |
| 6 | 489. | 494. | 500. | 505. | 511. | 516. | 521. | 527. |
| 7 | 123. | 129. | 135. | 141. | 147. | 153. | 160. | 166. |
| 8 | 401.6 | 403.4 | 405.2 | 407.0 | 408.7 | 410.5 | 412.3 | 414.1 |
| 9 | 56. | 63. | 69. | 75. | 82. | 88. | 95. | 101. |
| (v.c.0) 1 | 108.01 | 107.74 | 106.29 | 103.68 | 99.98 | 95.25 | 89.62 | 83.21 |
| 2 | 15.80 | 14.71 | 13.66 | 12.65 | 11.68 | 10.76 | 9.90 | 9.09 |
| 3 | 72.43 | 73.60 | 74.77 | 75.94 | 77.10 | 78.26 | 79.42 | 80.56 |
| 5 | .59 | .58 | .58 | .57 | .56 | .56 | .55 | .54 |
| 6 | .02 | .02 | .02 | .03 | .03 | .03 | .03 | .03 |
| 7 | .07 | .07 | .06 | .06 | .06 | .06 | .05 | .05 |
| 8 | 1.68 | 1.65 | 1.63 | 1.60 | 1.58 | 1.55 | 1.53 | 1.50 |
| 9 | .09 | .09 | .09 | .09 | .08 | .08 | .08 | .07 |
| (v.c.0) | 198.69 | 198.46 | 197.10 | 194.62 | 191.07 | 186.55 | 181.18 | 175.05 |
| (v.s.1) 1 | 6.84 | 7.68 | 8.40 | 8.99 | 9.44 | 9.74 | 9.91 | 9.98 |
| 2 | 132.20 | 125.48 | 118.82 | 112.21 | 105.67 | 99.24 | 92.89 | 86.69 |
| sec. 2 | .18 | .20 | .22 | .23 | .25 | .27 | .29 | .31 |
| 3 | 1.09 | 1.07 | 1.08 | 1.12 | 1.19 | 1.29 | 1.41 | 1.56 |
| 4 | .13 | .12 | .11 | .10 | .10 | .09 | .09 | .08 |
| 5 | .19 | .19 | .19 | .19 | .19 | .20 | .20 | .20 |
| 6 | .04 | .04 | .04 | .05 | .05 | .06 | .06 | .07 |
| 7 | .14 | .14 | .13 | .13 | .12 | .12 | .11 | .10 |
| 8 | 1.28 | 1.28 | 1.27 | 1.26 | 1.26 | 1.25 | 1.24 | 1.23 |
| 9 | .08 | .08 | .08 | .08 | .07 | .07 | .07 | .07 |
| Σ | 142.17 | 136.28 | 130.34 | 124.36 | 118.34 | 112.33 | 106.27 | 100.29 |
| Tab. XVII | -154.21 | -153.84 | -153.47 | -153.10 | -152.73 | -152.36 | -151.99 | -151.62 |
| (v.s.1) | -12.04 | -17.56 | -23.13 | -28.74 | -34.39 | -40.03 | -45.72 | -51.33 |
| (v.c.1) 1 | 5.64 | 6.03 | 6.38 | 6.67 | 6.85 | 6.91 | 6.82 | 6.59 |
| 2 | 5.01 | 6.00 | 7.31 | 8.90 | 10.80 | 13.00 | 15.49 | 18.25 |
| sec. 2 | .69 | .71 | .72 | .74 | .75 | .77 | .79 | .80 |
| 3 | 23.20 | 24.09 | 24.95 | 25.82 | 26.68 | 27.53 | 28.37 | 29.19 |
| 4 | .47 | .48 | .49 | .50 | .50 | .51 | .52 | .52 |
| 5 | .37 | .37 | .37 | .37 | .37 | .37 | .37 | .37 |
| 6 | .16 | .17 | .17 | .18 | .18 | .18 | .18 | .18 |
| 7 | .15 | .14 | .13 | .13 | .12 | .12 | .11 | .10 |
| 8 | 0.66 | 0.64 | 0.63 | 0.61 | 0.60 | 0.59 | 0.57 | 0.56 |
| 9 | .07 | .07 | .06 | .06 | .05 | .04 | .04 | .03 |
| Σ | 36.42 | 38.70 | 41.21 | 43.98 | 46.90 | 50.02 | 53.26 | 56.59 |
| Tab. XVII | -205.86 | -206.08 | -206.29 | -206.51 | -206.73 | -206.94 | -207.15 | -207.37 |
| (v.c.1) | -169.44 | -167.38 | -165.08 | -162.53 | -159.83 | -156.92 | -153.89 | -150.78 |
| (v.s.2) 1 | .06 | .10 | .15 | .21 | .26 | .31 | .34 | .37 |
| 2 | 54.12 | 49.37 | 44.78 | 40.37 | 36.15 | 32.11 | 28.30 | 24.70 |
| sec. 2 | .39 | .42 | .44 | .47 | .49 | .51 | .53 | .56 |
| 3 | 2.05 | 2.19 | 2.31 | 2.45 | 2.60 | 2.76 | 2.91 | 3.07 |
| Σ | 56.62 | 52.08 | 47.68 | 43.50 | 39.50 | 35.69 | 32.08 | 28.70 |
| Tab. XVII | -134.34 | -134.32 | -134.31 | -134.29 | -134.27 | -134.26 | -134.24 | -134.22 |
| (v.s.2) | -77.72 | -82.24 | -86.63 | -90.79 | -94.77 | -98.57 | -102.16 | -105.52 |

| Date, { | 1875, Dec. 15
1875.955. | 1876, Apr. 13
1876.283. | Aug. 11
1876.612. | Dec. 9
1867.940. | 1877, Apr. 8
1877.269. | Aug. 6
1877.597. | Dec. 4
1877.925. | 1878, Apr. 3
1878.254. |
|-----------|----------------------------|----------------------------|----------------------|---------------------|---------------------------|---------------------|---------------------|---------------------------|
| | o / | o / | o / | o / | o / | o / | o / | o / |
| <i>g</i> | 331 23.086 | 332 47.554 | 334 12.021 | 335 36.489 | 337 0.057 | 338 25.425 | 339 49.893 | 341 14.361 |
| (v.c.2) 1 | .31 | .30 | .29 | .29 | .29 | .30 | .30 | .31 |
| 2 | 25.34 | 28.93 | 32.71 | 36.73 | 40.93 | 45.35 | 49.93 | 54.69 |
| sec. 2 | .76 | .76 | .77 | .78 | .79 | .79 | .80 | .80 |
| 3 | 6.23 | 6.40 | 6.56 | 6.70 | 6.85 | 6.99 | 7.10 | 7.20 |
| Σ | 32.64 | 36.39 | 40.33 | 44.50 | 48.86 | 53.43 | 58.13 | 63.00 |
| Tab. XVII | -136.57 | -136.59 | -136.60 | -136.62 | -136.64 | -136.65 | -136.67 | -136.69 |
| (v.c.2) | -103.93 | -100.20 | -96.27 | -92.12 | -87.78 | -83.22 | -78.54 | -73.69 |
| (v.s.3) 2 | 6.92 | 6.89 | 6.84 | 6.78 | 6.70 | 6.61 | 6.51 | 6.37 |
| 3 | 0.87 | 0.91 | 0.94 | 0.98 | 1.01 | 1.04 | 1.08 | 1.11 |
| Tab. XVII | 7.79 | 7.80 | 7.78 | 7.76 | 7.71 | 7.65 | 7.59 | 7.48 |
| (v.s.3) | -8.92 | -8.92 | -8.91 | -8.91 | -8.91 | -8.91 | -8.91 | -8.91 |
| (v.s.3) | -1.13 | -1.12 | -1.13 | -1.15 | -1.20 | -1.26 | -1.32 | -1.43 |
| (v.c.3) 2 | 3.19 | 3.12 | 3.05 | 2.98 | 2.90 | 2.82 | 2.75 | 2.67 |
| 3 | 1.34 | 1.34 | 1.34 | 1.34 | 1.33 | 1.33 | 1.31 | 1.29 |
| Tab. XVII | 4.53 | 4.46 | 4.39 | 4.32 | 4.23 | 4.15 | 4.06 | 3.96 |
| (v.c.3) | -7.06 | -7.06 | -7.06 | -7.06 | -7.06 | -7.06 | -7.06 | -7.06 |
| (v.c.3) | -2.53 | -2.60 | -2.67 | -2.74 | -2.83 | -2.91 | -3.00 | -3.10 |
| (v.s.4) 2 | 1.48 | 1.48 | 1.47 | 1.45 | 1.43 | 1.40 | 3.36 | 1.33 |
| 3 | -0.88 | -0.88 | -0.88 | -0.89 | -0.89 | -0.91 | -0.93 | - .94 |
| (v.s.4) | +0.60 | +0.60 | +0.59 | +0.56 | +0.54 | +0.49 | +0.43 | +0.39 |
| (v.c.4) 2 | 0.83 | 0.77 | 0.72 | 0.66 | 0.61 | 0.57 | 0.51 | 0.47 |
| 3 | -1.00 | -1.02 | -1.03 | -1.04 | -1.05 | -1.06 | -1.07 | -1.08 |
| (v.c.4) | -0.17 | -0.25 | -0.31 | -0.38 | -0.44 | -0.49 | -0.56 | -0.61 |
| (ρ.c.0) 1 | 1230 | 1063 | 899 | 741 | 594 | 460 | 344 | 246 |
| 2 | 98 | 99 | 101 | 104 | 108 | 113 | 118 | 126 |
| 3 | 11 | 10 | 9 | 9 | 8 | 8 | 8 | 9 |
| Tab. XVII | 968 | 968 | 968 | 967 | 967 | 967 | 966 | 966 |
| (ρ.c.0) | 2307 | 2140 | 1977 | 1821 | 1677 | 1548 | 1436 | 1347 |
| (ρ.s.1) 1 | 248 | 240 | 230 | 221 | 211 | 203 | 194 | 187 |
| 2 | 2835 | 2822 | 2807 | 2789 | 2769 | 2748 | 2723 | 2696 |
| 3 | 173 | 168 | 165 | 161 | 158 | 155 | 151 | 148 |
| Σ | 3256 | 3230 | 3202 | 3171 | 3138 | 3106 | 3068 | 3031 |
| Tab. XVII | -1984 | -1987 | -1989 | -1991 | -1993 | -1996 | -1998 | 2000 |
| (ρ.s.1) | +1272 | 1243 | 1213 | 1180 | 1145 | 1110 | 1070 | 1031 |
| (ρ.c.1) 1 | 123 | 112 | 97 | 83 | 69 | 55 | 41 | 31 |
| 2 | 1266 | 1202 | 1138 | 1074 | 1009 | 947 | 886 | 827 |
| 3 | 66 | 66 | 67 | 67 | 68 | 70 | 72 | 74 |
| Σ | 1455 | 1380 | 1302 | 1224 | 1146 | 1072 | 999 | 932 |
| Tab. XVII | -1977 | -1981 | -1986 | -1990 | -1994 | -1998 | -2002 | 2006 |
| (ρ.c.1) | - 522 | - 601 | - 684 | - 766 | - 848 | - 926 | -1003 | -1074 |

| Date, { | 1875, Dec. 15
1875.955. | 1876, Apr. 13
1876.283. | Aug. 11
1876.612. | Dec. 9
1867.940. | 1877, Apr. 8
1877.269. | Aug. 6
1877.597. | Dec. 4
1877.925. | 1878, Apr. 3
1878.254. |
|--------------------|----------------------------|----------------------------|----------------------|---------------------|---------------------------|---------------------|---------------------|---------------------------|
| <i>g</i> | 331 23.086 | 332 47.554 | 334 12.021 | 335 36.489 | 337 0.957 | 338 25.425 | 339 49.893 | 341 14.361 |
| (<i>p.s.</i> 2) 2 | 174 | 181 | 189 | 196 | 204 | 213 | 223 | 232 |
| 3 | 26 | 25 | 25 | 24 | 24 | 24 | 23 | 23 |
| Σ | 200 | 206 | 214 | 220 | 228 | 237 | 246 | 255 |
| Tab.XVII | -459 | -459 | -459 | -459 | -460 | -460 | -460 | -460 |
| (<i>p.s.</i> 2) | -259 | -253 | -245 | -239 | -232 | -223 | -214 | -205 |
| (<i>p.c.</i> 2) 2 | 559 | 568 | 576 | 584 | 592 | 600 | 607 | 614 |
| 3 | 33 | 33 | 34 | 35 | 36 | 37 | 39 | 40 |
| Σ | 592 | 601 | 610 | 619 | 628 | 637 | 646 | 654 |
| Tab.XVII | -464 | -464 | -464 | -465 | -465 | -465 | -465 | -466 |
| (<i>p.c.</i> 2) | +123 | 137 | 146 | 154 | 163 | 172 | 181 | 188 |
| (<i>p.s.</i> 3) 2 | 28 | 31 | 34 | 39 | 43 | 47 | 52 | 57 |
| Tab.XVII | -101 | -101 | -101 | -101 | -101 | -101 | -101 | -101 |
| (<i>p.s.</i> 3) | - 73 | - 70 | - 67 | - 62 | - 58 | - 54 | - 49 | - 44 |
| (<i>p.c.</i> 3) 2 | 155 | 159 | 162 | 166 | 169 | 172 | 175 | 177 |
| Tab.XVII | -102 | -102 | -102 | -102 | -102 | -102 | -102 | -102 |
| (<i>p.c.</i> 3) | + 53 | 57 | 60 | 64 | 67 | 70 | 73 | 75 |
| (<i>b.c.</i> 0) 2 | 0.12 | 0.12 | 0.13 | 0.13 | 0.14 | 0.14 | 0.15 | 0.15 |
| 3 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 |
| Tab.XXIII | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 |
| (<i>b.c.</i> 0) | 0.30 | 0.30 | 0.31 | 0.31 | 0.32 | 0.32 | 0.33 | 0.33 |
| (<i>b.s.</i> 1) 1 | 0.30 | 0.22 | 0.16 | 0.12 | 0.08 | 0.06 | 0.05 | 0.06 |
| 2 | 0.12 | 0.09 | 0.06 | 0.05 | 0.04 | 0.04 | 0.05 | 0.06 |
| 3 | 1.11 | 1.11 | 1.12 | 1.12 | 1.13 | 1.13 | 1.14 | 1.14 |
| Σ | 1.53 | 1.42 | 1.34 | 1.29 | 1.25 | 1.23 | 1.24 | 1.26 |
| Tab.XXIII | -5.23 | -5.24 | -5.24 | -5.24 | -5.24 | -5.25 | -5.25 | -5.25 |
| (<i>b.s.</i> 1) | -3.70 | -3.82 | -3.90 | -3.95 | -3.99 | -4.02 | -4.01 | -3.99 |
| (<i>b.c.</i> 1) 1 | 1.21 | 1.14 | 1.06 | 0.98 | 0.90 | 0.80 | 0.70 | 0.61 |
| 2 | 0.96 | 1.06 | 1.17 | 1.28 | 1.39 | 1.51 | 1.63 | 1.75 |
| 3 | 1.01 | 1.00 | 1.00 | 0.99 | 0.99 | 0.99 | 0.98 | 0.98 |
| Σ | 3.13 | 3.20 | 3.23 | 3.25 | 3.28 | 3.30 | 3.31 | 3.34 |
| Tab.XXIII | -4.45 | -4.44 | -4.43 | -4.43 | -4.42 | -4.41 | -4.41 | -4.40 |
| (<i>b.c.</i> 1) | -1.27 | -1.24 | -1.20 | -1.18 | -1.14 | -1.11 | -1.10 | -1.06 |
| (<i>b.s.</i> 2) 2 | 0.02 | 0.02 | 0.03 | 0.04 | 0.04 | 0.05 | 0.06 | 0.07 |
| 3 | 0.16 | 0.16 | 0.16 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 |
| Tab.XXIII | -0.48 | -0.48 | -0.48 | -0.48 | -0.48 | -0.48 | -0.48 | -0.48 |
| (<i>b.s.</i> 2) | -0.30 | -0.30 | -0.29 | -0.29 | -0.29 | -0.28 | -0.27 | -0.26 |
| (<i>b.c.</i> 2) 2 | 0.31 | 0.32 | 0.33 | 0.34 | 0.34 | 0.35 | 0.36 | 0.36 |
| 3 | 0.10 | 0.10 | 0.10 | 0.10 | 0.11 | 0.11 | 0.11 | 0.11 |
| Tab.XXIII | -0.38 | -0.38 | -0.38 | -0.38 | -0.38 | -0.38 | -0.38 | -0.38 |
| (<i>b.c.</i> 2) | +0.03 | +0.04 | +0.05 | +0.06 | +0.07 | +0.08 | +0.09 | +0.09 |

| Date, { | 1875, Dec. 15
1875.955. | 1876, April 13
1876.283. | Aug. 11
1876.612. | Dec. 9
1876.940. | 1877, April 8
1877.269. | Aug. 6
1877.597. | Dec. 4
1877.925. | 1878, April 3
1878.254. |
|---------------------------|----------------------------|-----------------------------|----------------------|---------------------|----------------------------|---------------------|---------------------|----------------------------|
| <i>g</i> | 331 23.086 | 332 47.554 | 334 12.021 | 335 36.489 | 337 0.957 | 338 25.425 | 339 49.893 | 341 14.361 |
| log (v.s.1) | -1.0806 | -1.2445 | -1.3642 | -1.4585 | -1.5365 | -1.6024 | -1.6601 | -1.7104 |
| sin <i>g</i> | -9.6803 | -9.6601 | -9.6386 | -9.6159 | -9.5916 | -9.5655 | -9.5375 | -9.5073 |
| log (p.s.1) | +3.1045 | 3.0944 | 3.0838 | 3.0719 | 3.0588 | 3.0453 | 3.0294 | 3.0132 |
| log (v.c.1) | -2.2290 | -2.2237 | -2.2177 | -2.2109 | -2.2036 | -2.1957 | -2.1872 | -2.1784 |
| cos <i>g</i> | +9.9434 | 9.9491 | 9.9544 | 9.9594 | 9.9641 | 9.9685 | 9.9725 | 9.9763 |
| log (p.c.1) | -2.718 | -2.779 | -2.835 | -2.884 | -2.928 | -2.967 | -3.0013 | -3.0310 |
| log (v.s.2) | -1.8905 | -1.9151 | -1.9376 | -1.9580 | -1.9766 | -1.9937 | -2.0093 | -2.0233 |
| sin 2 <i>g</i> | -9.9247 | -9.9102 | -9.8941 | -9.8763 | -9.8566 | -9.8350 | -9.8111 | -9.7846 |
| log (p.s.2) | -2.413 | -2.403 | -2.389 | -2.378 | -2.365 | -2.348 | -2.330 | -2.312 |
| log (v.c.2) | -2.0167 | -2.0009 | -1.9835 | -1.9644 | -1.9434 | -1.9202 | -1.8951 | -1.8674 |
| cos 2 <i>g</i> | +9.7335 | 9.7649 | 9.7932 | 9.8189 | 9.8421 | 9.8630 | 9.8821 | 9.8994 |
| log (p.c.2) | +2.107 | 2.137 | 2.164 | 2.187 | 2.212 | 2.236 | 2.258 | 2.274 |
| <i>Δg</i> | 1 24 28.065 | 1 24 28.066 | 1 24 28.066 | 1 24 28.067 | 1 24 28.068 | 1 24 28.068 | 1 24 28.069 | 1 24 28.069 |
| <i>g</i> | 331 23 5.16 | 332 47 33.225 | 334 12 1.291 | 335 36 29.357 | 337 0 57.424 | 338 25 25.492 | 339 49 53.560 | 341 14 21.63 |
| <i>ω</i> | 95 11 41.00 | 95 11 51.40 | 95 12 1.81 | 95 12 12.21 | 95 12 22.61 | 95 12 33.01 | 95 12 43.42 | 95 12 53.82 |
| <i>E</i> | -2 42 49.15 | -2 35 33.18 | -2 28 10.63 | -2 20 41.76 | -2 13 6.93 | -2 5 26.43 | -1 57 40.55 | -1 49 49.66 |
| <i>c.0</i> | 3 18.69 | 3 18.46 | 3 17.10 | 3 14.62 | 3 11.07 | 3 6.55 | 3 1.18 | 2 55.05 |
| <i>s.1</i> | 5.77 | 8.03 | 10.07 | 11.87 | 13.43 | 14.72 | 15.76 | 16.51 |
| <i>c.1</i> | -2 28.72 | -2 28.86 | -2 28.62 | -2 28.00 | -2 27.13 | -2 25.93 | -2 24.43 | -2 22.80 |
| <i>s.2</i> | 1 5.34 | 1 6.88 | 1 7.87 | 1 8.28 | 1 8.12 | 1 7.40 | 1 6.13 | 1 4.26 |
| <i>c.2</i> | - 56.27 | - 58.31 | - 59.80 | -1 0.71 | -1 1.03 | -1 0.70 | - 59.87 | - 58.45 |
| (3+4) | 0.46 | 0.25 | 0.03 | - 0.19 | - 0.41 | - 0.63 | - 0.83 | - 1.00 |
| <i>u</i> | 63 53 2.28 | 65 24 57.89 | 66 56 59.12 | 68 29 5.68 | 70 1 17.15 | 71 33 33.48 | 73 5 54.37 | 74 38 19.36 |
| <i>θ</i> | 73 22 0.15 | 73 22 6.26 | 73 22 12.37 | 73 22 18.48 | 73 22 24.60 | 73 22 30.71 | 73 22 36.82 | 73 22 42.93 |
| <i>R</i> | 2.58 | 2.91 | 3.25 | 3.61 | 3.98 | 4.37 | 4.79 | 5.22 |
| Longitude | 137 15 5.01 | 138 47 7.06 | 140 19 14.74 | 141 51 27.77 | 143 23 45.73 | 144 56 8.56 | 146 28 35.98 | 148 1 7.51 |
| log <i>r</i> ₀ | 1.2647392 | 1.2644735 | 1.2642196 | 1.2639778 | 1.2637486 | 1.2635319 | 1.2633280 | 1.2631370 |
| <i>c.0</i> | 2307 | 2140 | 1977 | 1821 | 1677 | 1548 | 1436 | 1347 |
| <i>s.1</i> | -609 | -568 | -523 | -487 | -447 | -408 | -369 | -332 |
| <i>c.1</i> | -458 | -535 | -615 | -697 | -780 | -861 | -941 | -1017 |
| <i>s.2</i> | +218 | 206 | 192 | 179 | 167 | 152 | 138 | 125 |
| <i>c.2</i> | + 69 | 80 | 91 | 101 | 113 | 126 | 138 | 149 |
| <i>s.3</i> | + 73 | 69 | 65 | 60 | 54 | 49 | 43 | 37 |
| <i>c.3</i> | + 4 | 8 | 13 | 19 | 23 | 29 | 36 | 41 |
| log <i>r</i> | 1.2648996 | 1.2646135 | 1.2643391 | 1.2640774 | 1.2638293 | 1.2635954 | 1.2633761 | 1.2631720 |
| <i>β</i> ₀ | +41 36.64 | 42 8.48 | 42 38.54 | 43 6.78 | 43 33.14 | 43 57.62 | 44 20.23 | 44 41.19 |
| <i>c.0</i> | +0.30 | 0.30 | 0.31 | 0.31 | 0.32 | 0.32 | 0.33 | 0.33 |
| <i>s.1</i> | +1.77 | 1.75 | 1.70 | 1.63 | 1.57 | 1.48 | 1.38 | 1.29 |
| <i>c.1</i> | -1.11 | -1.10 | -1.08 | -1.07 | -1.05 | -1.03 | -1.03 | -1.00 |
| <i>s.2</i> | +0.25 | 0.24 | 0.23 | 0.22 | 0.21 | 0.19 | 0.17 | 0.16 |
| <i>c.2</i> | +0.02 | 0.03 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.07 |
| Latitude | +41 37.87 | 42 9.70 | 42 39.73 | 43 7.91 | 43 34.24 | 43 58.64 | 44 21.15 | 44 42.04 |

TABLE I.—CORRECTIONS OF ARGUMENTS FOR PAST AND FUTURE CENTURIES.

| Century. | g | | | ω | | | θ | | | Arg. 1 | | |
|----------|-----|----|-------|----------|----|-------|----------|-----|----|--------|---------|---------|
| | ° | ' | " | ° | ' | " | ° | ' | " | | | |
| 0J | 207 | 15 | 59.32 | 343 | 52 | 17.36 | +108.00 | 351 | 6 | 26.89 | -148.32 | 408.924 |
| 100 | 275 | 45 | 34.18 | 344 | 46 | 54.12 | 102.00 | 351 | 34 | 55.39 | -140.08 | 552.952 |
| 200 | 344 | 15 | 9.04 | 345 | 41 | 24.88 | 96.00 | 352 | 3 | 32.13 | -131.84 | 96.980 |
| 300 | 52 | 44 | 43.90 | 346 | 35 | 49.64 | 90.00 | 352 | 32 | 17.11 | -123.60 | 241.008 |
| 400 | 121 | 14 | 18.76 | 347 | 30 | 8.40 | 84.00 | 353 | 1 | 10.33 | -115.36 | 385.036 |
| 500 | 189 | 43 | 53.62 | 348 | 24 | 21.16 | + 78.00 | 353 | 30 | 11.79 | -107.12 | 529.064 |
| 600 | 258 | 13 | 28.48 | 349 | 18 | 27.92 | 72.00 | 353 | 59 | 21.49 | - 98.88 | 73.092 |
| 700 | 326 | 43 | 3.34 | 350 | 12 | 28.68 | 66.00 | 354 | 28 | 39.43 | - 90.64 | 217.120 |
| 800 | 35 | 12 | 38.20 | 351 | 6 | 23.44 | 60.00 | 354 | 58 | 5.61 | - 82.40 | 361.148 |
| 900 | 103 | 42 | 13.06 | 352 | 0 | 12.20 | 54.00 | 355 | 27 | 40.03 | - 74.16 | 505.176 |
| 1000 | 172 | 11 | 47.92 | 352 | 53 | 54.96 | + 48.00 | 355 | 57 | 22.69 | - 65.92 | 49.204 |
| 1100 | 240 | 41 | 22.78 | 353 | 47 | 31.72 | 42.00 | 356 | 27 | 13.59 | - 57.68 | 193.232 |
| 1200 | 309 | 10 | 57.64 | 354 | 41 | 2.48 | 36.00 | 356 | 57 | 12.73 | - 49.44 | 337.260 |
| 1300 | 17 | 40 | 32.50 | 355 | 34 | 27.24 | 30.00 | 357 | 27 | 20.11 | - 41.20 | 481.288 |
| 1400 | 86 | 10 | 7.36 | 356 | 27 | 46.00 | 24.00 | 357 | 57 | 35.73 | - 32.96 | 25.316 |
| 1500J | 154 | 39 | 42.22 | 357 | 20 | 58.76 | + 18.00 | 358 | 27 | 59.59 | - 24.72 | 169.344 |
| 1500G | 154 | 32 | 39.89 | 357 | 20 | 57.89 | 18.00 | 358 | 27 | 59.08 | - 24.72 | 168.155 |
| 1600 | 223 | 2 | 14.75 | 358 | 14 | 4.65 | 12.00 | 358 | 58 | 31.18 | - 16.48 | 312.183 |
| 1700 | 291 | 31 | 7.37 | 359 | 7 | 5.33 | 6.00 | 359 | 29 | 11.47 | - 8.24 | 456.092 |
| 1800 | 0 | 0 | 0.00 | 0 | 0 | 0.00 | 0.00 | 0 | 0 | 0.00 | 0.00 | 0.000 |
| 1900 | 68 | 28 | 52.63 | 0 | 52 | 48.67 | - 6.00 | 0 | 30 | 56.77 | + 8.24 | 143.908 |
| 2000 | 136 | 58 | 27.49 | 1 | 45 | 31.43 | - 12.00 | 1 | 2 | 1.83 | 16.48 | 287.936 |
| 2100 | 205 | 27 | 20.11 | 2 | 38 | 8.11 | - 18.00 | 1 | 33 | 15.08 | 24.72 | 431.845 |
| 2200 | 273 | 56 | 12.74 | 3 | 30 | 38.78 | - 24.00 | 2 | 4 | 36.57 | + 32.96 | 575.755 |

TABLE II.—ARGUMENTS FOR THE BEGINNING OF EACH FOURTH YEAR 1752—1948.

| Year. | g | | | ω | | | θ | | | Arg. 1 |
|-------|-----|----|-------|----------|----|-------|----------|----|-------|---------|
| | ° | ' | " | ° | ' | " | ° | ' | " | |
| 1752 | 160 | 15 | 8.10 | 94 | 6 | 10.48 | 72 | 43 | 42.30 | 162.101 |
| 1756 | 177 | 23 | 31.10 | 94 | 8 | 17.46 | 72 | 44 | 56.25 | 335.862 |
| 1760 | 194 | 31 | 54.09 | 94 | 10 | 24.43 | 72 | 46 | 10.22 | 509.623 |
| 1764 | 211 | 40 | 17.09 | 94 | 12 | 31.39 | 72 | 47 | 24.21 | 83.384 |
| 1768 | 228 | 48 | 40.08 | 94 | 14 | 38.34 | 72 | 48 | 38.20 | 257.145 |
| 1772 | 245 | 57 | 3.08 | 94 | 16 | 45.28 | 72 | 49 | 52.21 | 430.907 |
| 1776 | 263 | 5 | 26.07 | 94 | 18 | 52.22 | 72 | 51 | 6.23 | 4.668 |
| 1780 | 280 | 13 | 49.06 | 94 | 20 | 59.14 | 72 | 52 | 20.27 | 178.429 |
| 1784 | 297 | 22 | 12.06 | 94 | 23 | 6.05 | 72 | 53 | 34.32 | 352.190 |
| 1788 | 314 | 30 | 35.05 | 94 | 25 | 12.96 | 72 | 54 | 48.38 | 525.951 |
| 1792 | 331 | 38 | 58.05 | 94 | 27 | 19.85 | 72 | 56 | 2.46 | 99.712 |
| 1796 | 348 | 47 | 21.04 | 94 | 29 | 26.74 | 72 | 57 | 16.54 | 273.473 |
| 1800 | 5 | 55 | 1.80 | 94 | 31 | 33.53 | 72 | 58 | 30.59 | 447.115 |

TABLE I.—*Continued.*

| Century. | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---------|---------|-----|-----|-----|-----|-------|-----|
| 0J | 191.528 | 299.485 | 280 | 512 | 102 | 61 | 470.0 | 15 |
| 100 | 314.245 | 49.520 | 64 | 250 | 563 | 124 | 416.6 | 214 |
| 200 | 436.962 | 399.555 | 449 | 589 | 424 | 188 | 351.2 | 413 |
| 300 | 559.679 | 149.590 | 233 | 327 | 285 | 251 | 291.7 | 13 |
| 400 | 82.396 | 499.625 | 18 | 65 | 146 | 314 | 232.3 | 212 |
| 500 | 205.113 | 249.660 | 402 | 403 | 7 | 378 | 172.9 | 411 |
| 600 | 327.839 | 599.695 | 187 | 142 | 468 | 441 | 113.5 | 10 |
| 700 | 450.547 | 349.730 | 571 | 480 | 329 | 504 | 54.1 | 209 |
| 800 | 573.264 | 99.765 | 356 | 218 | 190 | 567 | 594.6 | 409 |
| 900 | 95.981 | 449.800 | 140 | 556 | 51 | 31 | 535.2 | 8 |
| 1000 | 218.698 | 199.835 | 524 | 295 | 512 | 94 | 475.8 | 207 |
| 1100 | 341.415 | 549.870 | 309 | 33 | 373 | 157 | 416.4 | 406 |
| 1200 | 464.132 | 299.905 | 93 | 371 | 234 | 221 | 357.0 | 5 |
| 1300 | 586.849 | 49.940 | 478 | 109 | 95 | 284 | 297.5 | 205 |
| 1400 | 109.566 | 399.975 | 262 | 448 | 556 | 347 | 238.1 | 404 |
| 1500J | 232.283 | 150.010 | 47 | 186 | 417 | 411 | 178.6 | 3 |
| 1500G | 231.921 | 149.914 | 47 | 186 | 417 | 410 | 178.3 | 2 |
| 1600 | 354.638 | 499.949 | 431 | 524 | 278 | 473 | 118.8 | 201 |
| 1700 | 477.319 | 249.975 | 216 | 262 | 139 | 537 | 59.4 | 401 |
| 1800 | 0.000 | 0.000 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1900 | 122.681 | 350.025 | 385 | 338 | 461 | 63 | 540.6 | 199 |
| 2000 | 245.398 | 100.060 | 169 | 76 | 322 | 127 | 481.2 | 398 |
| 2100 | 368.079 | 450.086 | 554 | 415 | 183 | 190 | 421.8 | 597 |
| 2200 | 490.760 | 200.111 | 338 | 153 | 44 | 253 | 362.3 | 196 |

TABLE II.—*Continued.*

| Year. | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|---------|---------|-----|-----|-----|-----|-------|-----|
| 1752 | 481.104 | 345.855 | 133 | 348 | 229 | 213 | 331.6 | 578 |
| 1756 | 534.013 | 359.856 | 172 | 362 | 296 | 287 | 353.2 | 58 |
| 1760 | 586.921 | 373.858 | 212 | 375 | 362 | 362 | 374.8 | 138 |
| 1764 | 39.830 | 387.859 | 251 | 389 | 429 | 436 | 396.4 | 218 |
| 1768 | 92.739 | 401.860 | 290 | 402 | 495 | 511 | 418.1 | 298 |
| 1772 | 145.647 | 415.862 | 330 | 416 | 562 | 585 | 439.7 | 377 |
| 1776 | 198.556 | 429.863 | 369 | 429 | 28 | 60 | 461.3 | 457 |
| 1780 | 251.465 | 443.865 | 408 | 443 | 95 | 134 | 482.9 | 537 |
| 1784 | 304.373 | 457.866 | 448 | 457 | 161 | 209 | 504.6 | 17 |
| 1788 | 357.282 | 471.868 | 487 | 470 | 227 | 283 | 526.2 | 97 |
| 1792 | 410.191 | 485.869 | 527 | 484 | 294 | 358 | 547.8 | 177 |
| 1796 | 463.100 | 499.870 | 566 | 497 | 360 | 433 | 569.4 | 257 |
| 1800 | 515.972 | 513.862 | 5 | 511 | 427 | 507 | 591.1 | 337 |

TABLE II.—Continued.

| Year. | g | | | ω | | | θ | | | Arg. l . |
|----------------------|-----|----|---------|----------|----|---------|----------|----|---------|------------|
| | ° | ' | " | ° | ' | " | ° | ' | " | |
| 1800 | 5 | 55 | 1.80 | 94 | 31 | 33.53 | 72 | 58 | 30.59 | 447.115 |
| 1804 | 23 | 3 | 24.80 | 94 | 33 | 40.39 | 72 | 59 | 44.71 | 20.876 |
| 1808 | 40 | 11 | 47.79 | 94 | 35 | 47.25 | 73 | 0 | 58.84 | 194.638 |
| 1812 | 57 | 20 | 10.79 | 94 | 37 | 54.09 | 73 | 2 | 12.98 | 368.399 |
| 1816 | 74 | 28 | 33.78 | 94 | 40 | 0.93 | 73 | 3 | 27.13 | 542.160 |
| 1820 | 91 | 36 | 56.77 | 94 | 42 | 7.76 | 73 | 4 | 41.30 | 115.921 |
| 1824 | 108 | 45 | 19.77 | 94 | 44 | 14.58 | 73 | 5 | 55.48 | 289.683 |
| 1828 | 125 | 53 | 42.76 | 94 | 46 | 21.38 | 73 | 7 | 9.67 | 463.444 |
| 1832 | 143 | 2 | 5.76 | 94 | 48 | 28.18 | 73 | 8 | 23.88 | 37.205 |
| 1836 | 160 | 10 | 28.75 | 94 | 50 | 34.97 | 73 | 9 | 38.10 | 210.966 |
| 1840 | 177 | 18 | 51.75 | 94 | 52 | 41.75 | 73 | 10 | 52.33 | 384.727 |
| 1844 | 194 | 27 | 14.74 | 94 | 54 | 48.52 | 73 | 12 | 6.58 | 558.488 |
| 1848 | 211 | 35 | 37.74 | 94 | 56 | 55.28 | 73 | 13 | 20.84 | 132.249 |
| 1852 | 228 | 44 | 0.73 | 94 | 59 | 2.03 | 73 | 14 | 35.11 | 306.010 |
| 1856 | 245 | 52 | 23.73 | 95 | 1 | 8.77 | 73 | 15 | 49.40 | 479.771 |
| 1860 | 263 | 0 | 46.72 | 95 | 3 | 15.50 | 73 | 17 | 3.70 | 53.532 |
| 1864 | 280 | 9 | 9.71 | 95 | 5 | 22.22 | 73 | 18 | 18.01 | 227.293 |
| 1868 | 297 | 17 | 32.71 | 95 | 7 | 28.94 | 73 | 19 | 32.34 | 401.054 |
| 1872 | 314 | 25 | 55.70 | 95 | 9 | 35.64 | 73 | 20 | 46.67 | 574.815 |
| 1876 | 331 | 34 | 18.70 | 95 | 11 | 42.33 | 73 | 22 | 1.03 | 148.576 |
| 1880 | 348 | 42 | 41.69 | 95 | 13 | 49.01 | 73 | 23 | 15.39 | 322.337 |
| 1884 | 5 | 51 | 4.69 | 95 | 15 | 55.69 | 73 | 24 | 29.77 | 496.098 |
| 1888 | 22 | 59 | 27.68 | 95 | 18 | 2.35 | 73 | 25 | 44.16 | 69.860 |
| 1892 | 40 | 7 | 50.68 | 95 | 20 | 9.01 | 73 | 26 | 58.57 | 243.621 |
| 1896 | 57 | 16 | 13.67 | 95 | 22 | 15.65 | 73 | 28 | 12.98 | 417.382 |
| 1900 | 74 | 23 | 54.43 | 95 | 24 | 22.20 | 73 | 29 | 27.36 | 591.024 |
| 1904 | 91 | 32 | 17.43 | 95 | 26 | 28.83 | 73 | 30 | 41.81 | 164.785 |
| 1908 | 108 | 40 | 40.42 | 95 | 28 | 35.44 | 73 | 31 | 56.26 | 338.546 |
| 1912 | 125 | 49 | 3.41 | 95 | 30 | 42.05 | 73 | 33 | 10.74 | 512.307 |
| 1916 | 142 | 57 | 26.41 | 95 | 32 | 48.64 | 73 | 34 | 25.22 | 86.068 |
| 1920 | 160 | 5 | 49.40 | 95 | 34 | 55.23 | 73 | 35 | 39.72 | 259.830 |
| 1924 | 177 | 14 | 12.40 | 95 | 37 | 1.81 | 73 | 36 | 54.23 | 433.591 |
| 1928 | 194 | 22 | 35.39 | 95 | 39 | 8.38 | 73 | 38 | 8.75 | 7.352 |
| 1932 | 211 | 30 | 58.39 | 95 | 41 | 14.94 | 73 | 39 | 23.29 | 181.113 |
| 1936 | 228 | 39 | 21.38 | 95 | 43 | 21.48 | 73 | 40 | 37.84 | 354.874 |
| 1940 | 245 | 47 | 44.38 | 95 | 45 | 28.02 | 73 | 41 | 52.40 | 528.635 |
| 1944 | 262 | 56 | 7.37 | 95 | 47 | 34.55 | 73 | 43 | 6.97 | 102.396 |
| 1948 | 280 | 4 | 30.36 | 95 | 49 | 41.07 | 73 | 44 | 21.56 | 276.158 |
| $\Delta_{120}^{(1)}$ | 1 | 24 | 28.007 | | | 10.411 | | | 6.100 | 14.2715 |
| Factor T | | | +0.222 | | | —0.020 | | | +0.027 | —0.0012 |
| $\Delta_{120}^{(2)}$ | | | +0.0007 | | | —0.0001 | | | +0.0001 | 0 |

TABLE II.—Continued.

| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|---------|---------|-----|-----|-----|-----|-------|-----|
| 1800 | 515.972 | 513.862 | 5 | 511 | 427 | 507 | 591.1 | 337 |
| 1804 | 568.881 | 527.863 | 45 | 524 | 493 | 582 | 12.7 | 417 |
| 1808 | 21.790 | 541.865 | 84 | 538 | 560 | 56 | 34.3 | 497 |
| 1812 | 74.698 | 555.866 | 124 | 551 | 26 | 131 | 55.9 | 577 |
| 1816 | 127.607 | 569.868 | 163 | 565 | 93 | 205 | 77.5 | 57 |
| 1820 | 180.516 | 583.869 | 202 | 578 | 159 | 280 | 99.2 | 137 |
| 1824 | 233.424 | 597.870 | 242 | 592 | 225 | 354 | 120.8 | 217 |
| 1828 | 286.333 | 11.872 | 281 | 6 | 292 | 429 | 142.4 | 297 |
| 1832 | 339.242 | 25.874 | 320 | 19 | 358 | 503 | 164.0 | 377 |
| 1836 | 392.150 | 39.875 | 360 | 32 | 425 | 578 | 185.7 | 457 |
| 1840 | 445.059 | 53.876 | 399 | 46 | 491 | 52 | 207.3 | 537 |
| 1844 | 497.968 | 67.878 | 439 | 59 | 557 | 127 | 228.9 | 17 |
| 1848 | 550.876 | 81.879 | 478 | 73 | 24 | 201 | 250.5 | 97 |
| 1852 | 3.785 | 95.880 | 517 | 87 | 90 | 276 | 272.1 | 177 |
| 1856 | 56.694 | 109.882 | 557 | 160 | 157 | 350 | 293.8 | 257 |
| 1860 | 109.602 | 123.883 | 596 | 114 | 223 | 425 | 315.4 | 337 |
| 1864 | 162.511 | 137.885 | 35 | 127 | 290 | 499 | 337.0 | 417 |
| 1868 | 215.420 | 151.886 | 75 | 141 | 356 | 574 | 358.6 | 497 |
| 1872 | 268.328 | 165.888 | 114 | 154 | 423 | 49 | 380.3 | 577 |
| 1876 | 321.237 | 179.889 | 153 | 168 | 489 | 123 | 401.9 | 57 |
| 1880 | 374.146 | 193.890 | 193 | 181 | 555 | 198 | 423.5 | 137 |
| 1884 | 427.054 | 207.892 | 232 | 195 | 22 | 272 | 445.1 | 217 |
| 1888 | 479.963 | 221.893 | 271 | 208 | 88 | 347 | 466.7 | 297 |
| 1892 | 532.872 | 235.895 | 311 | 222 | 155 | 421 | 488.4 | 376 |
| 1896 | 585.780 | 249.896 | 350 | 235 | 221 | 496 | 510.0 | 456 |
| 1900 | 38.653 | 263.889 | 390 | 249 | 287 | 570 | 531.6 | 536 |
| 1904 | 91.562 | 277.890 | 429 | 262 | 354 | 45 | 553.2 | 16 |
| 1908 | 144.470 | 291.891 | 468 | 276 | 420 | 119 | 574.9 | 96 |
| 1912 | 197.379 | 305.892 | 508 | 290 | 487 | 194 | 596.5 | 176 |
| 1916 | 250.288 | 319.894 | 547 | 303 | 553 | 268 | 18.1 | 256 |
| 1920 | 303.197 | 333.895 | 587 | 317 | 20 | 343 | 39.7 | 336 |
| 1924 | 356.105 | 347.897 | 26 | 330 | 86 | 417 | 61.4 | 416 |
| 1928 | 409.014 | 361.898 | 65 | 344 | 152 | 492 | 83.0 | 496 |
| 1932 | 461.923 | 375.900 | 105 | 357 | 219 | 566 | 104.6 | 576 |
| 1936 | 514.831 | 389.901 | 144 | 371 | 285 | 41 | 126.2 | 56 |
| 1940 | 567.740 | 403.902 | 184 | 384 | 352 | 116 | 147.8 | 136 |
| 1944 | 20.649 | 417.904 | 223 | 398 | 418 | 190 | 169.5 | 216 |
| 1948 | 73.557 | 431.905 | 262 | 411 | 485 | 265 | 191.1 | 296 |
| | 4.3457 | 1.1500 | 3.2 | 1.1 | 5.4 | 6.1 | 1.78 | 6.6 |
| | — .0001 | + .0002 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

TABLE III.—REDUCTION OF THE EPOCHS AND ARGUMENTS TO THE BEGINNING OF EACH MONTH
IN A CYCLE OF FOUR YEARS.

| | <i>g</i> | | | ω | | | θ | | | θ' | Arg. 1 | |
|---------------|----------|----|-------|----------|---|------|----------|---|------|-----------|--------|---------|
| | ° | ' | " | ° | ' | " | ° | ' | " | | | |
| Year 0 | | | | | | | | | | | | |
| January 0 | 0 | 0 | 0.00 | 0 | 0 | 0.00 | 0 | 0 | 0.00 | 0.00 | 0.000 | |
| February 0 | 0 | 21 | 49.24 | 0 | 0 | 2.69 | 0 | 0 | 1.58 | 0.00 | 3.687 | |
| March 0 | 0 | 42 | 14.00 | 0 | 0 | 5.21 | 0 | 0 | 3.05 | 0.00 | 7.136 | |
| April 0 | 0 | 1 | 4 | 3.24 | 0 | 0 | 7.89 | 0 | 0 | 4.63 | 0.01 | 10.823 |
| May 0 | 0 | 1 | 25 | 10.24 | 0 | 0 | 10.50 | 0 | 0 | 6.15 | 0.01 | 14.391 |
| June 0 | 0 | 1 | 46 | 59.48 | 0 | 0 | 13.19 | 0 | 0 | 7.73 | 0.01 | 18.078 |
| July 0 | 0 | 2 | 8 | 6.48 | 0 | 0 | 15.79 | 0 | 0 | 9.25 | 0.01 | 21.647 |
| August 0 | 0 | 2 | 29 | 55.71 | 0 | 0 | 18.48 | 0 | 0 | 10.83 | 0.01 | 25.333 |
| September 0 | 0 | 2 | 51 | 44.95 | 0 | 0 | 21.17 | 0 | 0 | 12.40 | 0.01 | 29.019 |
| October 0 | 0 | 3 | 12 | 51.95 | 0 | 0 | 23.77 | 0 | 0 | 13.93 | 0.02 | 32.587 |
| November 0 | 0 | 3 | 34 | 41.19 | 0 | 0 | 26.46 | 0 | 0 | 15.51 | 0.02 | 36.274 |
| December 0 | 0 | 3 | 55 | 48.19 | 0 | 0 | 29.06 | 0 | 0 | 17.03 | 0.02 | 39.842 |
| Year 1 | | | | | | | | | | | | |
| January 0 | 0 | 4 | 17 | 37.42 | 0 | 0 | 31.75 | 0 | 0 | 18.61 | 0.02 | 43.529 |
| February 0 | 0 | 4 | 39 | 26.66 | 0 | 0 | 34.44 | 0 | 0 | 20.18 | 0.02 | 47.216 |
| March 0 | 0 | 4 | 59 | 9.19 | 0 | 0 | 36.87 | 0 | 0 | 21.61 | 0.03 | 50.546 |
| April 0 | 0 | 5 | 20 | 58.43 | 0 | 0 | 39.56 | 0 | 0 | 23.18 | 0.03 | 54.233 |
| May 0 | 0 | 5 | 42 | 5.43 | 0 | 0 | 42.16 | 0 | 0 | 24.71 | 0.03 | 57.801 |
| June 0 | 0 | 6 | 3 | 54.67 | 0 | 0 | 44.85 | 0 | 0 | 26.28 | 0.03 | 61.488 |
| July 0 | 0 | 6 | 25 | 1.67 | 0 | 0 | 47.46 | 0 | 0 | 27.81 | 0.03 | 65.056 |
| August 0 | 0 | 6 | 46 | 50.90 | 0 | 0 | 50.14 | 0 | 0 | 29.38 | 0.04 | 68.742 |
| September 0 | 0 | 7 | 8 | 40.14 | 0 | 0 | 52.83 | 0 | 0 | 30.96 | 0.04 | 72.429 |
| October 0 | 0 | 7 | 29 | 47.14 | 0 | 0 | 55.44 | 0 | 0 | 32.48 | 0.04 | 75.997 |
| November 0 | 0 | 7 | 51 | 36.38 | 0 | 0 | 58.13 | 0 | 0 | 34.06 | 0.04 | 79.684 |
| December 0 | 0 | 8 | 12 | 43.38 | 0 | 1 | 0.73 | 0 | 0 | 35.59 | 0.04 | 83.252 |
| Year 2 | | | | | | | | | | | | |
| January 0 | 0 | 8 | 34 | 32.61 | 0 | 1 | 3.43 | 0 | 0 | 37.16 | 0.04 | 86.939 |
| February 0 | 0 | 8 | 56 | 21.85 | 0 | 1 | 6.11 | 0 | 0 | 38.74 | 0.05 | 90.626 |
| March 0 | 0 | 9 | 16 | 4.38 | 0 | 1 | 8.54 | 0 | 0 | 40.16 | 0.05 | 93.956 |
| April 0 | 0 | 9 | 37 | 53.62 | 0 | 1 | 11.23 | 0 | 0 | 41.74 | 0.05 | 97.643 |
| May 0 | 0 | 9 | 59 | 0.62 | 0 | 1 | 13.83 | 0 | 0 | 43.26 | 0.05 | 101.211 |
| June 0 | 0 | 10 | 20 | 49.86 | 0 | 1 | 16.52 | 0 | 0 | 44.84 | 0.05 | 104.898 |
| July 0 | 0 | 10 | 41 | 56.86 | 0 | 1 | 19.12 | 0 | 0 | 46.36 | 0.06 | 108.466 |
| August 0 | 0 | 11 | 3 | 46.09 | 0 | 1 | 21.81 | 0 | 0 | 47.94 | 0.06 | 112.153 |
| September 0 | 0 | 11 | 25 | 35.33 | 0 | 1 | 24.50 | 0 | 0 | 49.52 | 0.06 | 115.840 |
| October 0 | 0 | 11 | 46 | 42.33 | 0 | 1 | 27.10 | 0 | 0 | 51.04 | 0.06 | 119.407 |
| November 0 | 0 | 12 | 8 | 31.57 | 0 | 1 | 29.79 | 0 | 0 | 52.62 | 0.06 | 123.094 |
| December 0 | 0 | 12 | 29 | 38.57 | 0 | 1 | 32.40 | 0 | 0 | 54.14 | 0.07 | 126.662 |
| Year 3 | | | | | | | | | | | | |
| January 0 | 0 | 12 | 51 | 27.80 | 0 | 1 | 35.08 | 0 | 0 | 55.72 | 0.07 | 130.349 |
| February 0 | 0 | 13 | 13 | 17.04 | 0 | 1 | 37.77 | 0 | 0 | 57.29 | 0.07 | 134.036 |
| March 0 | 0 | 13 | 32 | 59.57 | 0 | 1 | 40.20 | 0 | 0 | 58.72 | 0.07 | 137.366 |
| April 0 | 0 | 13 | 54 | 48.81 | 0 | 1 | 42.89 | 0 | 1 | 0.29 | 0.07 | 141.053 |
| May 0 | 0 | 14 | 15 | 55.81 | 0 | 1 | 45.50 | 0 | 1 | 1.82 | 0.08 | 144.621 |
| June 0 | 0 | 14 | 37 | 45.05 | 0 | 1 | 48.18 | 0 | 1 | 3.40 | 0.08 | 148.308 |
| July 0 | 0 | 14 | 58 | 52.05 | 0 | 1 | 50.79 | 0 | 1 | 4.92 | 0.08 | 151.876 |
| August 0 | 0 | 15 | 20 | 41.28 | 0 | 1 | 53.48 | 0 | 1 | 6.50 | 0.08 | 155.563 |
| September 0 | 0 | 15 | 42 | 30.52 | 0 | 1 | 56.17 | 0 | 1 | 8.07 | 0.08 | 159.249 |
| October 0 | 0 | 16 | 3 | 37.52 | 0 | 1 | 58.77 | 0 | 1 | 9.60 | 0.08 | 162.817 |
| November 0 | 0 | 16 | 25 | 26.76 | 0 | 2 | 1.46 | 0 | 1 | 11.17 | 0.09 | 166.504 |
| December 0 | 0 | 16 | 46 | 33.76 | 0 | 2 | 4.06 | 0 | 1 | 12.70 | 0.09 | 170.072 |

TABLE III.—Continued.

| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---------------|--------|--------|----|----|----|----|------|----|
| Year 0 | | | | | | | | |
| January 0 | 0.000 | 0.000 | 0 | 0 | 0 | 0 | 0.0 | 0 |
| February 0 | 1.123 | 0.297 | 1 | 0 | 1 | 1 | 0.5 | 2 |
| March 0 | 2.209 | 0.575 | 2 | 1 | 3 | 3 | 0.9 | 3 |
| April 0 | 3.259 | 0.872 | 2 | 1 | 4 | 4 | 1.3 | 5 |
| May 0 | 4.382 | 1.159 | 3 | 1 | 5 | 6 | 1.8 | 7 |
| June 0 | 5.505 | 1.457 | 4 | 1 | 7 | 7 | 2.2 | 8 |
| July 0 | 6.591 | 1.745 | 5 | 2 | 8 | 9 | 2.7 | 10 |
| August 0 | 7.714 | 2.042 | 6 | 2 | 10 | 10 | 3.1 | 12 |
| September 0 | 8.837 | 2.339 | 7 | 2 | 11 | 12 | 3.6 | 13 |
| October 0 | 9.923 | 2.626 | 7 | 3 | 12 | 14 | 4.0 | 15 |
| November 0 | 11.046 | 2.923 | 8 | 3 | 14 | 15 | 4.5 | 17 |
| December 0 | 12.132 | 3.211 | 9 | 3 | 15 | 17 | 4.9 | 18 |
| Year 1 | | | | | | | | |
| January 0 | 13.254 | 3.508 | 10 | 3 | 17 | 19 | 5.4 | 20 |
| February 0 | 14.377 | 3.805 | 11 | 4 | 18 | 20 | 5.9 | 22 |
| March 0 | 15.391 | 4.073 | 11 | 4 | 19 | 22 | 6.3 | 23 |
| April 0 | 16.514 | 4.370 | 12 | 4 | 21 | 23 | 6.7 | 25 |
| May 0 | 17.600 | 4.658 | 13 | 4 | 22 | 25 | 7.2 | 27 |
| June 0 | 18.723 | 4.955 | 14 | 5 | 24 | 27 | 7.6 | 28 |
| July 0 | 19.809 | 5.242 | 15 | 5 | 25 | 28 | 8.1 | 30 |
| August 0 | 20.932 | 5.539 | 15 | 5 | 26 | 30 | 8.5 | 32 |
| September 0 | 22.054 | 5.836 | 16 | 6 | 28 | 31 | 9.0 | 33 |
| October 0 | 23.140 | 6.124 | 17 | 6 | 29 | 33 | 9.4 | 35 |
| November 0 | 24.263 | 6.421 | 18 | 6 | 30 | 34 | 9.9 | 37 |
| December 0 | 25.349 | 6.709 | 19 | 6 | 32 | 36 | 10.3 | 38 |
| Year 2 | | | | | | | | |
| January 0 | 26.472 | 7.006 | 20 | 7 | 33 | 37 | 10.8 | 40 |
| February 0 | 27.595 | 7.303 | 20 | 7 | 34 | 39 | 11.3 | 42 |
| March 0 | 28.609 | 7.571 | 21 | 7 | 36 | 40 | 11.7 | 43 |
| April 0 | 29.732 | 7.868 | 22 | 7 | 37 | 42 | 12.1 | 45 |
| May 0 | 30.818 | 8.156 | 23 | 8 | 38 | 44 | 12.6 | 47 |
| June 0 | 31.941 | 8.453 | 24 | 8 | 40 | 45 | 13.0 | 48 |
| July 0 | 33.027 | 8.741 | 25 | 8 | 41 | 47 | 13.5 | 50 |
| August 0 | 34.150 | 9.038 | 25 | 9 | 43 | 48 | 13.9 | 52 |
| September 0 | 35.272 | 9.335 | 26 | 9 | 44 | 50 | 14.4 | 53 |
| October 0 | 36.358 | 9.622 | 27 | 9 | 45 | 51 | 14.8 | 55 |
| November 0 | 37.481 | 9.919 | 28 | 9 | 47 | 53 | 15.3 | 57 |
| December 0 | 38.567 | 10.207 | 29 | 10 | 48 | 55 | 15.7 | 58 |
| Year 3 | | | | | | | | |
| January 0 | 39.690 | 10.504 | 30 | 10 | 50 | 56 | 16.2 | 60 |
| February 0 | 40.813 | 10.801 | 30 | 10 | 51 | 58 | 16.7 | 62 |
| March 0 | 41.827 | 11.070 | 31 | 11 | 52 | 59 | 17.1 | 63 |
| April 0 | 42.950 | 11.367 | 32 | 11 | 54 | 61 | 17.5 | 65 |
| May 0 | 44.036 | 11.654 | 33 | 11 | 55 | 62 | 18.0 | 67 |
| June 0 | 45.159 | 11.951 | 34 | 11 | 57 | 64 | 18.4 | 68 |
| July 0 | 46.245 | 12.239 | 35 | 12 | 58 | 65 | 18.9 | 70 |
| August 0 | 47.368 | 12.536 | 35 | 12 | 59 | 67 | 19.3 | 72 |
| September 0 | 48.490 | 12.833 | 36 | 12 | 61 | 68 | 19.8 | 73 |
| October 0 | 49.576 | 13.121 | 37 | 12 | 62 | 70 | 20.2 | 75 |
| November 0 | 50.699 | 13.418 | 38 | 13 | 64 | 72 | 20.7 | 77 |
| December 0 | 51.785 | 13.705 | 39 | 13 | 65 | 73 | 21.1 | 78 |

TABLE IV.—MOTION OF ARGUMENTS FOR DAYS.

| Days. | <i>g</i> | ω | θ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|----------|----------|----------|-------|-------|-------|---|---|---|---|-----|---|
| 1 | 0 42.23 | 0.09 | 0.05 | 0.119 | 0.036 | 0.010 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 24.47 | 0.17 | 0.10 | 0 238 | 0.072 | 0.019 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 2 6.70 | 0.26 | 0.15 | 0.357 | 0.109 | 0.029 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 2 48.93 | 0.35 | 0.20 | 0.476 | 0.145 | 0.039 | 0 | 0 | 0 | 0 | 0.1 | 0 |
| 5 | 3 31.17 | 0.43 | 0.25 | 0.595 | 0.181 | 0.048 | 0 | 0 | 0 | 0 | 0.1 | 0 |
| 6 | 4 13.40 | 0.52 | 0.30 | 0.714 | 0.217 | 0.058 | 0 | 0 | 0 | 0 | 0.1 | 0 |
| 7 | 4 55.63 | 0.61 | 0.36 | 0.833 | 0.253 | 0.067 | 0 | 0 | 0 | 0 | 0.1 | 0 |
| 8 | 5 37.87 | 0.69 | 0.41 | 0.951 | 0.290 | 0.077 | 0 | 0 | 0 | 0 | 0.1 | 0 |
| 9 | 6 20.10 | 0.78 | 0.46 | 1.070 | 0.326 | 0.086 | 0 | 0 | 0 | 0 | 0.1 | 0 |
| 10 | 7 2.33 | 0.87 | 0.51 | 1.189 | 0.362 | 0.096 | 0 | 0 | 1 | 1 | 0.1 | 1 |
| 11 | 7 44.57 | 0.95 | 0.56 | 1.308 | 0.398 | 0.105 | 0 | 0 | 1 | 1 | 0.2 | 1 |
| 12 | 8 26.80 | 1.04 | 0.61 | 1.427 | 0.434 | 0.115 | 0 | 0 | 1 | 1 | 0.2 | 1 |
| 13 | 9 9.03 | 1.13 | 0.66 | 1.546 | 0.471 | 0.125 | 0 | 0 | 1 | 1 | 0.2 | 1 |
| 14 | 9 51.27 | 1.21 | 0.71 | 1.665 | 0.507 | 0.134 | 0 | 0 | 1 | 1 | 0.2 | 1 |
| 15 | 10 33.50 | 1.30 | 0.76 | 1.784 | 0.543 | 0.144 | 0 | 0 | 1 | 1 | 0.2 | 1 |
| 16 | 11 15.73 | 1.39 | 0.81 | 1.903 | 0.579 | 0.153 | 0 | 0 | 1 | 1 | 0.2 | 1 |
| 17 | 11 57.97 | 1.47 | 0.86 | 2.022 | 0.616 | 0.163 | 0 | 0 | 1 | 1 | 0.2 | 1 |
| 18 | 12 40.20 | 1.56 | 0.91 | 2.141 | 0.652 | 0.173 | 0 | 0 | 1 | 1 | 0.3 | 1 |
| 19 | 13 22.43 | 1.65 | 0.97 | 2.260 | 0.688 | 0.182 | 1 | 0 | 1 | 1 | 0.3 | 1 |
| 20 | 14 4.67 | 1.74 | 1.02 | 2.378 | 0.724 | 0.192 | 1 | 0 | 1 | 1 | 0.3 | 1 |
| 21 | 14 46.90 | 1.82 | 1.07 | 2.497 | 0.760 | 0.201 | 1 | 0 | 1 | 1 | 0.3 | 1 |
| 22 | 15 29.14 | 1.91 | 1.12 | 2.616 | 0.797 | 0.211 | 1 | 0 | 1 | 1 | 0.3 | 1 |
| 23 | 16 11.37 | 2.00 | 1.17 | 2.735 | 0.833 | 0.220 | 1 | 0 | 1 | 1 | 0.3 | 1 |
| 24 | 16 53.60 | 2.08 | 1.22 | 2.854 | 0.869 | 0.230 | 1 | 0 | 1 | 1 | 0.3 | 1 |
| 25 | 17 35.84 | 2.17 | 1.27 | 2.973 | 0.905 | 0.240 | 1 | 0 | 1 | 1 | 0.4 | 1 |
| 26 | 18 18.07 | 2.26 | 1.32 | 3.092 | 0.941 | 0.249 | 1 | 0 | 1 | 1 | 0.4 | 1 |
| 27 | 19 0.30 | 2.34 | 1.37 | 3.211 | 0.978 | 0.259 | 1 | 0 | 1 | 1 | 0.4 | 1 |
| 28 | 19 42.54 | 2.43 | 1.42 | 3.330 | 1.014 | 0.268 | 1 | 0 | 1 | 1 | 0.4 | 1 |
| 29 | 20 24.77 | 2.52 | 1.47 | 3.449 | 1.050 | 0.278 | 1 | 0 | 1 | 1 | 0.4 | 1 |
| 30 | 21 7.00 | 2.60 | 1.52 | 3.568 | 1.086 | 0.288 | 1 | 0 | 1 | 1 | 0.4 | 1 |
| 31 | 21 49.24 | 2.69 | 1.57 | 3.687 | 1.123 | 0.297 | 1 | 0 | 1 | 1 | 0.5 | 1 |

TABLE V.—MOTION OF *g* FOR HOURS.

| Hours. | <i>g</i> | Hours. | <i>g</i> | Hours. | <i>g</i> | Hours. | <i>g</i> |
|--------|----------|--------|----------|--------|----------|--------|----------|
| 0 | 0.00 | 6 | 10.56 | 12 | 21.12 | 18 | 31.67 |
| 1 | 1.76 | 7 | 12.32 | 13 | 22.88 | 19 | 33.43 |
| 2 | 3.52 | 8 | 14.08 | 14 | 24.64 | 20 | 35.19 |
| 3 | 5.28 | 9 | 15.84 | 15 | 26.40 | 21 | 36.95 |
| 4 | 7.04 | 10 | 17.60 | 16 | 28.16 | 22 | 38.71 |
| 5 | 8.80 | 11 | 19.36 | 17 | 29.92 | 23 | 40.47 |
| 6 | 10.56 | 12 | 21.12 | 18 | 31.67 | 24 | 42.23 |

The period of arguments 1 to 9 is 600.

In January and February of those years which, though divisible by 4, are not leap years, namely, 1700, 1800, 1900, 2100, etc., Table IV must be entered with a number 1 greater than the real day of the month.

TABLE VI.—CORRECTIONS OF ARGUMENTS FOR TERMS OF LONG PERIOD.

| Year. | <i>g</i> | 1 | 2 | 3 | Year. | <i>g</i> | 1 | 2 | 3 |
|-------|-----------|--------|--------|--------|-------|----------------------|--------|--------|--------|
| 1000 | +36 51.00 | -0.614 | -1.024 | +1.792 | 1550 | +5 9.73 | -0.200 | -0.144 | +0.252 |
| 1010 | 36 6.49 | 0.621 | 1.003 | 1.756 | 1560 | 4 49.70 | 0.153 | 0.134 | 0.236 |
| 1020 | 35 22.14 | 0.629 | 0.983 | 1.720 | 1570 | 4 30.32 | 0.106 | 0.125 | 0.219 |
| 1030 | 34 37.97 | 0.638 | 0.962 | 1.684 | 1580 | 4 11.57 | 0.060 | 0.117 | 0.205 |
| 1040 | 33 54.00 | 0.648 | 0.942 | 1.649 | 1590 | 3 53.48 | -0.013 | 0.108 | 0.189 |
| 1050 | +33 10.25 | -0.659 | -0.922 | +1.614 | 1600 | +3 36.03 | +0.034 | -0.100 | +0.175 |
| 1060 | 32 26.71 | 0.672 | 0.902 | 1.579 | 1610 | 3 19.26 | 0.078 | 0.092 | 0.161 |
| 1070 | 31 43.41 | 0.686 | 0.882 | 1.544 | 1620 | 3 3.14 | 0.120 | 0.085 | 0.148 |
| 1080 | 31 0.37 | 0.702 | 0.862 | 1.509 | 1630 | 2 47.69 | 0.162 | 0.078 | 0.135 |
| 1090 | 30 17.61 | 0.718 | 0.842 | 1.474 | 1640 | 2 32.89 | 0.203 | 0.071 | 0.123 |
| 1100 | +29 35.15 | -0.735 | -0.822 | +1.439 | 1650 | +2 18.77 | +0.242 | -0.064 | +0.112 |
| 1110 | 28 53.01 | 0.752 | 0.803 | 1.405 | 1660 | 2 5.32 | 0.278 | 0.058 | 0.101 |
| 1120 | 28 11.20 | 0.770 | 0.783 | 1.370 | 1670 | 1 52.55 | 0.312 | 0.052 | 0.091 |
| 1130 | 27 29.71 | 0.790 | 0.764 | 1.337 | 1680 | 1 40.46 | 0.345 | 0.046 | 0.081 |
| 1140 | 26 48.56 | 0.809 | 0.745 | 1.304 | 1690 | 1 29.05 | 0.375 | 0.041 | 0.072 |
| 1150 | +26 7.74 | -0.829 | -0.726 | +1.270 | 1700 | +1 18.32 | +0.403 | -0.036 | +0.063 |
| 1160 | 25 27.27 | 0.848 | 0.707 | 1.237 | 1710 | 1 8.28 | 0.429 | 0.031 | 0.054 |
| 1170 | 24 47.16 | 0.866 | 0.689 | 1.206 | 1720 | 0 58.92 | 0.452 | 0.027 | 0.046 |
| 1180 | 24 7.42 | 0.883 | 0.670 | 1.172 | 1730 | 0 50.24 | 0.472 | 0.023 | 0.039 |
| 1190 | 23 28.06 | 0.899 | 0.652 | 1.141 | 1740 | 0 42.26 | 0.489 | 0.019 | 0.032 |
| 1200 | +22 49.09 | -0.914 | -0.634 | +1.110 | 1750 | +0 34.96 | +0.503 | -0.015 | +0.026 |
| 1210 | 22 10.51 | 0.927 | 0.616 | 1.078 | 1760 | 0 28.36 | 0.514 | 0.012 | 0.021 |
| 1220 | 21 32.34 | 0.939 | 0.598 | 1.046 | 1770 | 0 22.44 | 0.522 | 0.009 | 0.017 |
| 1230 | 20 54.57 | 0.950 | 0.581 | 1.017 | 1780 | 0 17.22 | 0.527 | 0.007 | 0.013 |
| 1240 | 20 17.23 | 0.960 | 0.564 | 0.986 | 1790 | +0 12.68 | 0.529 | 0.005 | 0.010 |
| 1250 | +19 40.31 | -0.967 | -0.547 | +0.957 | 1800 | +8.84 | +0.528 | -0.004 | +0.007 |
| 1260 | 19 3.83 | 0.972 | 0.530 | 0.928 | 1801 | 8.49 ⁻³⁵ | 0.528 | 0.004 | 0.007 |
| 1270 | 18 27.80 | 0.975 | 0.513 | 0.898 | 1802 | 8.15 ⁻³⁴ | 0.528 | 0.004 | 0.007 |
| 1280 | 17 52.21 | 0.977 | 0.497 | 0.870 | 1803 | 7.82 ⁻³³ | 0.527 | 0.004 | 0.007 |
| 1290 | 17 17.09 | 0.977 | 0.481 | 0.842 | 1804 | 7.50 ⁻³² | 0.527 | 0.004 | 0.007 |
| 1300 | +16 42.43 | -0.974 | -0.465 | +0.814 | 1805 | +7.18 | +0.527 | -0.003 | +0.006 |
| 1310 | 16 8.25 | 0.967 | 0.449 | 0.786 | 1806 | 6.87 ⁻³¹ | 0.526 | 0.003 | 0.006 |
| 1320 | 15 34.55 | 0.959 | 0.433 | 0.758 | 1807 | 6.57 ⁻³⁰ | 0.526 | 0.003 | 0.006 |
| 1330 | 15 1.34 | 0.948 | 0.417 | 0.730 | 1808 | 6.27 ⁻³⁰ | 0.525 | 0.003 | 0.006 |
| 1340 | 14 28.63 | 0.936 | 0.402 | 0.704 | 1809 | 5.98 ⁻²⁹ | 0.525 | 0.003 | 0.006 |
| 1350 | +13 56.44 | -0.921 | -0.387 | +0.677 | 1810 | +5.69 ⁻²⁹ | +0.524 | -0.003 | +0.005 |
| 1360 | 13 24.76 | 0.903 | 0.372 | 0.651 | 1811 | 5.41 ⁻²⁸ | 0.523 | 0.003 | 0.005 |
| 1370 | 12 53.61 | 0.883 | 0.358 | 0.626 | 1812 | 5.14 ⁻²⁷ | 0.523 | 0.003 | 0.005 |
| 1380 | 12 22.99 | 0.861 | 0.344 | 0.602 | 1813 | 4.88 ⁻²⁶ | 0.522 | 0.003 | 0.005 |
| 1390 | 11 52.90 | 0.837 | 0.330 | 0.578 | 1814 | 4.62 ⁻²⁶ | 0.522 | 0.002 | 0.005 |
| 1400 | +11 23.35 | -0.810 | -0.317 | +0.555 | 1815 | +4.37 ⁻²⁵ | +0.521 | -0.002 | +0.004 |
| 1410 | 10 54.35 | 0.780 | 0.303 | 0.530 | 1816 | 4.13 ⁻²⁴ | 0.520 | 0.002 | 0.004 |
| 1420 | 10 25.92 | 0.748 | 0.290 | 0.508 | 1817 | 3.89 ⁻²⁴ | 0.520 | 0.002 | 0.004 |
| 1430 | 9 58.04 | 0.714 | 0.277 | 0.485 | 1818 | 3.66 ⁻²³ | 0.519 | 0.002 | 0.004 |
| 1440 | 9 30.74 | 0.678 | 0.264 | 0.462 | 1819 | 3.44 ⁻²² | 0.518 | 0.002 | 0.004 |
| 1450 | + 9 4.01 | -0.641 | -0.252 | +0.441 | 1820 | +3.23 | +0.517 | -0.002 | +0.003 |
| 1460 | 8 37.88 | 0.601 | 0.240 | 0.420 | 1821 | 3.02 ⁻²¹ | 0.516 | 0.002 | 0.003 |
| 1470 | 8 12.34 | 0.560 | 0.228 | 0.399 | 1822 | 2.82 ⁻²⁰ | 0.515 | 0.002 | 0.003 |
| 1480 | 7 47.39 | 0.518 | 0.216 | 0.378 | 1823 | 2.63 ⁻¹⁹ | 0.514 | 0.002 | 0.003 |
| 1490 | 7 23.04 | 0.475 | 0.205 | 0.359 | 1824 | 2.44 ⁻¹⁹ | 0.513 | 0.002 | 0.003 |
| 1500 | + 6 59.29 | -0.431 | -0.194 | +0.340 | 1825 | +2.26 ⁻¹⁸ | +0.512 | -0.001 | +0.002 |
| 1510 | 6 36.14 | 0.386 | 0.183 | 0.322 | 1826 | 2.09 ⁻¹⁷ | 0.511 | 0.001 | 0.002 |
| 1520 | 6 13.61 | 0.340 | 0.172 | 0.303 | 1827 | 1.92 ⁻¹⁷ | 0.510 | 0.001 | 0.002 |
| 1530 | 5 51.69 | 0.293 | 0.162 | 0.284 | 1828 | 1.76 ⁻¹⁶ | 0.509 | 0.001 | 0.002 |
| 1540 | + 5 30.40 | -0.247 | -0.153 | +0.268 | 1829 | +1.61 ⁻¹⁵ | -0.508 | -0.001 | +0.002 |

TABLE VII.—EQUATION OF CENTRE AND PRINCIPAL TERM OF LOG. *r*.

| <i>g</i> | | | | | <i>E</i> | | | | | Log. <i>r</i> | | | | | | | | | | | | | | | | | | | | | | |
|----------|---|----|-------|-------|----------|--|--|--|-------|---------------|-----|--|--|--|------|-----|---|----|-------|--|--|--|--|-------|-------|------|--|--|--|------|-----|-----|
| ° ' " | | | | | " " | | | | | 1.26 | | | | | | | | | | | | | | | | | | | | | | |
| 0° | 0 | 0 | 00.00 | | | | | | | 18915 | | | | | 360° | 10° | 0 | 59 | 26.71 | | | | | | 22488 | | | | | 350° | 50' | 120 |
| 10' | 0 | 0 | 59.81 | 59.81 | | | | | 58.70 | 18916 | 1 | | | | 50' | 10' | 1 | 0 | 25.41 | | | | | 22608 | 120 | 50' | | | | | | |
| 20 | 0 | 1 | 59.62 | 59.81 | | | | | 58.67 | 18919 | 3 | | | | 40 | 20 | 1 | 1 | 24.08 | | | | | 22729 | 121 | 40 | | | | | | |
| 30 | 0 | 2 | 59.42 | 59.80 | | | | | 58.63 | 18924 | 5 | | | | 30 | 30 | 1 | 2 | 22.71 | | | | | 22852 | 123 | 30 | | | | | | |
| 40 | 0 | 3 | 59.23 | 59.81 | | | | | 58.59 | 18931 | 7 | | | | 20 | 40 | 1 | 3 | 21.30 | | | | | 22977 | 125 | 20 | | | | | | |
| 50 | 0 | 4 | 59.03 | 59.80 | | | | | 58.55 | 18940 | 9 | | | | 10 | 50 | 1 | 4 | 19.85 | | | | | 23104 | 127 | 10 | | | | | | |
| | | | | 59.79 | | | | | 58.52 | | 11 | | | | | | | | | | | | | | 130 | | | | | | | |
| 1° | 0 | 5 | 58.82 | | | | | | | 18951 | | | | | 359° | 11° | 1 | 5 | 18.37 | | | | | 23234 | | 349° | | | | | | |
| 10' | 0 | 6 | 58.61 | 59.79 | | | | | 58.48 | 18964 | 13 | | | | 50' | 10' | 1 | 6 | 16.85 | | | | | 23365 | 131 | 50' | | | | | | |
| 20 | 0 | 7 | 58.39 | 59.78 | | | | | 58.43 | 18979 | 15 | | | | 40 | 20 | 1 | 7 | 15.28 | | | | | 23498 | 133 | 40 | | | | | | |
| 30 | 0 | 8 | 58.18 | 59.79 | | | | | 58.40 | 18996 | 17 | | | | 30 | 30 | 1 | 8 | 13.68 | | | | | 23633 | 135 | 30 | | | | | | |
| 40 | 0 | 9 | 57.96 | 59.78 | | | | | 58.36 | 19015 | 19 | | | | 20 | 40 | 1 | 9 | 12.04 | | | | | 23770 | 137 | 20 | | | | | | |
| 50 | 0 | 10 | 57.74 | 59.78 | | | | | 58.32 | 19036 | 21 | | | | 10 | 50 | 1 | 10 | 10.36 | | | | | 23909 | 139 | 10 | | | | | | |
| | | | | 59.77 | | | | | 58.27 | | 23 | | | | | | | | | | | | | | 142 | | | | | | | |
| 2° | 0 | 11 | 57.51 | | | | | | | 19059 | | | | | 358° | 12° | 1 | 11 | 8.63 | | | | | 24051 | | 348° | | | | | | |
| 10' | 0 | 12 | 57.27 | 59.76 | | | | | 58.23 | 19084 | 25 | | | | 50' | 10' | 1 | 12 | 6.86 | | | | | 24194 | 143 | 50' | | | | | | |
| 20 | 0 | 13 | 57.02 | 59.75 | | | | | 58.18 | 19111 | 27 | | | | 40 | 20 | 1 | 13 | 5.04 | | | | | 24339 | 145 | 40 | | | | | | |
| 30 | 0 | 14 | 56.76 | 59.74 | | | | | 58.14 | 19140 | 29 | | | | 30 | 30 | 1 | 14 | 3.18 | | | | | 24485 | 146 | 30 | | | | | | |
| 40 | 0 | 15 | 56.49 | 59.73 | | | | | 58.10 | 19171 | 31 | | | | 20 | 40 | 1 | 15 | 1.28 | | | | | 24633 | 148 | 20 | | | | | | |
| 50 | 0 | 16 | 56.22 | 59.73 | | | | | 58.05 | 19204 | 33 | | | | 10 | 50 | 1 | 15 | 59.33 | | | | | 24784 | 151 | 10 | | | | | | |
| | | | | 59.71 | | | | | 58.00 | | 34 | | | | | | | | | | | | | | 153 | | | | | | | |
| 3° | 0 | 17 | 55.93 | | | | | | | 19238 | | | | | 357° | 13° | 1 | 16 | 57.33 | | | | | 24937 | | 347° | | | | | | |
| 10' | 0 | 18 | 55.63 | 59.70 | | | | | 57.96 | 19275 | 37 | | | | 50' | 10' | 1 | 17 | 55.29 | | | | | 25091 | 154 | 50' | | | | | | |
| 20 | 0 | 19 | 55.33 | 59.70 | | | | | 57.91 | 19314 | 39 | | | | 40 | 20 | 1 | 18 | 53.20 | | | | | 25247 | 156 | 40 | | | | | | |
| 30 | 0 | 20 | 55.01 | 59.68 | | | | | 57.86 | 19355 | 41 | | | | 30 | 30 | 1 | 19 | 51.06 | | | | | 25405 | 158 | 30 | | | | | | |
| 40 | 0 | 21 | 54.68 | 59.67 | | | | | 57.81 | 19397 | 42 | | | | 20 | 40 | 1 | 20 | 48.87 | | | | | 25566 | 161 | 20 | | | | | | |
| 50 | 0 | 22 | 54.34 | 59.66 | | | | | 57.76 | 19442 | 45 | | | | 10 | 50 | 1 | 21 | 46.63 | | | | | 25728 | 162 | 10 | | | | | | |
| | | | | 59.64 | | | | | 57.72 | | 47 | | | | | | | | | | | | | | 164 | | | | | | | |
| 4° | 0 | 23 | 53.98 | | | | | | | 19489 | | | | | 356° | 14° | 1 | 22 | 44.35 | | | | | 25892 | | 346° | | | | | | |
| 10' | 0 | 24 | 53.61 | 59.63 | | | | | 57.66 | 19538 | 49 | | | | 50' | 10' | 1 | 23 | 42.01 | | | | | 26057 | 165 | 50' | | | | | | |
| 20 | 0 | 25 | 53.22 | 59.61 | | | | | 57.62 | 19589 | 51 | | | | 40 | 20 | 1 | 24 | 39.63 | | | | | 26225 | 168 | 40 | | | | | | |
| 30 | 0 | 26 | 52.81 | 59.59 | | | | | 57.56 | 19642 | 53 | | | | 30 | 30 | 1 | 25 | 37.19 | | | | | 26395 | 170 | 30 | | | | | | |
| 40 | 0 | 27 | 52.39 | 59.58 | | | | | 57.51 | 19697 | 55 | | | | 20 | 40 | 1 | 26 | 34.70 | | | | | 26566 | 171 | 20 | | | | | | |
| 50 | 0 | 28 | 51.95 | 59.56 | | | | | 57.46 | 19754 | 57 | | | | 10 | 50 | 1 | 27 | 32.16 | | | | | 26740 | 174 | 10 | | | | | | |
| | | | | 59.54 | | | | | 57.41 | | 58 | | | | | | | | | | | | | | 176 | | | | | | | |
| 5° | 0 | 29 | 51.49 | | | | | | | 19812 | | | | | 355° | 15° | 1 | 28 | 29.57 | | | | | 26916 | | 345° | | | | | | |
| 10' | 0 | 30 | 51.01 | 59.52 | | | | | 57.35 | 19872 | 60 | | | | 50' | 10' | 1 | 29 | 26.92 | | | | | 27093 | 177 | 50' | | | | | | |
| 20 | 0 | 31 | 50.52 | 59.51 | | | | | 57.30 | 19934 | 62 | | | | 40 | 20 | 1 | 30 | 24.22 | | | | | 27271 | 178 | 40 | | | | | | |
| 30 | 0 | 32 | 50.01 | 59.49 | | | | | 57.25 | 19999 | 65 | | | | 30 | 30 | 1 | 31 | 21.47 | | | | | 27451 | 180 | 30 | | | | | | |
| 40 | 0 | 33 | 49.48 | 59.47 | | | | | 57.20 | 20065 | 66 | | | | 20 | 40 | 1 | 32 | 18.67 | | | | | 27634 | 183 | 20 | | | | | | |
| 50 | 0 | 34 | 48.93 | 59.45 | | | | | 57.13 | 20134 | 69 | | | | 10 | 50 | 1 | 33 | 15.80 | | | | | 27819 | 185 | 10 | | | | | | |
| | | | | 59.43 | | | | | 57.08 | | 71 | | | | | | | | | | | | | | 188 | | | | | | | |
| 6° | 0 | 35 | 48.36 | | | | | | | 20205 | | | | | 354° | 16° | 1 | 34 | 12.88 | | | | | 28007 | | 344° | | | | | | |
| 10' | 0 | 36 | 47.77 | 59.41 | | | | | 57.02 | 20278 | 73 | | | | 50' | 10' | 1 | 35 | 9.90 | | | | | 28196 | 189 | 50' | | | | | | |
| 20 | 0 | 37 | 47.15 | 59.38 | | | | | 56.97 | 20352 | 74 | | | | 40 | 20 | 1 | 36 | 6.87 | | | | | 28386 | 190 | 40 | | | | | | |
| 30 | 0 | 38 | 46.51 | 59.36 | | | | | 56.90 | 20428 | 76 | | | | 30 | 30 | 1 | 37 | 3.77 | | | | | 28578 | 192 | 30 | | | | | | |
| 40 | 0 | 39 | 45.84 | 59.33 | | | | | 56.85 | 20507 | 79 | | | | 20 | 40 | 1 | 38 | 0.62 | | | | | 28772 | 194 | 20 | | | | | | |
| 50 | 0 | 40 | 45.15 | 59.31 | | | | | 56.79 | 20588 | 81 | | | | 10 | 50 | 1 | 38 | 57.41 | | | | | 28968 | 196 | 10 | | | | | | |
| | | | | 59.29 | | | | | 56.73 | | 82 | | | | | | | | | | | | | | 199 | | | | | | | |
| 7° | 0 | 41 | 44.44 | | | | | | | 20670 | | | | | 353° | 17° | 1 | 39 | 54.14 | | | | | 29167 | | 343° | | | | | | |
| 10' | 0 | 42 | 43.70 | 59.26 | | | | | 56.67 | 20755 | 85 | | | | 50' | 10' | 1 | 40 | 50.81 | | | | | 29367 | 200 | 50' | | | | | | |
| 20 | 0 | 43 | 42.94 | 59.24 | | | | | 56.62 | 20841 | 86 | | | | 40 | 20 | 1 | 41 | 47.43 | | | | | 29568 | 201 | 40 | | | | | | |
| 30 | 0 | 44 | 42.15 | 59.21 | | | | | 56.55 | 20929 | 88 | | | | 30 | 30 | 1 | 42 | 43.98 | | | | | 29771 | 203 | 30 | | | | | | |
| 40 | 0 | 45 | 41.33 | 59.18 | | | | | 56.48 | 21020 | 91 | | | | 20 | 40 | 1 | 43 | 40.46 | | | | | 29976 | 205 | 20 | | | | | | |
| 50 | 0 | 46 | 40.48 | 59.15 | | | | | 56.43 | 21112 | 92 | | | | 10 | 50 | 1 | 44 | 36.89 | | | | | 30183 | 207 | 10 | | | | | | |
| | | | | 59.13 | | | | | 56.36 | | 94 | | | | | | | | | | | | | | 209 | | | | | | | |
| 8° | 0 | 47 | 39.61 | | | | | | | 21206 | | | | | 352° | 18° | 1 | 45 | 33.25 | | | | | 30392 | | 342° | | | | | | |
| 10' | 0 | 48 | 38.71 | 59.10 | | | | | 56.30 | 21302 | 96 | | | | 50' | 10' | 1 | 46 | 29.55 | | | | | 30603 | 211 | 50' | | | | | | |
| 20 | 0 | 49 | 37.78 | 59.07 | | | | | 56.23 | 21400 | 98 | | | | 40 | 20 | 1 | 47 | 25.78 | | | | | 30816 | 213 | 40 | | | | | | |
| 30 | 0 | 50 | 36.82 | 59.04 | | | | | 56.17 | 21499 | 99 | | | | 30 | 30 | 1 | 48 | 21.95 | | | | | 31030 | 214 | 30 | | | | | | |
| 40 | 0 | 51 | 35.83 | 59.01 | | | | | 56.10 | 21601 | 102 | | | | 20 | 40 | 1 | 49 | 18.05 | | | | | 31246 | 216 | 20 | | | | | | |
| 50 | 0 | 52 | 34.81 | 58.98 | | | | | 56.04 | 21705 | 104 | | | | 10 | 50 | 1 | 50 | 14.69 | | | | | 31465 | 219 | 10 | | | | | | |
| | | | | 58.95 | | | | | 55.97 | | 106 | | | | | | | | | | | | | | 221 | | | | | | | |
| 9° | 0 | 53 | 33.76 | | | | | | | 21811 | | | | | 351° | 19° | 1 | 51 | 10.03 | | | | | 31686 | | 341° | | | | | | |
| 10' | 0 | 54 | 32.67 | 58.91 | | | | | 55.90 | 21919 | 108 | | | | 50' | 10' | 1 | 52 | 5.96 | | | | | 31908 | 222 | 50' | | | | | | |
| 20 | 0 | 55 | 31.55 | 58.88 | | | | | 55.84 | 22029 | 110 | | | | 40 | 20 | 1 | 53 | 1.80 | | | | | 32132 | 224 | 40 | | | | | | |
| 30 | 0 | 56 | 30.39 | 58.84 | | | | | 55.77 | 22140 | 111 | | | | 30 | 30 | 1 | 53 | 57.57 | | | | | 32358 | 226 | 30 | | | | | | |
| 40 | 0 | 57 | 29.20 | 58.81 | | | | | 55.70 | 22254 | 114 | | | | 20 | 40 | 1 | 54 | 53.27 | | | | | 32585 | 227 | 20 | | | | | | |
| 50 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

TABLE VII.—Continued.

| g | | | | E | | | | Log. r | | | | g | | | | E | | | | Log. r | | | | | | | | | | | | | |
|-----|---|----|-------|-------|---|---|-----|--------|-----|---|-----|-----|-----|---|-----|-------|-------|---|-----|--------|-------|-----|-----|-----|-----|-----|-----|-------|-------|-------|-------|-----|-----|
| | | | | 1.26 | | | | | | | | | | | | 1.26 | | | | | | | | | | | | | | | | | |
| ° | ' | " | ''' | ° | ' | " | ''' | ° | ' | " | ''' | ° | ' | " | ''' | ° | ' | " | ''' | ° | ' | " | ''' | ° | ' | " | ''' | ° | ' | " | ''' | | |
| 20 | 1 | 56 | 44.47 | | | | | 33045 | | | | 340 | 30 | 2 | 49 | 51.28 | | | | | 50121 | | | | 330 | 50' | 2 | 50 | 41.63 | 50.35 | 50457 | 336 | 50' |
| 10' | 1 | 57 | 39.96 | 55.49 | | | | 33278 | 233 | | | 50' | 10' | 2 | 50 | 41.63 | 50.35 | | | | 50457 | 336 | 50' | 330 | 50' | 2 | 51 | 31.87 | 50.24 | 50795 | 338 | 40 | |
| 20 | 1 | 58 | 35.39 | 55.43 | | | | 33513 | 235 | | | 40 | 20 | 2 | 51 | 31.87 | 50.24 | | | | 50795 | 338 | 40 | 30 | 30 | 2 | 52 | 22.01 | 50.14 | 51134 | 339 | 30 | |
| 30 | 1 | 59 | 30.74 | 55.35 | | | | 33749 | 236 | | | 30 | 30 | 2 | 52 | 22.01 | 50.14 | | | | 51134 | 339 | 30 | 40 | 40 | 2 | 53 | 12.06 | 50.05 | 51475 | 341 | 20 | |
| 40 | 2 | 0 | 26.02 | 55.28 | | | | 33987 | 238 | | | 20 | 40 | 2 | 53 | 12.06 | 50.05 | | | | 51475 | 341 | 20 | 50 | 50 | 2 | 54 | 2.00 | 49.94 | 51818 | 343 | 10 | |
| 50 | 2 | 1 | 21.23 | 55.21 | | | | 34227 | 240 | | | 10 | 50 | 2 | 54 | 2.00 | 49.94 | | | | 51818 | 343 | 10 | | | | | 49.83 | | 52162 | 344 | | |
| | | | | 55.13 | | | | | 241 | | | | | | | | | | | | | | | | | | | | | 52507 | 345 | | |
| 21 | 2 | 2 | 16.36 | | | | | 34468 | | | | 339 | 31 | 2 | 54 | 51.83 | | | | | 52162 | | | | 329 | 50' | 2 | 55 | 41.56 | 49.73 | 52507 | 345 | 50' |
| 10' | 2 | 3 | 11.42 | 55.06 | | | | 34711 | 243 | | | 50' | 10' | 2 | 55 | 41.56 | 49.73 | | | | 52507 | 345 | 50' | 329 | 50' | 2 | 56 | 31.18 | 49.62 | 52854 | 347 | 40 | |
| 20 | 2 | 4 | 6.41 | 54.99 | | | | 34956 | 245 | | | 40 | 20 | 2 | 56 | 31.18 | 49.62 | | | | 52854 | 347 | 40 | 40 | 40 | 2 | 57 | 20.70 | 49.52 | 53203 | 349 | 30 | |
| 30 | 2 | 5 | 1.33 | 54.92 | | | | 35204 | 248 | | | 30 | 30 | 2 | 57 | 20.70 | 49.52 | | | | 53203 | 349 | 30 | 50 | 50 | 2 | 58 | 10.12 | 49.42 | 53553 | 350 | 20 | |
| 40 | 2 | 5 | 56.17 | 54.84 | | | | 35454 | 250 | | | 20 | 40 | 2 | 58 | 10.12 | 49.42 | | | | 53553 | 350 | 20 | | | | | 49.31 | | 53905 | 352 | 10 | |
| 50 | 2 | 6 | 50.93 | 54.76 | | | | 35705 | 251 | | | 10 | 50 | 2 | 58 | 59.43 | 49.20 | | | | 53905 | 352 | 10 | | | | | 49.20 | | | 353 | | |
| | | | | 54.68 | | | | | 252 | | | | | | | | | | | | | | | | | | | | | 54258 | | | |
| 22 | 2 | 7 | 45.61 | | | | | 35957 | | | | 338 | 32 | 2 | 59 | 48.63 | | | | | 54258 | | | | 328 | 50' | 3 | 0 | 37.73 | 49.10 | 54613 | 355 | 50' |
| 10' | 2 | 8 | 40.22 | 54.61 | | | | 36211 | 254 | | | 50' | 10' | 3 | 0 | 37.73 | 49.10 | | | | 54613 | 355 | 50' | 328 | 50' | 3 | 1 | 26.73 | 49.00 | 54970 | 357 | 40 | |
| 20 | 2 | 9 | 34.76 | 54.54 | | | | 36467 | 256 | | | 40 | 20 | 3 | 1 | 26.73 | 49.00 | | | | 54970 | 357 | 40 | 50 | 50 | 3 | 2 | 15.62 | 48.89 | 55329 | 359 | 30 | |
| 30 | 2 | 10 | 29.22 | 54.46 | | | | 36726 | 259 | | | 30 | 30 | 3 | 2 | 15.62 | 48.89 | | | | 55329 | 359 | 30 | | | | | 48.79 | | 55689 | 360 | 20 | |
| 40 | 2 | 11 | 23.60 | 54.38 | | | | 36986 | 260 | | | 20 | 40 | 3 | 3 | 4.41 | 48.79 | | | | 55689 | 360 | 20 | | | | | 48.68 | | 56050 | 361 | 10 | |
| 50 | 2 | 12 | 17.90 | 54.30 | | | | 37248 | 262 | | | 10 | 50 | 3 | 3 | 53.09 | 48.68 | | | | 56050 | 361 | 10 | | | | | 48.57 | | | 363 | | |
| | | | | 54.22 | | | | | 263 | | | | | | | | | | | | | | | | | | | | | 56413 | | | |
| 23 | 2 | 13 | 12.12 | | | | | 37511 | | | | 337 | 33 | 3 | 4 | 41.66 | | | | | 56413 | | | | 327 | 50' | 3 | 5 | 30.12 | 48.46 | 56777 | 364 | 50' |
| 10' | 2 | 14 | 6.26 | 54.14 | | | | 37776 | 265 | | | 50' | 10' | 3 | 5 | 30.12 | 48.46 | | | | 56777 | 364 | 50' | 327 | 50' | 3 | 6 | 18.47 | 48.35 | 57143 | 366 | 40 | |
| 20 | 2 | 15 | 0.32 | 54.06 | | | | 38043 | 267 | | | 40 | 20 | 3 | 6 | 18.47 | 48.35 | | | | 57143 | 366 | 40 | 50 | 50 | 3 | 7 | 6.72 | 48.25 | 57510 | 367 | 30 | |
| 30 | 2 | 15 | 54.30 | 53.98 | | | | 38311 | 268 | | | 30 | 30 | 3 | 7 | 6.72 | 48.25 | | | | 57510 | 367 | 30 | | | | | 48.13 | | 57879 | 369 | 20 | |
| 40 | 2 | 16 | 48.20 | 53.90 | | | | 38581 | 270 | | | 20 | 40 | 3 | 7 | 54.85 | 48.13 | | | | 57879 | 369 | 20 | | | | | 48.02 | | 58249 | 370 | 10 | |
| 50 | 2 | 17 | 42.02 | 53.82 | | | | 38853 | 272 | | | 10 | 50 | 3 | 8 | 42.87 | 48.02 | | | | 58249 | 370 | 10 | | | | | 47.91 | | | 372 | | |
| | | | | 53.73 | | | | | 275 | | | | | | | | | | | | | | | | | | | | | 58621 | | | |
| 24 | 2 | 18 | 35.75 | | | | | 39128 | | | | 336 | 34 | 3 | 9 | 30.78 | | | | | 58621 | | | | 326 | 50' | 3 | 10 | 18.58 | 47.80 | 58994 | 373 | 50' |
| 10' | 2 | 19 | 29.40 | 53.65 | | | | 39404 | 276 | | | 50' | 10' | 3 | 10 | 18.58 | 47.80 | | | | 58994 | 373 | 50' | 326 | 50' | 3 | 11 | 6.26 | 47.68 | 59369 | 375 | 40 | |
| 20 | 2 | 20 | 22.97 | 53.57 | | | | 39681 | 277 | | | 40 | 20 | 3 | 11 | 6.26 | 47.68 | | | | 59369 | 375 | 40 | 50 | 50 | 3 | 11 | 53.83 | 47.57 | 59746 | 377 | 30 | |
| 30 | 2 | 21 | 16.46 | 53.49 | | | | 39960 | 279 | | | 30 | 30 | 3 | 11 | 53.83 | 47.57 | | | | 59746 | 377 | 30 | | | | | 47.46 | | 60124 | 378 | 20 | |
| 40 | 2 | 22 | 9.87 | 53.41 | | | | 40241 | 281 | | | 20 | 40 | 3 | 12 | 41.29 | 47.46 | | | | 60124 | 378 | 20 | | | | | 47.35 | | 60503 | 379 | 10 | |
| 50 | 2 | 23 | 3.19 | 53.32 | | | | 40524 | 283 | | | 10 | 50 | 3 | 13 | 28.64 | 47.35 | | | | 60503 | 379 | 10 | | | | | 47.23 | | | 381 | | |
| | | | | 53.23 | | | | | 284 | | | | | | | | | | | | | | | | | | | | | 60884 | | | |
| 25 | 2 | 23 | 56.42 | | | | | 40803 | | | | 335 | 35 | 3 | 14 | 15.87 | | | | | 60884 | | | | 325 | 50' | 3 | 15 | 2.99 | 47.12 | 61266 | 382 | 50' |
| 10' | 2 | 24 | 49.57 | 53.15 | | | | 41094 | 286 | | | 50' | 10' | 3 | 15 | 2.99 | 47.12 | | | | 61266 | 382 | 50' | 325 | 50' | 3 | 15 | 50.00 | 47.01 | 61650 | 384 | 40 | |
| 20 | 2 | 25 | 42.63 | 53.06 | | | | 41382 | 288 | | | 40 | 20 | 3 | 15 | 50.00 | 47.01 | | | | 61650 | 384 | 40 | 50 | 50 | 3 | 16 | 36.89 | 46.89 | 62035 | 385 | 30 | |
| 30 | 2 | 26 | 35.61 | 52.98 | | | | 41671 | 289 | | | 30 | 30 | 3 | 16 | 36.89 | 46.89 | | | | 62035 | 385 | 30 | | | | | 46.78 | | 62422 | 387 | 20 | |
| 40 | 2 | 27 | 28.50 | 52.89 | | | | 41962 | 291 | | | 20 | 40 | 3 | 17 | 23.67 | 46.78 | | | | 62422 | 387 | 20 | | | | | 46.66 | | 62810 | 388 | 10 | |
| 50 | 2 | 28 | 21.39 | 52.80 | | | | 42255 | 293 | | | 10 | 50 | 3 | 18 | 10.33 | 46.66 | | | | 62810 | 388 | 10 | | | | | 46.55 | | | 390 | | |
| | | | | 52.71 | | | | | 295 | | | | | | | | | | | | | | | | | | | | | 63200 | | | |
| 26 | 2 | 29 | 14.01 | | | | | 42550 | | | | 334 | 36 | 3 | 18 | 56.88 | | | | | 63200 | | | | 324 | 50' | 3 | 19 | 43.31 | 46.43 | 63592 | 392 | 50' |
| 10' | 2 | 30 | 6.63 | 52.62 | | | | 42846 | 296 | | | 50' | 10' | 3 | 19 | 43.31 | 46.43 | | | | 63592 | 392 | 50' | 324 | 50' | 3 | 20 | 29.63 | 46.32 | 63985 | 393 | 40 | |
| 20 | 2 | 30 | 59.16 | 52.53 | | | | 43144 | 298 | | | 40 | 20 | 3 | 20 | 29.63 | 46.32 | | | | 63985 | 393 | 40 | 50 | 50 | 3 | 21 | 15.83 | 46.20 | 64380 | 395 | 30 | |
| 30 | 2 | 31 | 51.60 | 52.44 | | | | 43444 | 300 | | | 30 | 30 | 3 | 21 | 15.83 | 46.20 | | | | 64380 | 395 | 30 | | | | | 46.08 | | 64776 | 396 | 20 | |
| 40 | 2 | 32 | 43.95 | 52.35 | | | | 43745 | 301 | | | 20 | 40 | 3 | 22 | 1.91 | 46.08 | | | | 64776 | 396 | 20 | | | | | 45.97 | | 65173 | 397 | 10 | |
| 50 | 2 | 33 | 36.21 | 52.26 | | | | 44048 | 303 | | | 10 | 50 | 3 | 22 | 47.88 | 45.97 | | | | 65173 | 397 | 10 | | | | | 45.84 | | | 399 | | |
| | | | | 52.17 | | | | | 305 | | | | | | | | | | | | | | | | | | | | | 65572 | | | |
| 27 | 2 | 34 | 28.38 | | | | | 44353 | | | | 333 | 37 | 3 | 23 | 33.72 | | | | | 65572 | | | | 323 | 50' | 3 | 24 | 19.44 | 45.72 | 65972 | 400 | 50' |
| 10' | 2 | 35 | 20.46 | 52.08 | | | | 44659 | 306 | | | 50' | 10' | 3 | 24 | 19.44 | 45.72 | | | | 65972 | | | | | | | | | | | | |

TABLE VII.—Continued.

| g | | | | | E | | | | | Log. r | | | | |
|-----|---|----|-------|--------|-------|------|------|-----|----|--------|-------|-------|------|------|
| g | | | | | E | | | | | Log. r | | | | |
| g | | | | | E | | | | | Log. r | | | | |
| g | | | | | E | | | | | Log. r | | | | |
| 40° | 3 | 36 | 58.19 | 1.26 | 72984 | 425 | 320° | 50° | 4 | 16 | 34.27 | 00687 | 496 | 310° |
| 10' | 3 | 37 | 41.71 | 73409 | 426 | 50' | 10' | 4 | 17 | 9.62 | 35.35 | 01183 | 498 | 50' |
| 20 | 3 | 38 | 25.10 | 73835 | 427 | 40 | 20 | 4 | 17 | 44.82 | 35.20 | 01681 | 499 | 40 |
| 30 | 3 | 39 | 8.37 | 74262 | 429 | 30 | 30 | 4 | 18 | 19.88 | 35.06 | 02180 | 500 | 30 |
| 40 | 3 | 39 | 51.51 | 74691 | 430 | 20 | 40 | 4 | 18 | 54.80 | 34.92 | 02680 | 501 | 20 |
| 50 | 3 | 40 | 34.52 | 75121 | 432 | 10 | 50 | 4 | 19 | 29.57 | 34.77 | 03181 | 502 | 10 |
| | | | 42.88 | | | | | | | | 34.62 | | | |
| 41° | 3 | 41 | 17.40 | 75553 | 433 | 319° | 51° | 4 | 20 | 4.19 | 03683 | 503 | 309° | |
| 10' | 3 | 42 | 0.16 | 75986 | 434 | 50' | 10' | 4 | 20 | 38.67 | 34.48 | 04186 | 504 | 50' |
| 20 | 3 | 42 | 42.79 | 76420 | 436 | 40 | 20 | 4 | 21 | 13.00 | 34.33 | 04690 | 505 | 40 |
| 30 | 3 | 43 | 25.29 | 76856 | 437 | 30 | 30 | 4 | 21 | 47.18 | 34.18 | 05195 | 505 | 30 |
| 40 | 3 | 44 | 7.66 | 77293 | 438 | 20 | 40 | 4 | 22 | 21.22 | 34.04 | 05701 | 506 | 20 |
| 50 | 3 | 44 | 49.91 | 77731 | 440 | 10 | 50 | 4 | 22 | 55.11 | 33.89 | 06207 | 507 | 10 |
| | | | 42.11 | | | | | | | | 33.74 | | | |
| 42° | 3 | 45 | 32.02 | 78171 | 440 | 318° | 52° | 4 | 23 | 28.85 | 06714 | 508 | 308° | |
| 10' | 3 | 46 | 14.01 | 78611 | 442 | 50' | 10' | 4 | 24 | 2.44 | 33.59 | 07222 | 510 | 50' |
| 20 | 3 | 46 | 55.86 | 79053 | 443 | 40 | 20 | 4 | 24 | 35.88 | 33.44 | 07732 | 511 | 40 |
| 30 | 3 | 47 | 37.58 | 79496 | 445 | 30 | 30 | 4 | 25 | 9.18 | 33.30 | 08243 | 511 | 30 |
| 40 | 3 | 48 | 19.17 | 79941 | 446 | 20 | 40 | 4 | 25 | 42.32 | 33.14 | 08754 | 512 | 20 |
| 50 | 3 | 49 | 0.63 | 80387 | 448 | 10 | 50 | 4 | 26 | 15.32 | 32.84 | 09266 | 514 | 10 |
| | | | 41.33 | | | | | | | | | | | |
| 43° | 3 | 49 | 41.96 | 80835 | 449 | 317° | 53° | 4 | 26 | 48.16 | 09780 | 514 | 307° | |
| 10' | 3 | 50 | 23.16 | 81284 | 449 | 50' | 10' | 4 | 27 | 20.85 | 32.69 | 10294 | 515 | 50' |
| 20 | 3 | 51 | 4.22 | 81733 | 450 | 40 | 20 | 4 | 27 | 53.39 | 32.54 | 10809 | 516 | 40 |
| 30 | 3 | 51 | 45.15 | 82183 | 452 | 30 | 30 | 4 | 28 | 25.79 | 32.40 | 11325 | 517 | 30 |
| 40 | 3 | 52 | 25.94 | 82635 | 454 | 20 | 40 | 4 | 28 | 58.03 | 32.24 | 11842 | 519 | 20 |
| 50 | 3 | 53 | 6.60 | 83089 | 455 | 10 | 50 | 4 | 29 | 30.12 | 32.09 | 12361 | 520 | 10 |
| | | | 40.53 | | | | | | | | 31.94 | | | |
| 44° | 3 | 53 | 47.13 | 83544 | 456 | 316° | 54° | 4 | 30 | 2.06 | 12881 | 520 | 306° | |
| 10' | 3 | 54 | 27.52 | 84000 | 457 | 50' | 10' | 4 | 30 | 33.85 | 31.79 | 13401 | 521 | 50' |
| 20 | 3 | 55 | 7.78 | 84457 | 457 | 40 | 20 | 4 | 31 | 5.49 | 31.64 | 13922 | 522 | 40 |
| 30 | 3 | 55 | 47.90 | 84914 | 459 | 30 | 30 | 4 | 31 | 36.98 | 31.49 | 14444 | 522 | 30 |
| 40 | 3 | 56 | 27.89 | 85373 | 461 | 20 | 40 | 4 | 32 | 8.31 | 31.33 | 14966 | 524 | 20 |
| 50 | 3 | 57 | 7.75 | 85834 | 463 | 10 | 50 | 4 | 32 | 39.50 | 31.19 | 15490 | 525 | 10 |
| | | | 39.72 | | | | | | | | 31.03 | | | |
| 45° | 3 | 57 | 47.47 | 86297 | 463 | 315° | 55° | 4 | 33 | 10.53 | 16015 | 525 | 305° | |
| 10' | 3 | 58 | 27.05 | 86760 | 464 | 50' | 10' | 4 | 33 | 41.41 | 30.88 | 16540 | 527 | 50' |
| 20 | 3 | 59 | 6.50 | 87224 | 466 | 40 | 20 | 4 | 34 | 12.14 | 30.73 | 17067 | 528 | 40 |
| 30 | 3 | 59 | 45.81 | 87690 | 466 | 30 | 30 | 4 | 34 | 42.71 | 30.57 | 17595 | 528 | 30 |
| 40 | 4 | 0 | 24.98 | 88156 | 468 | 20 | 40 | 4 | 35 | 13.13 | 30.42 | 18123 | 529 | 20 |
| 50 | 4 | 1 | 4.01 | 88624 | 469 | 10 | 50 | 4 | 35 | 43.40 | 30.27 | 18652 | 530 | 10 |
| | | | 38.89 | | | | | | | | 30.12 | | | |
| 46° | 4 | 1 | 42.90 | 89093 | 471 | 314° | 56° | 4 | 36 | 13.52 | 19182 | 530 | 304° | |
| 10' | 4 | 2 | 21.65 | 89564 | 471 | 50' | 10' | 4 | 36 | 43.48 | 29.96 | 19712 | 531 | 50' |
| 20 | 4 | 3 | 0.27 | 90035 | 472 | 40 | 20 | 4 | 37 | 13.29 | 29.81 | 20243 | 533 | 40 |
| 30 | 4 | 3 | 38.75 | 90507 | 473 | 30 | 30 | 4 | 37 | 42.94 | 29.65 | 20776 | 533 | 30 |
| 40 | 4 | 4 | 17.09 | 90980 | 475 | 20 | 40 | 4 | 38 | 12.43 | 29.49 | 21309 | 533 | 20 |
| 50 | 4 | 4 | 55.30 | 91455 | 476 | 10 | 50 | 4 | 38 | 41.77 | 29.34 | 21842 | 534 | 10 |
| | | | 38.06 | | | | | | | | 29.19 | | | |
| 47° | 4 | 5 | 33.36 | 91931 | 477 | 313° | 57° | 4 | 39 | 10.96 | 22376 | 535 | 303° | |
| 10' | 4 | 6 | 11.29 | 92408 | 478 | 50' | 10' | 4 | 39 | 39.99 | 29.03 | 22911 | 537 | 50' |
| 20 | 4 | 6 | 49.07 | 92886 | 479 | 40 | 20 | 4 | 40 | 8.87 | 28.88 | 23448 | 538 | 40 |
| 30 | 4 | 7 | 26.72 | 93365 | 480 | 30 | 30 | 4 | 40 | 37.59 | 28.72 | 23986 | 538 | 30 |
| 40 | 4 | 8 | 4.22 | 93845 | 482 | 20 | 40 | 4 | 41 | 6.16 | 28.57 | 24524 | 539 | 20 |
| 50 | 4 | 8 | 41.58 | 94327 | 483 | 10 | 50 | 4 | 41 | 34.57 | 28.41 | 25063 | 540 | 10 |
| | | | 37.22 | | | | | | | | 28.26 | | | |
| 48° | 4 | 9 | 18.80 | 94810 | 484 | 312° | 58° | 4 | 42 | 2.83 | 25603 | 540 | 302° | |
| 10' | 4 | 9 | 55.88 | 95294 | 485 | 50' | 10' | 4 | 42 | 30.93 | 28.10 | 26143 | 541 | 50' |
| 20 | 4 | 10 | 32.81 | 95779 | 486 | 40 | 20 | 4 | 42 | 58.87 | 27.94 | 26684 | 542 | 40 |
| 30 | 4 | 11 | 9.60 | 96265 | 487 | 30 | 30 | 4 | 43 | 26.66 | 27.79 | 27226 | 542 | 30 |
| 40 | 4 | 11 | 46.25 | 96752 | 488 | 20 | 40 | 4 | 43 | 54.29 | 27.63 | 27768 | 543 | 20 |
| 50 | 4 | 12 | 22.76 | 97240 | 490 | 10 | 50 | 4 | 44 | 21.76 | 27.47 | 28311 | 544 | 10 |
| | | | 36.36 | | | | | | | | 27.31 | | | |
| 49° | 4 | 12 | 59.12 | 97730 | 490 | 311° | 59° | 4 | 44 | 49.07 | 28855 | 544 | 301° | |
| 10' | 4 | 13 | 35.34 | 98220 | 491 | 50' | 10' | 4 | 45 | 16.23 | 27.16 | 29399 | 545 | 50' |
| 20 | 4 | 14 | 11.41 | 98711 | 492 | 40 | 20 | 4 | 45 | 43.22 | 26.99 | 29944 | 547 | 40 |
| 30 | 4 | 14 | 47.34 | 99203 | 493 | 30 | 30 | 4 | 46 | 10.06 | 26.84 | 30491 | 547 | 30 |
| 40 | 4 | 15 | 23.13 | 99696 | 495 | 20 | 40 | 4 | 46 | 36.75 | 26.69 | 31038 | 549 | 20 |
| 50 | 4 | 15 | 58.77 | *00191 | 496 | 10 | 50 | 4 | 47 | 3.27 | 26.52 | 31587 | 550 | 10 |
| | | | 35.50 | | | | | | | | 26.37 | | | |
| 50° | 4 | 16 | 34.27 | *00687 | 496 | 310° | 60° | 4 | 47 | 29.64 | 32137 | 550 | 300° | |
| | | | | *1.27 | | | | | | | 1.27 | | | |

TABLE VII.—Continued.

| <i>g</i> | <i>E</i> | | | | Log. <i>r</i> | | <i>g</i> | <i>E</i> | | | | Log. <i>r</i> | | | |
|----------|----------|----|-------|-------|---------------|-----------------|----------|----------|---|----|-------|---------------|--------|-----------------|----------|
| | ° | ' | " | " | 1.27 | | | ° | ' | " | " | 1.27 | | | |
| 60° | 4 | 47 | 29.64 | | 32137 | 55° | 300° | 70° | 5 | 8 | 57.01 | | 66156 | | 290° |
| 10' | 4 | 47 | 55.85 | 26.21 | 32687 | 55° | 50' | 10' | 5 | 9 | 13.48 | 16.47 | 66738 | 58 ₂ | 50' |
| 20 | 4 | 48 | 21.90 | 26.05 | 33237 | 55° | 40 | 20 | 5 | 9 | 29.79 | 16.31 | 67320 | 58 ₂ | 40 |
| 30 | 4 | 48 | 47.79 | 25.89 | 33787 | 55° | 30 | 30 | 5 | 9 | 45.93 | 16.14 | 67903 | 58 ₃ | 30 |
| 40 | 4 | 49 | 13.53 | 25.74 | 34338 | 55 ¹ | 20 | 40 | 5 | 10 | 1.91 | 15.98 | 68487 | 58 ₄ | 20 |
| 50 | 4 | 49 | 39.10 | 25.57 | 34890 | 55 ² | 10 | 50 | 5 | 10 | 17.72 | 15.81 | 69071 | 58 ₄ | 10 |
| | | | | 25.42 | | 553 | | | | | | 15.64 | | 58 ₄ | |
| 61° | 4 | 50 | 4.52 | | 35443 | | 299° | 71° | 5 | 10 | 33.36 | | 69655 | | 289° |
| 10' | 4 | 50 | 29.78 | 25.26 | 35996 | 553 | 50' | 10' | 5 | 10 | 48.84 | 15.48 | 70239 | 58 ₄ | 50' |
| 20 | 4 | 50 | 54.88 | 25.10 | 36550 | 554 | 40 | 20 | 5 | 11 | 4.15 | 15.31 | 70824 | 58 ₅ | 40 |
| 30 | 4 | 51 | 19.82 | 24.94 | 37105 | 555 | 30 | 30 | 5 | 11 | 19.30 | 15.15 | 71409 | 58 ₅ | 30 |
| 40 | 4 | 51 | 44.60 | 24.78 | 37660 | 555 | 20 | 40 | 5 | 11 | 34.28 | 14.98 | 71994 | 58 ₅ | 20 |
| 50 | 4 | 52 | 9.22 | 24.62 | 38216 | 556 | 10 | 50 | 5 | 11 | 49.09 | 14.81 | 72579 | 58 ₅ | 10 |
| | | | | 24.46 | | 557 | | | | | | 14.65 | | 58 ₆ | |
| 62° | 4 | 52 | 33.68 | | 38773 | | 298° | 72° | 5 | 12 | 3.74 | | 73165 | | 288° |
| 10' | 4 | 52 | 57.98 | 24.30 | 39330 | 557 | 50' | 10' | 5 | 12 | 18.23 | 14.49 | 73751 | 58 ₆ | 50' |
| 20 | 4 | 53 | 22.11 | 24.13 | 39888 | 558 | 40 | 20 | 5 | 12 | 32.55 | 14.32 | 74337 | 58 ₆ | 40 |
| 30 | 4 | 53 | 46.09 | 23.98 | 40446 | 558 | 30 | 30 | 5 | 12 | 46.70 | 14.15 | 74924 | 58 ₇ | 30 |
| 40 | 4 | 54 | 9.90 | 23.81 | 41005 | 559 | 20 | 40 | 5 | 13 | 0.69 | 13.99 | 75511 | 58 ₇ | 20 |
| 50 | 4 | 54 | 33.56 | 23.66 | 41565 | 56 ⁰ | 10 | 50 | 5 | 13 | 14.51 | 13.82 | 76098 | 58 ₇ | 10 |
| | | | | 23.49 | | 561 | | | | | | 13.66 | | 58 ₇ | |
| 63° | 4 | 54 | 57.05 | | 42126 | | 297° | 73° | 5 | 13 | 28.17 | | 76685 | | 287° |
| 10' | 4 | 55 | 20.38 | 23.33 | 42687 | 561 | 50' | 10' | 5 | 13 | 41.66 | 13.49 | 77273 | 588 | 50' |
| 20 | 4 | 55 | 43.56 | 23.18 | 43249 | 562 | 40 | 20 | 5 | 13 | 54.98 | 13.32 | 77861 | 588 | 40 |
| 30 | 4 | 56 | 6.57 | 23.01 | 43811 | 562 | 30 | 30 | 5 | 14 | 8.13 | 13.15 | 78450 | 58 ₉ | 30 |
| 40 | 4 | 56 | 29.42 | 22.85 | 44374 | 563 | 20 | 40 | 5 | 14 | 21.12 | 12.99 | 79039 | 58 ₉ | 20 |
| 50 | 4 | 56 | 52.11 | 22.69 | 44938 | 564 | 10 | 50 | 5 | 14 | 33.94 | 12.82 | 79628 | 58 ₉ | 10 |
| | | | | 22.53 | | 565 | | | | | | 12.66 | | 58 ₉ | |
| 64° | 4 | 57 | 14.64 | | 45503 | | 296° | 74° | 5 | 14 | 46.60 | | 80217 | | 286° |
| 10' | 4 | 57 | 37.01 | 22.37 | 46068 | 565 | 50' | 10' | 5 | 14 | 59.09 | 12.49 | 80807 | 59 ⁰ | 50' |
| 20 | 4 | 57 | 59.21 | 22.20 | 46633 | 565 | 40 | 20 | 5 | 15 | 11.41 | 12.32 | 81397 | 59 ⁰ | 40 |
| 30 | 4 | 58 | 21.25 | 22.04 | 47199 | 566 | 30 | 30 | 5 | 15 | 23.57 | 12.16 | 81987 | 59 ⁰ | 30 |
| 40 | 4 | 58 | 43.13 | 21.88 | 47766 | 567 | 20 | 40 | 5 | 15 | 35.56 | 11.99 | 82577 | 59 ⁰ | 20 |
| 50 | 4 | 59 | 4.84 | 21.71 | 48333 | 567 | 10 | 50 | 5 | 15 | 47.39 | 11.83 | 83168 | 59 ¹ | 10 |
| | | | | 21.56 | | 568 | | | | | | 11.66 | | 59 ¹ | |
| 65° | 4 | 59 | 26.40 | | 48901 | | 295° | 75° | 5 | 15 | 59.05 | | 83759 | | 285° |
| 10' | 4 | 59 | 47.79 | 21.39 | 49469 | 568 | 50' | 10' | 5 | 16 | 10.54 | 11.49 | 84350 | 59 ¹ | 50' |
| 20 | 5 | 0 | 9.02 | 21.23 | 50038 | 569 | 40 | 20 | 5 | 16 | 21.87 | 11.33 | 84941 | 59 ¹ | 40 |
| 30 | 5 | 0 | 30.09 | 21.07 | 50607 | 569 | 30 | 30 | 5 | 16 | 33.03 | 11.16 | 85532 | 59 ¹ | 30 |
| 40 | 5 | 0 | 50.99 | 20.90 | 51177 | 57 ⁰ | 20 | 40 | 5 | 16 | 44.03 | 11.00 | 86124 | 59 ² | 20 |
| 50 | 5 | 1 | 11.73 | 20.74 | 51747 | 57 ⁰ | 10 | 50 | 5 | 16 | 54.86 | 10.83 | 86715 | 59 ² | 10 |
| | | | | 20.58 | | 571 | | | | | | 10.66 | | 59 ² | |
| 66° | 5 | 1 | 32.31 | | 52318 | | 294° | 76° | 5 | 17 | 5.52 | | 87307 | | 284° |
| 10' | 5 | 1 | 52.72 | 20.41 | 52889 | 571 | 50' | 10' | 5 | 17 | 16.02 | 10.50 | 87899 | 59 ² | 50' |
| 20 | 5 | 2 | 12.97 | 20.25 | 53461 | 572 | 40 | 20 | 5 | 17 | 26.35 | 10.33 | 88491 | 59 ² | 40 |
| 30 | 5 | 2 | 33.06 | 20.09 | 54033 | 572 | 30 | 30 | 5 | 17 | 36.51 | 10.16 | 89084 | 59 ₃ | 30 |
| 40 | 5 | 2 | 52.98 | 19.92 | 54606 | 573 | 20 | 40 | 5 | 17 | 46.51 | 10.00 | 89677 | 59 ₃ | 20 |
| 50 | 5 | 3 | 12.74 | 19.76 | 55179 | 573 | 10 | 50 | 5 | 17 | 56.34 | 9.83 | 90270 | 59 ₃ | 10 |
| | | | | 19.60 | | 574 | | | | | | 9.66 | | 59 ₃ | |
| 67° | 5 | 3 | 32.34 | | 55753 | | 293° | 77° | 5 | 18 | 6.00 | | 90863 | | 283° |
| 10' | 5 | 3 | 51.77 | 19.43 | 56327 | 574 | 50' | 10' | 5 | 18 | 15.50 | 9.50 | 91456 | 59 ₃ | 50' |
| 20 | 5 | 4 | 11.04 | 19.27 | 56901 | 574 | 40 | 20 | 5 | 18 | 24.82 | 9.32 | 92049 | 59 ₃ | 40 |
| 30 | 5 | 4 | 30.15 | 19.11 | 57476 | 575 | 30 | 30 | 5 | 18 | 33.99 | 9.17 | 92643 | 59 ₄ | 30 |
| 40 | 5 | 4 | 49.09 | 18.94 | 58051 | 575 | 20 | 40 | 5 | 18 | 42.98 | 8.99 | 93237 | 59 ₄ | 20 |
| 50 | 5 | 5 | 7.87 | 18.78 | 58627 | 576 | 10 | 50 | 5 | 18 | 51.81 | 8.83 | 93830 | 59 ₄ | 10 |
| | | | | 18.62 | | 577 | | | | | | 8.66 | | 59 ₄ | |
| 68° | 5 | 5 | 26.49 | | 59204 | | 292° | 78° | 5 | 19 | 0.47 | | 94424 | | 282° |
| 10' | 5 | 5 | 44.94 | 18.45 | 59781 | 577 | 50' | 10' | 5 | 19 | 8.97 | 8.50 | 95018 | 59 ₄ | 50' |
| 20 | 5 | 6 | 3.23 | 18.29 | 60358 | 577 | 40 | 20 | 5 | 19 | 17.30 | 8.33 | 95612 | 59 ₄ | 40 |
| 30 | 5 | 6 | 21.35 | 18.12 | 60936 | 578 | 30 | 30 | 5 | 19 | 25.46 | 8.16 | 96206 | 59 ₄ | 30 |
| 40 | 5 | 6 | 39.30 | 17.95 | 61514 | 578 | 20 | 40 | 5 | 19 | 33.46 | 8.00 | 96800 | 59 ₄ | 20 |
| 50 | 5 | 6 | 57.09 | 17.79 | 62093 | 579 | 10 | 50 | 5 | 19 | 41.29 | 7.83 | 97395 | 59 ₅ | 10 |
| | | | | 17.63 | | 579 | | | | | | 7.66 | | 59 ₅ | |
| 69° | 5 | 7 | 14.72 | | 62673 | | 291° | 79° | 5 | 19 | 48.95 | | 97990 | | 281° |
| 10' | 5 | 7 | 32.18 | 17.46 | 63252 | 579 | 50' | 10' | 5 | 19 | 56.45 | 7.50 | 98585 | 59 ₅ | 50' |
| 20 | 5 | 7 | 49.48 | 17.30 | 63832 | 58 ⁰ | 40 | 20 | 5 | 20 | 3.78 | 7.33 | 99179 | 59 ₄ | 40 |
| 30 | 5 | 8 | 6.61 | 17.13 | 64412 | 58 ⁰ | 30 | 30 | 5 | 20 | 10.94 | 7.16 | 99774 | 59 ₅ | 30 |
| 40 | 5 | 8 | 23.57 | 16.96 | 64993 | 58 ¹ | 20 | 40 | 5 | 20 | 17.94 | 7.00 | *00369 | 59 ₅ | 20 |
| 50 | 5 | 8 | 40.37 | 16.80 | 65574 | 58 ¹ | 10 | 50 | 5 | 20 | 24.77 | 6.83 | *00965 | 59 ₆ | 10 |
| | | | | 16.64 | | 58 ₂ | | | | | | 6.67 | | 59 ₅ | |
| 70° | 5 | 8 | 57.01 | | 66156 | | 290° | 80° | 5 | 20 | 31.44 | | *01560 | | 280° |
| | | | | | 1.27 | | <i>g</i> | | | | | | *1.28 | | <i>g</i> |

TABLE VII.—Continued.

| <i>g</i> | <i>E</i> | | | | Log. <i>r</i> | | <i>g</i> | <i>E</i> | | | | Log. <i>r</i> | |
|----------|----------|----|-------|------|---------------|-----|----------|----------|---|----|-------|---------------|----------|
| | ° | ' | '' | ''' | 1.28 | | | ° | ' | '' | ''' | 1.28 | |
| 80° | 5 | 20 | 31.44 | 6.50 | 01560 | 595 | 280° | 90° | 5 | 22 | 9.05 | 37187 | 270° |
| 10' | 5 | 20 | 37.94 | 6.34 | 02155 | 595 | 50' | 10' | 5 | 22 | 5.68 | 37776 | 589 |
| 20 | 5 | 20 | 44.28 | 6.17 | 02750 | 595 | 40 | 20 | 5 | 22 | 2.15 | 38365 | 589 |
| 30 | 5 | 20 | 50.45 | 6.01 | 03346 | 596 | 30 | 30 | 5 | 21 | 58.45 | 38954 | 589 |
| 40 | 5 | 20 | 56.46 | 5.84 | 03941 | 595 | 20 | 40 | 5 | 21 | 54.59 | 39543 | 589 |
| 50 | 5 | 21 | 2.30 | 5.66 | 04536 | 595 | 10 | 50 | 5 | 21 | 50.58 | 40131 | 588 |
| 81° | 5 | 21 | 7.96 | 5.50 | 05131 | 595 | 279° | 91° | 5 | 21 | 46.40 | 40719 | 588 |
| 10' | 5 | 21 | 13.46 | 5.34 | 05726 | 596 | 50' | 10' | 5 | 21 | 42.06 | 41307 | 588 |
| 20 | 5 | 21 | 18.80 | 5.17 | 06322 | 595 | 40 | 20 | 5 | 21 | 37.57 | 41894 | 587 |
| 30 | 5 | 21 | 23.97 | 5.00 | 06917 | 595 | 30 | 30 | 5 | 21 | 32.91 | 42482 | 588 |
| 40 | 5 | 21 | 28.97 | 4.84 | 07512 | 595 | 20 | 40 | 5 | 21 | 28.09 | 43069 | 587 |
| 50 | 5 | 21 | 33.81 | 4.67 | 08107 | 595 | 10 | 50 | 5 | 21 | 23.11 | 43655 | 586 |
| 82° | 5 | 21 | 38.48 | 4.51 | 08702 | 595 | 278° | 92° | 5 | 21 | 17.97 | 44242 | 587 |
| 10' | 5 | 21 | 42.99 | 4.34 | 09297 | 596 | 50' | 10' | 5 | 21 | 12.67 | 44828 | 586 |
| 20 | 5 | 21 | 47.33 | 4.17 | 09893 | 595 | 40 | 20 | 5 | 21 | 7.20 | 45414 | 586 |
| 30 | 5 | 21 | 51.50 | 4.01 | 10488 | 595 | 30 | 30 | 5 | 21 | 1.58 | 46000 | 586 |
| 40 | 5 | 21 | 55.51 | 3.85 | 11083 | 595 | 20 | 40 | 5 | 20 | 55.79 | 46585 | 585 |
| 50 | 5 | 21 | 59.36 | 3.68 | 11679 | 596 | 10 | 50 | 5 | 20 | 49.85 | 47170 | 585 |
| 83° | 5 | 22 | 3.04 | 3.52 | 12274 | 595 | 277° | 93° | 5 | 20 | 43.74 | 47755 | 585 |
| 10' | 5 | 22 | 6.56 | 3.35 | 12869 | 596 | 50' | 10' | 5 | 20 | 37.48 | 48339 | 584 |
| 20 | 5 | 22 | 9.91 | 3.18 | 13465 | 595 | 40 | 20 | 5 | 20 | 31.05 | 48923 | 584 |
| 30 | 5 | 22 | 13.09 | 3.02 | 14060 | 595 | 30 | 30 | 5 | 20 | 24.47 | 49507 | 584 |
| 40 | 5 | 22 | 16.11 | 2.86 | 14655 | 595 | 20 | 40 | 5 | 20 | 17.72 | 50090 | 583 |
| 50 | 5 | 22 | 18.97 | 2.69 | 15250 | 596 | 10 | 50 | 5 | 20 | 10.82 | 50673 | 583 |
| 84° | 5 | 22 | 21.66 | 2.53 | 15846 | 595 | 276° | 94° | 5 | 20 | 3.76 | 51256 | 582 |
| 10' | 5 | 22 | 24.19 | 2.36 | 16441 | 595 | 50' | 10' | 5 | 19 | 56.54 | 51838 | 582 |
| 20 | 5 | 22 | 26.55 | 2.20 | 17036 | 595 | 40 | 20 | 5 | 19 | 49.16 | 52420 | 582 |
| 30 | 5 | 22 | 28.75 | 2.03 | 17631 | 595 | 30 | 30 | 5 | 19 | 41.62 | 53002 | 582 |
| 40 | 5 | 22 | 30.78 | 1.86 | 18226 | 594 | 20 | 40 | 5 | 19 | 33.93 | 53583 | 581 |
| 50 | 5 | 22 | 32.64 | 1.70 | 18820 | 595 | 10 | 50 | 5 | 19 | 26.07 | 54163 | 580 |
| 85° | 5 | 22 | 34.34 | 1.53 | 19415 | 595 | 275° | 95° | 5 | 19 | 18.06 | 54743 | 580 |
| 10' | 5 | 22 | 35.87 | 1.37 | 20010 | 594 | 50' | 10' | 5 | 19 | 9.89 | 55323 | 579 |
| 20 | 5 | 22 | 37.24 | 1.20 | 20604 | 594 | 40 | 20 | 5 | 19 | 1.56 | 55902 | 579 |
| 30 | 5 | 22 | 38.44 | 1.04 | 21198 | 594 | 30 | 30 | 5 | 18 | 53.06 | 56481 | 579 |
| 40 | 5 | 22 | 39.48 | 0.87 | 21792 | 594 | 20 | 40 | 5 | 18 | 44.42 | 57060 | 578 |
| 50 | 5 | 22 | 40.35 | 0.71 | 22386 | 594 | 10 | 50 | 5 | 18 | 35.61 | 57638 | 578 |
| 86° | 5 | 22 | 41.06 | 0.54 | 22980 | 594 | 274° | 96° | 5 | 18 | 26.65 | 58216 | 578 |
| 10' | 5 | 22 | 41.60 | 0.39 | 23574 | 593 | 50' | 10' | 5 | 18 | 17.53 | 58794 | 577 |
| 20 | 5 | 22 | 41.99 | 0.22 | 24167 | 593 | 40 | 20 | 5 | 18 | 8.26 | 59371 | 577 |
| 30 | 5 | 22 | 42.21 | 0.05 | 24761 | 593 | 30 | 30 | 5 | 17 | 58.83 | 59948 | 577 |
| 40 | 5 | 22 | 42.26 | 0.11 | 25354 | 594 | 20 | 40 | 5 | 17 | 49.24 | 60525 | 576 |
| 50 | 5 | 22 | 42.15 | 0.27 | 25948 | 593 | 10 | 50 | 5 | 17 | 39.50 | 61101 | 576 |
| 87° | 5 | 22 | 41.88 | 0.43 | 26541 | 593 | 273° | 97° | 5 | 17 | 29.60 | 61677 | 575 |
| 10' | 5 | 22 | 41.45 | 0.60 | 27134 | 592 | 50' | 10' | 5 | 17 | 19.54 | 62252 | 575 |
| 20 | 5 | 22 | 40.85 | 0.76 | 27726 | 592 | 40 | 20 | 5 | 17 | 9.33 | 62827 | 575 |
| 30 | 5 | 22 | 40.09 | 0.93 | 28319 | 592 | 30 | 30 | 5 | 16 | 58.96 | 63401 | 574 |
| 40 | 5 | 22 | 39.16 | 1.09 | 28911 | 593 | 20 | 40 | 5 | 16 | 48.44 | 63975 | 574 |
| 50 | 5 | 22 | 38.07 | 1.25 | 29504 | 592 | 10 | 50 | 5 | 16 | 37.76 | 64548 | 573 |
| 88° | 5 | 22 | 36.82 | 1.42 | 30096 | 592 | 272° | 98° | 5 | 16 | 26.92 | 65120 | 572 |
| 10' | 5 | 22 | 35.40 | 1.58 | 30688 | 592 | 50' | 10' | 5 | 16 | 15.93 | 65692 | 572 |
| 20 | 5 | 22 | 33.82 | 1.74 | 31280 | 592 | 40 | 20 | 5 | 16 | 4.78 | 66264 | 572 |
| 30 | 5 | 22 | 32.08 | 1.91 | 31872 | 592 | 30 | 30 | 5 | 15 | 53.48 | 66836 | 571 |
| 40 | 5 | 22 | 30.17 | 2.07 | 32464 | 591 | 20 | 40 | 5 | 15 | 42.02 | 67407 | 571 |
| 50 | 5 | 22 | 28.10 | 2.24 | 33055 | 592 | 10 | 50 | 5 | 15 | 30.41 | 67977 | 570 |
| 89° | 5 | 22 | 25.86 | 2.39 | 33647 | 591 | 271° | 99° | 5 | 15 | 18.64 | 68547 | 570 |
| 10' | 5 | 22 | 23.47 | 2.56 | 34238 | 590 | 50' | 10' | 5 | 15 | 6.72 | 69117 | 569 |
| 20 | 5 | 22 | 20.91 | 2.72 | 34828 | 590 | 40 | 20 | 5 | 14 | 54.64 | 69686 | 569 |
| 30 | 5 | 22 | 18.19 | 2.88 | 35418 | 590 | 30 | 30 | 5 | 14 | 42.41 | 70255 | 568 |
| 40 | 5 | 22 | 15.31 | 3.05 | 36008 | 589 | 20 | 40 | 5 | 14 | 30.03 | 70823 | 568 |
| 50 | 5 | 22 | 12.26 | 3.21 | 36597 | 590 | 10 | 50 | 5 | 14 | 17.49 | 71391 | 567 |
| 90° | 5 | 22 | 9.05 | | 37187 | | 270° | 100° | 5 | 14 | 4.80 | 71958 | 567 |
| | | | | | 1.23 | | <i>g</i> | | | | | 1.23 | <i>g</i> |

TABLE VII.—Continued.

| g | | E | | | | Log. r | | g | | E | | | | Log. r | | |
|------|---|----|-------|-------|--------|--------|------|------|------|----|-------|-------|-------|--------|------|------|
| | ° | ' | " | " | 1.23 | | | | ° | ' | " | " | 1.29 | | | |
| 100° | 5 | 14 | 4.80 | | 71958 | | | 260° | 110° | 4 | 56 | 49.88 | | 04888 | | 250° |
| 10' | 5 | 13 | 51.96 | 12.84 | 72525 | 567 | 50' | 50' | 4 | 56 | 28.20 | 21.68 | 05416 | 528 | 50' | |
| 20 | 5 | 13 | 38.96 | 13.00 | 73091 | 566 | 40 | 20 | 4 | 56 | 6.39 | 21.81 | 05944 | 528 | 40 | |
| 30 | 5 | 13 | 25.81 | 13.15 | 73656 | 565 | 30 | 30 | 4 | 55 | 44.44 | 21.95 | 06471 | 527 | 30 | |
| 40 | 5 | 13 | 12.51 | 13.30 | 74221 | 565 | 20 | 40 | 4 | 55 | 22.34 | 22.10 | 06997 | 526 | 20 | |
| 50 | 5 | 12 | 59.06 | 13.45 | 74785 | 564 | 10 | 50 | 4 | 55 | 0.10 | 22.24 | 07522 | 525 | 10 | |
| | | | | 13.61 | | 563 | | | | | | 22.38 | | 525 | | |
| 101° | 5 | 12 | 45.45 | | 75348 | | 259° | 111° | 4 | 54 | 37.72 | | 08047 | | 249° | |
| 10' | 5 | 12 | 31.69 | 13.76 | 75911 | 563 | 50' | 50' | 4 | 54 | 15.21 | 22.51 | 08571 | 524 | 50' | |
| 20 | 5 | 12 | 17.78 | 13.91 | 76474 | 563 | 40 | 20 | 4 | 53 | 52.55 | 22.66 | 09094 | 523 | 40 | |
| 30 | 5 | 12 | 3.72 | 14.06 | 77036 | 562 | 30 | 30 | 4 | 53 | 29.76 | 22.79 | 09616 | 522 | 30 | |
| 40 | 5 | 11 | 49.50 | 14.22 | 77598 | 562 | 20 | 40 | 4 | 53 | 6.83 | 22.93 | 10137 | 521 | 20 | |
| 50 | 5 | 11 | 35.14 | 14.36 | 78159 | 561 | 10 | 50 | 4 | 52 | 43.76 | 23.07 | 10658 | 521 | 10 | |
| | | | | 14.52 | | 561 | | | | | | 23.21 | | 520 | | |
| 102° | 5 | 11 | 20.62 | | 78720 | | 258° | 112° | 4 | 52 | 20.55 | | 11178 | | 248° | |
| 10' | 5 | 11 | 5.95 | 14.67 | 79280 | 560 | 50' | 50' | 4 | 51 | 57.20 | 23.35 | 11697 | 519 | 50' | |
| 20 | 5 | 10 | 51.13 | 14.82 | 79839 | 559 | 40 | 20 | 4 | 51 | 33.72 | 23.48 | 12215 | 518 | 40 | |
| 30 | 5 | 10 | 36.16 | 14.97 | 80398 | 559 | 30 | 30 | 4 | 51 | 10.10 | 23.62 | 12732 | 517 | 30 | |
| 40 | 5 | 10 | 21.04 | 15.12 | 80956 | 558 | 20 | 40 | 4 | 50 | 46.34 | 23.76 | 13248 | 516 | 20 | |
| 50 | 5 | 10 | 5.77 | 15.27 | 81514 | 558 | 10 | 50 | 4 | 50 | 22.44 | 23.90 | 13764 | 516 | 10 | |
| | | | | 15.42 | | 557 | | | | | | 24.03 | | 515 | | |
| 103° | 5 | 9 | 50.35 | | 82071 | | 257° | 113° | 4 | 49 | 58.41 | | 14279 | | 247° | |
| 10' | 5 | 9 | 34.78 | 15.57 | 82628 | 557 | 50 | 50' | 4 | 49 | 34.24 | 24.17 | 14793 | 514 | 50' | |
| 20 | 5 | 9 | 19.06 | 15.72 | 83184 | 556 | 40 | 20 | 4 | 49 | 9.94 | 24.30 | 15306 | 513 | 40 | |
| 30 | 5 | 9 | 3.19 | 15.87 | 83739 | 555 | 30 | 30 | 4 | 48 | 45.50 | 24.44 | 15818 | 512 | 30 | |
| 40 | 5 | 8 | 47.17 | 16.02 | 84294 | 555 | 20 | 40 | 4 | 48 | 20.92 | 24.58 | 16329 | 511 | 20 | |
| 50 | 5 | 8 | 31.01 | 16.16 | 84849 | 555 | 10 | 50 | 4 | 47 | 56.21 | 24.71 | 16840 | 511 | 10 | |
| | | | | 16.32 | | 554 | | | | | | 24.85 | | 510 | | |
| 104° | 5 | 8 | 14.69 | | 85403 | | 256° | 114° | 4 | 47 | 31.36 | | 17350 | | 246° | |
| 10' | 5 | 7 | 58.22 | 16.47 | 85956 | 553 | 50' | 50' | 4 | 47 | 6.38 | 24.98 | 17859 | 509 | 50' | |
| 20 | 5 | 7 | 41.61 | 16.61 | 86508 | 552 | 40 | 20 | 4 | 46 | 41.26 | 25.12 | 18367 | 508 | 40 | |
| 30 | 5 | 7 | 24.85 | 16.76 | 87059 | 551 | 30 | 30 | 4 | 46 | 16.00 | 25.26 | 18874 | 507 | 30 | |
| 40 | 5 | 7 | 7.94 | 16.91 | 87610 | 551 | 20 | 40 | 4 | 45 | 50.61 | 25.39 | 19380 | 506 | 20 | |
| 50 | 5 | 6 | 50.88 | 17.06 | 88160 | 550 | 10 | 50 | 4 | 45 | 25.09 | 25.52 | 19886 | 506 | 10 | |
| | | | | 17.21 | | 549 | | | | | | 25.65 | | 505 | | |
| 105° | 5 | 6 | 33.67 | | 88709 | | 255° | 115° | 4 | 44 | 59.44 | | 20391 | | 245° | |
| 10' | 5 | 6 | 16.32 | 17.35 | 89258 | 549 | 50' | 50' | 4 | 44 | 33.66 | 25.78 | 20895 | 504 | 50' | |
| 20 | 5 | 5 | 58.82 | 17.50 | 89806 | 548 | 40 | 20 | 4 | 44 | 7.74 | 25.92 | 21398 | 503 | 40 | |
| 30 | 5 | 5 | 41.17 | 17.65 | 90354 | 548 | 30 | 30 | 4 | 43 | 41.69 | 26.05 | 21900 | 502 | 30 | |
| 40 | 5 | 5 | 23.37 | 17.80 | 90902 | 548 | 20 | 40 | 4 | 43 | 15.50 | 26.19 | 22401 | 501 | 20 | |
| 50 | 5 | 5 | 5.42 | 17.95 | 91449 | 547 | 10 | 50 | 4 | 42 | 49.19 | 26.31 | 22901 | 500 | 10 | |
| | | | | 18.09 | | 546 | | | | | | 26.45 | | 500 | | |
| 106° | 5 | 4 | 47.33 | | 91995 | | 254° | 116° | 4 | 42 | 22.74 | | 23401 | | 244° | |
| 10' | 5 | 4 | 29.10 | 18.23 | 92540 | 545 | 50' | 50' | 4 | 41 | 56.16 | 26.58 | 23899 | 498 | 50' | |
| 20 | 5 | 4 | 10.72 | 18.38 | 93085 | 545 | 40 | 20 | 4 | 41 | 29.45 | 26.71 | 24397 | 498 | 40 | |
| 30 | 5 | 3 | 52.19 | 18.53 | 93629 | 544 | 30 | 30 | 4 | 41 | 2.00 | 26.85 | 24894 | 497 | 30 | |
| 40 | 5 | 3 | 33.52 | 18.67 | 94172 | 543 | 20 | 40 | 4 | 40 | 35.62 | 26.98 | 25390 | 496 | 20 | |
| 50 | 5 | 3 | 14.70 | 18.82 | 94715 | 543 | 10 | 50 | 4 | 40 | 8.52 | 27.10 | 25885 | 495 | 10 | |
| | | | | 18.96 | | 542 | | | | | | 27.24 | | 495 | | |
| 107° | 5 | 2 | 55.74 | | 95257 | | 253° | 117° | 4 | 39 | 41.28 | | 26380 | | 243° | |
| 10' | 5 | 2 | 36.63 | 19.11 | 95798 | 541 | 50' | 50' | 4 | 39 | 13.91 | 27.37 | 26874 | 494 | 50' | |
| 20 | 5 | 2 | 17.38 | 19.25 | 96338 | 540 | 40 | 20 | 4 | 38 | 46.41 | 27.50 | 27366 | 492 | 40 | |
| 30 | 5 | 1 | 57.99 | 19.39 | 96878 | 540 | 30 | 30 | 4 | 38 | 18.78 | 27.63 | 27857 | 491 | 30 | |
| 40 | 5 | 1 | 38.45 | 19.54 | 97417 | 539 | 20 | 40 | 4 | 37 | 51.02 | 27.76 | 28348 | 491 | 20 | |
| 50 | 5 | 1 | 18.77 | 19.68 | 97955 | 538 | 10 | 50 | 4 | 37 | 23.13 | 27.89 | 28837 | 489 | 10 | |
| | | | | 19.83 | | 538 | | | | | | 28.01 | | 488 | | |
| 108° | 5 | 0 | 58.94 | | 98493 | | 252° | 118° | 4 | 36 | 55.12 | | 29325 | | 242° | |
| 10' | 5 | 0 | 38.97 | 19.97 | 99030 | 537 | 50' | 50' | 4 | 36 | 26.98 | 28.14 | 29812 | 487 | 50' | |
| 20 | 5 | 0 | 18.86 | 20.11 | 99566 | 536 | 40 | 20 | 4 | 35 | 58.71 | 28.27 | 30299 | 487 | 40 | |
| 30 | 4 | 59 | 58.60 | 20.26 | *00101 | 535 | 30 | 30 | 4 | 35 | 30.31 | 28.40 | 30785 | 486 | 30 | |
| 40 | 4 | 59 | 38.21 | 20.39 | *00636 | 535 | 20 | 40 | 4 | 35 | 1.78 | 28.53 | 31270 | 485 | 20 | |
| 50 | 4 | 59 | 17.67 | 20.54 | *01170 | 534 | 10 | 50 | 4 | 34 | 33.13 | 28.65 | 31754 | 484 | 10 | |
| | | | | 20.69 | | 533 | | | | | | 28.78 | | 483 | | |
| 109° | 4 | 58 | 56.98 | | *01703 | | 251° | 119° | 4 | 34 | 4.35 | | 32237 | | 241° | |
| 10' | 4 | 58 | 36.15 | 20.83 | *02236 | 533 | 50' | 50' | 4 | 33 | 35.44 | 28.91 | 32719 | 482 | 50' | |
| 20 | 4 | 58 | 15.18 | 20.97 | *02768 | 532 | 40 | 20 | 4 | 33 | 6.41 | 29.03 | 33200 | 481 | 40 | |
| 30 | 4 | 57 | 54.07 | 21.11 | *03299 | 531 | 30 | 30 | 4 | 32 | 37.25 | 29.16 | 33681 | 481 | 30 | |
| 40 | 4 | 57 | 32.81 | 21.26 | *03829 | 530 | 20 | 40 | 4 | 32 | 7.96 | 29.29 | 34161 | 480 | 20 | |
| 50 | 4 | 57 | 11.42 | 21.39 | *04359 | 530 | 10 | 50 | 4 | 31 | 38.55 | 29.41 | 34640 | 479 | 10 | |
| | | | | 21.54 | | 529 | | | | | | 29.54 | | 477 | | |
| 110° | 4 | 56 | 49.88 | | *04888 | | 250° | 120° | 4 | 31 | 9.01 | | 35117 | | 240° | |
| | | | | | *1.29 | | g | | | | | | 1.29 | | g | |

TABLE VII.—Continued.

| <i>g</i> | <i>E</i> | Log. <i>r</i> | <i>g</i> | <i>E</i> | Log. <i>r</i> |
|-------------|-------------|---------------|-------------|-------------|---------------|
| | | 1.29 | | | 1.29 |
| 120° | 4 31 9.01 | 29.66 | 240° | 3 57 57.75 | 61903 |
| 10' | 4 30 39.35 | 29.79 | 50' | 3 57 21.10 | 62317 414 |
| 20 | 4 30 9.56 | 29.91 | 40 | 3 56 44.35 | 62729 412 |
| 30 | 4 29 39.65 | 30.04 | 30 | 3 56 7.48 | 63140 411 |
| 40 | 4 29 9.61 | 30.16 | 20 | 3 55 30.51 | 63550 410 |
| 50 | 4 28 39.45 | 30.29 | 10 | 3 54 53.43 | 63959 409 |
| | | | | | 64366 407 |
| 121° | 4 28 9.16 | 30.41 | 239° | 3 54 16.25 | 64773 407 |
| 10' | 4 27 38.75 | 30.53 | 50' | 3 53 38.97 | 65179 406 |
| 20 | 4 27 8.22 | 30.66 | 40 | 3 53 1.58 | 65583 404 |
| 30 | 4 26 37.56 | 30.78 | 30 | 3 52 24.08 | 65987 404 |
| 40 | 4 26 6.78 | 30.90 | 20 | 3 51 46.48 | 66389 402 |
| 50 | 4 25 35.88 | 31.03 | 10 | 3 51 8.78 | 66790 401 |
| | | | | | 67190 400 |
| 122° | 4 25 4.85 | 31.14 | 238° | 3 50 30.97 | 67588 398 |
| 10' | 4 24 33.71 | 31.27 | 50' | 3 49 53.06 | 67985 397 |
| 20 | 4 24 2.44 | 31.39 | 40 | 3 49 15.04 | 68382 397 |
| 30 | 4 23 31.05 | 31.51 | 30 | 3 48 36.92 | 68777 395 |
| 40 | 4 22 59.54 | 31.63 | 20 | 3 47 58.70 | 69171 393 |
| 50 | 4 22 27.91 | 31.75 | 10 | 3 47 20.38 | 69564 391 |
| | | | | | 69955 391 |
| 123° | 4 21 56.16 | 31.87 | 237° | 3 46 41.95 | 70345 390 |
| 10' | 4 21 24.29 | 32.00 | 50' | 3 46 3.42 | 70734 389 |
| 20 | 4 20 52.29 | 32.11 | 40 | 3 45 24.79 | 71122 387 |
| 30 | 4 20 20.18 | 32.24 | 30 | 3 44 46.06 | 71509 386 |
| 40 | 4 19 47.94 | 32.35 | 20 | 3 44 7.23 | 71895 384 |
| 50 | 4 19 15.59 | 32.47 | 10 | 3 43 28.30 | 72279 383 |
| | | | | | 72662 383 |
| 124° | 4 18 43.12 | 32.59 | 236° | 3 42 49.27 | 73044 382 |
| 10' | 4 18 10.53 | 32.70 | 50' | 3 42 10.14 | 73425 381 |
| 20 | 4 17 37.83 | 32.82 | 40 | 3 41 30.91 | 73805 379 |
| 30 | 4 17 5.01 | 32.94 | 30 | 3 40 51.58 | 74184 377 |
| 40 | 4 16 32.07 | 33.06 | 20 | 3 40 12.15 | 74561 376 |
| 50 | 4 15 59.01 | 33.17 | 10 | 3 39 32.62 | 74937 375 |
| | | | | | 75312 374 |
| 125° | 4 15 25.84 | 33.29 | 235° | 3 38 53.00 | 75686 373 |
| 10' | 4 14 52.55 | 33.41 | 50' | 3 38 13.28 | 76059 371 |
| 20 | 4 14 19.14 | 33.52 | 40 | 3 37 33.46 | 76430 370 |
| 30 | 4 13 45.62 | 33.64 | 30 | 3 36 53.54 | 76800 369 |
| 40 | 4 13 11.98 | 33.76 | 20 | 3 36 13.52 | 77169 368 |
| 50 | 4 12 38.22 | 33.87 | 10 | 3 35 33.41 | 77537 366 |
| | | | | | 77903 365 |
| 126° | 4 12 4.35 | 33.98 | 234° | 3 34 53.21 | 78268 364 |
| 10' | 4 11 30.37 | 34.10 | 50' | 3 34 12.90 | 78632 363 |
| 20 | 4 10 56.27 | 34.21 | 40 | 3 33 32.50 | 78995 361 |
| 30 | 4 10 22.06 | 34.33 | 30 | 3 32 52.01 | 79356 360 |
| 40 | 4 9 47.73 | 34.44 | 20 | 3 32 11.42 | 79716 359 |
| 50 | 4 9 13.29 | 34.55 | 10 | 3 31 30.73 | 80075 358 |
| | | | | | 80433 356 |
| 127° | 4 8 38.74 | 34.67 | 233° | 3 30 49.95 | 80789 355 |
| 10' | 4 8 4.07 | 34.78 | 50' | 3 30 9.08 | 81144 354 |
| 20 | 4 7 29.29 | 34.89 | 40 | 3 29 28.11 | 81498 353 |
| 30 | 4 6 54.40 | 35.00 | 30 | 3 28 47.05 | 81851 351 |
| 40 | 4 6 19.40 | 35.11 | 20 | 3 28 5.90 | 82202 350 |
| 50 | 4 5 44.29 | 35.23 | 10 | 3 27 24.66 | 82552 349 |
| | | | | | 82901 348 |
| 128° | 4 5 9.06 | 35.34 | 232° | 3 26 43.32 | 83249 346 |
| 10' | 4 4 33.72 | 35.45 | 50' | 3 26 1.89 | 83595 345 |
| 20 | 4 3 58.27 | 35.55 | 40 | 3 25 20.37 | 83940 344 |
| 30 | 4 3 22.72 | 35.67 | 30 | 3 24 38.76 | 84284 343 |
| 40 | 4 2 47.05 | 35.78 | 20 | 3 23 57.05 | 84627 343 |
| 50 | 4 2 11.27 | 35.89 | 10 | 3 23 15.26 | 84970 343 |
| | | | | | 85313 343 |
| 129° | 4 1 35.38 | 36.00 | 231° | 3 22 33.38 | 85656 342 |
| 10' | 4 0 59.38 | 36.11 | 50' | 3 21 51.41 | 85999 341 |
| 20 | 4 0 23.27 | 36.22 | 40 | 3 21 9.35 | 86342 340 |
| 30 | 3 59 47.05 | 36.32 | 30 | 3 20 27.20 | 86685 339 |
| 40 | 3 59 10.73 | 36.44 | 20 | 3 19 44.96 | 87028 338 |
| 50 | 3 58 34.29 | 36.54 | 10 | 3 19 2.63 | 87371 337 |
| | | | | | 87714 337 |
| 130° | 3 57 57.75 | 61903 | 230° | 3 18 20.22 | 88057 336 |
| | 1.29 | <i>g</i> | | 1.29 | <i>g</i> |

TABLE VII.—Continued.

| <i>g</i> | | | | <i>E</i> | | | | Log. <i>r</i> | | <i>g</i> | | | | <i>E</i> | | | | Log. <i>r</i> | |
|----------|---|----|-------|----------|--------|-----|--|---------------|------|----------|----|-------|-------|----------|-----|------|--|---------------|--|
| ° ' " | | | | ° ' " | | | | 1.29 | | ° ' " | | | | ° ' " | | | | 1.30 | |
| 140° | 3 | 18 | 20.22 | 42.50 | 84627 | | | 220° | 150° | 2 | 33 | 26.93 | 47.14 | 02798 | 262 | 210° | | | |
| 10' | 3 | 17 | 37.72 | 42.59 | 84968 | 341 | | 50' | 10' | 2 | 32 | 39.79 | 47.20 | 03060 | 260 | 50' | | | |
| 20 | 3 | 16 | 55.13 | 42.68 | 85308 | 340 | | 40 | 20 | 2 | 31 | 52.59 | 47.27 | 03320 | 259 | 40 | | | |
| 30 | 3 | 16 | 12.45 | 42.77 | 85647 | 339 | | 30 | 30 | 2 | 31 | 5.32 | 47.34 | 03579 | 258 | 30 | | | |
| 40 | 3 | 15 | 29.68 | 42.85 | 85985 | 338 | | 20 | 40 | 2 | 30 | 17.98 | 47.40 | 03837 | 256 | 20 | | | |
| 50 | 3 | 14 | 46.83 | 42.94 | 86321 | 336 | | 10 | 50 | 2 | 29 | 30.58 | 47.47 | 04093 | 255 | 10 | | | |
| | | | | | | 335 | | | | | | | | | | | | | |
| 141° | 3 | 14 | 3.89 | 43.02 | 86656 | | | 219° | 151° | 2 | 28 | 43.11 | 47.53 | 04348 | 254 | 209° | | | |
| 10' | 3 | 13 | 20.87 | 43.11 | 86990 | 334 | | 50' | 10' | 2 | 27 | 55.58 | 47.60 | 04602 | 252 | 50' | | | |
| 20 | 3 | 12 | 37.76 | 43.20 | 87322 | 332 | | 40 | 20 | 2 | 27 | 7.98 | 47.66 | 04854 | 251 | 40 | | | |
| 30 | 3 | 11 | 54.56 | 43.28 | 87653 | 331 | | 30 | 30 | 2 | 26 | 20.32 | 47.73 | 05105 | 250 | 30 | | | |
| 40 | 3 | 11 | 11.28 | 43.36 | 87983 | 330 | | 20 | 40 | 2 | 25 | 32.59 | 47.79 | 05355 | 249 | 20 | | | |
| 50 | 3 | 10 | 27.92 | 43.45 | 88312 | 329 | | 10 | 50 | 2 | 24 | 44.80 | 47.85 | 05604 | 247 | 10 | | | |
| | | | | | | 327 | | | | | | | | | | | | | |
| 142° | 3 | 9 | 44.47 | 43.53 | 88639 | | | 218° | 152° | 2 | 23 | 56.95 | 47.91 | 05851 | 245 | 208° | | | |
| 10' | 3 | 9 | 0.94 | 43.62 | 88965 | 326 | | 50' | 10' | 2 | 23 | 9.04 | 47.98 | 06096 | 244 | 50' | | | |
| 20 | 3 | 8 | 17.32 | 43.70 | 89290 | 325 | | 40 | 20 | 2 | 22 | 21.06 | 48.04 | 06340 | 243 | 40 | | | |
| 30 | 3 | 7 | 33.62 | 43.79 | 89614 | 324 | | 30 | 30 | 2 | 21 | 33.02 | 48.10 | 06583 | 241 | 30 | | | |
| 40 | 3 | 6 | 49.83 | 43.86 | 89936 | 322 | | 20 | 40 | 2 | 20 | 44.92 | 48.16 | 06824 | 240 | 20 | | | |
| 50 | 3 | 6 | 5.97 | 43.95 | 90257 | 321 | | 10 | 50 | 2 | 19 | 56.76 | 48.22 | 07064 | 238 | 10 | | | |
| | | | | | | 320 | | | | | | | | | | | | | |
| 143° | 3 | 5 | 22.02 | 44.03 | 90577 | | | 217° | 153° | 2 | 19 | 8.54 | 48.28 | 07302 | 237 | 207° | | | |
| 10' | 3 | 4 | 37.99 | 44.11 | 90895 | 318 | | 50' | 10' | 2 | 18 | 20.26 | 48.34 | 07539 | 236 | 50' | | | |
| 20 | 3 | 3 | 53.88 | 44.19 | 91212 | 317 | | 40 | 20 | 2 | 17 | 31.92 | 48.41 | 07775 | 234 | 40 | | | |
| 30 | 3 | 3 | 9.69 | 44.27 | 91528 | 316 | | 30 | 30 | 2 | 16 | 43.51 | 48.46 | 08009 | 233 | 30 | | | |
| 40 | 3 | 2 | 25.42 | 44.34 | 91842 | 314 | | 20 | 40 | 2 | 15 | 55.05 | 48.52 | 08242 | 231 | 20 | | | |
| 50 | 3 | 1 | 41.08 | 44.43 | 92155 | 313 | | 10 | 50 | 2 | 15 | 6.53 | 48.58 | 08473 | 230 | 10 | | | |
| | | | | | | 311 | | | | | | | | | | | | | |
| 144° | 3 | 0 | 56.65 | 44.51 | 92466 | | | 216° | 154° | 2 | 14 | 17.95 | 48.64 | 08703 | 228 | 206° | | | |
| 10' | 3 | 0 | 12.14 | 44.59 | 92776 | 310 | | 50' | 10' | 2 | 13 | 29.31 | 48.69 | 08931 | 227 | 50' | | | |
| 20 | 2 | 59 | 27.55 | 44.66 | 93085 | 309 | | 40 | 20 | 2 | 12 | 40.62 | 48.76 | 09158 | 226 | 40 | | | |
| 30 | 2 | 58 | 42.89 | 44.75 | 93393 | 308 | | 30 | 30 | 2 | 11 | 51.86 | 48.81 | 09384 | 224 | 30 | | | |
| 40 | 2 | 57 | 58.14 | 44.82 | 93700 | 307 | | 20 | 40 | 2 | 11 | 3.05 | 48.87 | 09608 | 223 | 20 | | | |
| 50 | 2 | 57 | 13.32 | 44.90 | 94005 | 305 | | 10 | 50 | 2 | 10 | 14.18 | 48.92 | 09831 | 222 | 10 | | | |
| | | | | | | 304 | | | | | | | | | | | | | |
| 145° | 2 | 56 | 28.42 | 44.98 | 94309 | | | 215° | 155° | 2 | 9 | 25.26 | 48.98 | 10053 | 220 | 205° | | | |
| 10' | 2 | 55 | 43.44 | 45.05 | 94612 | 303 | | 50' | 10' | 2 | 8 | 36.28 | 49.04 | 10273 | 219 | 50' | | | |
| 20 | 2 | 54 | 58.39 | 45.13 | 94913 | 301 | | 40 | 20 | 2 | 7 | 47.24 | 49.09 | 10492 | 217 | 40 | | | |
| 30 | 2 | 54 | 13.26 | 45.21 | 95213 | 300 | | 30 | 30 | 2 | 6 | 58.15 | 49.14 | 10709 | 216 | 30 | | | |
| 40 | 2 | 53 | 28.05 | 45.28 | 95511 | 298 | | 20 | 40 | 2 | 6 | 9.01 | 49.20 | 10925 | 214 | 20 | | | |
| 50 | 2 | 52 | 42.77 | 45.36 | 95808 | 297 | | 10 | 50 | 2 | 5 | 19.81 | 49.26 | 11139 | 213 | 10 | | | |
| | | | | | | 296 | | | | | | | | | | | | | |
| 146° | 2 | 51 | 57.41 | 45.44 | 96104 | | | 214° | 156° | 2 | 4 | 30.55 | 49.31 | 11352 | 212 | 204° | | | |
| 10' | 2 | 51 | 11.97 | 45.51 | 96398 | 294 | | 50' | 10' | 2 | 3 | 41.24 | 49.36 | 11564 | 210 | 50' | | | |
| 20 | 2 | 50 | 26.46 | 45.59 | 96691 | 293 | | 40 | 20 | 2 | 2 | 51.88 | 49.41 | 11774 | 209 | 40 | | | |
| 30 | 2 | 49 | 40.87 | 45.66 | 96983 | 292 | | 30 | 30 | 2 | 2 | 2.47 | 49.47 | 11983 | 208 | 30 | | | |
| 40 | 2 | 48 | 55.21 | 45.73 | 97273 | 290 | | 20 | 40 | 2 | 1 | 13.00 | 49.52 | 12191 | 206 | 20 | | | |
| 50 | 2 | 48 | 9.48 | 45.81 | 97562 | 289 | | 10 | 50 | 2 | 0 | 23.48 | 49.57 | 12397 | 205 | 10 | | | |
| | | | | | | 288 | | | | | | | | | | | | | |
| 147° | 2 | 47 | 23.67 | 45.88 | 97850 | | | 213° | 157° | 1 | 59 | 33.91 | 49.62 | 12602 | 203 | 203° | | | |
| 10' | 2 | 46 | 37.79 | 45.96 | 98136 | 286 | | 50' | 10' | 1 | 58 | 44.29 | 49.68 | 12805 | 202 | 50' | | | |
| 20 | 2 | 45 | 51.83 | 46.03 | 98421 | 285 | | 40 | 20 | 1 | 57 | 54.61 | 49.73 | 13007 | 200 | 40 | | | |
| 30 | 2 | 45 | 5.80 | 46.10 | 98705 | 284 | | 30 | 30 | 1 | 57 | 4.88 | 49.78 | 13207 | 198 | 30 | | | |
| 40 | 2 | 44 | 19.70 | 46.17 | 98988 | 283 | | 20 | 40 | 1 | 56 | 15.10 | 49.82 | 13405 | 197 | 20 | | | |
| 50 | 2 | 43 | 33.53 | 46.24 | 99269 | 281 | | 10 | 50 | 1 | 55 | 25.28 | 49.88 | 13602 | 196 | 10 | | | |
| | | | | | | 280 | | | | | | | | | | | | | |
| 148° | 2 | 42 | 47.29 | 46.31 | 99549 | | | 212° | 158° | 1 | 54 | 35.40 | 49.93 | 13798 | 194 | 202° | | | |
| 10' | 2 | 42 | 0.98 | 46.39 | 99827 | 278 | | 50' | 10' | 1 | 53 | 45.47 | 49.97 | 13992 | 193 | 50' | | | |
| 20 | 2 | 41 | 14.59 | 46.45 | *00104 | 277 | | 40 | 20 | 1 | 52 | 55.50 | 50.02 | 14185 | 192 | 40 | | | |
| 30 | 2 | 40 | 28.14 | 46.53 | *00380 | 276 | | 30 | 30 | 1 | 52 | 5.48 | 50.07 | 14377 | 190 | 30 | | | |
| 40 | 2 | 39 | 41.61 | 46.60 | *00654 | 274 | | 20 | 40 | 1 | 51 | 15.41 | 50.12 | 14567 | 189 | 20 | | | |
| 50 | 2 | 38 | 55.01 | 46.66 | *00926 | 272 | | 10 | 50 | 1 | 50 | 25.29 | 50.17 | 14756 | 188 | 10 | | | |
| | | | | | | 271 | | | | | | | | | | | | | |
| 149° | 2 | 38 | 8.35 | 46.73 | *01197 | | | 211° | 159° | 1 | 49 | 35.12 | 50.21 | 14944 | 186 | 201° | | | |
| 10' | 2 | 37 | 21.62 | 46.81 | *01467 | 270 | | 50' | 10' | 1 | 48 | 44.91 | 50.26 | 15130 | 185 | 50' | | | |
| 20 | 2 | 36 | 34.81 | 46.87 | *01736 | 269 | | 40 | 20 | 1 | 47 | 54.65 | 50.31 | 15315 | 183 | 40 | | | |
| 30 | 2 | 35 | 47.94 | 46.93 | *02004 | 268 | | 30 | 30 | 1 | 47 | 4.34 | 50.36 | 15498 | 182 | 30 | | | |
| 40 | 2 | 35 | 1.01 | 47.01 | *02270 | 266 | | 20 | 40 | 1 | 46 | 13.98 | 50.40 | 15680 | 180 | 20 | | | |
| 50 | 2 | 34 | 14.00 | 47.07 | *02535 | 265 | | 10 | 50 | 1 | 45 | 23.58 | 50.44 | 15860 | 179 | 10 | | | |
| | | | | | | 263 | | | | | | | | | | | | | |
| 150° | 2 | 33 | 26.93 | | *02798 | | | 210° | 160° | 1 | 44 | 33.14 | | 16039 | | 200° | | | |
| | | | | | *1.30 | | | <i>g</i> | | | | | 1.30 | | | | | | |

TABLE VII.—Continued.

| <i>g</i> | <i>E</i> | | | | Log. <i>r</i> | <i>g</i> | <i>E</i> | | | | Log. <i>r</i> | | |
|----------|----------|-------|-------|---|---------------|----------|----------|------|-------|-------------|---------------|----------|------|
| | ° | ' | " | " | | | ° | ' | " | " | | | |
| | | | | | 1.30 | | | | | 1.30 | | | |
| 160° | 1 44 | 33.14 | | | 16039 | 200° | 170° | 0 52 | 57.32 | | 24087 | 190° | |
| 10' | 1 43 | 42.65 | 50.49 | | 16216 | 177 | 50' | 0 52 | 4.81 | 52.51 | 24176 | 89 | 50' |
| 20 | 1 42 | 52.11 | 50.54 | | 16392 | 176 | 40 | 0 51 | 12.28 | 52.53 | 24264 | 88 | 40 |
| 30 | 1 42 | 1.53 | 50.58 | | 16566 | 174 | 30 | 0 50 | 19.72 | 52.56 | 24350 | 86 | 30 |
| 40 | 1 41 | 10.91 | 50.62 | | 16739 | 173 | 20 | 0 49 | 27.14 | 52.58 | 24434 | 84 | 20 |
| 50 | 1 40 | 20.24 | 50.67 | | 16910 | 171 | 10 | 0 48 | 34.54 | 52.60 | 24517 | 83 | 10 |
| | | | 50.71 | | | 170 | | | | 52.62 | | 82 | |
| 161° | 1 39 | 29.53 | | | 17080 | 199° | 171° | 0 47 | 41.92 | | 24599 | 80 | 189° |
| 10' | 1 38 | 38.78 | 50.75 | | 17248 | 168 | 50' | 0 46 | 49.28 | 52.64 | 24679 | 79 | 50' |
| 20 | 1 37 | 47.98 | 50.80 | | 17415 | 167 | 40 | 0 45 | 56.62 | 52.66 | 24758 | 77 | 40 |
| 30 | 1 36 | 57.14 | 50.84 | | 17581 | 166 | 30 | 0 45 | 3.93 | 52.69 | 24835 | 75 | 30 |
| 40 | 1 36 | 6.25 | 50.89 | | 17745 | 164 | 20 | 0 44 | 11.23 | 52.70 | 24910 | 74 | 20 |
| 50 | 1 35 | 15.33 | 50.92 | | 17908 | 163 | 10 | 0 43 | 18.51 | 52.72 | 24984 | 73 | 10 |
| | | | 50.96 | | | 162 | | | | 52.74 | | 73 | |
| 162° | 1 34 | 24.37 | | | 18070 | 198° | 172° | 0 42 | 25.77 | | 25057 | 71 | 188° |
| 10' | 1 33 | 33.37 | 51.00 | | 18230 | 160 | 50' | 0 41 | 33.01 | 52.76 | 25128 | 71 | 50' |
| 20 | 1 32 | 42.33 | 51.04 | | 18388 | 158 | 40 | 0 40 | 40.24 | 52.77 | 25198 | 70 | 40 |
| 30 | 1 31 | 51.24 | 51.09 | | 18545 | 157 | 30 | 0 39 | 47.45 | 52.79 | 25267 | 69 | 30 |
| 40 | 1 31 | 0.12 | 51.12 | | 18700 | 155 | 20 | 0 38 | 54.64 | 52.81 | 25334 | 67 | 20 |
| 50 | 1 30 | 8.96 | 51.16 | | 18854 | 154 | 10 | 0 38 | 1.82 | 52.82 | 25399 | 65 | 10 |
| | | | 51.20 | | | 153 | | | | 52.84 | | 64 | |
| 163° | 1 29 | 17.76 | | | 19007 | 197° | 173° | 0 37 | 8.98 | | 25463 | 62 | 187° |
| 10' | 1 28 | 26.52 | 51.24 | | 19158 | 151 | 50' | 0 36 | 16.12 | 52.86 | 25525 | 61 | 50' |
| 20 | 1 27 | 35.25 | 51.27 | | 19308 | 150 | 40 | 0 35 | 23.25 | 52.87 | 25586 | 61 | 40 |
| 30 | 1 26 | 43.93 | 51.32 | | 19456 | 148 | 30 | 0 34 | 30.36 | 52.89 | 25645 | 59 | 30 |
| 40 | 1 25 | 52.58 | 51.35 | | 19603 | 147 | 20 | 0 33 | 37.45 | 52.91 | 25703 | 58 | 20 |
| 50 | 1 25 | 1.19 | 51.39 | | 19749 | 146 | 10 | 0 32 | 44.53 | 52.92 | 25759 | 56 | 10 |
| | | | 51.42 | | | 144 | | | | 52.93 | | 55 | |
| 164° | 1 24 | 9.77 | | | 19893 | 196° | 174° | 0 31 | 51.60 | | 25814 | 53 | 186° |
| 10' | 1 23 | 18.31 | 51.46 | | 20035 | 142 | 50' | 0 30 | 58.66 | 52.94 | 25867 | 53 | 50' |
| 20 | 1 22 | 26.81 | 51.50 | | 20176 | 141 | 40 | 0 30 | 5.70 | 52.96 | 25919 | 52 | 40 |
| 30 | 1 21 | 35.28 | 51.53 | | 20316 | 140 | 30 | 0 29 | 12.73 | 52.97 | 25969 | 50 | 30 |
| 40 | 1 20 | 43.71 | 51.57 | | 20454 | 138 | 20 | 0 28 | 19.75 | 52.98 | 26017 | 48 | 20 |
| 50 | 1 19 | 52.11 | 51.60 | | 20590 | 136 | 10 | 0 27 | 26.76 | 52.99 | 26064 | 47 | 10 |
| | | | 51.64 | | | 135 | | | | 53.00 | | 46 | |
| 165° | 1 19 | 0.47 | | | 20725 | 195° | 175° | 0 26 | 33.76 | | 26110 | 44 | 185° |
| 10' | 1 18 | 8.80 | 51.67 | | 20858 | 133 | 50' | 0 25 | 40.75 | 53.01 | 26154 | 44 | 50' |
| 20 | 1 17 | 17.10 | 51.70 | | 20990 | 132 | 40 | 0 24 | 47.72 | 53.03 | 26197 | 43 | 40 |
| 30 | 1 16 | 25.36 | 51.74 | | 21121 | 131 | 30 | 0 23 | 54.68 | 53.04 | 26239 | 42 | 30 |
| 40 | 1 15 | 33.59 | 51.77 | | 21250 | 129 | 20 | 0 23 | 1.64 | 53.04 | 26279 | 40 | 20 |
| 50 | 1 14 | 41.79 | 51.80 | | 21377 | 127 | 10 | 0 22 | 8.58 | 53.06 | 26318 | 39 | 10 |
| | | | 51.83 | | | 126 | | | | 53.07 | | 37 | |
| 166° | 1 13 | 49.96 | | | 21503 | 194° | 176° | 0 21 | 15.51 | | 26355 | 35 | 184° |
| 10' | 1 12 | 58.10 | 51.86 | | 21628 | 125 | 50' | 0 20 | 22.43 | 53.08 | 26390 | 35 | 50' |
| 20 | 1 12 | 6.20 | 51.90 | | 21751 | 123 | 40 | 0 19 | 29.35 | 53.08 | 26424 | 34 | 40 |
| 30 | 1 11 | 14.27 | 51.93 | | 21873 | 122 | 30 | 0 18 | 36.25 | 53.10 | 26456 | 32 | 30 |
| 40 | 1 10 | 22.31 | 51.96 | | 21993 | 120 | 20 | 0 17 | 43.15 | 53.10 | 26486 | 30 | 20 |
| 50 | 1 9 | 30.33 | 51.98 | | 22112 | 119 | 10 | 0 16 | 50.03 | 53.12 | 26515 | 29 | 10 |
| | | | 52.02 | | | 118 | | | | 53.12 | | 28 | |
| 167° | 1 8 | 38.31 | | | 22230 | 193° | 177° | 0 15 | 56.91 | | 26543 | 26 | 183° |
| 10' | 1 7 | 46.27 | 52.04 | | 22346 | 116 | 50' | 0 15 | 3.78 | 53.13 | 26569 | 26 | 50' |
| 20 | 1 6 | 54.19 | 52.08 | | 22460 | 114 | 40 | 0 14 | 10.65 | 53.13 | 26594 | 25 | 40 |
| 30 | 1 6 | 2.08 | 52.11 | | 22573 | 113 | 30 | 0 13 | 17.51 | 53.14 | 26617 | 23 | 30 |
| 40 | 1 5 | 9.94 | 52.14 | | 22684 | 111 | 20 | 0 12 | 24.36 | 53.15 | 26639 | 22 | 20 |
| 50 | 1 4 | 17.78 | 52.16 | | 22793 | 109 | 10 | 0 11 | 31.21 | 53.15 | 26659 | 20 | 10 |
| | | | 52.19 | | | 108 | | | | 53.15 | | 19 | |
| 168° | 1 3 | 25.59 | | | 22901 | 192° | 178° | 0 10 | 38.06 | | 26678 | 17 | 182° |
| 10' | 1 2 | 33.37 | 52.22 | | 23008 | 107 | 50' | 0 9 | 44.90 | 53.16 | 26695 | 16 | 50' |
| 20 | 1 1 | 41.13 | 52.24 | | 23113 | 105 | 40 | 0 8 | 51.74 | 53.16 | 26711 | 16 | 40 |
| 30 | 1 0 | 48.86 | 52.27 | | 23217 | 104 | 30 | 0 7 | 58.58 | 53.16 | 26726 | 15 | 30 |
| 40 | 0 59 | 56.57 | 52.29 | | 23320 | 103 | 20 | 0 7 | 5.41 | 53.17 | 26739 | 13 | 20 |
| 50 | 0 59 | 4.25 | 52.32 | | 23421 | 101 | 10 | 0 6 | 12.24 | 53.17 | 26750 | 11 | 10 |
| | | | 52.35 | | | 100 | | | | 53.17 | | 10 | |
| 169° | 0 58 | 11.90 | | | 23521 | 191° | 179° | 0 5 | 19.07 | | 26760 | 8 | 181° |
| 10' | 0 57 | 19.53 | 52.37 | | 23619 | 98 | 50' | 0 4 | 25.89 | 53.18 | 26768 | 8 | 50' |
| 20 | 0 56 | 27.13 | 52.40 | | 23716 | 97 | 40 | 0 3 | 32.72 | 53.17 | 26775 | 7 | 40 |
| 30 | 0 55 | 34.71 | 52.42 | | 23811 | 95 | 30 | 0 2 | 39.54 | 53.18 | 26780 | 5 | 30 |
| 40 | 0 54 | 42.27 | 52.44 | | 23904 | 93 | 20 | 0 1 | 46.36 | 53.18 | 26784 | 4 | 20 |
| 50 | 0 53 | 49.81 | 52.46 | | 23996 | 92 | 10 | 0 0 | 53.18 | 53.18 | 26786 | 2 | 10 |
| | | | 52.49 | | | 91 | | | | 53.18 | | 1 | |
| 170° | 0 52 | 57.32 | | | 24087 | 190° | 180° | 0 0 | 0.00 | | 26787 | 1 | 180° |
| | | | | | 1.30 | <i>g</i> | | | | | 1.30 | <i>g</i> | |

TABLE VIII, ARG. I.—ACTION OF JUPITER.

| Arg. | (v.c.0) | Diff. | (v.s.1) | (v.c.1) | (v.s.2) | (v.c.2) | (p.c.0) | (p.s.1) | (p.c.1) |
|------|---------|-------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | " | " | " | | | |
| 0 | 55.03 | | 0.14 | 4.42 | 0.10 | 0.11 | 2331 | 164 | 134 |
| 1 | 55.58 | +0.55 | 0.13 | 4.42 | 0.10 | 0.12 | 2331 | 166 | 134 |
| 2 | 56.13 | 0.55 | 0.12 | 4.41 | 0.10 | 0.12 | 2330 | 167 | 134 |
| 3 | 56.68 | 0.55 | 0.11 | 4.41 | 0.10 | 0.12 | 2330 | 169 | 134 |
| 4 | 57.23 | 0.55 | 0.11 | 4.41 | 0.10 | 0.12 | 2330 | 171 | 135 |
| 5 | 57.78 | | 0.10 | 4.40 | 0.10 | 0.13 | 2329 | 172 | 135 |
| 6 | 58.32 | +0.54 | 0.09 | 4.40 | 0.11 | 0.13 | 2328 | 174 | 135 |
| 7 | 58.87 | 0.55 | 0.09 | 4.39 | 0.11 | 0.13 | 2328 | 176 | 135 |
| 8 | 59.42 | 0.55 | 0.08 | 4.39 | 0.11 | 0.13 | 2327 | 177 | 135 |
| 9 | 59.97 | 0.55 | 0.08 | 4.38 | 0.11 | 0.14 | 2326 | 179 | 136 |
| 10 | 60.51 | | 0.08 | 4.38 | 0.11 | 0.14 | 2324 | 180 | 136 |
| 11 | 61.06 | +0.55 | 0.07 | 4.37 | 0.11 | 0.14 | 2323 | 182 | 136 |
| 12 | 61.60 | 0.54 | 0.07 | 4.37 | 0.11 | 0.15 | 2321 | 184 | 137 |
| 13 | 62.15 | 0.55 | 0.07 | 4.36 | 0.11 | 0.15 | 2320 | 185 | 137 |
| 14 | 62.69 | 0.54 | 0.08 | 4.36 | 0.11 | 0.15 | 2318 | 187 | 137 |
| 15 | 63.23 | | 0.08 | 4.35 | 0.11 | 0.15 | 2316 | 188 | 137 |
| 16 | 63.78 | +0.55 | 0.08 | 4.35 | 0.11 | 0.16 | 2314 | 190 | 137 |
| 17 | 64.32 | 0.54 | 0.09 | 4.34 | 0.11 | 0.16 | 2312 | 191 | 138 |
| 18 | 64.86 | 0.54 | 0.09 | 4.34 | 0.11 | 0.16 | 2310 | 193 | 138 |
| 19 | 65.40 | 0.54 | 0.10 | 4.33 | 0.11 | 0.17 | 2308 | 194 | 138 |
| 20 | 65.93 | | 0.11 | 4.33 | 0.11 | 0.17 | 2305 | 196 | 138 |
| 21 | 66.47 | +0.54 | 0.12 | 4.32 | 0.11 | 0.17 | 2303 | 198 | 139 |
| 22 | 67.00 | 0.53 | 0.13 | 4.32 | 0.11 | 0.17 | 2300 | 199 | 139 |
| 23 | 67.54 | 0.54 | 0.14 | 4.31 | 0.11 | 0.18 | 2297 | 200 | 139 |
| 24 | 68.07 | 0.53 | 0.15 | 4.30 | 0.11 | 0.18 | 2294 | 202 | 140 |
| 25 | 68.60 | | 0.16 | 4.30 | 0.10 | 0.18 | 2291 | 203 | 140 |
| 26 | 69.14 | +0.54 | 0.18 | 4.29 | 0.10 | 0.19 | 2288 | 205 | 140 |
| 27 | 69.66 | 0.52 | 0.20 | 4.29 | 0.10 | 0.19 | 2284 | 206 | 140 |
| 28 | 70.19 | 0.53 | 0.21 | 4.28 | 0.10 | 0.19 | 2281 | 208 | 140 |
| 29 | 70.72 | 0.53 | 0.23 | 4.28 | 0.10 | 0.20 | 2277 | 209 | 141 |
| 30 | 71.24 | | 0.25 | 4.27 | 0.10 | 0.20 | 2274 | 210 | 141 |
| 31 | 71.76 | +0.52 | 0.27 | 4.27 | 0.10 | 0.20 | 2270 | 212 | 141 |
| 32 | 72.28 | 0.52 | 0.29 | 4.26 | 0.10 | 0.21 | 2266 | 213 | 141 |
| 33 | 72.80 | 0.52 | 0.31 | 4.26 | 0.10 | 0.21 | 2262 | 214 | 142 |
| 34 | 73.32 | 0.52 | 0.33 | 4.25 | 0.10 | 0.21 | 2258 | 216 | 142 |
| 35 | 73.83 | | 0.35 | 4.25 | 0.10 | 0.22 | 2254 | 217 | 142 |
| 36 | 74.34 | +0.51 | 0.38 | 4.24 | 0.10 | 0.22 | 2249 | 218 | 142 |
| 37 | 74.85 | 0.51 | 0.40 | 4.24 | 0.10 | 0.22 | 2245 | 219 | 143 |
| 38 | 75.36 | 0.51 | 0.43 | 4.23 | 0.10 | 0.23 | 2240 | 221 | 143 |
| 39 | 75.87 | 0.51 | 0.46 | 4.23 | 0.09 | 0.23 | 2236 | 222 | 143 |
| 40 | 76.37 | | 0.49 | 4.22 | 0.09 | 0.23 | 2231 | 223 | 143 |
| 41 | 76.87 | +0.50 | 0.52 | 4.22 | 0.09 | 0.23 | 2226 | 224 | 144 |
| 42 | 77.37 | 0.50 | 0.55 | 4.22 | 0.09 | 0.24 | 2221 | 225 | 144 |
| 43 | 77.87 | 0.50 | 0.58 | 4.21 | 0.09 | 0.24 | 2216 | 227 | 144 |
| 44 | 78.37 | 0.50 | 0.61 | 4.21 | 0.09 | 0.24 | 2210 | 228 | 144 |
| 45 | 78.86 | | 0.64 | 4.21 | 0.09 | 0.25 | 2205 | 229 | 144 |
| 46 | 79.35 | +0.49 | 0.68 | 4.20 | 0.09 | 0.25 | 2200 | 230 | 145 |
| 47 | 79.84 | 0.49 | 0.71 | 4.20 | 0.09 | 0.25 | 2194 | 231 | 145 |
| 48 | 80.32 | 0.48 | 0.75 | 4.20 | 0.09 | 0.26 | 2188 | 232 | 145 |
| 49 | 80.80 | 0.48 | 0.79 | 4.20 | 0.09 | 0.26 | 2182 | 233 | 145 |
| 50 | 81.28 | | 0.82 | 4.20 | 0.08 | 0.26 | 2176 | 234 | 146 |
| 51 | 81.76 | +0.48 | 0.86 | 4.19 | 0.08 | 0.26 | 2170 | 235 | 146 |
| 52 | 82.23 | 0.47 | 0.90 | 4.19 | 0.08 | 0.26 | 2164 | 236 | 146 |
| 53 | 82.70 | 0.47 | 0.94 | 4.19 | 0.08 | 0.27 | 2158 | 237 | 146 |
| 54 | 83.17 | 0.47 | 0.98 | 4.19 | 0.08 | 0.27 | 2151 | 238 | 146 |
| 55 | 83.63 | | 1.02 | 4.19 | 0.07 | 0.27 | 2145 | 239 | 146 |
| 56 | 84.10 | +0.47 | 1.07 | 4.19 | 0.07 | 0.28 | 2138 | 240 | 146 |
| 57 | 84.55 | 0.45 | 1.11 | 4.19 | 0.07 | 0.28 | 2132 | 241 | 146 |
| 58 | 85.01 | 0.46 | 1.16 | 4.19 | 0.07 | 0.28 | 2125 | 242 | 147 |
| 59 | 85.46 | 0.45 | 1.20 | 4.19 | 0.07 | 0.28 | 2118 | 243 | 147 |
| 60 | 85.91 | | 1.25 | 4.19 | 0.07 | 0.29 | 2111 | 244 | 147 |

TABLE VIII, ARG. 1.—*Continued.*

| Arg. | (v.c.0) | Diff. | (v.s.1) | (v.c.1) | (v.s.2) | (v.c.2) | (p.c.0) | (p.s.1) | (p.c.1) |
|------|---------|-------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | " | " | " | | | |
| 60 | 85.91 | | 1.25 | 4.19 | 0.07 | 0.29 | 2111 | 244 | 147 |
| 61 | 86.36 | +0.45 | 1.30 | 4.19 | 0.07 | 0.29 | 2104 | 244 | 147 |
| 62 | 86.80 | 0.44 | 1.34 | 4.20 | 0.06 | 0.29 | 2097 | 245 | 148 |
| 63 | 87.24 | 0.44 | 1.39 | 4.20 | 0.06 | 0.29 | 2090 | 246 | 148 |
| 64 | 87.68 | 0.44 | 1.44 | 4.20 | 0.06 | 0.29 | 2082 | 247 | 148 |
| 65 | 88.11 | | 1.49 | 4.20 | 0.06 | 0.30 | 2075 | 248 | 148 |
| 66 | 88.53 | +0.42 | 1.54 | 4.21 | 0.06 | 0.30 | 2067 | 248 | 148 |
| 67 | 88.96 | 0.43 | 1.60 | 4.21 | 0.06 | 0.30 | 2059 | 249 | 148 |
| 68 | 89.38 | 0.42 | 1.65 | 4.21 | 0.05 | 0.30 | 2052 | 250 | 148 |
| 69 | 89.80 | 0.42 | 1.70 | 4.22 | 0.05 | 0.31 | 2044 | 250 | 148 |
| 70 | 90.21 | | 1.75 | 4.22 | 0.05 | 0.31 | 2036 | 251 | 149 |
| 71 | 90.62 | +0.41 | 1.81 | 4.23 | 0.05 | 0.31 | 2028 | 252 | 149 |
| 72 | 91.03 | 0.41 | 1.86 | 4.23 | 0.05 | 0.31 | 2020 | 252 | 149 |
| 73 | 91.43 | 0.40 | 1.92 | 4.24 | 0.05 | 0.31 | 2012 | 253 | 149 |
| 74 | 91.83 | 0.40 | 1.98 | 4.24 | 0.04 | 0.32 | 2003 | 253 | 149 |
| 75 | 92.22 | | 2.03 | 4.25 | 0.04 | 0.32 | 1995 | 254 | 149 |
| 76 | 92.62 | +0.40 | 2.09 | 4.26 | 0.04 | 0.32 | 1986 | 254 | 149 |
| 77 | 93.00 | 0.33 | 2.15 | 4.26 | 0.04 | 0.32 | 1978 | 255 | 149 |
| 78 | 93.38 | 0.38 | 2.21 | 4.27 | 0.04 | 0.32 | 1969 | 255 | 149 |
| 79 | 93.76 | 0.38 | 2.27 | 4.28 | 0.04 | 0.32 | 1960 | 256 | 149 |
| 80 | 94.14 | | 2.33 | 4.29 | 0.04 | 0.32 | 1952 | 256 | 149 |
| 81 | 94.51 | +0.37 | 2.39 | 4.30 | 0.03 | 0.32 | 1943 | 257 | 149 |
| 82 | 94.87 | 0.36 | 2.45 | 4.31 | 0.03 | 0.33 | 1934 | 257 | 149 |
| 83 | 95.23 | 0.36 | 2.51 | 4.32 | 0.03 | 0.33 | 1925 | 257 | 149 |
| 84 | 95.59 | 0.36 | 2.57 | 4.33 | 0.03 | 0.33 | 1916 | 258 | 149 |
| 85 | 95.95 | | 2.64 | 4.34 | 0.03 | 0.33 | 1906 | 258 | 149 |
| 86 | 96.30 | +0.35 | 2.70 | 4.35 | 0.03 | 0.33 | 1897 | 258 | 149 |
| 87 | 96.64 | 0.34 | 2.76 | 4.36 | 0.03 | 0.33 | 1888 | 259 | 149 |
| 88 | 96.98 | 0.34 | 2.83 | 4.37 | 0.03 | 0.33 | 1878 | 259 | 149 |
| 89 | 97.32 | 0.34 | 2.89 | 4.38 | 0.02 | 0.33 | 1869 | 259 | 149 |
| 90 | 97.65 | | 2.96 | 4.39 | 0.02 | 0.33 | 1859 | 260 | 148 |
| 91 | 97.97 | +0.32 | 3.02 | 4.41 | 0.02 | 0.33 | 1850 | 260 | 148 |
| 92 | 98.29 | 0.32 | 3.09 | 4.42 | 0.02 | 0.33 | 1840 | 260 | 148 |
| 93 | 98.61 | 0.32 | 3.15 | 4.43 | 0.02 | 0.33 | 1830 | 260 | 148 |
| 94 | 98.92 | 0.31 | 3.22 | 4.45 | 0.02 | 0.33 | 1820 | 260 | 148 |
| 95 | 99.23 | 0.31 | 3.29 | 4.46 | 0.02 | 0.33 | 1810 | 261 | 148 |
| 96 | 99.53 | +0.30 | 3.35 | 4.48 | 0.02 | 0.34 | 1800 | 261 | 148 |
| 97 | 99.83 | 0.30 | 3.42 | 4.49 | 0.02 | 0.34 | 1790 | 261 | 148 |
| 98 | 100.12 | 0.29 | 3.49 | 4.51 | 0.01 | 0.34 | 1780 | 261 | 147 |
| 99 | 100.41 | 0.29 | 3.56 | 4.52 | 0.01 | 0.34 | 1770 | 261 | 147 |
| 100 | 100.70 | | 3.63 | 4.54 | 0.01 | 0.34 | 1760 | 261 | 147 |
| 101 | 100.97 | +0.27 | 3.69 | 4.56 | 0.01 | 0.34 | 1750 | 261 | 147 |
| 102 | 101.25 | 0.28 | 3.76 | 4.57 | 0.01 | 0.34 | 1739 | 261 | 147 |
| 103 | 101.52 | 0.27 | 3.83 | 4.59 | 0.01 | 0.34 | 1729 | 261 | 146 |
| 104 | 101.78 | 0.26 | 3.90 | 4.61 | 0.01 | 0.34 | 1718 | 261 | 146 |
| 105 | 102.04 | 0.26 | 3.97 | 4.63 | 0.01 | 0.34 | 1708 | 261 | 146 |
| 106 | 102.29 | +0.25 | 4.04 | 4.65 | 0.01 | 0.34 | 1697 | 261 | 145 |
| 107 | 102.54 | 0.25 | 4.11 | 4.67 | 0.01 | 0.34 | 1686 | 261 | 145 |
| 108 | 102.78 | 0.24 | 4.18 | 4.69 | 0.01 | 0.34 | 1676 | 261 | 145 |
| 109 | 103.02 | 0.24 | 4.25 | 4.71 | 0.01 | 0.34 | 1665 | 261 | 145 |
| 110 | 103.25 | 0.23 | 4.32 | 4.73 | 0.01 | 0.34 | 1654 | 261 | 144 |
| 111 | 103.48 | +0.23 | 4.39 | 4.75 | 0.01 | 0.34 | 1643 | 261 | 144 |
| 112 | 103.70 | 0.22 | 4.46 | 4.77 | 0.01 | 0.34 | 1632 | 261 | 144 |
| 113 | 103.92 | 0.22 | 4.53 | 4.79 | 0.01 | 0.34 | 1622 | 260 | 143 |
| 114 | 104.13 | 0.21 | 4.60 | 4.81 | 0.01 | 0.34 | 1611 | 260 | 143 |
| 115 | 104.34 | 0.21 | 4.67 | 4.83 | 0.01 | 0.33 | 1600 | 260 | 142 |
| 116 | 104.54 | +0.20 | 4.74 | 4.85 | 0.01 | 0.33 | 1588 | 260 | 142 |
| 117 | 104.74 | 0.20 | 4.81 | 4.87 | 0.01 | 0.33 | 1577 | 260 | 142 |
| 118 | 104.93 | 0.19 | 4.88 | 4.90 | 0.01 | 0.33 | 1566 | 259 | 141 |
| 119 | 105.11 | 0.18 | 4.95 | 4.92 | 0.01 | 0.33 | 1555 | 259 | 141 |
| 120 | 105.29 | 0.18 | 5.01 | 4.94 | 0.02 | 0.33 | 1544 | 259 | 140 |

TABLE VIII, ARG. 1.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | (c.c.1) | (v.s.2) | (v.c.2) | (p.c.0) | (p.s.1) | (p.c.1) |
|------|---------|-------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | " | " | " | | | |
| 120 | 105.29 | | 5.01 | 4.94 | 0.02 | 0.33 | 1544 | 259 | 140 |
| 121 | 105.46 | +0.17 | 5.08 | 4.97 | 0.02 | 0.33 | 1533 | 259 | 140 |
| 122 | 105.63 | 0.17 | 5.15 | 4.99 | 0.02 | 0.33 | 1521 | 258 | 139 |
| 123 | 105.79 | 0.16 | 5.22 | 5.01 | 0.02 | 0.33 | 1510 | 258 | 139 |
| 124 | 105.95 | 0.16 | 5.29 | 5.04 | 0.02 | 0.33 | 1499 | 258 | 138 |
| | | 0.15 | | | | | | | |
| 125 | 106.10 | | 5.36 | 5.06 | 0.02 | 0.33 | 1487 | 257 | 138 |
| 126 | 106.25 | +0.15 | 5.43 | 5.09 | 0.02 | 0.33 | 1476 | 257 | 137 |
| 127 | 106.39 | 0.14 | 5.50 | 5.11 | 0.02 | 0.33 | 1464 | 257 | 137 |
| 128 | 106.53 | 0.14 | 5.57 | 5.14 | 0.02 | 0.33 | 1453 | 256 | 136 |
| 129 | 106.65 | 0.12 | 5.64 | 5.16 | 0.02 | 0.33 | 1441 | 256 | 135 |
| | | 0.13 | | | | | | | |
| 130 | 106.78 | | 5.71 | 5.19 | 0.02 | 0.32 | 1430 | 256 | 135 |
| 131 | 106.89 | +0.11 | 5.78 | 5.21 | 0.03 | 0.32 | 1418 | 255 | 134 |
| 132 | 107.01 | 0.12 | 5.85 | 5.24 | 0.03 | 0.32 | 1407 | 255 | 134 |
| 133 | 107.11 | 0.10 | 5.91 | 5.26 | 0.03 | 0.32 | 1395 | 254 | 133 |
| 134 | 107.22 | 0.11 | 5.98 | 5.29 | 0.03 | 0.32 | 1384 | 254 | 132 |
| | | 0.09 | | | | | | | |
| 135 | 107.31 | | 6.05 | 5.32 | 0.03 | 0.32 | 1372 | 254 | 132 |
| 136 | 107.40 | +0.09 | 6.11 | 5.34 | 0.04 | 0.32 | 1360 | 253 | 131 |
| 137 | 107.49 | 0.09 | 6.18 | 5.37 | 0.04 | 0.32 | 1349 | 253 | 131 |
| 138 | 107.57 | 0.08 | 6.25 | 5.40 | 0.04 | 0.32 | 1337 | 252 | 130 |
| 139 | 107.64 | 0.07 | 6.31 | 5.42 | 0.04 | 0.32 | 1325 | 252 | 129 |
| | | 0.06 | | | | | | | |
| 140 | 107.70 | | 6.38 | 5.45 | 0.04 | 0.32 | 1314 | 251 | 129 |
| 141 | 107.76 | +0.06 | 6.44 | 5.48 | 0.05 | 0.31 | 1302 | 251 | 128 |
| 142 | 107.82 | 0.06 | 6.51 | 5.51 | 0.05 | 0.31 | 1290 | 250 | 127 |
| 143 | 107.87 | 0.05 | 6.57 | 5.53 | 0.05 | 0.31 | 1278 | 250 | 126 |
| 144 | 107.91 | 0.04 | 6.64 | 5.56 | 0.05 | 0.31 | 1267 | 249 | 126 |
| | | 0.04 | | | | | | | |
| 145 | 107.95 | | 6.70 | 5.59 | 0.05 | 0.31 | 1255 | 249 | 125 |
| 146 | 107.98 | +0.03 | 6.76 | 5.61 | 0.06 | 0.31 | 1243 | 248 | 124 |
| 147 | 108.01 | 0.03 | 6.83 | 5.64 | 0.06 | 0.31 | 1231 | 248 | 123 |
| 148 | 108.03 | 0.02 | 6.89 | 5.67 | 0.06 | 0.31 | 1220 | 247 | 123 |
| 149 | 108.05 | 0.02 | 6.95 | 5.69 | 0.06 | 0.31 | 1208 | 247 | 122 |
| | | 0.00 | | | | | | | |
| 150 | 108.05 | | 7.01 | 5.72 | 0.07 | 0.31 | 1196 | 246 | 121 |
| 151 | 108.06 | +0.01 | 7.08 | 5.75 | 0.07 | 0.31 | 1184 | 246 | 120 |
| 152 | 108.05 | -0.01 | 7.14 | 5.78 | 0.07 | 0.31 | 1172 | 245 | 119 |
| 153 | 108.04 | 0.01 | 7.20 | 5.80 | 0.07 | 0.30 | 1161 | 244 | 118 |
| 154 | 108.03 | 0.01 | 7.25 | 5.83 | 0.08 | 0.30 | 1149 | 244 | 118 |
| | | 0.02 | | | | | | | |
| 155 | 108.01 | | 7.31 | 5.86 | 0.08 | 0.30 | 1137 | 243 | 117 |
| 156 | 107.98 | -0.03 | 7.37 | 5.88 | 0.08 | 0.30 | 1126 | 243 | 116 |
| 157 | 107.95 | 0.03 | 7.43 | 5.91 | 0.09 | 0.30 | 1114 | 242 | 115 |
| 158 | 107.91 | 0.04 | 7.49 | 5.94 | 0.09 | 0.30 | 1102 | 241 | 114 |
| 159 | 107.87 | 0.04 | 7.54 | 5.97 | 0.09 | 0.30 | 1090 | 241 | 113 |
| | | 0.05 | | | | | | | |
| 160 | 107.82 | | 7.60 | 5.99 | 0.10 | 0.30 | 1079 | 240 | 112 |
| 161 | 107.77 | -0.05 | 7.66 | 6.02 | 0.10 | 0.30 | 1067 | 240 | 112 |
| 162 | 107.70 | 0.07 | 7.71 | 6.04 | 0.10 | 0.30 | 1055 | 239 | 111 |
| 163 | 107.64 | 0.06 | 7.77 | 6.07 | 0.11 | 0.30 | 1044 | 238 | 110 |
| 164 | 107.56 | 0.08 | 7.82 | 6.10 | 0.11 | 0.29 | 1032 | 238 | 109 |
| | | 0.07 | | | | | | | |
| 165 | 107.49 | | 7.87 | 6.12 | 0.11 | 0.29 | 1020 | 237 | 108 |
| 166 | 107.40 | -0.09 | 7.93 | 6.15 | 0.12 | 0.29 | 1009 | 236 | 107 |
| 167 | 107.31 | 0.09 | 7.98 | 6.17 | 0.12 | 0.29 | 997 | 236 | 106 |
| 168 | 107.22 | 0.09 | 8.03 | 6.20 | 0.12 | 0.29 | 986 | 235 | 105 |
| 169 | 107.11 | 0.11 | 8.08 | 6.22 | 0.13 | 0.29 | 974 | 234 | 104 |
| | | 0.10 | | | | | | | |
| 170 | 107.01 | | 8.13 | 6.25 | 0.13 | 0.29 | 963 | 234 | 103 |
| 171 | 106.89 | -0.12 | 8.18 | 6.27 | 0.14 | 0.29 | 951 | 233 | 102 |
| 172 | 106.77 | 0.12 | 8.23 | 6.29 | 0.14 | 0.29 | 940 | 232 | 101 |
| 173 | 106.65 | 0.12 | 8.28 | 6.32 | 0.14 | 0.29 | 928 | 232 | 100 |
| 174 | 106.52 | 0.13 | 8.33 | 6.34 | 0.15 | 0.29 | 917 | 231 | 99 |
| | | 0.14 | | | | | | | |
| 175 | 106.38 | | 8.37 | 6.36 | 0.15 | 0.29 | 906 | 230 | 98 |
| 176 | 106.24 | -0.14 | 8.42 | 6.39 | 0.15 | 0.29 | 894 | 230 | 97 |
| 177 | 106.10 | 0.14 | 8.46 | 6.41 | 0.16 | 0.29 | 883 | 229 | 96 |
| 178 | 105.94 | 0.16 | 8.51 | 6.43 | 0.16 | 0.29 | 872 | 228 | 95 |
| 179 | 105.78 | 0.16 | 8.55 | 6.45 | 0.17 | 0.29 | 861 | 228 | 94 |
| | | 0.16 | | | | | | | |
| 180 | 105.62 | | 8.60 | 6.47 | 0.17 | 0.29 | 850 | 227 | 93 |

TABLE VIII, ARG. I.—Continued.

| Arg. | (v.c.0) | Diff | (v.s.1) | (v.c.1) | (v.s.2) | (v.c.2) | (p.c.0) | (p.s.1) | (p.c.1) |
|------|---------|-------|---------|---------|---------|---------|---------|---------|---------|
| 180 | 105.62 | " | 8.60 | 6.47 | 0.17 | 0.29 | 850 | 227 | 93 |
| 181 | 105.45 | —0.17 | 8.64 | 6.50 | 0.17 | 0.29 | 839 | 226 | 92 |
| 182 | 105.27 | 0.18 | 8.68 | 6.52 | 0.18 | 0.29 | 828 | 226 | 91 |
| 183 | 105.09 | 0.18 | 8.72 | 6.54 | 0.18 | 0.29 | 816 | 225 | 90 |
| 184 | 104.90 | 0.19 | 8.76 | 6.56 | 0.19 | 0.29 | 805 | 224 | 89 |
| 185 | 104.71 | " | 8.80 | 6.57 | 0.19 | 0.29 | 794 | 224 | 88 |
| 186 | 104.51 | —0.20 | 8.84 | 6.59 | 0.19 | 0.29 | 784 | 223 | 87 |
| 187 | 104.31 | 0.20 | 8.88 | 6.61 | 0.20 | 0.29 | 773 | 222 | 86 |
| 188 | 104.10 | 0.21 | 8.92 | 6.63 | 0.20 | 0.29 | 762 | 222 | 85 |
| 189 | 103.89 | 0.21 | 8.96 | 6.65 | 0.21 | 0.29 | 751 | 221 | 84 |
| 190 | 103.67 | 0.22 | 8.99 | 6.67 | 0.21 | 0.29 | 740 | 221 | 83 |
| 191 | 103.44 | —0.23 | 9.03 | 6.68 | 0.21 | 0.29 | 730 | 220 | 82 |
| 192 | 103.21 | 0.23 | 9.07 | 6.70 | 0.22 | 0.29 | 719 | 219 | 81 |
| 193 | 102.98 | 0.23 | 9.10 | 6.71 | 0.22 | 0.29 | 708 | 219 | 80 |
| 194 | 102.74 | 0.24 | 9.13 | 6.73 | 0.22 | 0.29 | 698 | 218 | 79 |
| 195 | 102.49 | 0.25 | 9.17 | 6.74 | 0.23 | 0.29 | 688 | 217 | 78 |
| 196 | 102.24 | —0.25 | 9.20 | 6.76 | 0.23 | 0.29 | 677 | 216 | 77 |
| 197 | 101.98 | 0.26 | 9.23 | 6.77 | 0.24 | 0.29 | 667 | 216 | 76 |
| 198 | 101.72 | 0.26 | 9.26 | 6.78 | 0.24 | 0.29 | 657 | 215 | 75 |
| 199 | 101.45 | 0.27 | 9.29 | 6.79 | 0.24 | 0.29 | 646 | 215 | 74 |
| 200 | 101.18 | 0.27 | 9.32 | 6.81 | 0.25 | 0.29 | 636 | 214 | 73 |
| 201 | 100.90 | —0.28 | 9.35 | 6.82 | 0.25 | 0.29 | 626 | 213 | 72 |
| 202 | 100.62 | 0.28 | 9.38 | 6.83 | 0.25 | 0.29 | 616 | 213 | 71 |
| 203 | 100.33 | 0.29 | 9.41 | 6.84 | 0.26 | 0.29 | 606 | 212 | 70 |
| 204 | 100.04 | 0.29 | 9.43 | 6.85 | 0.26 | 0.29 | 596 | 211 | 69 |
| 205 | 99.74 | 0.30 | 9.46 | 6.86 | 0.26 | 0.29 | 586 | 211 | 68 |
| 206 | 99.44 | —0.30 | 9.48 | 6.86 | 0.27 | 0.29 | 577 | 210 | 67 |
| 207 | 99.13 | 0.31 | 9.51 | 6.87 | 0.27 | 0.29 | 567 | 210 | 66 |
| 208 | 98.82 | 0.31 | 9.53 | 6.88 | 0.27 | 0.29 | 557 | 209 | 65 |
| 209 | 98.50 | 0.32 | 9.55 | 6.88 | 0.28 | 0.29 | 548 | 208 | 64 |
| 210 | 98.18 | 0.32 | 9.58 | 6.89 | 0.28 | 0.29 | 538 | 208 | 63 |
| 211 | 97.85 | —0.33 | 9.60 | 6.89 | 0.28 | 0.29 | 529 | 207 | 62 |
| 212 | 97.52 | 0.33 | 9.62 | 6.90 | 0.29 | 0.29 | 520 | 206 | 61 |
| 213 | 97.18 | 0.34 | 9.64 | 6.90 | 0.29 | 0.29 | 510 | 206 | 60 |
| 214 | 96.84 | 0.34 | 9.66 | 6.90 | 0.29 | 0.29 | 501 | 205 | 59 |
| 215 | 96.49 | 0.35 | 9.68 | 6.91 | 0.30 | 0.29 | 492 | 205 | 58 |
| 216 | 96.14 | —0.35 | 9.70 | 6.91 | 0.30 | 0.29 | 483 | 204 | 57 |
| 217 | 95.79 | 0.35 | 9.71 | 6.91 | 0.30 | 0.29 | 474 | 204 | 56 |
| 218 | 95.43 | 0.36 | 9.73 | 6.91 | 0.31 | 0.29 | 464 | 203 | 55 |
| 219 | 95.06 | 0.37 | 9.75 | 6.91 | 0.31 | 0.30 | 456 | 202 | 54 |
| 220 | 94.69 | 0.37 | 9.76 | 6.91 | 0.31 | 0.30 | 448 | 202 | 53 |
| 221 | 94.32 | —0.37 | 9.78 | 6.90 | 0.31 | 0.30 | 439 | 201 | 52 |
| 222 | 93.94 | 0.38 | 9.79 | 6.90 | 0.32 | 0.30 | 430 | 201 | 51 |
| 223 | 93.56 | 0.38 | 9.81 | 6.90 | 0.32 | 0.30 | 422 | 200 | 50 |
| 224 | 93.17 | 0.39 | 9.82 | 6.90 | 0.32 | 0.30 | 413 | 200 | 49 |
| 225 | 92.78 | 0.39 | 9.83 | 6.89 | 0.33 | 0.30 | 405 | 199 | 48 |
| 226 | 92.39 | —0.39 | 9.85 | 6.88 | 0.33 | 0.30 | 398 | 198 | 47 |
| 227 | 91.99 | 0.40 | 9.86 | 6.88 | 0.33 | 0.30 | 390 | 198 | 46 |
| 228 | 91.59 | 0.40 | 9.87 | 6.87 | 0.33 | 0.30 | 382 | 197 | 46 |
| 229 | 91.18 | 0.41 | 9.88 | 6.86 | 0.34 | 0.30 | 374 | 197 | 45 |
| 230 | 90.77 | 0.41 | 9.89 | 6.85 | 0.34 | 0.30 | 365 | 196 | 44 |
| 231 | 90.36 | —0.41 | 9.90 | 6.84 | 0.34 | 0.30 | 357 | 196 | 43 |
| 232 | 89.94 | 0.42 | 9.91 | 6.83 | 0.34 | 0.30 | 349 | 195 | 42 |
| 233 | 89.51 | 0.43 | 9.91 | 6.82 | 0.34 | 0.30 | 342 | 194 | 41 |
| 234 | 89.09 | 0.42 | 9.92 | 6.81 | 0.35 | 0.30 | 334 | 194 | 41 |
| 235 | 88.65 | 0.44 | 9.93 | 6.80 | 0.35 | 0.30 | 327 | 194 | 40 |
| 236 | 88.22 | —0.43 | 9.94 | 6.79 | 0.35 | 0.30 | 320 | 193 | 39 |
| 237 | 87.78 | 0.44 | 9.94 | 6.78 | 0.35 | 0.30 | 312 | 192 | 38 |
| 238 | 87.34 | 0.44 | 9.95 | 6.76 | 0.35 | 0.30 | 305 | 192 | 38 |
| 239 | 86.89 | 0.45 | 9.95 | 6.74 | 0.36 | 0.30 | 298 | 191 | 37 |
| 240 | 86.44 | 0.45 | 9.96 | 6.73 | 0.36 | 0.31 | 291 | 191 | 36 |

TABLE VIII, ARG. 1.—Continued.

| Arg. | (r.c.0) | Diff. | (r.s.1) | (v.c.1) | (v.s.2) | (v.c.2) | (p.c.0) | (p.s.1) | (p.c.1) |
|------|---------|-------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | " | " | " | | | |
| 240 | 86.44 | | 9.96 | 6.73 | 0.36 | 0.31 | 291 | 191 | 36 |
| 241 | 85.99 | —0.45 | 9.96 | 6.71 | 0.36 | 0.31 | 284 | 190 | 35 |
| 242 | 85.53 | 0.46 | 9.96 | 6.69 | 0.36 | 0.31 | 278 | 190 | 35 |
| 243 | 85.08 | 0.45 | 9.97 | 6.67 | 0.36 | 0.31 | 271 | 189 | 34 |
| 244 | 84.61 | 0.47 | 9.97 | 6.66 | 0.37 | 0.31 | 265 | 189 | 33 |
| | | 0.46 | | | | | | | |
| 245 | 84.15 | | 9.97 | 6.64 | 0.37 | 0.31 | 258 | 188 | 33 |
| 246 | 83.68 | —0.47 | 9.97 | 6.61 | 0.37 | 0.31 | 252 | 188 | 32 |
| 247 | 83.21 | 0.47 | 9.98 | 6.59 | 0.37 | 0.31 | 246 | 187 | 31 |
| 248 | 82.73 | 0.48 | 9.98 | 6.57 | 0.37 | 0.31 | 240 | 187 | 31 |
| 249 | 82.25 | 0.48 | 9.98 | 6.55 | 0.37 | 0.31 | 234 | 186 | 30 |
| | | 0.48 | | | | | | | |
| 250 | 81.77 | | 9.98 | 6.53 | 0.37 | 0.31 | 228 | 186 | 29 |
| 251 | 81.28 | —0.49 | 9.98 | 6.51 | 0.38 | 0.31 | 222 | 185 | 29 |
| 252 | 80.80 | 0.48 | 9.98 | 6.48 | 0.38 | 0.31 | 216 | 185 | 28 |
| 253 | 80.30 | 0.50 | 9.97 | 6.46 | 0.38 | 0.31 | 211 | 184 | 28 |
| 254 | 79.81 | 0.49 | 9.97 | 6.43 | 0.38 | 0.31 | 205 | 184 | 27 |
| | | 0.50 | | | | | | | |
| 255 | 79.31 | | 9.97 | 6.40 | 0.38 | 0.31 | 200 | 183 | 27 |
| 256 | 78.82 | —0.49 | 9.97 | 6.37 | 0.38 | 0.30 | 195 | 183 | 26 |
| 257 | 78.31 | 0.51 | 9.96 | 6.34 | 0.38 | 0.30 | 190 | 182 | 26 |
| 258 | 77.81 | 0.50 | 9.96 | 6.31 | 0.38 | 0.30 | 184 | 182 | 25 |
| 259 | 77.30 | 0.51 | 9.96 | 6.28 | 0.38 | 0.30 | 179 | 181 | 25 |
| | | 0.51 | | | | | | | |
| 260 | 76.79 | | 9.95 | 6.25 | 0.38 | 0.30 | 175 | 181 | 24 |
| 261 | 76.28 | —0.51 | 9.95 | 6.22 | 0.38 | 0.30 | 170 | 180 | 24 |
| 262 | 75.77 | 0.51 | 9.95 | 6.19 | 0.38 | 0.30 | 165 | 180 | 24 |
| 263 | 75.25 | 0.52 | 9.94 | 6.16 | 0.38 | 0.30 | 161 | 179 | 23 |
| 264 | 74.73 | 0.52 | 9.94 | 6.13 | 0.38 | 0.30 | 156 | 179 | 23 |
| | | 0.52 | | | | | | | |
| 265 | 74.21 | | 9.93 | 6.10 | 0.38 | 0.30 | 152 | 178 | 22 |
| 266 | 73.69 | —0.52 | 9.93 | 6.06 | 0.38 | 0.30 | 148 | 178 | 22 |
| 267 | 73.16 | 0.53 | 9.92 | 6.03 | 0.38 | 0.30 | 144 | 177 | 22 |
| 268 | 72.64 | 0.52 | 9.92 | 5.99 | 0.38 | 0.30 | 140 | 177 | 21 |
| 269 | 72.11 | 0.53 | 9.91 | 5.96 | 0.38 | 0.30 | 137 | 176 | 21 |
| | | 0.54 | | | | | | | |
| 270 | 71.57 | | 9.90 | 5.92 | 0.39 | 0.30 | 133 | 176 | 21 |
| 271 | 71.04 | —0.53 | 9.90 | 5.88 | 0.39 | 0.30 | 129 | 175 | 20 |
| 272 | 70.51 | 0.53 | 9.89 | 5.85 | 0.39 | 0.30 | 126 | 175 | 20 |
| 273 | 69.97 | 0.54 | 9.88 | 5.81 | 0.39 | 0.30 | 122 | 174 | 20 |
| 274 | 69.43 | 0.54 | 9.88 | 5.77 | 0.39 | 0.30 | 119 | 174 | 20 |
| | | 0.54 | | | | | | | |
| 275 | 68.89 | | 9.87 | 5.73 | 0.39 | 0.30 | 116 | 174 | 20 |
| 276 | 68.35 | —0.54 | 9.86 | 5.69 | 0.39 | 0.29 | 113 | 173 | 19 |
| 277 | 67.81 | 0.54 | 9.85 | 5.65 | 0.39 | 0.29 | 110 | 172 | 19 |
| 278 | 67.26 | 0.55 | 9.84 | 5.61 | 0.39 | 0.29 | 107 | 172 | 19 |
| 279 | 66.72 | 0.54 | 9.84 | 5.57 | 0.39 | 0.29 | 105 | 171 | 19 |
| | | 0.55 | | | | | | | |
| 280 | 66.17 | | 9.83 | 5.52 | 0.39 | 0.29 | 102 | 171 | 19 |
| 281 | 65.62 | —0.55 | 9.82 | 5.48 | 0.39 | 0.29 | 100 | 170 | 19 |
| 282 | 65.07 | 0.55 | 9.81 | 5.44 | 0.39 | 0.29 | 97 | 170 | 18 |
| 283 | 64.52 | 0.55 | 9.80 | 5.40 | 0.39 | 0.29 | 95 | 169 | 18 |
| 284 | 63.97 | 0.55 | 9.79 | 5.35 | 0.39 | 0.29 | 93 | 169 | 18 |
| | | 0.56 | | | | | | | |
| 285 | 63.41 | | 9.78 | 5.31 | 0.39 | 0.28 | 91 | 168 | 18 |
| 286 | 62.86 | —0.55 | 9.77 | 5.26 | 0.39 | 0.28 | 90 | 168 | 18 |
| 287 | 62.31 | 0.55 | 9.77 | 5.22 | 0.39 | 0.28 | 88 | 167 | 18 |
| 288 | 61.75 | 0.56 | 9.76 | 5.17 | 0.39 | 0.28 | 86 | 167 | 18 |
| 289 | 61.19 | 0.56 | 9.75 | 5.13 | 0.38 | 0.28 | 85 | 166 | 18 |
| | | 0.55 | | | | | | | |
| 290 | 60.64 | | 9.74 | 5.08 | 0.38 | 0.28 | 84 | 166 | 18 |
| 291 | 60.08 | —0.56 | 9.73 | 5.03 | 0.38 | 0.27 | 82 | 165 | 19 |
| 292 | 59.52 | 0.56 | 9.72 | 4.99 | 0.38 | 0.27 | 81 | 164 | 19 |
| 293 | 58.96 | 0.56 | 9.71 | 4.94 | 0.38 | 0.27 | 80 | 164 | 19 |
| 294 | 58.40 | 0.56 | 9.70 | 4.89 | 0.38 | 0.27 | 80 | 163 | 19 |
| | | 0.56 | | | | | | | |
| 295 | 57.84 | | 9.69 | 4.84 | 0.38 | 0.27 | 79 | 163 | 19 |
| 296 | 57.28 | —0.56 | 9.68 | 4.80 | 0.38 | 0.27 | 78 | 162 | 19 |
| 297 | 56.72 | 0.56 | 9.67 | 4.75 | 0.38 | 0.26 | 78 | 162 | 19 |
| 298 | 56.16 | 0.56 | 9.66 | 4.70 | 0.38 | 0.26 | 77 | 161 | 20 |
| 299 | 55.60 | 0.56 | 9.65 | 4.65 | 0.38 | 0.26 | 77 | 161 | 20 |
| | | 0.56 | | | | | | | |
| 300 | 55.04 | | 9.64 | 4.60 | 0.38 | 0.26 | 77 | 160 | 20 |

TABLE VIII, ARG. I.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | (v.c.1) | (v.s.2) | (v.c.2) | (p.c.0) | (p.s.1) | (p.c.1) |
|------|---------|-------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | " | " | " | | | |
| 300 | 55.04 | | 9.64 | 4.60 | 0.38 | 0.26 | 77 | 160 | 20 |
| 301 | 54.48 | —0.56 | 9.63 | 4.55 | 0.38 | 0.26 | 77 | 159 | 20 |
| 302 | 53.92 | 0.56 | 9.62 | 4.50 | 0.38 | 0.25 | 77 | 159 | 20 |
| 303 | 53.36 | 0.56 | 9.61 | 4.45 | 0.38 | 0.25 | 77 | 158 | 21 |
| 304 | 52.80 | 0.56 | 9.60 | 4.40 | 0.38 | 0.25 | 78 | 158 | 21 |
| 305 | 52.24 | —0.56 | 9.59 | 4.35 | 0.38 | 0.25 | 78 | 157 | 21 |
| 306 | 51.68 | 0.56 | 9.58 | 4.30 | 0.38 | 0.25 | 79 | 156 | 22 |
| 307 | 51.12 | 0.56 | 9.57 | 4.25 | 0.38 | 0.24 | 80 | 156 | 22 |
| 308 | 50.56 | 0.56 | 9.56 | 4.20 | 0.38 | 0.24 | 81 | 155 | 22 |
| 309 | 50.00 | 0.56 | 9.55 | 4.15 | 0.38 | 0.24 | 82 | 154 | 23 |
| 310 | 49.44 | | 9.54 | 4.10 | 0.38 | 0.24 | 83 | 154 | 23 |
| 311 | 48.88 | —0.56 | 9.53 | 4.05 | 0.38 | 0.23 | 84 | 153 | 24 |
| 312 | 48.33 | 0.55 | 9.52 | 4.01 | 0.38 | 0.23 | 85 | 152 | 24 |
| 313 | 47.77 | 0.56 | 9.51 | 3.94 | 0.38 | 0.23 | 87 | 152 | 24 |
| 314 | 47.22 | 0.55 | 9.50 | 3.89 | 0.38 | 0.23 | 88 | 151 | 25 |
| 315 | 46.66 | | 9.49 | 3.84 | 0.38 | 0.23 | 90 | 150 | 25 |
| 316 | 46.11 | —0.55 | 9.48 | 3.79 | 0.38 | 0.22 | 92 | 150 | 26 |
| 317 | 45.56 | 0.55 | 9.47 | 3.74 | 0.38 | 0.22 | 94 | 149 | 26 |
| 318 | 45.01 | 0.55 | 9.46 | 3.69 | 0.38 | 0.22 | 96 | 148 | 27 |
| 319 | 44.46 | 0.55 | 9.45 | 3.63 | 0.38 | 0.22 | 98 | 148 | 27 |
| 320 | 43.91 | | 9.44 | 3.58 | 0.38 | 0.21 | 100 | 147 | 28 |
| 321 | 43.36 | —0.55 | 9.43 | 3.53 | 0.38 | 0.21 | 103 | 146 | 28 |
| 322 | 42.81 | 0.55 | 9.42 | 3.48 | 0.38 | 0.21 | 105 | 146 | 29 |
| 323 | 42.27 | 0.54 | 9.41 | 3.43 | 0.38 | 0.21 | 108 | 145 | 29 |
| 324 | 41.72 | 0.55 | 9.40 | 3.38 | 0.38 | 0.20 | 111 | 144 | 30 |
| 325 | 41.18 | | 9.40 | 3.33 | 0.38 | 0.20 | 114 | 144 | 30 |
| 326 | 40.64 | —0.54 | 9.39 | 3.28 | 0.38 | 0.20 | 117 | 143 | 31 |
| 327 | 40.10 | 0.54 | 9.38 | 3.23 | 0.38 | 0.20 | 120 | 142 | 32 |
| 328 | 39.56 | 0.54 | 9.37 | 3.18 | 0.38 | 0.19 | 123 | 141 | 32 |
| 329 | 39.03 | 0.53 | 9.36 | 3.14 | 0.37 | 0.19 | 127 | 140 | 33 |
| 330 | 38.49 | —0.54 | 9.35 | 3.07 | 0.37 | 0.19 | 130 | 140 | 34 |
| 331 | 37.96 | 0.53 | 9.34 | 3.02 | 0.37 | 0.19 | 134 | 139 | 34 |
| 332 | 37.43 | 0.53 | 9.33 | 2.98 | 0.37 | 0.18 | 138 | 138 | 35 |
| 333 | 36.90 | 0.53 | 9.32 | 2.93 | 0.37 | 0.18 | 141 | 137 | 36 |
| 334 | 36.37 | 0.53 | 9.31 | 2.88 | 0.37 | 0.18 | 145 | 136 | 36 |
| 335 | 35.85 | | 9.30 | 2.83 | 0.37 | 0.18 | 150 | 136 | 37 |
| 336 | 35.33 | —0.52 | 9.29 | 2.78 | 0.37 | 0.17 | 154 | 135 | 38 |
| 337 | 34.81 | 0.52 | 9.29 | 2.73 | 0.37 | 0.17 | 158 | 134 | 38 |
| 338 | 34.29 | 0.52 | 9.28 | 2.69 | 0.37 | 0.17 | 162 | 133 | 39 |
| 339 | 33.78 | 0.51 | 9.27 | 2.64 | 0.37 | 0.17 | 167 | 132 | 40 |
| 340 | 33.26 | | 9.26 | 2.59 | 0.37 | 0.17 | 172 | 132 | 41 |
| 341 | 32.75 | —0.51 | 9.25 | 2.54 | 0.37 | 0.16 | 176 | 131 | 42 |
| 342 | 32.24 | 0.51 | 9.24 | 2.50 | 0.37 | 0.16 | 181 | 130 | 42 |
| 343 | 31.74 | 0.50 | 9.23 | 2.45 | 0.37 | 0.16 | 186 | 129 | 43 |
| 344 | 31.23 | 0.51 | 9.22 | 2.41 | 0.37 | 0.16 | 191 | 128 | 44 |
| 345 | 30.73 | | 9.21 | 2.36 | 0.37 | 0.15 | 197 | 127 | 45 |
| 346 | 30.24 | —0.49 | 9.20 | 2.31 | 0.37 | 0.15 | 202 | 126 | 46 |
| 347 | 29.74 | 0.50 | 9.20 | 2.27 | 0.37 | 0.15 | 207 | 126 | 46 |
| 348 | 29.25 | 0.49 | 9.19 | 2.23 | 0.37 | 0.15 | 213 | 125 | 47 |
| 349 | 28.76 | 0.49 | 9.18 | 2.18 | 0.37 | 0.15 | 218 | 124 | 48 |
| 350 | 28.27 | | 9.17 | 2.14 | 0.37 | 0.14 | 224 | 123 | 49 |
| 351 | 27.79 | —0.48 | 9.16 | 2.09 | 0.37 | 0.14 | 230 | 122 | 50 |
| 352 | 27.31 | 0.48 | 9.15 | 2.05 | 0.37 | 0.14 | 236 | 121 | 51 |
| 353 | 26.83 | 0.48 | 9.14 | 2.01 | 0.37 | 0.14 | 242 | 120 | 51 |
| 354 | 26.36 | 0.47 | 9.13 | 1.97 | 0.37 | 0.14 | 248 | 119 | 52 |
| 355 | 25.89 | | 9.12 | 1.93 | 0.37 | 0.14 | 255 | 118 | 53 |
| 356 | 25.42 | —0.47 | 9.12 | 1.89 | 0.37 | 0.13 | 261 | 117 | 54 |
| 357 | 24.96 | 0.46 | 9.11 | 1.85 | 0.37 | 0.13 | 268 | 116 | 55 |
| 358 | 24.50 | 0.46 | 9.10 | 1.81 | 0.37 | 0.13 | 274 | 115 | 56 |
| 359 | 24.04 | 0.46 | 9.09 | 1.77 | 0.37 | 0.13 | 281 | 114 | 57 |
| 360 | 23.58 | | 9.08 | 1.73 | 0.38 | 0.13 | 288 | 113 | 58 |

TABLE VIII, ARG. 1.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | (v.c.1) | (v.s.2) | (v.c.2) | (p.c.0) | (p.s.1) | (p.c.1) |
|------|---------|-------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | " | " | " | | | |
| 360 | 23.58 | | 9.08 | 1.73 | 0.38 | 0.13 | 288 | 113 | 58 |
| 361 | 23.13 | —0.45 | 9.07 | 1.70 | 0.38 | 0.13 | 294 | 112 | 58 |
| 362 | 22.69 | 0.44 | 9.06 | 1.66 | 0.38 | 0.13 | 301 | 111 | 59 |
| 363 | 22.24 | 0.45 | 9.05 | 1.62 | 0.38 | 0.13 | 308 | 110 | 60 |
| 364 | 21.80 | 0.44 | 9.04 | 1.59 | 0.38 | 0.12 | 316 | 109 | 61 |
| | | 0.43 | | | | | | | |
| 365 | 21.37 | | 9.03 | 1.55 | 0.37 | 0.12 | 323 | 108 | 62 |
| 366 | 20.94 | —0.43 | 9.02 | 1.52 | 0.37 | 0.12 | 330 | 107 | 63 |
| 367 | 20.51 | 0.43 | 9.01 | 1.48 | 0.37 | 0.12 | 338 | 106 | 64 |
| 368 | 20.08 | 0.43 | 9.00 | 1.45 | 0.37 | 0.12 | 345 | 105 | 65 |
| 369 | 19.66 | 0.42 | 8.99 | 1.42 | 0.37 | 0.11 | 353 | 104 | 66 |
| | | 0.42 | | | | | | | |
| 370 | 19.24 | | 8.98 | 1.39 | 0.37 | 0.11 | 361 | 103 | 66 |
| 371 | 18.83 | —0.41 | 8.97 | 1.36 | 0.37 | 0.11 | 369 | 102 | 67 |
| 372 | 18.42 | 0.41 | 8.96 | 1.32 | 0.37 | 0.11 | 377 | 101 | 68 |
| 373 | 18.02 | 0.40 | 8.95 | 1.29 | 0.37 | 0.11 | 385 | 100 | 69 |
| 374 | 17.62 | 0.40 | 8.94 | 1.27 | 0.37 | 0.11 | 393 | 99 | 70 |
| | | 0.40 | | | | | | | |
| 375 | 17.22 | | 8.93 | 1.24 | 0.37 | 0.11 | 401 | 98 | 71 |
| 376 | 16.83 | —0.39 | 8.92 | 1.21 | 0.37 | 0.11 | 410 | 97 | 72 |
| 377 | 16.44 | 0.39 | 8.91 | 1.18 | 0.37 | 0.11 | 418 | 96 | 73 |
| 378 | 16.06 | 0.38 | 8.89 | 1.16 | 0.37 | 0.11 | 426 | 95 | 74 |
| 379 | 15.68 | 0.38 | 8.88 | 1.13 | 0.37 | 0.11 | 435 | 94 | 75 |
| | | 0.38 | | | | | | | |
| 380 | 15.30 | | 8.87 | 1.11 | 0.37 | 0.10 | 444 | 93 | 76 |
| 381 | 14.93 | —0.37 | 8.86 | 1.08 | 0.37 | 0.10 | 452 | 92 | 77 |
| 382 | 14.57 | 0.36 | 8.85 | 1.06 | 0.37 | 0.10 | 461 | 91 | 78 |
| 383 | 14.21 | 0.36 | 8.83 | 1.04 | 0.37 | 0.10 | 470 | 90 | 78 |
| 384 | 13.85 | 0.36 | 8.82 | 1.02 | 0.37 | 0.10 | 479 | 89 | 79 |
| | | 0.35 | | | | | | | |
| 385 | 13.50 | | 8.81 | 1.00 | 0.37 | 0.10 | 488 | 88 | 80 |
| 386 | 13.15 | —0.35 | 8.79 | 0.98 | 0.37 | 0.10 | 497 | 87 | 81 |
| 387 | 12.80 | 0.35 | 8.78 | 0.96 | 0.37 | 0.10 | 506 | 86 | 82 |
| 388 | 12.47 | 0.33 | 8.77 | 0.94 | 0.37 | 0.10 | 514 | 85 | 83 |
| 389 | 12.13 | 0.34 | 8.75 | 0.92 | 0.37 | 0.10 | 525 | 84 | 84 |
| | | 0.32 | | | | | | | |
| 390 | 11.81 | | 8.74 | 0.90 | 0.37 | 0.10 | 534 | 82 | 85 |
| 391 | 11.48 | —0.33 | 8.72 | 0.88 | 0.37 | 0.10 | 544 | 81 | 86 |
| 392 | 11.16 | 0.32 | 8.71 | 0.87 | 0.37 | 0.10 | 554 | 80 | 86 |
| 393 | 10.85 | 0.31 | 8.70 | 0.85 | 0.36 | 0.10 | 564 | 79 | 87 |
| 394 | 10.54 | 0.31 | 8.68 | 0.84 | 0.36 | 0.10 | 573 | 78 | 88 |
| | | 0.31 | | | | | | | |
| 395 | 10.23 | | 8.67 | 0.83 | 0.36 | 0.10 | 583 | 77 | 89 |
| 396 | 9.93 | —0.30 | 8.65 | 0.82 | 0.36 | 0.10 | 593 | 76 | 90 |
| 397 | 9.64 | 0.29 | 8.63 | 0.80 | 0.36 | 0.10 | 603 | 75 | 91 |
| 398 | 9.35 | 0.29 | 8.62 | 0.79 | 0.36 | 0.10 | 613 | 74 | 92 |
| 399 | 9.07 | 0.28 | 8.60 | 0.78 | 0.36 | 0.10 | 623 | 73 | 93 |
| | | 0.28 | | | | | | | |
| 400 | 8.79 | | 8.58 | 0.77 | 0.36 | 0.10 | 633 | 72 | 93 |
| 401 | 8.52 | —0.27 | 8.57 | 0.76 | 0.36 | 0.10 | 643 | 71 | 94 |
| 402 | 8.25 | 0.27 | 8.55 | 0.76 | 0.36 | 0.10 | 653 | 70 | 95 |
| 403 | 7.98 | 0.27 | 8.53 | 0.75 | 0.36 | 0.10 | 664 | 69 | 96 |
| 404 | 7.73 | 0.25 | 8.51 | 0.74 | 0.35 | 0.10 | 674 | 68 | 97 |
| | | 0.26 | | | | | | | |
| 405 | 7.47 | | 8.49 | 0.74 | 0.35 | 0.10 | 684 | 67 | 98 |
| 406 | 7.22 | —0.25 | 8.47 | 0.74 | 0.35 | 0.10 | 695 | 66 | 98 |
| 407 | 6.98 | 0.24 | 8.46 | 0.73 | 0.35 | 0.10 | 706 | 65 | 99 |
| 408 | 6.75 | 0.23 | 8.44 | 0.73 | 0.35 | 0.10 | 716 | 64 | 100 |
| 409 | 6.51 | 0.24 | 8.42 | 0.73 | 0.34 | 0.10 | 727 | 63 | 101 |
| | | 0.22 | | | | | | | |
| 410 | 6.29 | | 8.39 | 0.73 | 0.34 | 0.10 | 737 | 62 | 101 |
| 411 | 6.07 | —0.22 | 8.37 | 0.73 | 0.34 | 0.10 | 748 | 61 | 102 |
| 412 | 5.85 | 0.22 | 8.35 | 0.73 | 0.34 | 0.10 | 759 | 60 | 103 |
| 413 | 5.64 | 0.21 | 8.33 | 0.73 | 0.34 | 0.10 | 770 | 59 | 104 |
| 414 | 5.44 | 0.20 | 8.31 | 0.73 | 0.34 | 0.10 | 781 | 58 | 104 |
| | | 0.20 | | | | | | | |
| 415 | 5.24 | | 8.28 | 0.73 | 0.34 | 0.10 | 792 | 57 | 105 |
| 416 | 5.04 | —0.20 | 8.26 | 0.74 | 0.33 | 0.10 | 803 | 56 | 106 |
| 417 | 4.86 | 0.18 | 8.24 | 0.74 | 0.33 | 0.10 | 814 | 55 | 107 |
| 418 | 4.67 | 0.19 | 8.21 | 0.74 | 0.33 | 0.10 | 825 | 54 | 107 |
| 419 | 4.50 | 0.17 | 8.19 | 0.75 | 0.33 | 0.10 | 836 | 53 | 108 |
| | | 0.17 | | | | | | | |
| 420 | 4.33 | | 8.17 | 0.76 | 0.33 | 0.10 | 847 | 52 | 109 |

TABLE VIII, ARG. 1.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | (v.c.1) | (v.s.2) | (v.c.2) | (p.c.0) | (p.s.1) | (p.c.1) |
|------|---------|-------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | " | " | " | | | |
| 420 | 4.33 | | 8.17 | 0.76 | 0.33 | 0.10 | 847 | 52 | 109 |
| 421 | 4.16 | -0.17 | 8.14 | 0.76 | 0.32 | 0.10 | 858 | 51 | 110 |
| 422 | 4.00 | 0.16 | 8.11 | 0.77 | 0.32 | 0.11 | 870 | 50 | 110 |
| 423 | 3.85 | 0.15 | 8.09 | 0.78 | 0.32 | 0.11 | 881 | 50 | 111 |
| 424 | 3.70 | 0.15 | 8.06 | 0.79 | 0.32 | 0.11 | 892 | 49 | 111 |
| | | 0.14 | | | | | | | |
| 425 | 3.56 | | 8.04 | 0.80 | 0.32 | 0.11 | 904 | 48 | 112 |
| 426 | 3.42 | -0.14 | 8.01 | 0.81 | 0.31 | 0.11 | 916 | 47 | 113 |
| 427 | 3.29 | 0.13 | 7.98 | 0.82 | 0.31 | 0.11 | 927 | 46 | 113 |
| 428 | 3.16 | 0.13 | 7.95 | 0.83 | 0.31 | 0.11 | 938 | 45 | 114 |
| 429 | 3.04 | 0.12 | 7.92 | 0.85 | 0.31 | 0.11 | 950 | 44 | 114 |
| | | 0.11 | | | | | | | |
| 430 | 2.93 | | 7.89 | 0.86 | 0.31 | 0.11 | 961 | 44 | 115 |
| 431 | 2.82 | -0.11 | 7.86 | 0.87 | 0.30 | 0.11 | 973 | 43 | 116 |
| 432 | 2.72 | 0.10 | 7.83 | 0.89 | 0.30 | 0.11 | 984 | 42 | 116 |
| 433 | 2.62 | 0.10 | 7.80 | 0.90 | 0.30 | 0.11 | 996 | 41 | 117 |
| 434 | 2.53 | 0.09 | 7.77 | 0.92 | 0.29 | 0.11 | 1008 | 40 | 118 |
| | | 0.08 | | | | | | | |
| 435 | 2.45 | | 7.74 | 0.94 | 0.29 | 0.11 | 1019 | 40 | 118 |
| 436 | 2.37 | -0.08 | 7.71 | 0.95 | 0.29 | 0.12 | 1031 | 39 | 119 |
| 437 | 2.29 | 0.08 | 7.67 | 0.97 | 0.29 | 0.12 | 1043 | 38 | 119 |
| 438 | 2.23 | 0.06 | 7.64 | 0.99 | 0.28 | 0.12 | 1054 | 38 | 120 |
| 439 | 2.16 | 0.07 | 7.60 | 1.01 | 0.28 | 0.12 | 1066 | 37 | 120 |
| | | 0.05 | | | | | | | |
| 440 | 2.11 | | 7.57 | 1.03 | 0.28 | 0.12 | 1078 | 36 | 121 |
| 441 | 2.06 | -0.05 | 7.54 | 1.05 | 0.28 | 0.12 | 1089 | 35 | 121 |
| 442 | 2.02 | 0.04 | 7.50 | 1.07 | 0.27 | 0.12 | 1100 | 35 | 122 |
| 443 | 1.98 | 0.04 | 7.47 | 1.09 | 0.27 | 0.12 | 1113 | 34 | 122 |
| 444 | 1.95 | 0.03 | 7.43 | 1.11 | 0.27 | 0.12 | 1124 | 33 | 122 |
| | | 0.03 | | | | | | | |
| 445 | 1.92 | | 7.39 | 1.14 | 0.26 | 0.12 | 1137 | 33 | 123 |
| 446 | 1.90 | -0.02 | 7.36 | 1.16 | 0.26 | 0.12 | 1149 | 32 | 123 |
| 447 | 1.88 | 0.02 | 7.32 | 1.18 | 0.26 | 0.12 | 1160 | 32 | 124 |
| 448 | 1.87 | 0.01 | 7.28 | 1.21 | 0.26 | 0.12 | 1172 | 31 | 124 |
| 449 | 1.87 | 0.00 | 7.24 | 1.23 | 0.25 | 0.12 | 1184 | 30 | 125 |
| | | 0.00 | | | | | | | |
| 450 | 1.87 | | 7.20 | 1.26 | 0.25 | 0.12 | 1196 | 30 | 125 |
| 451 | 1.88 | +0.01 | 7.16 | 1.28 | 0.25 | 0.12 | 1208 | 30 | 125 |
| 452 | 1.90 | 0.02 | 7.12 | 1.31 | 0.24 | 0.12 | 1220 | 29 | 126 |
| 453 | 1.92 | 0.02 | 7.08 | 1.33 | 0.24 | 0.12 | 1232 | 28 | 126 |
| 454 | 1.95 | 0.03 | 7.04 | 1.36 | 0.24 | 0.12 | 1243 | 28 | 127 |
| | | 0.03 | | | | | | | |
| 455 | 1.98 | | 7.00 | 1.39 | 0.23 | 0.12 | 1255 | 28 | 127 |
| 456 | 2.01 | +0.03 | 6.96 | 1.42 | 0.23 | 0.12 | 1267 | 27 | 127 |
| 457 | 2.06 | 0.05 | 6.91 | 1.45 | 0.23 | 0.12 | 1279 | 27 | 128 |
| 458 | 2.11 | 0.05 | 6.87 | 1.47 | 0.22 | 0.12 | 1291 | 26 | 128 |
| 459 | 2.16 | 0.05 | 6.83 | 1.50 | 0.22 | 0.12 | 1302 | 26 | 128 |
| | | 0.07 | | | | | | | |
| 460 | 2.23 | | 6.78 | 1.53 | 0.22 | 0.12 | 1314 | 26 | 128 |
| 461 | 2.29 | +0.06 | 6.74 | 1.56 | 0.21 | 0.12 | 1326 | 25 | 129 |
| 462 | 2.37 | 0.08 | 6.69 | 1.59 | 0.21 | 0.12 | 1338 | 25 | 129 |
| 463 | 2.44 | 0.07 | 6.65 | 1.62 | 0.21 | 0.12 | 1350 | 25 | 129 |
| 464 | 2.53 | 0.09 | 6.60 | 1.65 | 0.20 | 0.12 | 1361 | 25 | 130 |
| | | 0.09 | | | | | | | |
| 465 | 2.62 | | 6.56 | 1.68 | 0.20 | 0.12 | 1373 | 24 | 130 |
| 466 | 2.71 | +0.09 | 6.51 | 1.72 | 0.20 | 0.12 | 1385 | 24 | 130 |
| 467 | 2.82 | 0.11 | 6.46 | 1.75 | 0.19 | 0.13 | 1396 | 24 | 130 |
| 468 | 2.92 | 0.10 | 6.41 | 1.78 | 0.19 | 0.13 | 1408 | 24 | 130 |
| 469 | 3.04 | 0.12 | 6.37 | 1.81 | 0.19 | 0.13 | 1420 | 23 | 131 |
| | | 0.11 | | | | | | | |
| 470 | 3.15 | | 6.32 | 1.84 | 0.18 | 0.13 | 1431 | 23 | 131 |
| 471 | 3.28 | +0.13 | 6.27 | 1.88 | 0.18 | 0.13 | 1443 | 23 | 131 |
| 472 | 3.41 | 0.13 | 6.22 | 1.91 | 0.18 | 0.12 | 1455 | 23 | 131 |
| 473 | 3.55 | 0.14 | 6.17 | 1.94 | 0.17 | 0.12 | 1466 | 23 | 131 |
| 474 | 3.69 | 0.14 | 6.12 | 1.98 | 0.17 | 0.12 | 1478 | 23 | 132 |
| | | 0.14 | | | | | | | |
| 475 | 3.83 | | 6.07 | 2.01 | 0.17 | 0.12 | 1489 | 23 | 132 |
| 476 | 3.98 | +0.15 | 6.02 | 2.04 | 0.16 | 0.12 | 1501 | 23 | 132 |
| 477 | 4.14 | 0.16 | 5.96 | 2.07 | 0.16 | 0.12 | 1512 | 23 | 132 |
| 478 | 4.31 | 0.17 | 5.91 | 2.11 | 0.16 | 0.12 | 1524 | 23 | 132 |
| 479 | 4.48 | 0.17 | 5.86 | 2.14 | 0.15 | 0.12 | 1535 | 23 | 132 |
| | | 0.17 | | | | | | | |
| 480 | 4.65 | | 5.81 | 2.18 | 0.15 | 0.12 | 1546 | 23 | 132 |

TABLE VIII, ARG. 1.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | (v.c.1) | (v.s.2) | (v.c.2) | (p.c.0) | (p.s.1) | (p.c.1) |
|------|---------|-------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | " | " | " | | | |
| 480 | 4.65 | | 5.81 | 2.18 | 0.15 | 0.12 | 1546 | 23 | 132 |
| 481 | 4.83 | +0.18 | 5.75 | 2.21 | 0.15 | 0.12 | 1558 | 23 | 132 |
| 482 | 5.02 | 0.19 | 5.70 | 2.24 | 0.15 | 0.12 | 1569 | 23 | 132 |
| 483 | 5.21 | 0.19 | 5.65 | 2.28 | 0.14 | 0.12 | 1580 | 23 | 132 |
| 484 | 5.40 | 0.19 | 5.59 | 2.31 | 0.14 | 0.12 | 1591 | 23 | 133 |
| | | 0.21 | | | | | | | |
| 485 | 5.61 | | 5.54 | 2.35 | 0.14 | 0.12 | 1602 | 24 | 133 |
| 486 | 5.81 | +0.20 | 5.48 | 2.38 | 0.13 | 0.12 | 1613 | 24 | 133 |
| 487 | 6.03 | 0.22 | 5.43 | 2.42 | 0.13 | 0.12 | 1624 | 24 | 133 |
| 488 | 6.24 | 0.21 | 5.37 | 2.45 | 0.13 | 0.11 | 1635 | 24 | 134 |
| 489 | 6.47 | 0.23 | 5.32 | 2.48 | 0.12 | 0.11 | 1646 | 25 | 134 |
| | | 0.23 | | | | | | | |
| 490 | 6.70 | | 5.26 | 2.52 | 0.12 | 0.11 | 1657 | 25 | 133 |
| 491 | 6.93 | +0.23 | 5.20 | 2.55 | 0.12 | 0.11 | 1668 | 25 | 133 |
| 492 | 7.17 | 0.24 | 5.15 | 2.59 | 0.12 | 0.11 | 1679 | 25 | 133 |
| 493 | 7.41 | 0.24 | 5.09 | 2.62 | 0.11 | 0.11 | 1690 | 25 | 133 |
| 494 | 7.66 | 0.25 | 5.03 | 2.66 | 0.11 | 0.11 | 1700 | 26 | 133 |
| | | 0.26 | | | | | | | |
| 495 | 7.92 | | 4.98 | 2.69 | 0.11 | 0.11 | 1711 | 27 | 133 |
| 496 | 8.18 | +0.26 | 4.92 | 2.72 | 0.11 | 0.11 | 1721 | 27 | 133 |
| 497 | 8.44 | 0.26 | 4.86 | 2.76 | 0.10 | 0.11 | 1732 | 28 | 133 |
| 498 | 8.71 | 0.27 | 4.80 | 2.79 | 0.10 | 0.11 | 1742 | 28 | 133 |
| 499 | 8.99 | 0.28 | 4.75 | 2.83 | 0.10 | 0.11 | 1753 | 28 | 133 |
| | | 0.27 | | | | | | | |
| 500 | 9.26 | | 4.69 | 2.86 | 0.10 | 0.11 | 1763 | 29 | 133 |
| 501 | 9.55 | +0.29 | 4.63 | 2.89 | 0.10 | 0.11 | 1774 | 30 | 133 |
| 502 | 9.84 | 0.29 | 4.57 | 2.93 | 0.09 | 0.10 | 1784 | 30 | 133 |
| 503 | 10.13 | 0.29 | 4.51 | 2.96 | 0.09 | 0.10 | 1794 | 31 | 133 |
| 504 | 10.43 | 0.30 | 4.44 | 2.99 | 0.09 | 0.10 | 1804 | 31 | 133 |
| | | 0.31 | | | | | | | |
| 505 | 10.74 | | 4.39 | 3.02 | 0.09 | 0.10 | 1814 | 32 | 133 |
| 506 | 11.05 | +0.31 | 4.33 | 3.06 | 0.09 | 0.10 | 1824 | 32 | 133 |
| 507 | 11.36 | 0.31 | 4.28 | 3.09 | 0.08 | 0.10 | 1834 | 33 | 133 |
| 508 | 11.68 | 0.32 | 4.22 | 3.12 | 0.08 | 0.10 | 1844 | 34 | 133 |
| 509 | 12.00 | 0.32 | 4.16 | 3.15 | 0.08 | 0.10 | 1853 | 35 | 133 |
| | | 0.33 | | | | | | | |
| 510 | 12.33 | | 4.10 | 3.18 | 0.08 | 0.09 | 1863 | 35 | 133 |
| 511 | 12.66 | +0.33 | 4.04 | 3.21 | 0.08 | 0.09 | 1873 | 36 | 133 |
| 512 | 13.00 | 0.34 | 3.98 | 3.25 | 0.07 | 0.09 | 1882 | 37 | 133 |
| 513 | 13.34 | 0.34 | 3.92 | 3.28 | 0.07 | 0.09 | 1892 | 38 | 133 |
| 514 | 13.68 | 0.34 | 3.86 | 3.31 | 0.07 | 0.09 | 1901 | 39 | 133 |
| | | 0.35 | | | | | | | |
| 515 | 14.03 | | 3.80 | 3.34 | 0.07 | 0.09 | 1910 | 40 | 133 |
| 516 | 14.39 | +0.36 | 3.74 | 3.37 | 0.07 | 0.09 | 1920 | 40 | 133 |
| 517 | 14.75 | 0.36 | 3.68 | 3.40 | 0.07 | 0.09 | 1929 | 41 | 133 |
| 518 | 15.11 | 0.36 | 3.62 | 3.43 | 0.07 | 0.09 | 1938 | 42 | 133 |
| 519 | 15.48 | 0.37 | 3.56 | 3.46 | 0.07 | 0.08 | 1947 | 43 | 133 |
| | | 0.37 | | | | | | | |
| 520 | 15.85 | | 3.50 | 3.49 | 0.06 | 0.08 | 1956 | 44 | 132 |
| 521 | 16.23 | +0.38 | 3.44 | 3.51 | 0.06 | 0.08 | 1964 | 45 | 132 |
| 522 | 16.61 | 0.38 | 3.38 | 3.54 | 0.06 | 0.08 | 1973 | 46 | 132 |
| 523 | 16.99 | 0.38 | 3.32 | 3.57 | 0.06 | 0.08 | 1982 | 47 | 132 |
| 524 | 17.38 | 0.39 | 3.27 | 3.60 | 0.06 | 0.08 | 1990 | 48 | 132 |
| | | 0.39 | | | | | | | |
| 525 | 17.77 | | 3.21 | 3.62 | 0.06 | 0.08 | 1999 | 49 | 132 |
| 526 | 18.17 | +0.40 | 3.15 | 3.65 | 0.06 | 0.08 | 2007 | 50 | 132 |
| 527 | 18.57 | 0.40 | 3.09 | 3.67 | 0.06 | 0.08 | 2016 | 51 | 132 |
| 528 | 18.97 | 0.40 | 3.03 | 3.70 | 0.06 | 0.07 | 2024 | 52 | 132 |
| 529 | 19.38 | 0.41 | 2.97 | 3.73 | 0.05 | 0.07 | 2032 | 53 | 132 |
| | | 0.41 | | | | | | | |
| 530 | 19.79 | | 2.91 | 3.75 | 0.05 | 0.07 | 2040 | 55 | 132 |
| 531 | 20.21 | +0.42 | 2.86 | 3.77 | 0.05 | 0.07 | 2048 | 56 | 132 |
| 532 | 20.62 | 0.41 | 2.80 | 3.80 | 0.05 | 0.07 | 2056 | 57 | 132 |
| 533 | 21.05 | 0.43 | 2.75 | 3.82 | 0.05 | 0.07 | 2063 | 58 | 132 |
| 534 | 21.47 | 0.42 | 2.69 | 3.84 | 0.05 | 0.07 | 2071 | 59 | 131 |
| | | 0.43 | | | | | | | |
| 535 | 21.90 | | 2.63 | 3.87 | 0.05 | 0.07 | 2079 | 61 | 131 |
| 536 | 22.34 | +0.44 | 2.57 | 3.89 | 0.05 | 0.07 | 2086 | 62 | 131 |
| 537 | 22.77 | 0.43 | 2.52 | 3.91 | 0.05 | 0.07 | 2094 | 63 | 131 |
| 538 | 23.21 | 0.44 | 2.46 | 3.93 | 0.05 | 0.07 | 2101 | 64 | 131 |
| 539 | 23.66 | 0.45 | 2.40 | 3.96 | 0.05 | 0.06 | 2108 | 66 | 131 |
| | | 0.44 | | | | | | | |
| 540 | 24.10 | | 2.35 | 3.98 | 0.05 | 0.06 | 2115 | 67 | 131 |

TABLE VIII, ARG. 1.—*Concluded.*

| Arg. | (v.c.0) | Diff. | (v.s.1) | (v.c.1) | (v.s.2) | (v.c.2) | (p.c.0) | (p.s.1) | (p.c.1) |
|------|---------|-------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | " | " | " | | | |
| 540 | 24.10 | | 2.35 | 3.98 | 0.05 | 0.06 | 2115 | 67 | 131 |
| 541 | 24.56 | +0.46 | 2.29 | 4.00 | 0.05 | 0.06 | 2122 | 68 | 131 |
| 542 | 25.01 | 0.45 | 2.24 | 4.02 | 0.05 | 0.06 | 2129 | 70 | 131 |
| 543 | 25.47 | 0.46 | 2.18 | 4.04 | 0.05 | 0.06 | 2135 | 71 | 130 |
| 544 | 25.93 | 0.46 | 2.13 | 4.06 | 0.05 | 0.06 | 2142 | 73 | 130 |
| 545 | 26.39 | | 2.08 | 4.07 | 0.05 | 0.06 | 2149 | 74 | 130 |
| 546 | 26.85 | +0.46 | 2.02 | 4.09 | 0.05 | 0.06 | 2155 | 75 | 130 |
| 547 | 27.32 | 0.47 | 1.97 | 4.11 | 0.05 | 0.06 | 2162 | 77 | 130 |
| 548 | 27.79 | 0.47 | 1.92 | 4.13 | 0.05 | 0.06 | 2168 | 78 | 130 |
| 549 | 28.27 | 0.48 | 1.87 | 4.14 | 0.05 | 0.06 | 2174 | 80 | 130 |
| 550 | 28.75 | | 1.82 | 4.16 | 0.05 | 0.06 | 2180 | 81 | 130 |
| 551 | 29.23 | +0.48 | 1.77 | 4.17 | 0.05 | 0.06 | 2186 | 83 | 130 |
| 552 | 29.71 | 0.48 | 1.72 | 4.19 | 0.06 | 0.06 | 2191 | 84 | 130 |
| 553 | 30.20 | 0.49 | 1.67 | 4.20 | 0.06 | 0.06 | 2197 | 86 | 130 |
| 554 | 30.69 | 0.49 | 1.62 | 4.22 | 0.06 | 0.06 | 2203 | 87 | 130 |
| 555 | 31.18 | | 1.57 | 4.23 | 0.06 | 0.06 | 2208 | 89 | 130 |
| 556 | 31.67 | +0.49 | 1.52 | 4.25 | 0.06 | 0.06 | 2214 | 90 | 130 |
| 557 | 32.16 | 0.49 | 1.48 | 4.26 | 0.06 | 0.06 | 2219 | 92 | 130 |
| 558 | 32.66 | 0.50 | 1.43 | 4.27 | 0.06 | 0.06 | 2224 | 94 | 130 |
| 559 | 33.16 | 0.50 | 1.38 | 4.28 | 0.06 | 0.06 | 2229 | 95 | 130 |
| 560 | 33.67 | | 1.34 | 4.29 | 0.06 | 0.06 | 2234 | 97 | 130 |
| 561 | 34.17 | +0.50 | 1.29 | 4.31 | 0.06 | 0.06 | 2238 | 98 | 130 |
| 562 | 34.68 | 0.51 | 1.25 | 4.32 | 0.07 | 0.06 | 2243 | 100 | 130 |
| 563 | 35.19 | 0.51 | 1.20 | 4.32 | 0.07 | 0.06 | 2248 | 101 | 130 |
| 564 | 35.70 | 0.51 | 1.16 | 4.33 | 0.07 | 0.06 | 2252 | 103 | 130 |
| 565 | 36.21 | | 1.12 | 4.34 | 0.07 | 0.06 | 2256 | 105 | 130 |
| 566 | 36.73 | +0.52 | 1.08 | 4.35 | 0.07 | 0.06 | 2261 | 106 | 130 |
| 567 | 37.25 | 0.52 | 1.03 | 4.36 | 0.07 | 0.06 | 2265 | 108 | 130 |
| 568 | 37.76 | 0.51 | 0.99 | 4.37 | 0.07 | 0.06 | 2269 | 110 | 130 |
| 569 | 38.29 | 0.53 | 0.96 | 4.38 | 0.07 | 0.06 | 2272 | 111 | 130 |
| 570 | 38.81 | | 0.92 | 4.38 | 0.07 | 0.06 | 2276 | 113 | 130 |
| 571 | 39.34 | +0.53 | 0.88 | 4.39 | 0.07 | 0.06 | 2280 | 115 | 130 |
| 572 | 39.86 | 0.52 | 0.84 | 4.40 | 0.07 | 0.07 | 2283 | 116 | 130 |
| 573 | 40.39 | 0.53 | 0.80 | 4.40 | 0.08 | 0.07 | 2287 | 118 | 130 |
| 574 | 40.92 | 0.53 | 0.77 | 4.41 | 0.08 | 0.07 | 2290 | 120 | 131 |
| 575 | 41.45 | | 0.73 | 4.41 | 0.08 | 0.07 | 2293 | 121 | 131 |
| 576 | 41.98 | +0.53 | 0.70 | 4.42 | 0.08 | 0.07 | 2296 | 123 | 131 |
| 577 | 42.52 | 0.54 | 0.67 | 4.42 | 0.08 | 0.07 | 2299 | 125 | 131 |
| 578 | 43.05 | 0.53 | 0.63 | 4.42 | 0.08 | 0.07 | 2302 | 126 | 131 |
| 579 | 43.59 | 0.54 | 0.60 | 4.43 | 0.08 | 0.07 | 2304 | 128 | 131 |
| 580 | 44.12 | | 0.57 | 4.43 | 0.08 | 0.07 | 2307 | 130 | 131 |
| 581 | 44.66 | +0.54 | 0.54 | 4.43 | 0.08 | 0.08 | 2309 | 132 | 131 |
| 582 | 45.20 | 0.54 | 0.51 | 4.44 | 0.09 | 0.08 | 2312 | 133 | 131 |
| 583 | 45.74 | 0.54 | 0.48 | 4.44 | 0.09 | 0.08 | 2314 | 135 | 131 |
| 584 | 46.28 | 0.54 | 0.46 | 4.44 | 0.09 | 0.08 | 2316 | 137 | 132 |
| 585 | 46.83 | 0.55 | 0.43 | 4.44 | 0.09 | 0.08 | 2318 | 138 | 132 |
| 586 | 47.37 | +0.54 | 0.40 | 4.44 | 0.09 | 0.08 | 2319 | 140 | 132 |
| 587 | 47.91 | 0.54 | 0.38 | 4.44 | 0.09 | 0.09 | 2321 | 142 | 132 |
| 588 | 48.46 | 0.55 | 0.36 | 4.44 | 0.09 | 0.09 | 2322 | 144 | 132 |
| 589 | 49.00 | 0.54 | 0.33 | 4.44 | 0.09 | 0.09 | 2324 | 145 | 132 |
| 590 | 49.55 | | 0.31 | 4.44 | 0.09 | 0.09 | 2325 | 147 | 132 |
| 591 | 50.10 | +0.55 | 0.29 | 4.44 | 0.09 | 0.09 | 2326 | 149 | 132 |
| 592 | 50.64 | 0.54 | 0.27 | 4.44 | 0.10 | 0.09 | 2327 | 150 | 133 |
| 593 | 51.19 | 0.55 | 0.25 | 4.44 | 0.10 | 0.10 | 2328 | 152 | 133 |
| 594 | 51.74 | 0.55 | 0.23 | 4.43 | 0.10 | 0.10 | 2329 | 154 | 133 |
| 595 | 52.29 | | 0.22 | 4.43 | 0.10 | 0.10 | 2330 | 155 | 133 |
| 596 | 52.84 | +0.55 | 0.20 | 4.43 | 0.10 | 0.10 | 2330 | 157 | 133 |
| 597 | 53.38 | 0.54 | 0.18 | 4.43 | 0.10 | 0.11 | 2330 | 159 | 133 |
| 598 | 53.93 | 0.55 | 0.17 | 4.43 | 0.10 | 0.11 | 2331 | 161 | 134 |
| 599 | 54.48 | 0.55 | 0.16 | 4.42 | 0.10 | 0.11 | 2331 | 162 | 134 |
| 600 | 55.03 | 0.55 | 0.14 | 4.42 | 0.10 | 0.11 | 2331 | 164 | 134 |

TABLE IX, ARG. 2.—ACTION OF SATURN.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | Sec.var. | (v.c.1) | Diff. | Sec.var. | (v.s.2) | Diff. | Sec.var. | (v.c.2) | Diff. | Sec.var. |
|------|---------|-------|---------|-------|----------|---------|-------|----------|---------|-------|----------|---------|-------|----------|
| | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 0 | 38.54 | | 139.94 | | 2.53 | 293.78 | | 0.56 | 182.62 | | 1.77 | 244.67 | | 0.12 |
| 1 | 38.63 | +0.09 | 141.38 | +1.44 | 2.52 | 293.81 | +0.03 | 0.55 | 183.80 | +1.18 | 1.76 | 244.10 | -0.57 | 0.12 |
| 2 | 38.72 | 0.09 | 142.82 | 1.44 | 2.50 | 293.83 | +0.02 | 0.54 | 184.96 | 1.16 | 1.74 | 243.52 | 0.58 | 0.12 |
| 3 | 38.81 | 0.09 | 144.26 | 1.44 | 2.49 | 293.83 | 0.00 | 0.53 | 186.12 | 1.16 | 1.73 | 242.93 | 0.59 | 0.11 |
| 4 | 38.90 | 0.09 | 145.70 | 1.44 | 2.48 | 293.82 | -0.01 | 0.52 | 187.27 | 1.15 | 1.71 | 242.32 | 0.61 | 0.11 |
| | | 0.08 | | 1.44 | | | 0.03 | | | 1.15 | | | 0.61 | |
| 5 | 38.98 | | 147.14 | | 2.47 | 293.79 | | 0.51 | 188.42 | | 1.70 | 241.71 | | 0.11 |
| 6 | 39.07 | +0.09 | 148.58 | +1.44 | 2.46 | 293.75 | -0.04 | 0.50 | 189.56 | +1.14 | 1.69 | 241.09 | -0.62 | 0.11 |
| 7 | 39.16 | 0.09 | 150.01 | 1.43 | 2.45 | 293.70 | 0.05 | 0.49 | 190.69 | 1.13 | 1.67 | 240.46 | 0.63 | 0.11 |
| 8 | 39.24 | 0.08 | 151.45 | 1.44 | 2.44 | 293.63 | 0.07 | 0.48 | 191.82 | 1.13 | 1.66 | 239.81 | 0.65 | 0.10 |
| 9 | 39.33 | 0.09 | 152.89 | 1.44 | 2.43 | 293.54 | 0.09 | 0.47 | 192.94 | 1.12 | 1.64 | 239.15 | 0.66 | 0.10 |
| | | 0.08 | | 1.43 | | | 0.10 | | | 1.11 | | | 0.67 | |
| 10 | 39.41 | | 154.32 | | 2.42 | 293.44 | | 0.46 | 194.05 | | 1.63 | 238.48 | | 0.10 |
| 11 | 39.50 | +0.09 | 155.76 | +1.44 | 2.41 | 293.33 | -0.11 | 0.45 | 195.16 | +1.11 | 1.61 | 237.80 | -0.68 | 0.10 |
| 12 | 39.58 | 0.08 | 157.19 | 1.43 | 2.40 | 293.20 | 0.13 | 0.44 | 196.26 | 1.10 | 1.60 | 237.11 | 0.69 | 0.10 |
| 13 | 39.67 | 0.09 | 158.62 | 1.43 | 2.39 | 293.06 | 0.14 | 0.43 | 197.35 | 1.09 | 1.58 | 236.41 | 0.70 | 0.10 |
| 14 | 39.75 | 0.08 | 160.05 | 1.43 | 2.38 | 292.90 | 0.16 | 0.42 | 198.43 | 1.08 | 1.57 | 235.69 | 0.72 | 0.10 |
| | | 0.09 | | 1.43 | | | 0.17 | | | 1.08 | | | 0.73 | |
| 15 | 39.84 | +0.08 | 161.48 | +1.42 | 2.36 | 292.73 | -0.18 | 0.42 | 199.51 | +1.07 | 1.55 | 234.96 | -0.74 | 0.10 |
| 16 | 39.92 | 0.09 | 162.90 | 1.42 | 2.35 | 292.55 | 0.21 | 0.41 | 200.58 | 1.06 | 1.53 | 234.22 | 0.74 | 0.09 |
| 17 | 40.01 | 0.09 | 164.32 | 1.42 | 2.34 | 292.34 | 0.21 | 0.40 | 201.64 | 1.05 | 1.52 | 233.48 | 0.75 | 0.09 |
| 18 | 40.10 | 0.09 | 165.74 | 1.42 | 2.33 | 292.13 | 0.21 | 0.39 | 202.69 | 1.05 | 1.50 | 232.73 | 0.75 | 0.09 |
| 19 | 40.18 | 0.08 | 167.16 | 1.42 | 2.32 | 291.90 | 0.23 | 0.38 | 203.74 | 1.05 | 1.49 | 231.96 | 0.77 | 0.09 |
| | | 0.09 | | 1.42 | | | 0.25 | | | 1.04 | | | 0.78 | |
| 20 | 40.27 | +0.08 | 168.58 | +1.41 | 2.31 | 291.65 | -0.26 | 0.37 | 204.78 | +1.03 | 1.47 | 231.18 | -0.79 | 0.09 |
| 21 | 40.35 | 0.09 | 169.99 | 1.41 | 2.30 | 291.39 | 0.27 | 0.36 | 205.81 | 1.02 | 1.46 | 230.39 | 0.79 | 0.09 |
| 22 | 40.44 | 0.08 | 171.40 | 1.41 | 2.29 | 291.12 | 0.27 | 0.35 | 206.83 | 1.02 | 1.44 | 229.60 | 0.79 | 0.09 |
| 23 | 40.52 | 0.10 | 172.81 | 1.41 | 2.27 | 290.83 | 0.29 | 0.35 | 207.84 | 1.01 | 1.43 | 228.80 | 0.80 | 0.10 |
| 24 | 40.62 | 0.09 | 174.22 | 1.40 | 2.26 | 290.53 | 0.30 | 0.34 | 208.85 | 1.01 | 1.41 | 227.98 | 0.82 | 0.10 |
| | | 0.09 | | 1.40 | | | 0.31 | | | 1.00 | | | 0.83 | |
| 25 | 40.71 | +0.09 | 175.62 | +1.40 | 2.25 | 290.22 | -0.33 | 0.33 | 209.85 | +0.99 | 1.40 | 227.15 | -0.84 | 0.10 |
| 26 | 40.80 | 0.09 | 177.02 | 1.40 | 2.24 | 289.89 | 0.34 | 0.32 | 210.84 | 0.98 | 1.39 | 226.31 | 0.85 | 0.10 |
| 27 | 40.89 | 0.09 | 178.42 | 1.39 | 2.23 | 289.55 | 0.34 | 0.31 | 211.82 | 0.97 | 1.37 | 225.46 | 0.86 | 0.10 |
| 28 | 40.98 | 0.09 | 179.81 | 1.39 | 2.21 | 289.19 | 0.36 | 0.30 | 212.79 | 0.97 | 1.36 | 224.60 | 0.86 | 0.11 |
| 29 | 41.07 | 0.09 | 181.20 | 1.39 | 2.20 | 288.82 | 0.37 | 0.30 | 213.76 | 0.97 | 1.34 | 223.74 | 0.86 | 0.11 |
| | | 0.09 | | 1.39 | | | 0.38 | | | 0.95 | | | 0.87 | |
| 30 | 41.16 | +0.09 | 182.59 | +1.38 | 2.19 | 288.44 | -0.40 | 0.29 | 214.71 | +0.95 | 1.33 | 222.87 | -0.88 | 0.11 |
| 31 | 41.25 | 0.10 | 183.97 | 1.38 | 2.18 | 288.04 | 0.41 | 0.28 | 215.66 | 0.94 | 1.32 | 221.99 | 0.89 | 0.11 |
| 32 | 41.35 | 0.09 | 185.35 | 1.38 | 2.17 | 287.63 | 0.41 | 0.28 | 216.60 | 0.94 | 1.30 | 221.10 | 0.90 | 0.11 |
| 33 | 41.44 | 0.10 | 186.73 | 1.37 | 2.15 | 287.20 | 0.43 | 0.27 | 217.53 | 0.93 | 1.29 | 220.20 | 0.90 | 0.12 |
| 34 | 41.54 | 0.10 | 188.10 | 1.37 | 2.14 | 286.76 | 0.44 | 0.26 | 218.44 | 0.91 | 1.27 | 219.28 | 0.92 | 0.12 |
| | | 0.10 | | 1.37 | | | 0.46 | | | 0.91 | | | 0.92 | |
| 35 | 41.64 | +0.10 | 189.47 | +1.36 | 2.13 | 286.30 | -0.46 | 0.26 | 219.35 | +0.90 | 1.26 | 218.36 | -0.93 | 0.12 |
| 36 | 41.74 | 0.09 | 190.83 | 1.36 | 2.11 | 285.84 | 0.49 | 0.25 | 220.25 | 0.89 | 1.25 | 217.43 | 0.94 | 0.12 |
| 37 | 41.83 | 0.10 | 192.19 | 1.36 | 2.10 | 285.35 | 0.49 | 0.24 | 221.14 | 0.89 | 1.23 | 216.49 | 0.94 | 0.12 |
| 38 | 41.93 | 0.10 | 193.55 | 1.35 | 2.09 | 284.86 | 0.49 | 0.23 | 222.03 | 0.89 | 1.22 | 215.55 | 0.94 | 0.13 |
| 39 | 42.03 | 0.10 | 194.90 | 1.35 | 2.07 | 284.35 | 0.51 | 0.23 | 222.90 | 0.87 | 1.20 | 214.59 | 0.96 | 0.13 |
| | | 0.10 | | 1.35 | | | 0.51 | | | 0.86 | | | 0.97 | |
| 40 | 42.13 | +0.11 | 196.25 | +1.34 | 2.06 | 283.84 | -0.53 | 0.22 | 223.76 | +0.85 | 1.19 | 213.62 | -0.97 | 0.13 |
| 41 | 42.24 | 0.10 | 197.59 | 1.34 | 2.05 | 283.31 | 0.55 | 0.22 | 224.61 | 0.84 | 1.18 | 212.65 | 0.98 | 0.13 |
| 42 | 42.34 | 0.10 | 198.93 | 1.34 | 2.03 | 282.76 | 0.55 | 0.21 | 225.45 | 0.84 | 1.16 | 211.67 | 0.98 | 0.14 |
| 43 | 42.44 | 0.11 | 200.26 | 1.33 | 2.02 | 282.20 | 0.56 | 0.21 | 226.28 | 0.83 | 1.15 | 210.68 | 0.99 | 0.14 |
| 44 | 42.55 | 0.11 | 201.59 | 1.33 | 2.00 | 281.63 | 0.57 | 0.20 | 227.11 | 0.83 | 1.13 | 209.68 | 1.00 | 0.14 |
| | | 0.11 | | 1.32 | | | 0.59 | | | 0.81 | | | 1.01 | |
| 45 | 42.66 | +0.11 | 202.91 | +1.32 | 1.99 | 281.04 | -0.60 | 0.20 | 227.92 | +0.80 | 1.12 | 208.67 | -1.01 | 0.15 |
| 46 | 42.77 | 0.10 | 204.23 | 1.31 | 1.97 | 280.44 | 0.62 | 0.19 | 228.72 | 0.79 | 1.10 | 207.66 | 1.02 | 0.15 |
| 47 | 42.87 | 0.11 | 205.54 | 1.31 | 1.96 | 279.82 | 0.62 | 0.19 | 229.51 | 0.78 | 1.09 | 206.64 | 1.03 | 0.15 |
| 48 | 42.98 | 0.11 | 206.85 | 1.30 | 1.94 | 279.20 | 0.64 | 0.18 | 230.29 | 0.78 | 1.07 | 205.61 | 1.04 | 0.15 |
| 49 | 43.09 | 0.11 | 208.15 | 1.29 | 1.93 | 278.56 | 0.65 | 0.18 | 231.07 | 0.76 | 1.06 | 204.57 | 1.05 | 0.16 |
| | | 0.11 | | 1.29 | | | 0.65 | | | 0.76 | | | 1.05 | |
| 50 | 43.20 | +0.12 | 209.44 | +1.29 | 1.92 | 277.91 | -0.67 | 0.17 | 231.83 | +0.75 | 1.04 | 203.52 | -1.05 | 0.16 |
| 51 | 43.32 | 0.11 | 210.73 | 1.28 | 1.91 | 277.24 | 0.68 | 0.17 | 232.58 | 0.74 | 1.03 | 202.47 | 1.06 | 0.17 |
| 52 | 43.43 | 0.11 | 212.01 | 1.28 | 1.89 | 276.56 | 0.69 | 0.16 | 233.32 | 0.73 | 1.01 | 201.41 | 1.07 | 0.17 |
| 53 | 43.54 | 0.11 | 213.29 | 1.28 | 1.88 | 275.87 | 0.70 | 0.16 | 234.05 | 0.72 | 1.00 | 200.34 | 1.07 | 0.18 |
| 54 | 43.65 | 0.12 | 214.57 | 1.27 | 1.87 | 275.17 | 0.72 | 0.16 | 234.77 | 0.71 | 0.99 | 199.26 | 1.08 | 0.18 |
| | | 0.12 | | 1.27 | | | 0.72 | | | 0.71 | | | 1.08 | |
| 55 | 43.77 | +0.11 | 215.84 | +1.26 | 1.85 | 274.45 | -0.73 | 0.15 | 235.48 | +0.70 | 0.97 | 198.18 | -1.09 | 0.19 |
| 56 | 43.88 | 0.12 | 217.10 | 1.25 | 1.84 | 273.72 | 0.74 | 0.15 | 236.18 | 0.69 | 0.96 | 197.09 | 1.09 | 0.20 |
| 57 | 44.00 | 0.12 | 218.35 | 1.25 | 1.82 | 272.98 | 0.75 | 0.15 | 236.87 | 0.67 | 0.95 | 196.00 | 1.10 | 0.20 |
| 58 | 44.12 | 0.12 | 219.60 | 1.24 | 1.81 | 272.23 | 0.75 | 0.15 | 237.54 | 0.66 | 0.94 | 194.90 | 1.11 | 0.21 |
| 59 | 44.24 | 0.11 | 220.84 | 1.23 | 1.79 | 271.46 | 0.77 | 0.14 | 238.20 | 0.65 | 0.92 | 193.79 | 1.12 | 0.21 |
| | | 0.11 | | 1.23 | | | 0.77 | | | 0.65 | | | 1.12 | |
| 60 | 44.35 | | 222.07 | | 1.78 | 270.69 | | 0.14 | 238.85 | | 0.91 | 192.67 | | 0.22 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | (p.s.3) | (p.c.3) |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0 | 11.40 | 14.53 | 1.58 | 1.68 | 1678 | 182 | 1513 | 711 | 306 | 137 | 85 |
| 1 | 11.52 | 14.48 | 1.59 | 1.66 | 1679 | 182 | 1526 | 711 | 302 | 137 | 85 |
| 2 | 11.64 | 14.43 | 1.60 | 1.65 | 1680 | 183 | 1539 | 710 | 298 | 137 | 85 |
| 3 | 11.75 | 14.38 | 1.61 | 1.64 | 1680 | 184 | 1551 | 709 | 295 | 137 | 85 |
| 4 | 11.87 | 14.33 | 1.62 | 1.63 | 1681 | 185 | 1564 | 708 | 291 | 137 | 85 |
| 5 | 11.98 | 14.28 | 1.63 | 1.62 | 1682 | 187 | 1577 | 707 | 288 | 137 | 85 |
| 6 | 12.10 | 14.22 | 1.64 | 1.61 | 1682 | 188 | 1590 | 706 | 284 | 137 | 85 |
| 7 | 12.21 | 14.16 | 1.64 | 1.60 | 1683 | 190 | 1603 | 704 | 281 | 138 | 84 |
| 8 | 12.32 | 14.10 | 1.65 | 1.59 | 1683 | 191 | 1615 | 703 | 278 | 138 | 84 |
| 9 | 12.43 | 14.04 | 1.66 | 1.58 | 1682 | 193 | 1628 | 701 | 274 | 138 | 84 |
| 10 | 12.54 | 13.97 | 1.67 | 1.56 | 1682 | 195 | 1641 | 700 | 271 | 138 | 84 |
| 11 | 12.65 | 13.91 | 1.68 | 1.55 | 1681 | 197 | 1654 | 699 | 267 | 138 | 84 |
| 12 | 12.76 | 13.84 | 1.69 | 1.54 | 1680 | 199 | 1666 | 698 | 264 | 138 | 84 |
| 13 | 12.87 | 13.77 | 1.69 | 1.53 | 1679 | 201 | 1679 | 696 | 260 | 138 | 84 |
| 14 | 12.97 | 13.70 | 1.70 | 1.52 | 1678 | 203 | 1691 | 695 | 257 | 137 | 83 |
| 15 | 13.07 | 13.63 | 1.71 | 1.50 | 1677 | 206 | 1704 | 694 | 253 | 137 | 83 |
| 16 | 13.18 | 13.56 | 1.71 | 1.49 | 1675 | 208 | 1716 | 692 | 250 | 137 | 83 |
| 17 | 13.28 | 13.49 | 1.72 | 1.48 | 1674 | 211 | 1729 | 691 | 246 | 137 | 83 |
| 18 | 13.38 | 13.41 | 1.73 | 1.47 | 1672 | 214 | 1741 | 689 | 243 | 137 | 82 |
| 19 | 13.48 | 13.33 | 1.74 | 1.45 | 1670 | 217 | 1754 | 687 | 239 | 137 | 82 |
| 20 | 13.58 | 13.25 | 1.74 | 1.44 | 1668 | 220 | 1766 | 685 | 236 | 137 | 82 |
| 21 | 13.68 | 13.17 | 1.75 | 1.43 | 1666 | 223 | 1778 | 683 | 233 | 137 | 82 |
| 22 | 13.78 | 13.09 | 1.75 | 1.42 | 1664 | 227 | 1790 | 681 | 230 | 137 | 81 |
| 23 | 13.87 | 13.01 | 1.76 | 1.40 | 1662 | 230 | 1802 | 679 | 226 | 137 | 81 |
| 24 | 13.97 | 12.92 | 1.76 | 1.39 | 1660 | 234 | 1814 | 677 | 223 | 137 | 81 |
| 25 | 14.06 | 12.83 | 1.77 | 1.37 | 1658 | 237 | 1826 | 675 | 220 | 137 | 81 |
| 26 | 14.15 | 12.74 | 1.78 | 1.36 | 1656 | 241 | 1838 | 673 | 217 | 137 | 81 |
| 27 | 14.24 | 12.65 | 1.78 | 1.35 | 1653 | 245 | 1850 | 671 | 214 | 137 | 80 |
| 28 | 14.33 | 12.56 | 1.79 | 1.34 | 1651 | 249 | 1862 | 668 | 211 | 137 | 80 |
| 29 | 14.41 | 12.47 | 1.79 | 1.32 | 1648 | 253 | 1874 | 666 | 208 | 137 | 80 |
| 30 | 14.50 | 12.37 | 1.79 | 1.31 | 1645 | 258 | 1886 | 664 | 205 | 137 | 79 |
| 31 | 14.58 | 12.28 | 1.80 | 1.30 | 1642 | 262 | 1898 | 662 | 202 | 137 | 78 |
| 32 | 14.66 | 12.19 | 1.80 | 1.28 | 1638 | 267 | 1910 | 659 | 199 | 137 | 78 |
| 33 | 14.74 | 12.09 | 1.80 | 1.27 | 1635 | 272 | 1922 | 657 | 196 | 137 | 78 |
| 34 | 14.82 | 11.99 | 1.81 | 1.25 | 1631 | 277 | 1934 | 654 | 193 | 137 | 78 |
| 35 | 14.90 | 11.89 | 1.81 | 1.24 | 1627 | 282 | 1946 | 652 | 190 | 137 | 77 |
| 36 | 14.97 | 11.79 | 1.81 | 1.22 | 1623 | 287 | 1958 | 650 | 187 | 137 | 77 |
| 37 | 15.05 | 11.69 | 1.81 | 1.21 | 1619 | 292 | 1970 | 647 | 184 | 137 | 77 |
| 38 | 15.12 | 11.59 | 1.82 | 1.20 | 1615 | 297 | 1982 | 645 | 181 | 137 | 76 |
| 39 | 15.19 | 11.49 | 1.82 | 1.18 | 1611 | 302 | 1993 | 642 | 179 | 137 | 76 |
| 40 | 15.26 | 11.38 | 1.82 | 1.17 | 1608 | 308 | 2005 | 640 | 176 | 137 | 76 |
| 41 | 15.33 | 11.28 | 1.82 | 1.15 | 1602 | 313 | 2016 | 637 | 173 | 137 | 76 |
| 42 | 15.39 | 11.17 | 1.82 | 1.14 | 1597 | 319 | 2028 | 635 | 171 | 137 | 76 |
| 43 | 15.46 | 11.06 | 1.82 | 1.13 | 1592 | 325 | 2039 | 632 | 168 | 137 | 75 |
| 44 | 15.52 | 10.95 | 1.82 | 1.11 | 1587 | 331 | 2051 | 630 | 166 | 137 | 74 |
| 45 | 15.58 | 10.84 | 1.82 | 1.10 | 1582 | 337 | 2062 | 627 | 163 | 137 | 74 |
| 46 | 15.64 | 10.73 | 1.82 | 1.08 | 1577 | 343 | 2073 | 624 | 160 | 137 | 74 |
| 47 | 15.70 | 10.62 | 1.82 | 1.07 | 1572 | 349 | 2084 | 621 | 158 | 137 | 73 |
| 48 | 15.76 | 10.51 | 1.82 | 1.06 | 1566 | 355 | 2095 | 619 | 155 | 137 | 73 |
| 49 | 15.81 | 10.40 | 1.82 | 1.04 | 1561 | 362 | 2106 | 616 | 153 | 137 | 73 |
| 50 | 15.86 | 10.28 | 1.82 | 1.03 | 1556 | 368 | 2117 | 613 | 150 | 137 | 72 |
| 51 | 15.91 | 10.17 | 1.82 | 1.01 | 1550 | 375 | 2128 | 610 | 148 | 137 | 72 |
| 52 | 15.96 | 10.05 | 1.82 | 1.00 | 1545 | 381 | 2138 | 607 | 145 | 137 | 72 |
| 53 | 16.01 | 9.94 | 1.81 | 0.98 | 1539 | 388 | 2149 | 604 | 143 | 137 | 72 |
| 54 | 16.05 | 9.82 | 1.81 | 0.97 | 1533 | 394 | 2159 | 601 | 140 | 137 | 72 |
| 55 | 16.09 | 9.70 | 1.81 | 0.96 | 1527 | 401 | 2170 | 598 | 138 | 137 | 71 |
| 56 | 16.13 | 9.59 | 1.81 | 0.94 | 1521 | 408 | 2180 | 595 | 135 | 137 | 71 |
| 57 | 16.17 | 9.47 | 1.80 | 0.93 | 1515 | 415 | 2191 | 592 | 133 | 137 | 71 |
| 58 | 16.20 | 9.35 | 1.80 | 0.92 | 1509 | 422 | 2201 | 589 | 131 | 137 | 71 |
| 59 | 16.23 | 9.23 | 1.80 | 0.90 | 1502 | 430 | 2212 | 586 | 129 | 137 | 71 |
| 60 | 16.26 | 9.11 | 1.80 | 0.89 | 1496 | 437 | 2222 | 583 | 127 | 137 | 71 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | Sec.var. | (v.c.1) | Diff. | Sec.var. | (v.s.2) | Diff. | Sec.var. | (v.c.2) | Diff. | Sec.var. |
|------|---------|-------|---------|-------|----------|---------|-------|----------|---------|-------|----------|---------|-------|----------|
| | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 60 | 44.35 | | 222.07 | | 1.78 | 270.69 | | 0.14 | 238.85 | | 0.91 | 192.67 | | 0.22 |
| 61 | 44.47 | +0.12 | 223.30 | +1.23 | 1.76 | 269.89 | -0.80 | 0.13 | 239.49 | +0.64 | 0.90 | 191.55 | -1.12 | 0.23 |
| 62 | 44.59 | 0.12 | 224.52 | 1.22 | 1.75 | 269.09 | 0.80 | 0.13 | 240.12 | 0.63 | 0.98 | 190.42 | 1.13 | 0.23 |
| 63 | 44.71 | 0.12 | 225.73 | 1.21 | 1.73 | 268.28 | 0.81 | 0.13 | 240.74 | 0.62 | 0.87 | 189.29 | 1.13 | 0.24 |
| 64 | 44.83 | 0.12 | 226.94 | 1.20 | 1.72 | 267.45 | 0.83 | 0.13 | 241.35 | 0.61 | 0.86 | 188.14 | 1.15 | 0.25 |
| 65 | 44.95 | | 228.14 | | 1.70 | 266.61 | | 0.12 | 241.95 | | 0.84 | 186.99 | | 0.25 |
| 66 | 45.07 | +0.12 | 229.33 | +1.19 | 1.69 | 265.76 | -0.85 | 0.12 | 242.53 | +0.58 | 0.83 | 185.83 | -1.16 | 0.26 |
| 67 | 45.19 | 0.12 | 230.51 | 1.18 | 1.67 | 264.89 | 0.87 | 0.12 | 243.10 | 0.57 | 0.82 | 184.67 | 1.16 | 0.27 |
| 68 | 45.31 | 0.12 | 231.69 | 1.18 | 1.66 | 264.02 | 0.87 | 0.12 | 243.67 | 0.57 | 0.81 | 183.51 | 1.16 | 0.28 |
| 69 | 45.44 | 0.13 | 232.86 | 1.17 | 1.64 | 263.13 | 0.89 | 0.11 | 244.22 | 0.55 | 0.79 | 182.34 | 1.17 | 0.28 |
| 70 | 45.56 | 0.12 | 234.02 | 1.16 | 1.63 | 262.23 | 0.90 | 0.11 | 244.76 | 0.54 | 0.78 | 181.16 | 1.18 | 0.29 |
| 71 | 45.68 | +0.12 | 235.17 | +1.15 | 1.62 | 261.32 | -0.91 | 0.11 | 245.28 | +0.52 | 0.77 | 179.97 | -1.19 | 0.30 |
| 72 | 45.80 | 0.12 | 236.31 | 1.14 | 1.60 | 260.39 | 0.93 | 0.11 | 245.79 | 0.51 | 0.75 | 178.78 | 1.19 | 0.31 |
| 73 | 45.92 | 0.12 | 237.45 | 1.14 | 1.59 | 259.46 | 0.93 | 0.11 | 246.29 | 0.50 | 0.74 | 177.59 | 1.19 | 0.32 |
| 74 | 46.04 | 0.12 | 238.58 | 1.13 | 1.57 | 258.52 | 0.94 | 0.11 | 246.78 | 0.49 | 0.73 | 176.40 | 1.19 | 0.33 |
| 75 | 46.16 | | 239.70 | | 1.56 | 257.56 | | 0.11 | 247.26 | | 0.71 | 175.20 | | 0.34 |
| 76 | 46.29 | +0.13 | 240.81 | +1.11 | 1.55 | 256.59 | -0.97 | 0.11 | 247.73 | +0.47 | 0.70 | 173.99 | -1.21 | 0.34 |
| 77 | 46.41 | 0.12 | 241.91 | 1.10 | 1.53 | 255.61 | 0.98 | 0.11 | 248.18 | 0.45 | 0.69 | 172.78 | 1.21 | 0.35 |
| 78 | 46.53 | 0.12 | 243.00 | 1.09 | 1.52 | 254.62 | 0.99 | 0.11 | 248.62 | 0.44 | 0.68 | 171.56 | 1.22 | 0.36 |
| 79 | 46.65 | 0.12 | 244.09 | 1.09 | 1.50 | 253.62 | 1.00 | 0.11 | 249.05 | 0.43 | 0.66 | 170.34 | 1.22 | 0.37 |
| 80 | 46.77 | 0.12 | 245.17 | 1.08 | 1.49 | 252.61 | 1.01 | 0.11 | 249.47 | 0.42 | 0.65 | 169.11 | 1.23 | 0.38 |
| 81 | 46.89 | +0.12 | 246.24 | +1.07 | 1.48 | 251.59 | -1.02 | 0.11 | 249.87 | +0.40 | 0.64 | 167.88 | -1.23 | 0.39 |
| 82 | 47.01 | 0.12 | 247.30 | 1.06 | 1.46 | 250.56 | 1.03 | 0.11 | 250.26 | 0.39 | 0.63 | 166.65 | 1.23 | 0.40 |
| 83 | 47.13 | 0.12 | 248.35 | 1.05 | 1.45 | 249.51 | 1.05 | 0.11 | 250.64 | 0.38 | 0.62 | 165.41 | 1.24 | 0.41 |
| 84 | 47.24 | 0.11 | 249.38 | 1.03 | 1.43 | 248.45 | 1.06 | 0.11 | 251.01 | 0.37 | 0.61 | 164.17 | 1.24 | 0.42 |
| 85 | 47.36 | 0.12 | 250.41 | 1.03 | 1.42 | 247.38 | 1.07 | 0.11 | 251.37 | 0.36 | 0.60 | 162.92 | 1.25 | 0.43 |
| 86 | 47.48 | +0.12 | 251.43 | +1.02 | 1.40 | 246.30 | -1.08 | 0.12 | 251.71 | +0.34 | 0.58 | 161.67 | -1.25 | 0.43 |
| 87 | 47.59 | 0.11 | 252.44 | 1.01 | 1.39 | 245.21 | 1.09 | 0.12 | 252.04 | 0.33 | 0.57 | 160.42 | 1.25 | 0.44 |
| 88 | 47.71 | 0.12 | 253.44 | 1.00 | 1.38 | 244.12 | 1.09 | 0.12 | 252.36 | 0.32 | 0.56 | 159.16 | 1.26 | 0.45 |
| 89 | 47.82 | 0.11 | 254.43 | 0.99 | 1.36 | 243.01 | 1.11 | 0.12 | 252.67 | 0.31 | 0.55 | 157.90 | 1.26 | 0.46 |
| 90 | 47.94 | 0.12 | 255.41 | 0.98 | 1.35 | 241.89 | 1.12 | 0.12 | 252.96 | 0.29 | 0.54 | 156.63 | 1.27 | 0.47 |
| 91 | 48.05 | +0.11 | 256.38 | +0.97 | 1.33 | 240.76 | -1.13 | 0.12 | 253.24 | +0.28 | 0.53 | 155.36 | -1.27 | 0.48 |
| 92 | 48.16 | 0.11 | 257.34 | 0.96 | 1.32 | 239.62 | 1.14 | 0.12 | 253.50 | 0.26 | 0.52 | 154.09 | 1.27 | 0.49 |
| 93 | 48.27 | 0.11 | 258.29 | 0.95 | 1.30 | 238.48 | 1.14 | 0.13 | 253.75 | 0.25 | 0.51 | 152.82 | 1.27 | 0.50 |
| 94 | 48.38 | 0.11 | 259.23 | 0.94 | 1.29 | 237.33 | 1.15 | 0.13 | 253.98 | 0.23 | 0.50 | 151.55 | 1.27 | 0.51 |
| 95 | 48.49 | 0.11 | 260.16 | 0.93 | 1.28 | 236.16 | 1.17 | 0.13 | 254.21 | 0.23 | 0.49 | 150.27 | 1.28 | 0.52 |
| 96 | 48.59 | +0.10 | 261.08 | +0.92 | 1.26 | 234.98 | -1.18 | 0.13 | 254.43 | +0.22 | 0.48 | 148.99 | -1.28 | 0.54 |
| 97 | 48.70 | 0.11 | 261.99 | 0.91 | 1.25 | 233.79 | 1.19 | 0.13 | 254.63 | 0.20 | 0.47 | 147.71 | 1.28 | 0.55 |
| 98 | 48.80 | 0.10 | 262.88 | 0.89 | 1.23 | 232.60 | 1.19 | 0.13 | 254.82 | 0.19 | 0.46 | 146.42 | 1.29 | 0.56 |
| 99 | 48.91 | 0.11 | 263.76 | 0.88 | 1.22 | 231.39 | 1.21 | 0.14 | 254.99 | 0.17 | 0.45 | 145.13 | 1.29 | 0.57 |
| 100 | 49.01 | 0.10 | 264.63 | 0.87 | 1.20 | 230.18 | 1.21 | 0.14 | 255.15 | 0.16 | 0.44 | 143.84 | 1.29 | 0.58 |
| 101 | 49.11 | +0.10 | 265.49 | +0.86 | 1.19 | 228.96 | -1.22 | 0.14 | 255.30 | +0.15 | 0.43 | 142.55 | -1.29 | 0.59 |
| 102 | 49.21 | 0.10 | 266.34 | 0.85 | 1.17 | 227.73 | 1.23 | 0.15 | 255.43 | 0.13 | 0.42 | 141.26 | 1.29 | 0.60 |
| 103 | 49.31 | 0.10 | 267.18 | 0.84 | 1.16 | 226.50 | 1.23 | 0.15 | 255.55 | 0.12 | 0.41 | 139.97 | 1.29 | 0.61 |
| 104 | 49.40 | 0.09 | 268.00 | 0.82 | 1.14 | 225.25 | 1.25 | 0.16 | 255.66 | 0.11 | 0.40 | 138.67 | 1.30 | 0.62 |
| 105 | 49.50 | 0.10 | 268.81 | 0.81 | 1.13 | 224.00 | 1.26 | 0.16 | 255.75 | 0.09 | 0.39 | 137.37 | 1.30 | 0.63 |
| 106 | 49.59 | +0.09 | 269.61 | +0.80 | 1.11 | 222.73 | -1.26 | 0.16 | 255.83 | +0.08 | 0.39 | 136.07 | -1.30 | 0.65 |
| 107 | 49.68 | 0.09 | 270.41 | 0.80 | 1.10 | 221.46 | 1.27 | 0.17 | 255.90 | 0.07 | 0.38 | 134.77 | 1.30 | 0.66 |
| 108 | 49.77 | 0.09 | 271.19 | 0.78 | 1.09 | 220.18 | 1.28 | 0.17 | 255.95 | 0.05 | 0.37 | 133.47 | 1.30 | 0.67 |
| 109 | 49.85 | 0.08 | 271.96 | 0.77 | 1.07 | 218.88 | 1.30 | 0.18 | 255.99 | 0.04 | 0.36 | 132.17 | 1.30 | 0.68 |
| 110 | 49.94 | 0.08 | 272.71 | 0.75 | 1.06 | 217.58 | 1.30 | 0.18 | 256.02 | 0.03 | 0.35 | 130.87 | 1.30 | 0.69 |
| 111 | 50.02 | +0.08 | 273.45 | +0.74 | 1.05 | 216.28 | -1.30 | 0.18 | 256.04 | +0.02 | 0.34 | 129.57 | -1.30 | 0.70 |
| 112 | 50.11 | 0.09 | 274.18 | 0.73 | 1.03 | 214.97 | 1.31 | 0.19 | 256.04 | 0.00 | 0.33 | 128.26 | 1.31 | 0.71 |
| 113 | 50.19 | 0.08 | 274.90 | 0.72 | 1.02 | 213.65 | 1.32 | 0.19 | 256.03 | -0.01 | 0.33 | 126.96 | 1.30 | 0.73 |
| 114 | 50.26 | 0.07 | 275.60 | 0.70 | 1.00 | 212.33 | 1.32 | 0.20 | 256.00 | 0.03 | 0.32 | 125.66 | 1.30 | 0.74 |
| 115 | 50.34 | 0.08 | 276.29 | 0.69 | 0.99 | 211.01 | 1.34 | 0.20 | 256.03 | 0.04 | 0.31 | 124.36 | 1.30 | 0.75 |
| 116 | 50.41 | +0.07 | 276.97 | +0.68 | 0.98 | 209.65 | -1.34 | 0.21 | 255.96 | +0.05 | 0.30 | 123.05 | -1.31 | 0.76 |
| 117 | 50.48 | 0.07 | 277.63 | 0.66 | 0.97 | 208.31 | 1.34 | 0.21 | 255.91 | 0.07 | 0.29 | 121.75 | 1.30 | 0.77 |
| 118 | 50.55 | 0.07 | 278.28 | 0.65 | 0.96 | 206.96 | 1.35 | 0.22 | 255.84 | 0.08 | 0.28 | 120.45 | 1.30 | 0.78 |
| 119 | 50.62 | 0.07 | 278.92 | 0.64 | 0.94 | 205.60 | 1.36 | 0.22 | 255.76 | 0.09 | 0.28 | 119.15 | 1.30 | 0.80 |
| 120 | 50.69 | 0.07 | 279.55 | 0.63 | 0.93 | 204.23 | 1.37 | 0.23 | 255.67 | 0.11 | 0.27 | 117.85 | 1.30 | 0.81 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | (p.s.3) | (p.c.3) |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | " | | | | | | | |
| 60 | 16.26 | 9.11 | 1.80 | 0.89 | 1496 | 437 | 2222 | 583 | 127 | 137 | 71 |
| 61 | 16.29 | 9.00 | 1.79 | 0.88 | 1489 | 445 | 2233 | 580 | 125 | 137 | 70 |
| 62 | 16.32 | 8.88 | 1.79 | 0.87 | 1483 | 453 | 2243 | 577 | 123 | 137 | 70 |
| 63 | 16.35 | 8.76 | 1.78 | 0.85 | 1477 | 460 | 2254 | 574 | 121 | 137 | 70 |
| 64 | 16.38 | 8.64 | 1.78 | 0.84 | 1470 | 468 | 2264 | 570 | 119 | 137 | 69 |
| 65 | 16.41 | 8.52 | 1.78 | 0.83 | 1464 | 476 | 2275 | 567 | 117 | 136 | 69 |
| 66 | 16.43 | 8.40 | 1.77 | 0.81 | 1457 | 484 | 2285 | 564 | 115 | 136 | 68 |
| 67 | 16.45 | 8.28 | 1.77 | 0.80 | 1451 | 492 | 2295 | 560 | 113 | 136 | 68 |
| 68 | 16.47 | 8.16 | 1.76 | 0.79 | 1444 | 501 | 2306 | 557 | 112 | 136 | 67 |
| 69 | 16.49 | 8.04 | 1.75 | 0.78 | 1438 | 509 | 2316 | 553 | 110 | 136 | 67 |
| 70 | 16.50 | 7.91 | 1.75 | 0.76 | 1431 | 518 | 2326 | 550 | 108 | 136 | 66 |
| 71 | 16.51 | 7.79 | 1.74 | 0.75 | 1424 | 527 | 2336 | 546 | 107 | 136 | 66 |
| 72 | 16.52 | 7.67 | 1.74 | 0.74 | 1417 | 536 | 2346 | 543 | 105 | 136 | 66 |
| 73 | 16.53 | 7.55 | 1.73 | 0.73 | 1410 | 545 | 2356 | 540 | 103 | 135 | 65 |
| 74 | 16.54 | 7.43 | 1.72 | 0.71 | 1403 | 554 | 2365 | 536 | 102 | 135 | 65 |
| 75 | 16.54 | 7.31 | 1.72 | 0.70 | 1396 | 563 | 2375 | 533 | 100 | 135 | 64 |
| 76 | 16.54 | 7.19 | 1.71 | 0.69 | 1389 | 572 | 2384 | 530 | 99 | 135 | 64 |
| 77 | 16.54 | 7.07 | 1.70 | 0.68 | 1382 | 581 | 2394 | 526 | 98 | 135 | 64 |
| 78 | 16.54 | 6.95 | 1.70 | 0.67 | 1374 | 590 | 2403 | 523 | 96 | 135 | 63 |
| 79 | 16.53 | 6.83 | 1.69 | 0.65 | 1367 | 599 | 2413 | 519 | 95 | 135 | 63 |
| 80 | 16.53 | 6.71 | 1.68 | 0.64 | 1360 | 608 | 2422 | 516 | 94 | 135 | 62 |
| 81 | 16.52 | 6.60 | 1.67 | 0.63 | 1353 | 618 | 2431 | 512 | 93 | 135 | 62 |
| 82 | 16.52 | 6.48 | 1.66 | 0.62 | 1346 | 627 | 2440 | 508 | 92 | 134 | 61 |
| 83 | 16.51 | 6.36 | 1.65 | 0.61 | 1339 | 637 | 2449 | 505 | 91 | 134 | 61 |
| 84 | 16.50 | 6.24 | 1.64 | 0.60 | 1332 | 646 | 2457 | 501 | 89 | 134 | 60 |
| 85 | 16.48 | 6.13 | 1.63 | 0.59 | 1325 | 656 | 2466 | 498 | 88 | 134 | 60 |
| 86 | 16.46 | 6.01 | 1.62 | 0.58 | 1318 | 664 | 2475 | 494 | 87 | 134 | 59 |
| 87 | 16.44 | 5.89 | 1.61 | 0.57 | 1311 | 676 | 2483 | 490 | 86 | 134 | 59 |
| 88 | 16.42 | 5.78 | 1.60 | 0.56 | 1304 | 684 | 2492 | 487 | 85 | 134 | 58 |
| 89 | 16.40 | 5.66 | 1.59 | 0.55 | 1296 | 696 | 2500 | 483 | 84 | 133 | 58 |
| 90 | 16.38 | 5.55 | 1.58 | 0.54 | 1289 | 706 | 2509 | 480 | 84 | 133 | 57 |
| 91 | 16.35 | 5.44 | 1.57 | 0.53 | 1281 | 717 | 2517 | 476 | 83 | 133 | 57 |
| 92 | 16.33 | 5.32 | 1.56 | 0.52 | 1273 | 727 | 2525 | 472 | 82 | 133 | 56 |
| 93 | 16.30 | 5.21 | 1.55 | 0.51 | 1266 | 738 | 2534 | 469 | 82 | 132 | 56 |
| 94 | 16.27 | 5.09 | 1.54 | 0.50 | 1258 | 749 | 2542 | 465 | 81 | 132 | 55 |
| 95 | 16.24 | 4.98 | 1.53 | 0.50 | 1250 | 760 | 2550 | 461 | 81 | 132 | 55 |
| 96 | 16.20 | 4.87 | 1.52 | 0.49 | 1242 | 771 | 2558 | 457 | 80 | 131 | 54 |
| 97 | 16.17 | 4.76 | 1.51 | 0.48 | 1234 | 782 | 2566 | 454 | 80 | 131 | 54 |
| 98 | 16.13 | 4.64 | 1.50 | 0.48 | 1226 | 793 | 2574 | 450 | 80 | 131 | 53 |
| 99 | 16.09 | 4.54 | 1.49 | 0.47 | 1218 | 804 | 2582 | 446 | 79 | 130 | 53 |
| 100 | 16.05 | 4.44 | 1.48 | 0.46 | 1210 | 815 | 2590 | 442 | 79 | 130 | 52 |
| 101 | 16.01 | 4.33 | 1.46 | 0.46 | 1202 | 826 | 2598 | 438 | 79 | 130 | 51 |
| 102 | 15.96 | 4.23 | 1.45 | 0.45 | 1194 | 838 | 2605 | 435 | 79 | 129 | 51 |
| 103 | 15.92 | 4.13 | 1.44 | 0.44 | 1186 | 849 | 2613 | 431 | 78 | 129 | 50 |
| 104 | 15.87 | 4.02 | 1.43 | 0.44 | 1178 | 861 | 2620 | 428 | 78 | 129 | 50 |
| 105 | 15.83 | 3.92 | 1.42 | 0.43 | 1170 | 872 | 2627 | 424 | 78 | 128 | 49 |
| 106 | 15.78 | 3.82 | 1.41 | 0.43 | 1162 | 883 | 2634 | 420 | 78 | 128 | 49 |
| 107 | 15.73 | 3.72 | 1.39 | 0.42 | 1154 | 895 | 2641 | 416 | 78 | 128 | 48 |
| 108 | 15.68 | 3.62 | 1.38 | 0.42 | 1146 | 907 | 2647 | 413 | 78 | 127 | 48 |
| 109 | 15.62 | 3.52 | 1.37 | 0.41 | 1138 | 919 | 2654 | 409 | 78 | 127 | 47 |
| 110 | 15.56 | 3.43 | 1.36 | 0.41 | 1130 | 931 | 2661 | 405 | 78 | 126 | 46 |
| 111 | 15.51 | 3.34 | 1.34 | 0.40 | 1122 | 943 | 2668 | 402 | 78 | 126 | 46 |
| 112 | 15.45 | 3.24 | 1.33 | 0.40 | 1114 | 955 | 2674 | 398 | 79 | 125 | 45 |
| 113 | 15.39 | 3.15 | 1.32 | 0.40 | 1106 | 967 | 2681 | 394 | 79 | 125 | 45 |
| 114 | 15.33 | 3.06 | 1.31 | 0.39 | 1098 | 979 | 2687 | 391 | 79 | 124 | 44 |
| 115 | 15.27 | 2.97 | 1.29 | 0.39 | 1090 | 991 | 2693 | 387 | 80 | 124 | 44 |
| 116 | 15.21 | 2.88 | 1.28 | 0.38 | 1082 | 1003 | 2699 | 383 | 80 | 123 | 44 |
| 117 | 15.14 | 2.80 | 1.26 | 0.38 | 1074 | 1016 | 2705 | 380 | 80 | 122 | 43 |
| 118 | 15.08 | 2.71 | 1.25 | 0.38 | 1066 | 1028 | 2711 | 376 | 81 | 122 | 43 |
| 119 | 15.01 | 2.62 | 1.24 | 0.38 | 1059 | 1041 | 2716 | 373 | 81 | 121 | 42 |
| 120 | 14.94 | 2.54 | 1.23 | 0.37 | 1051 | 1053 | 2722 | 370 | 82 | 120 | 42 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | Sec.var. | (v.c.1) | Diff. | Sec.var. | (v.s.2) | Diff. | Sec.var. | (v.c.2) | Diff. | Sec.var. |
|------|---------|-------|---------|-------|----------|---------|-------|----------|---------|-------|----------|---------|-------|----------|
| | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 120 | 50.69 | | 279.55 | | 0.93 | 204.23 | | 0.23 | 255.56 | | 0.27 | 117.85 | | 0.81 |
| 121 | 50.76 | +0.07 | 280.16 | +0.61 | 0.92 | 202.86 | -1.37 | 0.24 | 255.44 | -0.12 | 0.26 | 116.55 | -1.30 | 0.82 |
| 122 | 50.82 | 0.06 | 280.76 | 0.60 | 0.90 | 201.48 | 1.38 | 0.24 | 255.31 | 0.13 | 0.26 | 115.25 | 1.30 | 0.84 |
| 123 | 50.88 | 0.06 | 281.35 | 0.59 | 0.89 | 200.10 | 1.38 | 0.25 | 255.16 | 0.15 | 0.25 | 113.95 | 1.30 | 0.85 |
| 124 | 50.93 | 0.05 | 281.92 | 0.57 | 0.88 | 198.71 | 1.39 | 0.25 | 254.00 | 0.16 | 0.25 | 112.65 | 1.30 | 0.87 |
| | | 0.05 | | 0.56 | | | 1.40 | | | 0.17 | | | 1.29 | |
| 125 | 50.98 | | 282.48 | | 0.87 | 197.31 | | 0.26 | 254.83 | | 0.24 | 111.36 | | 0.88 |
| 126 | 51.03 | +0.05 | 283.02 | +0.54 | 0.85 | 195.91 | -1.40 | 0.27 | 254.64 | -0.19 | 0.23 | 110.07 | -1.29 | 0.89 |
| 127 | 51.08 | 0.05 | 283.55 | 0.53 | 0.84 | 194.50 | 1.41 | 0.27 | 254.44 | 0.20 | 0.23 | 108.77 | 1.30 | 0.91 |
| 128 | 51.13 | 0.05 | 284.07 | 0.52 | 0.83 | 193.09 | 1.41 | 0.28 | 254.23 | 0.21 | 0.22 | 107.48 | 1.29 | 0.92 |
| 129 | 51.17 | 0.04 | 284.57 | 0.50 | 0.81 | 191.67 | 1.42 | 0.28 | 254.00 | 0.23 | 0.22 | 106.19 | 1.29 | 0.94 |
| | | 0.05 | | 0.49 | | | 1.42 | | | 0.24 | | | 1.28 | |
| 130 | 51.22 | | 285.06 | | 0.80 | 190.25 | | 0.29 | 253.76 | | 0.21 | 104.91 | | 0.95 |
| 131 | 51.26 | +0.04 | 285.53 | +0.47 | 0.79 | 188.82 | -1.43 | 0.30 | 253.51 | -0.25 | 0.21 | 103.63 | -1.28 | 0.96 |
| 132 | 51.30 | 0.04 | 285.99 | 0.46 | 0.73 | 187.39 | 1.43 | 0.31 | 253.24 | 0.27 | 0.20 | 102.35 | 1.28 | 0.98 |
| 133 | 51.34 | 0.04 | 286.44 | 0.45 | 0.76 | 185.95 | 1.44 | 0.32 | 252.96 | 0.28 | 0.20 | 101.07 | 1.28 | 0.99 |
| 134 | 51.37 | 0.03 | 286.87 | 0.43 | 0.75 | 184.51 | 1.44 | 0.33 | 252.67 | 0.29 | 0.19 | 99.79 | 1.28 | 1.01 |
| | | 0.03 | | 0.42 | | | 1.44 | | | 0.31 | | | 1.27 | |
| 135 | 51.40 | | 287.29 | | 0.74 | 183.07 | | 0.33 | 252.36 | | 0.19 | 98.52 | | 1.02 |
| 136 | 51.43 | +0.03 | 287.69 | +0.40 | 0.73 | 181.62 | -1.45 | 0.34 | 252.04 | -0.32 | 0.18 | 97.25 | -1.27 | 1.03 |
| 137 | 51.45 | 0.02 | 288.08 | 0.39 | 0.72 | 180.17 | 1.45 | 0.35 | 251.71 | 0.33 | 0.18 | 95.99 | 1.26 | 1.05 |
| 138 | 51.47 | 0.02 | 288.45 | 0.37 | 0.70 | 178.72 | 1.45 | 0.36 | 251.36 | 0.35 | 0.17 | 94.73 | 1.26 | 1.06 |
| 139 | 51.49 | 0.02 | 288.81 | 0.36 | 0.69 | 177.26 | 1.46 | 0.37 | 251.00 | 0.36 | 0.17 | 93.47 | 1.26 | 1.08 |
| | | 0.02 | | 0.35 | | | 1.46 | | | 0.37 | | | 1.26 | |
| 140 | 51.51 | | 289.16 | | 0.68 | 175.80 | | 0.38 | 250.63 | | 0.16 | 92.21 | | 1.09 |
| 141 | 51.52 | +0.01 | 289.49 | +0.33 | 0.67 | 174.33 | -1.47 | 0.39 | 250.24 | -0.39 | 0.16 | 90.96 | -1.25 | 1.10 |
| 142 | 51.53 | 0.01 | 289.80 | 0.31 | 0.66 | 172.86 | 1.47 | 0.40 | 249.84 | 0.40 | 0.15 | 89.71 | 1.25 | 1.12 |
| 143 | 51.54 | 0.01 | 290.10 | 0.30 | 0.65 | 171.39 | 1.47 | 0.41 | 249.43 | 0.41 | 0.15 | 88.46 | 1.25 | 1.13 |
| 144 | 51.55 | 0.01 | 290.39 | 0.29 | 0.63 | 169.92 | 1.47 | 0.42 | 249.01 | 0.42 | 0.14 | 87.22 | 1.24 | 1.15 |
| | | 0.00 | | 0.27 | | | 1.48 | | | 0.44 | | | 1.24 | |
| 145 | 51.55 | | 290.66 | | 0.62 | 168.44 | | 0.42 | 248.57 | | 0.14 | 85.98 | | 1.16 |
| 146 | 51.55 | 0.00 | 290.92 | +0.26 | 0.61 | 166.96 | -1.48 | 0.43 | 248.12 | -0.45 | 0.14 | 84.75 | -1.23 | 1.17 |
| 147 | 51.55 | 0.00 | 291.16 | 0.24 | 0.60 | 165.48 | 1.48 | 0.44 | 247.66 | 0.46 | 0.13 | 83.52 | 1.23 | 1.19 |
| 148 | 51.54 | -0.01 | 291.38 | 0.22 | 0.58 | 164.00 | 1.48 | 0.45 | 247.18 | 0.48 | 0.13 | 82.29 | 1.23 | 1.20 |
| 149 | 51.53 | 0.01 | 291.59 | 0.21 | 0.57 | 162.52 | 1.48 | 0.46 | 246.69 | 0.49 | 0.12 | 81.07 | 1.22 | 1.22 |
| | | 0.01 | | 0.20 | | | 1.49 | | | 0.50 | | | 1.22 | |
| 150 | 51.52 | | 291.79 | | 0.56 | 161.03 | | 0.47 | 246.19 | | 0.12 | 79.85 | | 1.23 |
| 151 | 51.50 | -0.02 | 291.97 | +0.18 | 0.55 | 159.54 | -1.49 | 0.48 | 245.68 | -0.51 | 0.12 | 78.64 | -1.21 | 1.24 |
| 152 | 51.49 | 0.01 | 292.13 | 0.16 | 0.54 | 158.05 | 1.49 | 0.49 | 245.15 | 0.53 | 0.12 | 77.44 | 1.20 | 1.26 |
| 153 | 51.47 | 0.02 | 292.28 | 0.15 | 0.53 | 156.56 | 1.49 | 0.50 | 244.61 | 0.54 | 0.11 | 76.24 | 1.20 | 1.27 |
| 154 | 51.45 | 0.02 | 292.42 | 0.14 | 0.52 | 155.07 | 1.49 | 0.51 | 244.06 | 0.55 | 0.11 | 75.04 | 1.20 | 1.29 |
| | | 0.02 | | 0.12 | | | 1.49 | | | 0.56 | | | 1.19 | |
| 155 | 51.43 | | 292.54 | | 0.51 | 153.58 | | 0.52 | 243.50 | | 0.11 | 73.85 | | 1.30 |
| 156 | 51.40 | -0.03 | 292.64 | +0.10 | 0.50 | 152.09 | -1.49 | 0.54 | 242.92 | -0.58 | 0.11 | 72.67 | -1.18 | 1.31 |
| 157 | 51.37 | 0.03 | 292.73 | 0.09 | 0.49 | 150.60 | 1.49 | 0.55 | 242.33 | 0.59 | 0.11 | 71.49 | 1.18 | 1.33 |
| 158 | 51.34 | 0.03 | 292.80 | 0.07 | 0.48 | 149.11 | 1.49 | 0.56 | 241.73 | 0.60 | 0.10 | 70.32 | 1.17 | 1.34 |
| 159 | 51.30 | 0.04 | 292.86 | 0.06 | 0.47 | 147.62 | 1.49 | 0.57 | 241.12 | 0.61 | 0.10 | 69.15 | 1.17 | 1.36 |
| | | 0.04 | | 0.04 | | | 1.50 | | | 0.63 | | | 1.16 | |
| 160 | 51.26 | | 292.90 | | 0.46 | 146.12 | | 0.58 | 240.49 | | 0.10 | 67.99 | | 1.37 |
| 161 | 51.22 | -0.04 | 292.93 | +0.03 | 0.45 | 144.63 | -1.49 | 0.59 | 239.85 | -0.64 | 0.10 | 66.84 | -1.15 | 1.39 |
| 162 | 51.17 | 0.05 | 292.94 | 0.01 | 0.44 | 143.13 | 1.50 | 0.60 | 239.20 | 0.65 | 0.10 | 65.69 | 1.15 | 1.40 |
| 163 | 51.12 | 0.05 | 292.94 | 0.00 | 0.43 | 141.63 | 1.50 | 0.61 | 238.54 | 0.66 | 0.10 | 64.55 | 1.14 | 1.42 |
| 164 | 51.07 | 0.05 | 292.92 | -0.02 | 0.42 | 140.14 | 1.49 | 0.62 | 237.87 | 0.67 | 0.10 | 63.41 | 1.14 | 1.43 |
| | | 0.06 | | 0.04 | | | 1.50 | | | 0.69 | | | 1.13 | |
| 165 | 51.01 | | 292.88 | | 0.41 | 138.64 | | 0.64 | 237.18 | | 0.09 | 62.28 | | 1.45 |
| 166 | 50.95 | -0.06 | 292.83 | -0.05 | 0.40 | 137.15 | -1.49 | 0.65 | 236.48 | -0.70 | 0.09 | 61.16 | -1.12 | 1.47 |
| 167 | 50.89 | 0.06 | 292.76 | 0.07 | 0.40 | 135.66 | 1.49 | 0.66 | 235.77 | 0.71 | 0.09 | 60.05 | 1.11 | 1.48 |
| 168 | 50.82 | 0.07 | 292.68 | 0.08 | 0.39 | 134.17 | 1.49 | 0.67 | 235.05 | 0.72 | 0.09 | 58.94 | 1.11 | 1.50 |
| 169 | 50.76 | 0.06 | 292.58 | 0.10 | 0.38 | 132.68 | 1.49 | 0.68 | 234.32 | 0.73 | 0.09 | 57.84 | 1.10 | 1.51 |
| | | 0.07 | | 0.11 | | | 1.48 | | | 0.74 | | | 1.09 | |
| 170 | 50.69 | | 292.47 | | 0.37 | 131.20 | | 0.69 | 233.58 | | 0.09 | 56.75 | | 1.53 |
| 171 | 50.62 | -0.07 | 292.34 | -0.13 | 0.36 | 129.71 | -1.49 | 0.70 | 232.82 | -0.76 | 0.09 | 55.67 | -1.08 | 1.54 |
| 172 | 50.54 | 0.08 | 292.19 | 0.15 | 0.35 | 128.23 | 1.48 | 0.71 | 232.05 | 0.77 | 0.09 | 54.59 | 1.08 | 1.56 |
| 173 | 50.46 | 0.08 | 292.03 | 0.16 | 0.35 | 126.75 | 1.48 | 0.73 | 231.27 | 0.78 | 0.10 | 53.52 | 1.07 | 1.57 |
| 174 | 50.37 | 0.09 | 291.86 | 0.17 | 0.34 | 125.27 | 1.48 | 0.74 | 230.48 | 0.79 | 0.10 | 52.45 | 1.07 | 1.59 |
| | | 0.09 | | 0.19 | | | 1.48 | | | 0.80 | | | 1.05 | |
| 175 | 50.28 | | 291.67 | | 0.33 | 123.79 | | 0.75 | 229.68 | | 0.10 | 51.40 | | 1.60 |
| 176 | 50.19 | -0.09 | 291.46 | -0.21 | 0.32 | 122.32 | -1.47 | 0.76 | 228.87 | -0.81 | 0.10 | 50.35 | -1.05 | 1.61 |
| 177 | 50.10 | 0.09 | 291.24 | 0.22 | 0.31 | 120.85 | 1.47 | 0.77 | 228.05 | 0.82 | 0.10 | 49.31 | 1.04 | 1.63 |
| 178 | 50.01 | 0.09 | 291.01 | 0.23 | 0.31 | 119.38 | 1.47 | 0.79 | 227.22 | 0.83 | 0.11 | 48.28 | 1.03 | 1.64 |
| 179 | 49.91 | 0.10 | 290.76 | 0.25 | 0.30 | 118.91 | 1.47 | 0.80 | 226.37 | 0.85 | 0.11 | 47.26 | 1.02 | 1.66 |
| | | 0.10 | | 0.27 | | | 1.46 | | | 0.86 | | | 1.01 | |
| 180 | 49.81 | | 290.49 | | 0.29 | 116.45 | | 0.81 | 225.51 | | 0.11 | 46.25 | | 1.67 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | (p.s.3) | (p.c.3) |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | " | | | | | | | |
| 120 | 14.94 | 2.54 | 1.23 | 0.37 | 1051 | 1053 | 2722 | 370 | 82 | 120 | 42 |
| 121 | 14.87 | 2.46 | 1.22 | 0.37 | 1043 | 1065 | 2727 | 366 | 82 | 120 | 42 |
| 122 | 14.80 | 2.38 | 1.20 | 0.37 | 1035 | 1078 | 2732 | 362 | 83 | 120 | 41 |
| 123 | 14.73 | 2.30 | 1.19 | 0.37 | 1028 | 1090 | 2737 | 359 | 83 | 119 | 41 |
| 124 | 14.66 | 2.22 | 1.18 | 0.37 | 1020 | 1103 | 2742 | 355 | 84 | 119 | 40 |
| 125 | 14.59 | 2.15 | 1.17 | 0.37 | 1012 | 1116 | 2747 | 352 | 85 | 119 | 40 |
| 126 | 14.52 | 2.07 | 1.15 | 0.37 | 1004 | 1129 | 2751 | 348 | 85 | 118 | 39 |
| 127 | 14.44 | 2.00 | 1.14 | 0.37 | 996 | 1141 | 2756 | 344 | 86 | 118 | 39 |
| 128 | 14.37 | 1.93 | 1.13 | 0.37 | 989 | 1154 | 2760 | 341 | 87 | 117 | 38 |
| 129 | 14.29 | 1.86 | 1.11 | 0.37 | 981 | 1167 | 2765 | 337 | 88 | 117 | 38 |
| 130 | 14.21 | 1.79 | 1.10 | 0.37 | 973 | 1180 | 2769 | 334 | 89 | 116 | 37 |
| 131 | 14.13 | 1.72 | 1.09 | 0.37 | 965 | 1193 | 2773 | 330 | 90 | 115 | 37 |
| 132 | 14.06 | 1.66 | 1.07 | 0.37 | 958 | 1206 | 2777 | 327 | 91 | 115 | 36 |
| 133 | 13.98 | 1.59 | 1.06 | 0.37 | 950 | 1219 | 2781 | 324 | 92 | 114 | 36 |
| 134 | 13.90 | 1.53 | 1.05 | 0.38 | 942 | 1232 | 2785 | 321 | 93 | 113 | 35 |
| 135 | 13.82 | 1.47 | 1.04 | 0.38 | 935 | 1245 | 2789 | 318 | 94 | 112 | 35 |
| 136 | 13.74 | 1.41 | 1.02 | 0.38 | 927 | 1258 | 2793 | 314 | 95 | 112 | 35 |
| 137 | 13.66 | 1.35 | 1.01 | 0.38 | 920 | 1271 | 2796 | 311 | 96 | 111 | 34 |
| 138 | 13.57 | 1.30 | 1.00 | 0.39 | 912 | 1284 | 2800 | 308 | 97 | 110 | 34 |
| 139 | 13.49 | 1.24 | 0.99 | 0.39 | 905 | 1298 | 2803 | 305 | 99 | 109 | 33 |
| 140 | 13.40 | 1.19 | 0.97 | 0.39 | 897 | 1311 | 2806 | 302 | 100 | 108 | 33 |
| 141 | 13.32 | 1.14 | 0.96 | 0.40 | 889 | 1325 | 2809 | 298 | 101 | 108 | 33 |
| 142 | 13.24 | 1.09 | 0.95 | 0.40 | 882 | 1338 | 2811 | 295 | 103 | 107 | 32 |
| 143 | 13.15 | 1.04 | 0.94 | 0.41 | 875 | 1352 | 2814 | 292 | 104 | 106 | 32 |
| 144 | 13.07 | 0.99 | 0.93 | 0.41 | 867 | 1366 | 2816 | 289 | 106 | 105 | 32 |
| 145 | 12.98 | 0.94 | 0.91 | 0.41 | 859 | 1379 | 2819 | 286 | 107 | 105 | 31 |
| 146 | 12.90 | 0.90 | 0.90 | 0.42 | 852 | 1393 | 2821 | 282 | 109 | 104 | 31 |
| 147 | 12.81 | 0.86 | 0.89 | 0.42 | 844 | 1407 | 2823 | 279 | 110 | 103 | 31 |
| 148 | 12.73 | 0.82 | 0.88 | 0.43 | 837 | 1420 | 2825 | 276 | 112 | 102 | 30 |
| 149 | 12.64 | 0.78 | 0.87 | 0.44 | 829 | 1433 | 2826 | 273 | 113 | 101 | 30 |
| 150 | 12.55 | 0.74 | 0.86 | 0.44 | 822 | 1447 | 2828 | 270 | 115 | 100 | 30 |
| 151 | 12.46 | 0.70 | 0.85 | 0.45 | 815 | 1461 | 2829 | 267 | 117 | 99 | 30 |
| 152 | 12.38 | 0.67 | 0.84 | 0.46 | 807 | 1474 | 2831 | 264 | 118 | 98 | 29 |
| 153 | 12.29 | 0.64 | 0.83 | 0.46 | 800 | 1488 | 2832 | 261 | 120 | 97 | 29 |
| 154 | 12.20 | 0.61 | 0.82 | 0.47 | 793 | 1502 | 2834 | 258 | 122 | 96 | 29 |
| 155 | 12.11 | 0.58 | 0.81 | 0.48 | 786 | 1515 | 2835 | 256 | 124 | 95 | 28 |
| 156 | 12.03 | 0.55 | 0.80 | 0.49 | 778 | 1529 | 2836 | 253 | 126 | 94 | 28 |
| 157 | 11.94 | 0.52 | 0.79 | 0.49 | 771 | 1543 | 2837 | 250 | 128 | 93 | 28 |
| 158 | 11.86 | 0.50 | 0.78 | 0.50 | 764 | 1556 | 2837 | 247 | 129 | 92 | 28 |
| 159 | 11.77 | 0.47 | 0.77 | 0.51 | 756 | 1569 | 2838 | 244 | 131 | 91 | 27 |
| 160 | 11.68 | 0.45 | 0.76 | 0.52 | 749 | 1583 | 2838 | 242 | 133 | 90 | 27 |
| 161 | 11.59 | 0.43 | 0.75 | 0.52 | 742 | 1597 | 2838 | 239 | 135 | 90 | 27 |
| 162 | 11.51 | 0.41 | 0.74 | 0.53 | 735 | 1611 | 2838 | 236 | 137 | 89 | 27 |
| 163 | 11.42 | 0.40 | 0.73 | 0.54 | 728 | 1624 | 2837 | 233 | 139 | 88 | 27 |
| 164 | 11.33 | 0.38 | 0.72 | 0.55 | 721 | 1638 | 2837 | 230 | 141 | 87 | 27 |
| 165 | 11.24 | 0.37 | 0.71 | 0.56 | 714 | 1652 | 2836 | 228 | 143 | 86 | 27 |
| 166 | 11.16 | 0.36 | 0.71 | 0.57 | 707 | 1665 | 2835 | 225 | 145 | 85 | 27 |
| 167 | 11.07 | 0.35 | 0.70 | 0.58 | 700 | 1679 | 2834 | 222 | 147 | 84 | 27 |
| 168 | 10.99 | 0.34 | 0.69 | 0.59 | 693 | 1693 | 2833 | 219 | 149 | 83 | 27 |
| 169 | 10.91 | 0.33 | 0.68 | 0.60 | 686 | 1706 | 2833 | 216 | 151 | 82 | 27 |
| 170 | 10.82 | 0.32 | 0.68 | 0.61 | 679 | 1720 | 2831 | 214 | 153 | 82 | 27 |
| 171 | 10.74 | 0.32 | 0.67 | 0.62 | 672 | 1734 | 2829 | 211 | 156 | 81 | 27 |
| 172 | 10.66 | 0.31 | 0.66 | 0.63 | 665 | 1747 | 2828 | 208 | 158 | 80 | 27 |
| 173 | 10.57 | 0.31 | 0.66 | 0.64 | 659 | 1761 | 2826 | 206 | 160 | 79 | 27 |
| 174 | 10.49 | 0.31 | 0.65 | 0.65 | 652 | 1775 | 2824 | 204 | 162 | 78 | 28 |
| 175 | 10.41 | 0.31 | 0.65 | 0.66 | 645 | 1788 | 2822 | 202 | 165 | 77 | 28 |
| 176 | 10.33 | 0.31 | 0.64 | 0.67 | 638 | 1802 | 2820 | 199 | 167 | 76 | 28 |
| 177 | 10.25 | 0.31 | 0.63 | 0.68 | 632 | 1815 | 2817 | 197 | 170 | 75 | 28 |
| 178 | 10.17 | 0.32 | 0.63 | 0.69 | 625 | 1829 | 2815 | 195 | 173 | 74 | 28 |
| 179 | 10.09 | 0.32 | 0.63 | 0.70 | 618 | 1842 | 2812 | 192 | 175 | 73 | 28 |
| 180 | 10.01 | 0.33 | 0.62 | 0.71 | 611 | 1855 | 2810 | 190 | 178 | 72 | 28 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | Sec.var. | (v.c.1) | Diff. | Sec.var. | (v.s.2) | Diff. | Sec.var. | (v.c.2) | Diff. | Sec.var. |
|------|---------|-------|---------|-------|----------|---------|-------|----------|---------|-------|----------|---------|-------|----------|
| | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 180 | 49.81 | | 290.49 | | 0.29 | 116.45 | | 0.81 | 225.51 | | 0.11 | 46.25 | | 1.67 |
| 181 | 49.71 | -0.10 | 290.20 | -0.29 | 0.28 | 114.99 | -1.46 | 0.82 | 224.64 | -0.87 | 0.11 | 45.24 | -1.01 | 1.68 |
| 182 | 49.60 | 0.11 | 289.90 | 0.30 | 0.28 | 113.53 | 1.46 | 0.84 | 223.77 | 0.87 | 0.11 | 44.24 | 1.00 | 1.70 |
| 183 | 49.49 | 0.11 | 289.59 | 0.31 | 0.27 | 112.08 | 1.45 | 0.85 | 222.89 | 0.88 | 0.12 | 43.25 | 0.99 | 1.71 |
| 184 | 49.37 | 0.12 | 289.26 | 0.33 | 0.26 | 110.63 | 1.45 | 0.86 | 221.99 | 0.90 | 0.12 | 42.28 | 0.97 | 1.73 |
| | | 0.12 | | 0.35 | | | 1.45 | | | 0.91 | | | 0.97 | |
| 185 | 49.25 | | 288.91 | | 0.26 | 109.18 | | 0.88 | 221.08 | | 0.12 | 41.31 | | 1.74 |
| 186 | 49.13 | -0.12 | 288.55 | -0.36 | 0.25 | 107.74 | -1.44 | 0.89 | 220.16 | -0.92 | 0.12 | 40.35 | -0.96 | 1.75 |
| 187 | 49.00 | 0.13 | 288.17 | 0.38 | 0.24 | 106.30 | 1.44 | 0.90 | 219.23 | 0.93 | 0.12 | 39.40 | 0.95 | 1.77 |
| 188 | 48.87 | 0.13 | 287.78 | 0.39 | 0.24 | 104.87 | 1.43 | 0.91 | 218.29 | 0.94 | 0.13 | 38.46 | 0.94 | 1.78 |
| 189 | 48.74 | 0.13 | 287.37 | 0.41 | 0.23 | 103.44 | 1.43 | 0.93 | 217.34 | 0.95 | 0.13 | 37.53 | 0.93 | 1.80 |
| | | 0.13 | | 0.42 | | | 1.42 | | | 0.96 | | | 0.92 | |
| 190 | 48.61 | | 286.95 | | 0.22 | 102.02 | | 0.94 | 216.38 | | 0.13 | 36.61 | | 1.81 |
| 191 | 48.47 | -0.14 | 286.51 | -0.44 | 0.21 | 100.60 | -1.42 | 0.95 | 215.41 | -0.97 | 0.13 | 35.70 | -0.91 | 1.82 |
| 192 | 48.33 | 0.14 | 286.05 | 0.46 | 0.21 | 99.19 | 1.41 | 0.97 | 214.44 | 0.97 | 0.14 | 34.80 | 0.90 | 1.84 |
| 193 | 48.19 | 0.14 | 285.58 | 0.47 | 0.20 | 97.78 | 1.41 | 0.98 | 213.46 | 0.98 | 0.14 | 33.91 | 0.89 | 1.85 |
| 194 | 48.05 | 0.14 | 285.10 | 0.48 | 0.20 | 96.38 | 1.40 | 1.00 | 212.46 | 1.00 | 0.14 | 33.02 | 0.89 | 1.87 |
| | | 0.15 | | 0.50 | | | 1.40 | | | 1.01 | | | 0.87 | |
| 195 | 47.90 | | 284.60 | | 0.19 | 94.98 | | 1.01 | 211.45 | | 0.15 | 32.15 | | 1.88 |
| 196 | 47.75 | -0.15 | 284.09 | -0.51 | 0.19 | 93.59 | -1.39 | 1.02 | 210.43 | -1.02 | 0.15 | 31.29 | -0.86 | 1.90 |
| 197 | 47.60 | 0.15 | 283.56 | 0.53 | 0.18 | 92.20 | 1.39 | 1.04 | 209.40 | 1.03 | 0.15 | 30.44 | 0.85 | 1.91 |
| 198 | 47.44 | 0.16 | 283.01 | 0.55 | 0.18 | 90.82 | 1.38 | 1.05 | 208.37 | 1.03 | 0.15 | 29.60 | 0.84 | 1.93 |
| 199 | 47.28 | 0.16 | 282.45 | 0.56 | 0.17 | 89.44 | 1.38 | 1.07 | 207.33 | 1.04 | 0.16 | 28.77 | 0.83 | 1.94 |
| | | 0.16 | | 0.57 | | | 1.37 | | | 1.05 | | | 0.82 | |
| 200 | 47.12 | | 281.88 | | 0.17 | 88.07 | | 1.08 | 206.28 | | 0.16 | 27.95 | | 1.96 |
| 201 | 46.95 | -0.17 | 281.29 | -0.59 | 0.17 | 86.71 | -1.36 | 1.09 | 205.22 | -1.06 | 0.17 | 27.14 | -0.81 | 1.97 |
| 202 | 46.78 | 0.17 | 280.68 | 0.61 | 0.16 | 85.36 | 1.35 | 1.11 | 204.15 | 1.07 | 0.17 | 26.34 | 0.80 | 1.99 |
| 203 | 46.61 | 0.17 | 280.06 | 0.62 | 0.16 | 84.01 | 1.35 | 1.12 | 203.07 | 1.08 | 0.18 | 25.55 | 0.79 | 2.00 |
| 204 | 46.44 | 0.17 | 279.43 | 0.63 | 0.16 | 82.66 | 1.35 | 1.14 | 201.98 | 1.09 | 0.18 | 24.78 | 0.77 | 2.01 |
| | | 0.18 | | 0.65 | | | 1.34 | | | 1.09 | | | 0.77 | |
| 205 | 46.26 | | 278.78 | | 0.15 | 81.32 | | 1.15 | 200.89 | | 0.19 | 24.01 | | 2.03 |
| 206 | 46.08 | -0.18 | 278.12 | -0.66 | 0.15 | 79.99 | -1.33 | 1.16 | 199.79 | -1.10 | 0.20 | 23.26 | -0.75 | 2.04 |
| 207 | 45.90 | 0.18 | 277.44 | 0.68 | 0.15 | 78.67 | 1.32 | 1.18 | 198.68 | 1.11 | 0.20 | 22.52 | 0.74 | 2.05 |
| 208 | 45.71 | 0.19 | 276.75 | 0.69 | 0.14 | 77.36 | 1.31 | 1.19 | 197.56 | 1.12 | 0.21 | 21.79 | 0.73 | 2.06 |
| 209 | 45.52 | 0.19 | 276.04 | 0.71 | 0.14 | 76.05 | 1.31 | 1.21 | 196.43 | 1.13 | 0.21 | 21.07 | 0.72 | 2.08 |
| | | 0.19 | | 0.72 | | | 1.30 | | | 1.13 | | | 0.71 | |
| 210 | 45.33 | | 275.32 | | 0.14 | 74.75 | | 1.22 | 195.30 | | 0.22 | 20.36 | | 2.09 |
| 211 | 45.14 | -0.19 | 274.58 | -0.74 | 0.13 | 73.46 | -1.29 | 1.23 | 194.16 | -1.14 | 0.23 | 19.66 | -0.70 | 2.10 |
| 212 | 44.94 | 0.20 | 273.83 | 0.75 | 0.13 | 72.18 | 1.28 | 1.25 | 193.01 | 1.15 | 0.23 | 18.97 | 0.69 | 2.12 |
| 213 | 44.74 | 0.20 | 273.06 | 0.77 | 0.13 | 70.91 | 1.27 | 1.26 | 191.85 | 1.16 | 0.24 | 18.30 | 0.67 | 2.13 |
| 214 | 44.54 | 0.20 | 272.28 | 0.78 | 0.13 | 69.64 | 1.27 | 1.28 | 190.69 | 1.16 | 0.25 | 17.64 | 0.66 | 2.14 |
| | | 0.20 | | 0.79 | | | 1.27 | | | 1.17 | | | 0.64 | |
| 215 | 44.34 | | 271.49 | | 0.12 | 68.37 | | 1.29 | 189.52 | | 0.26 | 17.00 | | 2.16 |
| 216 | 44.13 | -0.21 | 270.68 | -0.81 | 0.12 | 67.12 | -1.25 | 1.31 | 188.34 | -1.18 | 0.27 | 16.37 | -0.63 | 2.17 |
| 217 | 43.92 | 0.21 | 269.86 | 0.82 | 0.12 | 65.88 | 1.24 | 1.32 | 187.15 | 1.19 | 0.28 | 15.74 | 0.63 | 2.18 |
| 218 | 43.71 | 0.21 | 269.03 | 0.83 | 0.12 | 64.65 | 1.23 | 1.34 | 185.96 | 1.19 | 0.28 | 15.13 | 0.61 | 2.19 |
| 219 | 43.50 | 0.21 | 268.18 | 0.85 | 0.12 | 63.42 | 1.23 | 1.35 | 184.76 | 1.20 | 0.29 | 14.53 | 0.60 | 2.21 |
| | | 0.22 | | 0.86 | | | 1.22 | | | 1.20 | | | 0.58 | |
| 220 | 43.28 | | 267.32 | | 0.11 | 62.20 | | 1.37 | 183.56 | | 0.29 | 13.95 | | 2.22 |
| 221 | 43.06 | -0.22 | 266.44 | -0.88 | 0.11 | 60.99 | -1.21 | 1.38 | 182.35 | -1.21 | 0.30 | 13.38 | -0.57 | 2.23 |
| 222 | 42.84 | 0.22 | 265.55 | 0.89 | 0.11 | 59.80 | 1.19 | 1.40 | 181.14 | 1.21 | 0.31 | 12.82 | 0.56 | 2.25 |
| 223 | 42.62 | 0.22 | 264.65 | 0.90 | 0.11 | 58.61 | 1.19 | 1.41 | 179.92 | 1.22 | 0.32 | 12.27 | 0.55 | 2.26 |
| 224 | 42.40 | 0.22 | 263.73 | 0.92 | 0.11 | 57.43 | 1.18 | 1.43 | 178.69 | 1.23 | 0.33 | 11.73 | 0.54 | 2.27 |
| | | 0.23 | | 0.93 | | | 1.17 | | | 1.23 | | | 0.52 | |
| 225 | 42.17 | | 262.80 | | 0.11 | 56.26 | | 1.44 | 177.46 | | 0.34 | 11.21 | | 2.29 |
| 226 | 41.94 | -0.23 | 261.85 | -0.95 | 0.11 | 55.10 | -1.16 | 1.45 | 176.22 | -1.24 | 0.34 | 10.70 | -0.51 | 2.30 |
| 227 | 41.71 | 0.23 | 260.89 | 0.96 | 0.11 | 53.96 | 1.14 | 1.47 | 174.97 | 1.25 | 0.35 | 10.20 | 0.50 | 2.31 |
| 228 | 41.48 | 0.23 | 259.92 | 0.97 | 0.11 | 52.82 | 1.14 | 1.48 | 173.72 | 1.25 | 0.36 | 9.72 | 0.48 | 2.32 |
| 229 | 41.24 | 0.24 | 258.94 | 0.98 | 0.11 | 51.69 | 1.13 | 1.50 | 172.46 | 1.26 | 0.37 | 9.25 | 0.47 | 2.34 |
| | | 0.24 | | 1.00 | | | 1.12 | | | 1.27 | | | 0.46 | |
| 230 | 41.00 | | 257.94 | | 0.11 | 50.57 | | 1.51 | 171.19 | | 0.38 | 8.79 | | 2.35 |
| 231 | 40.76 | -0.24 | 256.93 | -1.01 | 0.11 | 49.46 | -1.11 | 1.52 | 169.92 | -1.27 | 0.39 | 8.35 | -0.44 | 2.36 |
| 232 | 40.52 | 0.24 | 255.92 | 1.01 | 0.11 | 48.37 | 1.09 | 1.54 | 168.65 | 1.27 | 0.40 | 7.92 | 0.43 | 2.37 |
| 233 | 40.28 | 0.24 | 254.89 | 1.03 | 0.11 | 47.28 | 1.09 | 1.55 | 167.37 | 1.28 | 0.41 | 7.50 | 0.42 | 2.38 |
| 234 | 40.03 | 0.25 | 253.85 | 1.04 | 0.11 | 46.20 | 1.08 | 1.57 | 166.09 | 1.28 | 0.42 | 7.09 | 0.41 | 2.39 |
| | | 0.25 | | 1.06 | | | 1.07 | | | 1.29 | | | 0.39 | |
| 235 | 39.78 | | 252.79 | | 0.11 | 45.13 | | 1.58 | 164.80 | | 0.42 | 6.70 | | 2.41 |
| 236 | 39.53 | -0.25 | 251.72 | -1.07 | 0.11 | 44.08 | -1.05 | 1.60 | 163.51 | -1.29 | 0.43 | 6.32 | -0.38 | 2.42 |
| 237 | 39.28 | 0.25 | 250.64 | 1.08 | 0.12 | 43.05 | 1.03 | 1.61 | 162.22 | 1.29 | 0.44 | 5.96 | 0.36 | 2.43 |
| 238 | 39.03 | 0.25 | 249.55 | 1.09 | 0.12 | 42.02 | 1.03 | 1.63 | 160.92 | 1.30 | 0.45 | 5.61 | 0.35 | 2.44 |
| 239 | 38.77 | 0.26 | 248.45 | 1.10 | 0.12 | 41.00 | 1.02 | 1.64 | 159.62 | 1.30 | 0.46 | 5.27 | 0.34 | 2.45 |
| | | 0.26 | | 1.12 | | | 1.02 | | | 1.31 | | | 0.32 | |
| 240 | 38.51 | | 247.33 | | 0.12 | 39.98 | | 1.66 | 158.31 | | 0.47 | 4.95 | | 2.46 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | (p.s.3) | (p.c.3) |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | " | | | | | | | |
| 180 | 10.01 | 0.33 | 0.62 | 0.71 | 611 | 1855 | 2810 | 190 | 178 | 72 | 28 |
| 181 | 9.93 | 0.34 | 0.62 | 0.72 | 604 | 1869 | 2807 | 188 | 180 | 71 | 28 |
| 182 | 9.85 | 0.35 | 0.61 | 0.73 | 598 | 1882 | 2804 | 186 | 183 | 70 | 28 |
| 183 | 9.78 | 0.36 | 0.61 | 0.75 | 591 | 1895 | 2801 | 184 | 185 | 69 | 28 |
| 184 | 9.70 | 0.37 | 0.61 | 0.76 | 584 | 1908 | 2798 | 182 | 188 | 68 | 28 |
| 185 | 9.63 | 0.38 | 0.60 | 0.77 | 577 | 1921 | 2794 | 180 | 190 | 67 | 29 |
| 186 | 9.55 | 0.39 | 0.60 | 0.78 | 571 | 1935 | 2791 | 178 | 192 | 66 | 29 |
| 187 | 9.47 | 0.41 | 0.60 | 0.79 | 564 | 1948 | 2787 | 176 | 195 | 65 | 29 |
| 188 | 9.40 | 0.42 | 0.60 | 0.80 | 557 | 1961 | 2783 | 174 | 197 | 64 | 29 |
| 189 | 9.33 | 0.44 | 0.59 | 0.81 | 551 | 1974 | 2779 | 172 | 200 | 63 | 30 |
| 190 | 9.26 | 0.45 | 0.59 | 0.83 | 544 | 1987 | 2775 | 171 | 202 | 62 | 30 |
| 191 | 9.19 | 0.47 | 0.59 | 0.84 | 538 | 2000 | 2771 | 169 | 205 | 61 | 30 |
| 192 | 9.12 | 0.50 | 0.59 | 0.85 | 531 | 2013 | 2766 | 167 | 207 | 60 | 30 |
| 193 | 9.05 | 0.52 | 0.59 | 0.86 | 525 | 2025 | 2762 | 165 | 210 | 59 | 31 |
| 194 | 8.98 | 0.54 | 0.59 | 0.87 | 519 | 2038 | 2757 | 163 | 212 | 58 | 31 |
| 195 | 8.91 | 0.56 | 0.59 | 0.88 | 512 | 2051 | 2752 | 161 | 215 | 57 | 32 |
| 196 | 8.84 | 0.59 | 0.59 | 0.90 | 506 | 2064 | 2747 | 159 | 218 | 56 | 32 |
| 197 | 8.78 | 0.61 | 0.59 | 0.91 | 500 | 2076 | 2742 | 157 | 220 | 55 | 32 |
| 198 | 8.71 | 0.64 | 0.59 | 0.92 | 494 | 2089 | 2737 | 156 | 223 | 54 | 33 |
| 199 | 8.65 | 0.66 | 0.59 | 0.93 | 488 | 2101 | 2731 | 154 | 225 | 53 | 33 |
| 200 | 8.59 | 0.69 | 0.59 | 0.94 | 482 | 2113 | 2726 | 152 | 228 | 52 | 34 |
| 201 | 8.53 | 0.72 | 0.59 | 0.95 | 476 | 2126 | 2720 | 151 | 231 | 51 | 34 |
| 202 | 8.47 | 0.75 | 0.59 | 0.97 | 470 | 2138 | 2714 | 150 | 233 | 50 | 35 |
| 203 | 8.41 | 0.78 | 0.59 | 0.98 | 464 | 2150 | 2708 | 148 | 236 | 49 | 35 |
| 204 | 8.36 | 0.81 | 0.60 | 0.99 | 459 | 2162 | 2702 | 147 | 239 | 48 | 36 |
| 205 | 8.30 | 0.84 | 0.60 | 1.00 | 453 | 2174 | 2695 | 146 | 242 | 47 | 36 |
| 206 | 8.25 | 0.87 | 0.60 | 1.01 | 447 | 2186 | 2689 | 144 | 244 | 47 | 37 |
| 207 | 8.19 | 0.90 | 0.61 | 1.02 | 441 | 2198 | 2682 | 142 | 247 | 46 | 37 |
| 208 | 8.14 | 0.93 | 0.61 | 1.03 | 436 | 2210 | 2675 | 141 | 250 | 45 | 38 |
| 209 | 8.08 | 0.96 | 0.61 | 1.04 | 430 | 2222 | 2668 | 140 | 253 | 44 | 39 |
| 210 | 8.03 | 1.00 | 0.62 | 1.05 | 425 | 2234 | 2661 | 138 | 256 | 43 | 39 |
| 211 | 7.98 | 1.03 | 0.62 | 1.06 | 419 | 2245 | 2654 | 137 | 259 | 43 | 40 |
| 212 | 7.93 | 1.06 | 0.62 | 1.07 | 413 | 2257 | 2646 | 135 | 262 | 42 | 41 |
| 213 | 7.88 | 1.10 | 0.63 | 1.08 | 408 | 2268 | 2639 | 134 | 265 | 41 | 42 |
| 214 | 7.84 | 1.13 | 0.63 | 1.09 | 402 | 2280 | 2631 | 133 | 268 | 40 | 43 |
| 215 | 7.79 | 1.17 | 0.64 | 1.10 | 397 | 2291 | 2624 | 132 | 271 | 39 | 43 |
| 216 | 7.74 | 1.21 | 0.64 | 1.11 | 392 | 2302 | 2616 | 130 | 274 | 39 | 44 |
| 217 | 7.70 | 1.24 | 0.65 | 1.12 | 386 | 2313 | 2608 | 129 | 277 | 38 | 44 |
| 218 | 7.65 | 1.28 | 0.66 | 1.13 | 380 | 2324 | 2600 | 128 | 280 | 37 | 45 |
| 219 | 7.61 | 1.32 | 0.66 | 1.14 | 375 | 2335 | 2592 | 127 | 283 | 36 | 46 |
| 220 | 7.57 | 1.35 | 0.67 | 1.15 | 370 | 2346 | 2584 | 126 | 286 | 35 | 46 |
| 221 | 7.53 | 1.39 | 0.68 | 1.16 | 364 | 2357 | 2575 | 125 | 289 | 35 | 47 |
| 222 | 7.49 | 1.43 | 0.69 | 1.17 | 359 | 2367 | 2567 | 125 | 292 | 34 | 47 |
| 223 | 7.45 | 1.46 | 0.69 | 1.18 | 354 | 2378 | 2558 | 124 | 295 | 33 | 48 |
| 224 | 7.41 | 1.50 | 0.70 | 1.19 | 349 | 2389 | 2550 | 123 | 298 | 32 | 49 |
| 225 | 7.38 | 1.54 | 0.71 | 1.20 | 344 | 2399 | 2541 | 122 | 301 | 32 | 50 |
| 226 | 7.34 | 1.58 | 0.71 | 1.20 | 339 | 2409 | 2532 | 121 | 304 | 31 | 50 |
| 227 | 7.31 | 1.62 | 0.72 | 1.21 | 334 | 2420 | 2523 | 121 | 307 | 30 | 51 |
| 228 | 7.28 | 1.66 | 0.73 | 1.22 | 329 | 2430 | 2513 | 120 | 310 | 30 | 52 |
| 229 | 7.25 | 1.70 | 0.74 | 1.23 | 324 | 2440 | 2504 | 119 | 313 | 29 | 53 |
| 230 | 7.22 | 1.74 | 0.75 | 1.24 | 319 | 2450 | 2494 | 118 | 316 | 28 | 54 |
| 231 | 7.19 | 1.78 | 0.75 | 1.24 | 314 | 2460 | 2484 | 118 | 318 | 28 | 54 |
| 232 | 7.17 | 1.81 | 0.76 | 1.25 | 310 | 2470 | 2474 | 117 | 321 | 27 | 55 |
| 233 | 7.14 | 1.85 | 0.77 | 1.25 | 305 | 2480 | 2464 | 116 | 324 | 27 | 56 |
| 234 | 7.11 | 1.89 | 0.78 | 1.26 | 300 | 2489 | 2454 | 116 | 327 | 26 | 57 |
| 235 | 7.09 | 1.93 | 0.79 | 1.27 | 296 | 2499 | 2444 | 115 | 330 | 25 | 58 |
| 236 | 7.07 | 1.97 | 0.80 | 1.27 | 291 | 2508 | 2433 | 114 | 333 | 25 | 59 |
| 237 | 7.05 | 2.00 | 0.81 | 1.28 | 287 | 2518 | 2423 | 114 | 336 | 24 | 60 |
| 238 | 7.03 | 2.04 | 0.82 | 1.28 | 283 | 2527 | 2412 | 113 | 339 | 23 | 61 |
| 239 | 7.01 | 2.08 | 0.83 | 1.29 | 278 | 2536 | 2401 | 113 | 342 | 22 | 62 |
| 240 | 6.99 | 2.12 | 0.84 | 1.30 | 274 | 2545 | 2390 | 113 | 345 | 21 | 63 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | Sec.var. | (v.c.1) | Diff. | Sec.var. | (v.s.2) | Diff. | Sec.var. | (v.c.2) | Diff. | Sec.var. |
|------|---------|-------|---------|-------|----------|---------|-------|----------|---------|-------|----------|---------|-------|----------|
| | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 240 | 38.51 | -0.26 | 247.33 | -1.13 | 0.12 | 39.98 | -1.00 | 1.66 | 158.31 | -1.31 | 0.47 | 4.95 | -0.31 | 2.46 |
| 241 | 38.25 | 0.26 | 246.20 | 1.14 | 0.12 | 38.98 | 0.98 | 1.67 | 157.00 | 1.31 | 0.48 | 4.64 | 0.30 | 2.47 |
| 242 | 37.99 | 0.26 | 245.06 | 1.15 | 0.12 | 38.00 | 0.97 | 1.69 | 155.69 | 1.32 | 0.49 | 4.34 | 0.28 | 2.48 |
| 243 | 37.73 | 0.27 | 243.91 | 1.16 | 0.13 | 37.03 | 0.96 | 1.70 | 154.37 | 1.32 | 0.50 | 4.06 | 0.27 | 2.49 |
| 244 | 37.46 | 0.26 | 242.75 | 1.18 | 0.13 | 36.07 | 0.95 | 1.71 | 153.05 | 1.32 | 0.51 | 3.79 | 0.25 | 2.50 |
| 245 | 37.20 | -0.27 | 241.57 | -1.18 | 0.13 | 35.12 | -0.93 | 1.73 | 151.73 | -1.33 | 0.53 | 3.54 | -0.24 | 2.51 |
| 246 | 36.93 | 0.27 | 240.39 | 1.19 | 0.13 | 34.19 | 0.92 | 1.74 | 150.40 | 1.33 | 0.54 | 3.30 | 0.23 | 2.52 |
| 247 | 36.66 | 0.27 | 239.20 | 1.21 | 0.13 | 33.27 | 0.91 | 1.75 | 149.07 | 1.33 | 0.55 | 3.07 | 0.21 | 2.53 |
| 248 | 36.39 | 0.27 | 237.99 | 1.22 | 0.14 | 32.36 | 0.90 | 1.76 | 147.74 | 1.33 | 0.56 | 2.86 | 0.20 | 2.54 |
| 249 | 36.12 | 0.27 | 236.77 | 1.23 | 0.14 | 31.46 | 0.89 | 1.78 | 146.41 | 1.34 | 0.57 | 2.66 | 0.18 | 2.55 |
| 250 | 35.85 | -0.27 | 235.54 | -1.24 | 0.14 | 30.57 | -0.87 | 1.79 | 145.07 | -1.34 | 0.58 | 2.48 | -0.17 | 2.56 |
| 251 | 35.58 | 0.27 | 234.30 | 1.25 | 0.14 | 29.70 | 0.86 | 1.80 | 143.73 | 1.34 | 0.59 | 2.31 | 0.15 | 2.57 |
| 252 | 35.31 | 0.28 | 233.05 | 1.25 | 0.15 | 28.84 | 0.85 | 1.82 | 142.39 | 1.34 | 0.60 | 2.16 | 0.14 | 2.58 |
| 253 | 35.03 | 0.28 | 231.80 | 1.26 | 0.15 | 27.99 | 0.84 | 1.83 | 141.05 | 1.34 | 0.61 | 2.02 | 0.13 | 2.59 |
| 254 | 34.75 | 0.28 | 230.54 | 1.28 | 0.16 | 27.15 | 0.82 | 1.85 | 139.71 | 1.35 | 0.62 | 1.89 | 0.11 | 2.60 |
| 255 | 34.47 | -0.28 | 229.26 | -1.28 | 0.16 | 26.33 | -0.81 | 1.86 | 138.36 | -1.35 | 0.64 | 1.78 | -0.10 | 2.61 |
| 256 | 34.19 | 0.28 | 227.98 | 1.30 | 0.17 | 25.52 | 0.79 | 1.88 | 137.01 | 1.35 | 0.65 | 1.68 | 0.08 | 2.62 |
| 257 | 33.91 | 0.28 | 226.68 | 1.31 | 0.17 | 24.73 | 0.78 | 1.89 | 135.66 | 1.35 | 0.66 | 1.60 | 0.07 | 2.63 |
| 258 | 33.63 | 0.29 | 225.37 | 1.31 | 0.17 | 23.95 | 0.77 | 1.91 | 134.31 | 1.35 | 0.67 | 1.53 | 0.06 | 2.64 |
| 259 | 33.34 | 0.28 | 224.06 | 1.32 | 0.18 | 23.18 | 0.75 | 1.92 | 132.96 | 1.35 | 0.68 | 1.47 | 0.04 | 2.64 |
| 260 | 33.06 | -0.28 | 222.74 | -1.33 | 0.18 | 22.43 | -0.74 | 1.94 | 131.61 | -1.35 | 0.69 | 1.43 | -0.03 | 2.65 |
| 261 | 32.78 | 0.29 | 221.41 | 1.34 | 0.18 | 21.69 | 0.72 | 1.95 | 130.26 | 1.36 | 0.70 | 1.40 | -0.01 | 2.66 |
| 262 | 32.49 | 0.28 | 220.07 | 1.35 | 0.19 | 20.97 | 0.71 | 1.97 | 128.90 | 1.35 | 0.71 | 1.39 | 0.01 | 2.67 |
| 263 | 32.21 | 0.28 | 218.72 | 1.35 | 0.19 | 20.26 | 0.71 | 1.98 | 127.55 | 1.35 | 0.73 | 1.40 | 0.02 | 2.67 |
| 264 | 31.92 | 0.29 | 217.36 | 1.36 | 0.20 | 19.56 | 0.68 | 1.99 | 126.20 | 1.35 | 0.74 | 1.42 | 0.02 | 2.68 |
| 265 | 31.63 | 0.29 | 215.99 | 1.37 | 0.20 | 18.88 | 0.65 | 2.01 | 124.85 | 1.35 | 0.75 | 1.45 | 0.03 | 2.69 |
| 266 | 31.34 | -0.29 | 214.62 | -1.37 | 0.21 | 18.21 | -0.67 | 2.02 | 123.50 | -1.35 | 0.76 | 1.50 | -0.05 | 2.70 |
| 267 | 31.05 | 0.29 | 213.24 | 1.38 | 0.21 | 17.56 | 0.65 | 2.03 | 122.14 | 1.36 | 0.77 | 1.56 | 0.06 | 2.71 |
| 268 | 30.76 | 0.29 | 211.85 | 1.39 | 0.22 | 16.92 | 0.64 | 2.04 | 120.79 | 1.35 | 0.79 | 1.63 | 0.07 | 2.71 |
| 269 | 30.47 | 0.29 | 210.45 | 1.40 | 0.22 | 16.29 | 0.63 | 2.06 | 119.44 | 1.35 | 0.80 | 1.72 | 0.09 | 2.72 |
| 270 | 30.18 | -0.30 | 209.05 | -1.41 | 0.23 | 15.68 | -0.60 | 2.07 | 118.09 | -1.35 | 0.81 | 1.82 | -0.12 | 2.73 |
| 271 | 29.88 | 0.29 | 207.64 | 1.42 | 0.24 | 15.08 | 0.58 | 2.08 | 116.74 | 1.35 | 0.82 | 1.94 | 0.13 | 2.74 |
| 272 | 29.59 | 0.30 | 206.22 | 1.43 | 0.24 | 14.50 | 0.56 | 2.10 | 115.39 | 1.35 | 0.84 | 2.07 | 0.14 | 2.74 |
| 273 | 29.29 | 0.29 | 204.79 | 1.43 | 0.25 | 13.94 | 0.55 | 2.11 | 114.04 | 1.35 | 0.85 | 2.21 | 0.16 | 2.75 |
| 274 | 29.00 | 0.29 | 203.36 | 1.44 | 0.25 | 13.39 | 0.54 | 2.13 | 112.70 | 1.34 | 0.87 | 2.37 | 0.18 | 2.75 |
| 275 | 28.71 | -0.30 | 201.92 | -1.44 | 0.26 | 12.85 | -0.52 | 2.14 | 111.35 | -1.34 | 0.88 | 2.55 | -0.19 | 2.76 |
| 276 | 28.41 | 0.29 | 200.48 | 1.45 | 0.27 | 12.33 | 0.50 | 2.15 | 110.01 | 1.34 | 0.89 | 2.74 | 0.21 | 2.77 |
| 277 | 28.12 | 0.29 | 199.03 | 1.46 | 0.27 | 11.83 | 0.49 | 2.17 | 108.67 | 1.34 | 0.91 | 2.95 | 0.22 | 2.77 |
| 278 | 27.83 | 0.29 | 197.57 | 1.46 | 0.28 | 11.34 | 0.48 | 2.18 | 107.33 | 1.34 | 0.92 | 3.17 | 0.23 | 2.78 |
| 279 | 27.54 | 0.29 | 196.11 | 1.47 | 0.28 | 10.86 | 0.46 | 2.20 | 105.99 | 1.33 | 0.94 | 3.40 | 0.25 | 2.78 |
| 280 | 27.25 | -0.30 | 194.64 | -1.47 | 0.29 | 10.40 | -0.45 | 2.21 | 104.66 | -1.33 | 0.95 | 3.65 | -0.26 | 2.79 |
| 281 | 26.95 | 0.29 | 193.17 | 1.48 | 0.30 | 9.95 | 0.43 | 2.22 | 103.33 | 1.33 | 0.96 | 3.91 | 0.28 | 2.79 |
| 282 | 26.66 | 0.30 | 191.69 | 1.49 | 0.31 | 9.52 | 0.42 | 2.23 | 102.00 | 1.33 | 0.98 | 4.19 | 0.29 | 2.80 |
| 283 | 26.36 | 0.29 | 190.20 | 1.49 | 0.32 | 9.10 | 0.40 | 2.24 | 100.67 | 1.33 | 0.99 | 4.48 | 0.30 | 2.80 |
| 284 | 26.07 | 0.29 | 188.71 | 1.49 | 0.33 | 8.70 | 0.39 | 2.25 | 99.34 | 1.32 | 1.01 | 4.78 | 0.32 | 2.81 |
| 285 | 25.78 | -0.30 | 187.22 | -1.50 | 0.34 | 8.31 | -0.37 | 2.27 | 98.02 | -1.32 | 1.02 | 5.10 | -0.33 | 2.81 |
| 286 | 25.48 | 0.29 | 185.72 | 1.51 | 0.34 | 7.94 | 0.35 | 2.28 | 96.70 | 1.31 | 1.03 | 5.43 | 0.35 | 2.82 |
| 287 | 25.19 | 0.30 | 184.21 | 1.51 | 0.35 | 7.59 | 0.34 | 2.29 | 95.39 | 1.31 | 1.05 | 5.78 | 0.36 | 2.82 |
| 288 | 24.89 | 0.29 | 182.70 | 1.51 | 0.36 | 7.25 | 0.32 | 2.30 | 94.08 | 1.31 | 1.06 | 6.14 | 0.37 | 2.83 |
| 289 | 24.60 | 0.29 | 181.19 | 1.52 | 0.37 | 6.93 | 0.30 | 2.31 | 92.77 | 1.30 | 1.08 | 6.51 | 0.39 | 2.83 |
| 290 | 24.31 | -0.29 | 179.67 | -1.52 | 0.38 | 6.63 | -0.29 | 2.32 | 91.47 | -1.30 | 1.09 | 6.90 | -0.40 | 2.84 |
| 291 | 24.02 | 0.29 | 178.15 | 1.52 | 0.39 | 6.34 | 0.27 | 2.33 | 90.17 | 1.30 | 1.10 | 7.30 | 0.42 | 2.84 |
| 292 | 23.73 | 0.29 | 176.63 | 1.53 | 0.40 | 6.07 | 0.26 | 2.34 | 88.87 | 1.29 | 1.12 | 7.72 | 0.43 | 2.85 |
| 293 | 23.44 | 0.29 | 175.10 | 1.53 | 0.41 | 5.81 | 0.24 | 2.36 | 87.58 | 1.29 | 1.13 | 8.15 | 0.44 | 2.85 |
| 294 | 23.15 | 0.29 | 173.57 | 1.53 | 0.42 | 5.57 | 0.23 | 2.37 | 86.29 | 1.28 | 1.15 | 8.59 | 0.46 | 2.86 |
| 295 | 22.86 | -0.29 | 172.04 | -1.54 | 0.43 | 5.34 | -0.21 | 2.38 | 85.01 | -1.27 | 1.16 | 9.05 | -0.47 | 2.86 |
| 296 | 22.57 | 0.28 | 170.50 | 1.54 | 0.43 | 5.13 | 0.20 | 2.39 | 83.74 | 1.27 | 1.17 | 9.52 | 0.49 | 2.87 |
| 297 | 22.29 | 0.29 | 168.96 | 1.54 | 0.44 | 4.93 | 0.18 | 2.40 | 82.47 | 1.27 | 1.19 | 10.01 | 0.50 | 2.87 |
| 298 | 22.00 | 0.29 | 167.42 | 1.54 | 0.45 | 4.75 | 0.16 | 2.42 | 81.20 | 1.27 | 1.20 | 10.51 | 0.51 | 2.87 |
| 299 | 21.71 | 0.29 | 165.88 | 1.55 | 0.46 | 4.59 | 0.15 | 2.43 | 79.93 | 1.26 | 1.22 | 11.02 | 0.53 | 2.88 |
| 300 | 21.42 | | 164.33 | | 0.47 | 4.44 | | 2.44 | 78.67 | | 1.23 | 11.55 | | 2.88 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | (p.s.3) | (p.c.3) |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| " | " | " | " | " | | | | | | | |
| 240 | 6.99 | 2.12 | 0.84 | 1.30 | 274 | 2545 | 2390 | 113 | 345 | 21 | 63 |
| 241 | 6.97 | 2.16 | 0.85 | 1.31 | 270 | 2554 | 2379 | 113 | 348 | 21 | 64 |
| 242 | 6.96 | 2.20 | 0.86 | 1.31 | 265 | 2562 | 2368 | 113 | 350 | 20 | 65 |
| 243 | 6.94 | 2.24 | 0.87 | 1.31 | 261 | 2571 | 2357 | 112 | 353 | 19 | 66 |
| 244 | 6.92 | 2.28 | 0.88 | 1.31 | 257 | 2579 | 2346 | 112 | 356 | 19 | 67 |
| 245 | 6.90 | 2.31 | 0.90 | 1.32 | 253 | 2587 | 2334 | 112 | 359 | 18 | 68 |
| 246 | 6.88 | 2.35 | 0.91 | 1.32 | 249 | 2595 | 2322 | 112 | 362 | 18 | 69 |
| 247 | 6.87 | 2.38 | 0.92 | 1.32 | 245 | 2603 | 2311 | 112 | 365 | 17 | 70 |
| 248 | 6.85 | 2.42 | 0.93 | 1.32 | 242 | 2611 | 2299 | 112 | 368 | 16 | 72 |
| 249 | 6.84 | 2.45 | 0.94 | 1.33 | 238 | 2618 | 2287 | 111 | 371 | 16 | 73 |
| 250 | 6.83 | 2.48 | 0.95 | 1.33 | 234 | 2626 | 2275 | 111 | 374 | 15 | 74 |
| 251 | 6.82 | 2.52 | 0.96 | 1.33 | 230 | 2633 | 2263 | 111 | 377 | 15 | 75 |
| 252 | 6.82 | 2.55 | 0.97 | 1.33 | 227 | 2641 | 2251 | 111 | 380 | 14 | 76 |
| 253 | 6.81 | 2.58 | 0.98 | 1.33 | 223 | 2648 | 2239 | 111 | 383 | 14 | 77 |
| 254 | 6.80 | 2.61 | 0.99 | 1.34 | 219 | 2656 | 2226 | 111 | 386 | 13 | 78 |
| 255 | 6.79 | 2.64 | 1.00 | 1.34 | 215 | 2663 | 2214 | 111 | 389 | 13 | 79 |
| 256 | 6.79 | 2.67 | 1.02 | 1.34 | 212 | 2670 | 2201 | 111 | 392 | 13 | 80 |
| 257 | 6.78 | 2.70 | 1.03 | 1.34 | 208 | 2677 | 2189 | 112 | 395 | 12 | 82 |
| 258 | 6.78 | 2.73 | 1.04 | 1.34 | 204 | 2684 | 2176 | 112 | 398 | 12 | 83 |
| 259 | 6.77 | 2.76 | 1.05 | 1.34 | 201 | 2691 | 2164 | 112 | 401 | 12 | 84 |
| 260 | 6.77 | 2.79 | 1.06 | 1.34 | 198 | 2698 | 2151 | 112 | 404 | 11 | 85 |
| 261 | 6.77 | 2.82 | 1.07 | 1.34 | 195 | 2704 | 2138 | 112 | 407 | 11 | 86 |
| 262 | 6.77 | 2.85 | 1.09 | 1.33 | 192 | 2711 | 2125 | 113 | 410 | 11 | 87 |
| 263 | 6.77 | 2.88 | 1.10 | 1.33 | 189 | 2717 | 2112 | 113 | 413 | 11 | 89 |
| 264 | 6.77 | 2.90 | 1.11 | 1.33 | 186 | 2723 | 2098 | 113 | 416 | 11 | 90 |
| 265 | 6.77 | 2.93 | 1.12 | 1.33 | 183 | 2729 | 2085 | 114 | 419 | 11 | 91 |
| 266 | 6.77 | 2.95 | 1.13 | 1.32 | 180 | 2735 | 2072 | 114 | 422 | 11 | 92 |
| 267 | 6.77 | 2.98 | 1.14 | 1.32 | 177 | 2740 | 2058 | 115 | 425 | 11 | 93 |
| 268 | 6.77 | 3.00 | 1.15 | 1.32 | 174 | 2745 | 2045 | 115 | 427 | 11 | 94 |
| 269 | 6.77 | 3.03 | 1.16 | 1.32 | 172 | 2751 | 2031 | 116 | 430 | 11 | 96 |
| 270 | 6.77 | 3.05 | 1.17 | 1.31 | 169 | 2756 | 2018 | 116 | 433 | 11 | 97 |
| 271 | 6.77 | 3.07 | 1.18 | 1.31 | 166 | 2761 | 2004 | 117 | 436 | 11 | 98 |
| 272 | 6.77 | 3.09 | 1.20 | 1.31 | 164 | 2766 | 1990 | 117 | 439 | 10 | 99 |
| 273 | 6.78 | 3.11 | 1.21 | 1.30 | 161 | 2771 | 1977 | 118 | 442 | 10 | 100 |
| 274 | 6.78 | 3.13 | 1.22 | 1.30 | 159 | 2776 | 1963 | 119 | 444 | 10 | 102 |
| 275 | 6.78 | 3.15 | 1.23 | 1.29 | 156 | 2781 | 1949 | 119 | 447 | 10 | 103 |
| 276 | 6.79 | 3.17 | 1.24 | 1.28 | 154 | 2785 | 1935 | 120 | 450 | 10 | 104 |
| 277 | 6.79 | 3.19 | 1.25 | 1.28 | 151 | 2790 | 1921 | 121 | 452 | 10 | 105 |
| 278 | 6.80 | 3.20 | 1.26 | 1.27 | 149 | 2794 | 1907 | 121 | 455 | 10 | 106 |
| 279 | 6.81 | 3.22 | 1.27 | 1.27 | 146 | 2798 | 1892 | 122 | 457 | 10 | 107 |
| 280 | 6.81 | 3.23 | 1.28 | 1.26 | 144 | 2802 | 1878 | 123 | 460 | 10 | 108 |
| 281 | 6.82 | 3.25 | 1.28 | 1.25 | 142 | 2806 | 1864 | 124 | 463 | 10 | 110 |
| 282 | 6.83 | 3.26 | 1.29 | 1.25 | 139 | 2809 | 1849 | 124 | 465 | 10 | 111 |
| 283 | 6.83 | 3.27 | 1.30 | 1.24 | 137 | 2813 | 1835 | 125 | 468 | 10 | 112 |
| 284 | 6.84 | 3.28 | 1.31 | 1.23 | 135 | 2816 | 1820 | 126 | 471 | 11 | 113 |
| 285 | 6.84 | 3.29 | 1.32 | 1.22 | 133 | 2819 | 1805 | 127 | 474 | 11 | 115 |
| 286 | 6.85 | 3.30 | 1.33 | 1.22 | 131 | 2822 | 1791 | 127 | 476 | 11 | 116 |
| 287 | 6.85 | 3.31 | 1.34 | 1.21 | 129 | 2825 | 1776 | 128 | 479 | 11 | 117 |
| 288 | 6.86 | 3.32 | 1.34 | 1.20 | 127 | 2828 | 1761 | 129 | 482 | 11 | 118 |
| 289 | 6.86 | 3.32 | 1.35 | 1.19 | 126 | 2830 | 1746 | 130 | 484 | 12 | 120 |
| 290 | 6.87 | 3.33 | 1.36 | 1.18 | 124 | 2833 | 1731 | 131 | 487 | 12 | 121 |
| 291 | 6.88 | 3.33 | 1.37 | 1.17 | 123 | 2835 | 1716 | 132 | 490 | 12 | 122 |
| 292 | 6.88 | 3.34 | 1.38 | 1.16 | 121 | 2837 | 1702 | 133 | 492 | 12 | 123 |
| 293 | 6.89 | 3.34 | 1.38 | 1.15 | 119 | 2838 | 1686 | 134 | 495 | 13 | 125 |
| 294 | 6.89 | 3.35 | 1.39 | 1.14 | 118 | 2840 | 1671 | 136 | 497 | 13 | 126 |
| 295 | 6.90 | 3.35 | 1.40 | 1.13 | 116 | 2841 | 1656 | 137 | 500 | 13 | 127 |
| 296 | 6.90 | 3.35 | 1.40 | 1.12 | 115 | 2843 | 1641 | 138 | 503 | 13 | 128 |
| 297 | 6.90 | 3.35 | 1.41 | 1.11 | 114 | 2844 | 1626 | 139 | 505 | 14 | 130 |
| 298 | 6.91 | 3.35 | 1.42 | 1.10 | 112 | 2845 | 1611 | 141 | 508 | 14 | 131 |
| 299 | 6.91 | 3.35 | 1.42 | 1.09 | 111 | 2847 | 1596 | 142 | 510 | 14 | 132 |
| 300 | 6.92 | 3.35 | 1.43 | 1.08 | 110 | 2848 | 1581 | 143 | 512 | 15 | 133 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.c.0) Diff. | (v.s.1) Diff. Sec.var. | (v.c.1) Diff. Sec.var. | (v.s.2) Diff. Sec.var. | (v.c.2) Diff. Sec.var. | | | | |
|------|---------------|------------------------|------------------------|------------------------|------------------------|-------|------|-------|------|
| " | " | " | " | " | " | | | | |
| 300 | 21.42 | 164.33 | 0.47 | 4.44 | 2.44 | 78.67 | 1.23 | 11.55 | 2.88 |
| 301 | 21.13 | 162.78 | 1.55 | 4.31 | 2.45 | 77.42 | 1.24 | 12.09 | 2.88 |
| 302 | 20.85 | 161.23 | 1.55 | 4.19 | 2.46 | 76.17 | 1.26 | 12.64 | 2.88 |
| 303 | 20.57 | 159.68 | 1.55 | 4.09 | 2.47 | 74.93 | 1.27 | 13.21 | 2.89 |
| 304 | 20.29 | 158.12 | 1.56 | 4.01 | 2.48 | 73.70 | 1.29 | 13.79 | 2.89 |
| 305 | 20.01 | 156.57 | 1.55 | 3.94 | 2.49 | 72.47 | 1.30 | 14.38 | 2.89 |
| 306 | 19.73 | 155.01 | 1.56 | 3.89 | 2.50 | 71.25 | 1.31 | 14.99 | 2.89 |
| 307 | 19.45 | 153.46 | 1.55 | 3.86 | 2.51 | 70.03 | 1.33 | 15.61 | 2.89 |
| 308 | 19.17 | 151.90 | 1.56 | 3.85 | 2.52 | 68.82 | 1.34 | 16.24 | 2.90 |
| 309 | 18.90 | 150.34 | 1.56 | 3.84 | 2.53 | 67.62 | 1.36 | 16.88 | 2.90 |
| 310 | 18.63 | 148.79 | 1.55 | 3.85 | 2.54 | 66.42 | 1.37 | 17.54 | 2.90 |
| 311 | 18.36 | 147.23 | 1.56 | 3.88 | 2.55 | 65.23 | 1.39 | 18.21 | 2.90 |
| 312 | 18.09 | 145.67 | 1.56 | 3.92 | 2.56 | 64.05 | 1.40 | 18.90 | 2.90 |
| 313 | 17.82 | 144.12 | 1.55 | 3.98 | 2.57 | 62.87 | 1.42 | 19.60 | 2.90 |
| 314 | 17.55 | 142.56 | 1.56 | 4.06 | 2.58 | 61.70 | 1.43 | 20.31 | 2.90 |
| 315 | 17.28 | 141.00 | 1.56 | 4.16 | 2.59 | 60.54 | 1.45 | 21.03 | 2.91 |
| 316 | 17.02 | 139.44 | 1.56 | 4.27 | 2.59 | 59.39 | 1.47 | 21.76 | 2.91 |
| 317 | 16.75 | 137.88 | 1.56 | 4.40 | 2.60 | 58.24 | 1.48 | 22.51 | 2.91 |
| 318 | 16.49 | 136.33 | 1.55 | 4.55 | 2.61 | 57.10 | 1.50 | 23.27 | 2.91 |
| 319 | 16.23 | 134.77 | 1.56 | 4.71 | 2.62 | 55.97 | 1.51 | 24.04 | 2.91 |
| 320 | 15.97 | 133.22 | 1.55 | 4.89 | 2.63 | 54.85 | 1.53 | 24.82 | 2.91 |
| 321 | 15.71 | 131.67 | 1.55 | 5.08 | 2.64 | 53.74 | 1.54 | 25.62 | 2.91 |
| 322 | 15.46 | 130.12 | 1.55 | 5.29 | 2.65 | 52.64 | 1.56 | 26.42 | 2.91 |
| 323 | 15.21 | 128.57 | 1.55 | 5.51 | 2.65 | 51.54 | 1.57 | 27.24 | 2.90 |
| 324 | 14.96 | 127.02 | 1.55 | 5.75 | 2.66 | 50.45 | 1.59 | 28.07 | 2.90 |
| 325 | 14.71 | 125.48 | 1.54 | 6.00 | 2.67 | 49.37 | 1.60 | 28.92 | 2.90 |
| 326 | 14.47 | 123.94 | 1.54 | 6.27 | 2.68 | 48.30 | 1.61 | 29.77 | 2.90 |
| 327 | 14.23 | 122.41 | 1.53 | 6.56 | 2.69 | 47.23 | 1.63 | 30.63 | 2.90 |
| 328 | 13.99 | 120.88 | 1.53 | 6.87 | 2.69 | 46.17 | 1.64 | 31.51 | 2.89 |
| 329 | 13.75 | 119.35 | 1.53 | 7.19 | 2.70 | 45.13 | 1.66 | 32.40 | 2.89 |
| 330 | 13.51 | 117.82 | 1.53 | 7.52 | 2.71 | 44.10 | 1.67 | 33.30 | 2.89 |
| 331 | 13.28 | 116.30 | 1.52 | 7.87 | 2.72 | 43.07 | 1.68 | 34.21 | 2.89 |
| 332 | 13.04 | 114.78 | 1.52 | 8.24 | 2.72 | 42.06 | 1.70 | 35.13 | 2.89 |
| 333 | 12.81 | 113.26 | 1.52 | 8.62 | 2.73 | 41.06 | 1.71 | 36.06 | 2.88 |
| 334 | 12.58 | 111.74 | 1.52 | 9.02 | 2.74 | 40.06 | 1.73 | 37.01 | 2.88 |
| 335 | 12.35 | 110.23 | 1.51 | 9.44 | 2.75 | 39.07 | 1.74 | 37.97 | 2.88 |
| 336 | 12.13 | 108.72 | 1.51 | 9.87 | 2.75 | 38.10 | 1.75 | 38.94 | 2.88 |
| 337 | 11.91 | 107.22 | 1.50 | 10.32 | 2.76 | 37.14 | 1.77 | 39.91 | 2.88 |
| 338 | 11.69 | 105.73 | 1.49 | 10.78 | 2.77 | 36.19 | 1.78 | 40.89 | 2.87 |
| 339 | 11.47 | 104.24 | 1.49 | 11.26 | 2.77 | 35.24 | 1.80 | 41.89 | 2.87 |
| 340 | 11.26 | 102.75 | 1.49 | 11.75 | 2.78 | 34.30 | 1.81 | 42.90 | 2.87 |
| 341 | 11.05 | 101.27 | 1.48 | 12.26 | 2.78 | 33.38 | 1.82 | 43.92 | 2.87 |
| 342 | 10.84 | 99.80 | 1.47 | 12.79 | 2.79 | 32.47 | 1.84 | 44.95 | 2.86 |
| 343 | 10.64 | 98.33 | 1.47 | 13.33 | 2.79 | 31.57 | 1.85 | 45.99 | 2.86 |
| 344 | 10.44 | 96.86 | 1.47 | 13.89 | 2.80 | 30.68 | 1.87 | 47.03 | 2.86 |
| 345 | 10.24 | 95.40 | 1.46 | 14.46 | 2.80 | 29.80 | 1.88 | 48.09 | 2.85 |
| 346 | 10.05 | 93.95 | 1.45 | 15.05 | 2.81 | 28.93 | 1.90 | 49.15 | 2.85 |
| 347 | 9.85 | 92.50 | 1.45 | 15.65 | 2.81 | 28.08 | 1.91 | 50.22 | 2.85 |
| 348 | 9.66 | 91.06 | 1.44 | 16.26 | 2.82 | 27.23 | 1.93 | 51.30 | 2.85 |
| 349 | 9.47 | 89.63 | 1.43 | 16.89 | 2.82 | 26.39 | 1.94 | 52.40 | 2.84 |
| 350 | 9.28 | 88.21 | 1.42 | 17.54 | 2.83 | 25.57 | 1.96 | 53.50 | 2.84 |
| 351 | 9.10 | 86.79 | 1.42 | 18.20 | 2.83 | 24.76 | 1.97 | 54.61 | 2.83 |
| 352 | 8.92 | 85.38 | 1.41 | 18.88 | 2.84 | 23.97 | 1.99 | 55.73 | 2.83 |
| 353 | 8.74 | 83.98 | 1.40 | 19.57 | 2.84 | 23.18 | 2.00 | 56.86 | 2.82 |
| 354 | 8.57 | 82.58 | 1.40 | 20.28 | 2.84 | 22.40 | 2.01 | 57.99 | 2.82 |
| 355 | 8.40 | 81.19 | 1.39 | 21.00 | 2.85 | 21.64 | 2.03 | 59.14 | 2.81 |
| 356 | 8.23 | 79.81 | 1.38 | 21.74 | 2.85 | 20.89 | 2.04 | 60.29 | 2.80 |
| 357 | 8.07 | 78.44 | 1.37 | 22.49 | 2.85 | 20.16 | 2.05 | 61.45 | 2.80 |
| 358 | 7.91 | 77.07 | 1.37 | 23.25 | 2.85 | 19.43 | 2.06 | 62.62 | 2.79 |
| 359 | 7.75 | 75.71 | 1.36 | 24.03 | 2.86 | 18.71 | 2.08 | 63.80 | 2.79 |
| 360 | 7.60 | 74.36 | 1.35 | 24.82 | 2.86 | 18.01 | 2.09 | 64.98 | 2.78 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | (p.s.3) | (p.c.3) |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | | | | | | | | |
| 300 | 6.92 | 3.35 | 1.43 | 1.08 | 110 | 2848 | 1581 | 143 | 512 | 15 | 133 |
| 301 | 6.92 | 3.35 | 1.43 | 1.07 | 109 | 2849 | 1566 | 144 | 515 | 15 | 134 |
| 302 | 6.93 | 3.35 | 1.44 | 1.06 | 108 | 2849 | 1551 | 146 | 517 | 15 | 135 |
| 303 | 6.93 | 3.34 | 1.44 | 1.05 | 107 | 2850 | 1535 | 147 | 519 | 16 | 137 |
| 304 | 6.93 | 3.34 | 1.45 | 1.04 | 106 | 2850 | 1520 | 149 | 522 | 16 | 138 |
| 305 | 6.94 | 3.34 | 1.45 | 1.03 | 105 | 2850 | 1505 | 150 | 524 | 17 | 139 |
| 306 | 6.94 | 3.33 | 1.46 | 1.01 | 104 | 2850 | 1490 | 152 | 526 | 17 | 140 |
| 307 | 6.94 | 3.33 | 1.46 | 1.00 | 103 | 2850 | 1474 | 153 | 528 | 18 | 141 |
| 308 | 6.94 | 3.32 | 1.47 | 0.99 | 103 | 2850 | 1459 | 154 | 531 | 18 | 142 |
| 309 | 6.95 | 3.31 | 1.47 | 0.98 | 102 | 2849 | 1444 | 156 | 533 | 19 | 143 |
| 310 | 6.95 | 3.31 | 1.47 | 0.96 | 102 | 2849 | 1429 | 157 | 535 | 20 | 144 |
| 311 | 6.95 | 3.30 | 1.47 | 0.95 | 101 | 2848 | 1413 | 158 | 538 | 20 | 145 |
| 312 | 6.95 | 3.29 | 1.48 | 0.94 | 101 | 2847 | 1398 | 160 | 540 | 21 | 146 |
| 313 | 6.95 | 3.28 | 1.48 | 0.93 | 100 | 2847 | 1383 | 162 | 542 | 22 | 147 |
| 314 | 6.94 | 3.27 | 1.48 | 0.92 | 100 | 2846 | 1368 | 163 | 545 | 23 | 148 |
| 315 | 6.94 | 3.26 | 1.48 | 0.90 | 99 | 2845 | 1352 | 165 | 547 | 24 | 149 |
| 316 | 6.94 | 3.25 | 1.48 | 0.89 | 99 | 2843 | 1337 | 167 | 549 | 24 | 150 |
| 317 | 6.94 | 3.23 | 1.48 | 0.88 | 99 | 2842 | 1322 | 168 | 551 | 25 | 151 |
| 318 | 6.93 | 3.22 | 1.48 | 0.86 | 98 | 2840 | 1307 | 170 | 554 | 26 | 152 |
| 319 | 6.93 | 3.21 | 1.48 | 0.85 | 98 | 2838 | 1291 | 171 | 556 | 27 | 153 |
| 320 | 6.92 | 3.20 | 1.48 | 0.84 | 98 | 2836 | 1276 | 173 | 558 | 27 | 154 |
| 321 | 6.92 | 3.18 | 1.48 | 0.83 | 98 | 2834 | 1261 | 175 | 560 | 28 | 155 |
| 322 | 6.91 | 3.17 | 1.48 | 0.81 | 98 | 2831 | 1246 | 176 | 562 | 29 | 156 |
| 323 | 6.91 | 3.15 | 1.48 | 0.80 | 98 | 2828 | 1231 | 178 | 564 | 29 | 157 |
| 324 | 6.90 | 3.14 | 1.48 | 0.79 | 99 | 2825 | 1216 | 180 | 566 | 30 | 158 |
| 325 | 6.89 | 3.12 | 1.48 | 0.77 | 99 | 2822 | 1202 | 181 | 568 | 31 | 159 |
| 326 | 6.88 | 3.11 | 1.48 | 0.76 | 99 | 2819 | 1187 | 183 | 570 | 32 | 159 |
| 327 | 6.87 | 3.10 | 1.47 | 0.75 | 100 | 2815 | 1172 | 185 | 572 | 33 | 160 |
| 328 | 6.86 | 3.08 | 1.47 | 0.74 | 100 | 2812 | 1157 | 186 | 573 | 33 | 161 |
| 329 | 6.85 | 3.06 | 1.47 | 0.72 | 101 | 2808 | 1143 | 188 | 575 | 34 | 162 |
| 330 | 6.83 | 3.04 | 1.46 | 0.71 | 101 | 2804 | 1128 | 190 | 577 | 35 | 163 |
| 331 | 6.82 | 3.03 | 1.46 | 0.70 | 102 | 2800 | 1113 | 192 | 579 | 36 | 164 |
| 332 | 6.81 | 3.01 | 1.46 | 0.69 | 103 | 2796 | 1098 | 193 | 581 | 37 | 165 |
| 333 | 6.79 | 2.99 | 1.45 | 0.67 | 103 | 2792 | 1084 | 195 | 583 | 38 | 165 |
| 334 | 6.77 | 2.97 | 1.45 | 0.66 | 104 | 2788 | 1069 | 197 | 585 | 39 | 166 |
| 335 | 6.75 | 2.96 | 1.44 | 0.65 | 105 | 2784 | 1054 | 199 | 587 | 40 | 167 |
| 336 | 6.74 | 2.94 | 1.44 | 0.64 | 106 | 2779 | 1039 | 201 | 589 | 41 | 168 |
| 337 | 6.72 | 2.92 | 1.43 | 0.63 | 107 | 2775 | 1025 | 203 | 591 | 42 | 169 |
| 338 | 6.70 | 2.90 | 1.43 | 0.61 | 108 | 2770 | 1010 | 204 | 592 | 43 | 169 |
| 339 | 6.68 | 2.88 | 1.42 | 0.60 | 109 | 2765 | 996 | 206 | 594 | 44 | 170 |
| 340 | 6.66 | 2.87 | 1.42 | 0.59 | 110 | 2760 | 981 | 208 | 596 | 45 | 171 |
| 341 | 6.64 | 2.85 | 1.41 | 0.58 | 111 | 2755 | 967 | 210 | 598 | 46 | 171 |
| 342 | 6.62 | 2.83 | 1.40 | 0.57 | 112 | 2750 | 953 | 212 | 599 | 47 | 172 |
| 343 | 6.59 | 2.81 | 1.40 | 0.56 | 114 | 2744 | 938 | 214 | 601 | 48 | 173 |
| 344 | 6.57 | 2.79 | 1.39 | 0.55 | 115 | 2739 | 924 | 217 | 603 | 49 | 173 |
| 345 | 6.55 | 2.78 | 1.38 | 0.53 | 116 | 2733 | 910 | 219 | 605 | 50 | 174 |
| 346 | 6.52 | 2.76 | 1.37 | 0.52 | 117 | 2727 | 896 | 221 | 606 | 51 | 174 |
| 347 | 6.50 | 2.74 | 1.36 | 0.51 | 119 | 2721 | 882 | 223 | 608 | 52 | 175 |
| 348 | 6.47 | 2.72 | 1.36 | 0.50 | 121 | 2715 | 868 | 226 | 610 | 53 | 176 |
| 349 | 6.44 | 2.70 | 1.35 | 0.49 | 122 | 2709 | 855 | 228 | 611 | 54 | 176 |
| 350 | 6.41 | 2.69 | 1.34 | 0.48 | 124 | 2703 | 841 | 230 | 613 | 55 | 177 |
| 351 | 6.37 | 2.67 | 1.33 | 0.47 | 126 | 2696 | 828 | 232 | 614 | 57 | 177 |
| 352 | 6.34 | 2.65 | 1.32 | 0.46 | 128 | 2689 | 814 | 235 | 616 | 58 | 178 |
| 353 | 6.31 | 2.64 | 1.31 | 0.45 | 130 | 2682 | 801 | 237 | 617 | 59 | 178 |
| 354 | 6.28 | 2.62 | 1.30 | 0.44 | 132 | 2675 | 788 | 239 | 619 | 60 | 178 |
| 355 | 6.24 | 2.60 | 1.29 | 0.43 | 134 | 2668 | 775 | 241 | 620 | 61 | 179 |
| 356 | 6.21 | 2.59 | 1.28 | 0.42 | 136 | 2660 | 762 | 244 | 621 | 62 | 179 |
| 357 | 6.18 | 2.57 | 1.26 | 0.41 | 139 | 2653 | 749 | 246 | 623 | 63 | 180 |
| 358 | 6.14 | 2.56 | 1.25 | 0.40 | 141 | 2645 | 736 | 248 | 624 | 64 | 180 |
| 359 | 6.10 | 2.54 | 1.24 | 0.39 | 144 | 2637 | 723 | 250 | 626 | 65 | 180 |
| 360 | 6.07 | 2.53 | 1.23 | 0.38 | 146 | 2629 | 710 | 252 | 627 | 67 | 181 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (c.c.0) | Diff. | (v.s.1) | Diff. | Sec.var. | (v.c.1) | Diff. | Sec.var. | (v.s.2) | Diff. | Sec.var. | (v.c.2) | Diff. | Sec.var. |
|------|---------|-------|---------|-------|----------|---------|-------|----------|---------|-------|----------|---------|-------|----------|
| | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 360 | 7.60 | | 74.36 | | 1.22 | 24.82 | | 2.86 | 18.01 | | 2.09 | 64.98 | | 2.78 |
| 361 | 7.45 | -0.15 | 73.02 | -1.34 | 1.23 | 25.63 | +0.81 | 2.86 | 17.33 | -0.68 | 2.10 | 66.17 | +1.19 | 2.77 |
| 362 | 7.30 | 0.15 | 71.69 | 1.33 | 1.25 | 26.45 | 0.82 | 2.87 | 16.66 | 0.67 | 2.12 | 67.37 | 1.20 | 2.77 |
| 363 | 7.15 | 0.15 | 70.37 | 1.32 | 1.26 | 27.29 | 0.84 | 2.87 | 16.00 | 0.66 | 2.13 | 68.58 | 1.21 | 2.76 |
| 364 | 7.01 | 0.14 | 69.05 | 1.32 | 1.28 | 28.14 | 0.85 | 2.87 | 15.35 | 0.65 | 2.15 | 69.79 | 1.21 | 2.75 |
| | | 0.14 | | 1.30 | | | 0.86 | | | 0.64 | | | 1.22 | |
| 365 | 6.87 | | 67.75 | | 1.29 | 29.00 | | 2.88 | 14.71 | | 2.16 | 71.01 | | 2.74 |
| 366 | 6.74 | -0.13 | 66.46 | -1.29 | 1.31 | 29.88 | +0.88 | 2.88 | 14.09 | -0.62 | 2.17 | 72.23 | +1.22 | 2.74 |
| 367 | 6.61 | 0.13 | 65.18 | 1.28 | 1.32 | 30.78 | 0.90 | 2.88 | 13.48 | 0.61 | 2.18 | 73.46 | 1.23 | 2.73 |
| 368 | 6.48 | 0.13 | 63.90 | 1.28 | 1.34 | 31.69 | 0.91 | 2.88 | 12.88 | 0.60 | 2.19 | 74.70 | 1.24 | 2.72 |
| 369 | 6.36 | 0.12 | 62.63 | 1.27 | 1.35 | 32.61 | 0.92 | 2.89 | 12.29 | 0.59 | 2.21 | 75.95 | 1.25 | 2.71 |
| | | 0.12 | | 1.25 | | | 0.93 | | | 0.57 | | | 1.26 | |
| 370 | 6.24 | | 61.38 | | 1.37 | 33.54 | | 2.89 | 11.72 | | 2.22 | 77.21 | | 2.71 |
| 371 | 6.12 | -0.12 | 60.14 | -1.24 | 1.38 | 34.49 | +0.95 | 2.89 | 11.17 | -0.55 | 2.23 | 78.47 | +1.26 | 2.70 |
| 372 | 6.00 | 0.12 | 58.91 | 1.23 | 1.40 | 35.45 | 0.96 | 2.89 | 10.63 | 0.54 | 2.25 | 79.74 | 1.27 | 2.69 |
| 373 | 5.89 | 0.11 | 57.68 | 1.23 | 1.41 | 36.42 | 0.97 | 2.89 | 10.10 | 0.53 | 2.26 | 81.01 | 1.27 | 2.68 |
| 374 | 5.78 | 0.11 | 56.46 | 1.22 | 1.43 | 37.40 | 0.98 | 2.89 | 9.59 | 0.51 | 2.27 | 82.28 | 1.27 | 2.67 |
| | | 0.10 | | 1.20 | | | 1.00 | | | 0.50 | | | 1.28 | |
| 375 | 5.68 | | 55.26 | | 1.44 | 38.40 | | 2.89 | 9.09 | | 2.29 | 83.56 | | 2.66 |
| 376 | 5.57 | -0.11 | 54.07 | -1.19 | 1.45 | 39.41 | +1.01 | 2.89 | 8.60 | -0.49 | 2.30 | 84.85 | +1.29 | 2.66 |
| 377 | 5.47 | 0.10 | 52.89 | 1.18 | 1.47 | 40.43 | 1.02 | 2.89 | 8.12 | 0.48 | 2.31 | 86.14 | 1.29 | 2.65 |
| 378 | 5.38 | 0.09 | 51.72 | 1.17 | 1.48 | 41.47 | 1.04 | 2.89 | 7.66 | 0.46 | 2.32 | 87.43 | 1.29 | 2.64 |
| 379 | 5.30 | 0.08 | 50.57 | 1.15 | 1.50 | 42.52 | 1.05 | 2.89 | 7.21 | 0.45 | 2.34 | 88.73 | 1.30 | 2.63 |
| | | 0.08 | | 1.15 | | | 1.06 | | | 0.43 | | | 1.31 | |
| 380 | 5.22 | | 49.42 | | 1.51 | 43.58 | | 2.89 | 6.78 | | 2.35 | 90.04 | | 2.62 |
| 381 | 5.14 | -0.08 | 48.29 | -1.13 | 1.52 | 44.65 | +1.07 | 2.89 | 6.36 | -0.42 | 2.36 | 91.35 | +1.31 | 2.61 |
| 382 | 5.07 | 0.07 | 47.17 | 1.12 | 1.54 | 45.74 | 1.09 | 2.89 | 5.96 | 0.40 | 2.37 | 92.67 | 1.32 | 2.60 |
| 383 | 4.99 | 0.08 | 46.06 | 1.11 | 1.55 | 46.84 | 1.10 | 2.89 | 5.57 | 0.39 | 2.38 | 93.99 | 1.32 | 2.59 |
| 384 | 4.91 | 0.08 | 44.96 | 1.10 | 1.57 | 47.95 | 1.11 | 2.89 | 5.19 | 0.38 | 2.39 | 95.31 | 1.32 | 2.58 |
| | | 0.07 | | 1.09 | | | 1.12 | | | 0.36 | | | 1.33 | |
| 385 | 4.84 | | 43.87 | | 1.58 | 49.07 | | 2.89 | 4.83 | | 2.40 | 96.64 | | 2.57 |
| 386 | 4.77 | -0.07 | 42.80 | -1.07 | 1.60 | 50.20 | +1.13 | 2.89 | 4.48 | -0.35 | 2.42 | 97.97 | +1.33 | 2.57 |
| 387 | 4.71 | 0.06 | 41.74 | 1.06 | 1.61 | 51.34 | 1.14 | 2.88 | 4.15 | 0.33 | 2.43 | 99.30 | 1.33 | 2.56 |
| 388 | 4.65 | 0.06 | 40.69 | 1.05 | 1.63 | 52.50 | 1.16 | 2.88 | 3.83 | 0.32 | 2.44 | 100.64 | 1.34 | 2.55 |
| 389 | 4.60 | 0.05 | 39.65 | 1.04 | 1.64 | 53.67 | 1.17 | 2.88 | 3.52 | 0.31 | 2.45 | 101.98 | 1.34 | 2.54 |
| | | 0.04 | | 1.02 | | | 1.18 | | | 0.29 | | | 1.35 | |
| 390 | 4.56 | | 38.63 | | 1.66 | 54.85 | | 2.88 | 3.23 | | 2.46 | 103.33 | | 2.53 |
| 391 | 4.52 | -0.04 | 37.62 | -1.01 | 1.67 | 56.04 | +1.19 | 2.88 | 2.96 | -0.27 | 2.47 | 104.68 | +1.35 | 2.52 |
| 392 | 4.48 | 0.04 | 36.63 | 0.99 | 1.69 | 57.25 | 1.21 | 2.88 | 2.70 | 0.26 | 2.48 | 106.03 | 1.35 | 2.51 |
| 393 | 4.45 | 0.03 | 35.65 | 0.98 | 1.70 | 58.46 | 1.21 | 2.87 | 2.45 | 0.25 | 2.49 | 107.38 | 1.35 | 2.50 |
| 394 | 4.41 | 0.04 | 34.67 | 0.98 | 1.71 | 59.68 | 1.22 | 2.87 | 2.22 | 0.23 | 2.50 | 108.74 | 1.36 | 2.49 |
| | | 0.03 | | 0.96 | | | 1.23 | | | 0.21 | | | 1.36 | |
| 395 | 4.38 | | 33.71 | | 1.73 | 60.91 | | 2.87 | 2.01 | | 2.51 | 110.10 | | 2.47 |
| 396 | 4.35 | -0.03 | 32.77 | -0.94 | 1.74 | 62.15 | +1.24 | 2.87 | 1.81 | -0.20 | 2.52 | 111.46 | +1.36 | 2.46 |
| 397 | 4.33 | 0.02 | 31.85 | 0.92 | 1.75 | 63.40 | 1.25 | 2.87 | 1.62 | 0.19 | 2.53 | 112.82 | 1.36 | 2.45 |
| 398 | 4.31 | 0.02 | 30.94 | 0.91 | 1.76 | 64.67 | 1.27 | 2.86 | 1.45 | 0.17 | 2.54 | 114.18 | 1.36 | 2.44 |
| 399 | 4.30 | 0.01 | 30.03 | 0.91 | 1.78 | 65.95 | 1.28 | 2.86 | 1.29 | 0.16 | 2.55 | 115.55 | 1.37 | 2.43 |
| | | 0.01 | | 0.89 | | | 1.28 | | | 0.15 | | | 1.37 | |
| 400 | 4.29 | | 29.14 | | 1.79 | 67.23 | | 2.86 | 1.14 | | 2.56 | 116.92 | | 2.42 |
| 401 | 4.28 | -0.01 | 28.27 | -0.87 | 1.80 | 68.52 | +1.29 | 2.86 | 1.01 | -0.13 | 2.57 | 118.29 | +1.37 | 2.41 |
| 402 | 4.28 | 0.00 | 27.42 | 0.85 | 1.82 | 69.82 | 1.30 | 2.85 | 0.90 | 0.11 | 2.58 | 119.66 | 1.37 | 2.40 |
| 403 | 4.28 | 0.00 | 26.57 | 0.85 | 1.83 | 71.13 | 1.31 | 2.85 | 0.80 | 0.10 | 2.59 | 121.03 | 1.37 | 2.39 |
| 404 | 4.28 | 0.00 | 25.73 | 0.84 | 1.85 | 72.45 | 1.32 | 2.84 | 0.72 | 0.08 | 2.60 | 122.41 | 1.38 | 2.38 |
| | | +0.01 | | 0.82 | | | 1.33 | | | 0.07 | | | 1.37 | |
| 405 | 4.29 | | 24.91 | | 1.86 | 73.78 | | 2.84 | 0.65 | | 2.60 | 123.78 | | 2.36 |
| 406 | 4.30 | +0.01 | 24.11 | -0.80 | 1.88 | 75.12 | +1.34 | 2.84 | 0.60 | -0.05 | 2.61 | 125.16 | +1.38 | 2.35 |
| 407 | 4.32 | 0.02 | 23.33 | 0.78 | 1.89 | 76.47 | 1.35 | 2.83 | 0.56 | 0.04 | 2.62 | 126.53 | 1.37 | 2.34 |
| 408 | 4.34 | 0.02 | 22.56 | 0.77 | 1.91 | 77.83 | 1.36 | 2.83 | 0.53 | 0.03 | 2.63 | 127.91 | 1.38 | 2.33 |
| 409 | 4.36 | 0.02 | 21.80 | 0.76 | 1.92 | 79.19 | 1.36 | 2.82 | 0.52 | 0.01 | 2.64 | 129.28 | 1.37 | 2.32 |
| | | 0.03 | | 0.74 | | | 1.37 | | | 0.00 | | | 1.37 | |
| 410 | 4.39 | | 21.06 | | 1.94 | 80.56 | | 2.82 | 0.52 | | 2.65 | 130.65 | | 2.31 |
| 411 | 4.42 | +0.03 | 20.33 | -0.73 | 1.95 | 81.94 | +1.38 | 2.82 | 0.54 | +0.02 | 2.66 | 132.02 | +1.37 | 2.30 |
| 412 | 4.46 | 0.04 | 19.62 | 0.71 | 1.97 | 83.33 | 1.39 | 2.81 | 0.57 | 0.03 | 2.67 | 133.40 | 1.38 | 2.29 |
| 413 | 4.50 | 0.04 | 18.93 | 0.69 | 1.98 | 84.73 | 1.40 | 2.81 | 0.62 | 0.05 | 2.67 | 134.77 | 1.37 | 2.27 |
| 414 | 4.54 | 0.04 | 18.25 | 0.68 | 1.99 | 86.14 | 1.41 | 2.80 | 0.68 | 0.06 | 2.68 | 136.14 | 1.37 | 2.26 |
| | | 0.05 | | 0.67 | | | 1.41 | | | 0.08 | | | 1.37 | |
| 415 | 4.59 | | 17.58 | | 2.01 | 87.55 | | 2.80 | 0.76 | | 2.69 | 137.51 | | 2.25 |
| 416 | 4.64 | +0.05 | 16.93 | -0.65 | 2.02 | 88.97 | +1.42 | 2.79 | 0.85 | +0.09 | 2.70 | 138.88 | +1.37 | 2.24 |
| 417 | 4.70 | 0.06 | 16.30 | 0.63 | 2.03 | 90.39 | 1.42 | 2.79 | 0.96 | 0.11 | 2.71 | 140.25 | 1.37 | 2.22 |
| 418 | 4.76 | 0.06 | 15.67 | 0.63 | 2.04 | 91.82 | 1.43 | 2.78 | 1.08 | 0.12 | 2.71 | 141.62 | 1.37 | 2.21 |
| 419 | 4.82 | 0.06 | 15.06 | 0.61 | 2.06 | 93.26 | 1.44 | 2.78 | 1.22 | 0.14 | 2.72 | 142.99 | 1.37 | 2.20 |
| | | 0.06 | | 0.59 | | | 1.45 | | | 0.15 | | | 1.36 | |
| 420 | 4.88 | | 14.47 | | 2.07 | 94.71 | | 2.77 | 1.37 | | 2.73 | 144.35 | | 2.19 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | (p.s.3) | (p.c.3) |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 360 | 6.07 | 2.53 | 1.23 | 0.38 | 146 | 2629 | 710 | 252 | 627 | 67 | 181 |
| 361 | 6.02 | 2.51 | 1.22 | 0.38 | 148 | 2621 | 697 | 254 | 628 | 68 | 181 |
| 362 | 5.98 | 2.50 | 1.21 | 0.37 | 151 | 2613 | 685 | 256 | 629 | 69 | 182 |
| 363 | 5.94 | 2.49 | 1.20 | 0.36 | 153 | 2605 | 672 | 259 | 631 | 70 | 182 |
| 364 | 5.90 | 2.48 | 1.19 | 0.35 | 156 | 2596 | 660 | 261 | 632 | 71 | 182 |
| 365 | 5.85 | 2.46 | 1.17 | 0.35 | 159 | 2588 | 647 | 263 | 633 | 72 | 183 |
| 366 | 5.81 | 2.45 | 1.16 | 0.34 | 162 | 2579 | 635 | 265 | 634 | 74 | 183 |
| 367 | 5.76 | 2.44 | 1.15 | 0.34 | 164 | 2570 | 623 | 267 | 635 | 75 | 183 |
| 368 | 5.72 | 2.44 | 1.14 | 0.33 | 167 | 2562 | 610 | 270 | 636 | 76 | 183 |
| 369 | 5.67 | 2.43 | 1.12 | 0.32 | 170 | 2553 | 598 | 272 | 638 | 77 | 184 |
| 370 | 5.63 | 2.42 | 1.11 | 0.32 | 173 | 2544 | 586 | 274 | 639 | 78 | 184 |
| 371 | 5.58 | 2.41 | 1.10 | 0.31 | 176 | 2535 | 574 | 276 | 640 | 79 | 184 |
| 372 | 5.53 | 2.41 | 1.08 | 0.31 | 180 | 2526 | 562 | 279 | 641 | 80 | 184 |
| 373 | 5.48 | 2.40 | 1.07 | 0.30 | 183 | 2516 | 550 | 281 | 642 | 81 | 184 |
| 374 | 5.43 | 2.39 | 1.05 | 0.30 | 186 | 2507 | 539 | 283 | 643 | 82 | 184 |
| 375 | 5.38 | 2.39 | 1.04 | 0.29 | 190 | 2498 | 528 | 285 | 645 | 83 | 184 |
| 376 | 5.33 | 2.39 | 1.03 | 0.29 | 194 | 2488 | 517 | 288 | 646 | 84 | 184 |
| 377 | 5.28 | 2.38 | 1.01 | 0.29 | 197 | 2479 | 506 | 290 | 647 | 85 | 184 |
| 378 | 5.22 | 2.38 | 1.00 | 0.28 | 201 | 2469 | 495 | 292 | 648 | 86 | 184 |
| 379 | 5.17 | 2.38 | 0.98 | 0.28 | 205 | 2460 | 485 | 294 | 649 | 87 | 184 |
| 380 | 5.12 | 2.38 | 0.97 | 0.28 | 208 | 2450 | 474 | 297 | 650 | 88 | 184 |
| 381 | 5.07 | 2.38 | 0.96 | 0.28 | 212 | 2440 | 463 | 299 | 651 | 89 | 184 |
| 382 | 5.01 | 2.38 | 0.94 | 0.27 | 216 | 2429 | 453 | 302 | 652 | 90 | 184 |
| 383 | 4.96 | 2.39 | 0.93 | 0.27 | 220 | 2419 | 443 | 304 | 653 | 91 | 184 |
| 384 | 4.90 | 2.39 | 0.91 | 0.27 | 224 | 2408 | 433 | 306 | 654 | 92 | 184 |
| 385 | 4.84 | 2.39 | 0.90 | 0.27 | 228 | 2397 | 423 | 309 | 655 | 94 | 184 |
| 386 | 4.79 | 2.40 | 0.88 | 0.27 | 232 | 2386 | 413 | 312 | 656 | 95 | 184 |
| 387 | 4.73 | 2.40 | 0.86 | 0.27 | 236 | 2374 | 404 | 314 | 657 | 96 | 184 |
| 388 | 4.67 | 2.41 | 0.85 | 0.27 | 240 | 2363 | 394 | 317 | 657 | 97 | 183 |
| 389 | 4.61 | 2.42 | 0.84 | 0.27 | 245 | 2351 | 384 | 319 | 658 | 98 | 183 |
| 390 | 4.55 | 2.43 | 0.82 | 0.27 | 249 | 2340 | 375 | 322 | 659 | 99 | 183 |
| 391 | 4.49 | 2.44 | 0.81 | 0.27 | 253 | 2328 | 366 | 324 | 660 | 100 | 183 |
| 392 | 4.43 | 2.46 | 0.79 | 0.27 | 258 | 2316 | 357 | 327 | 661 | 102 | 183 |
| 393 | 4.37 | 2.47 | 0.78 | 0.27 | 262 | 2305 | 348 | 329 | 662 | 103 | 182 |
| 394 | 4.31 | 2.48 | 0.76 | 0.27 | 266 | 2293 | 339 | 332 | 662 | 104 | 182 |
| 395 | 4.24 | 2.49 | 0.75 | 0.27 | 271 | 2281 | 330 | 334 | 663 | 105 | 182 |
| 396 | 4.18 | 2.51 | 0.73 | 0.28 | 276 | 2269 | 322 | 336 | 664 | 106 | 182 |
| 397 | 4.12 | 2.53 | 0.72 | 0.28 | 280 | 2257 | 313 | 339 | 665 | 107 | 182 |
| 398 | 4.06 | 2.55 | 0.70 | 0.28 | 285 | 2245 | 304 | 341 | 666 | 109 | 181 |
| 399 | 4.00 | 2.57 | 0.69 | 0.28 | 290 | 2232 | 296 | 344 | 667 | 110 | 181 |
| 400 | 3.93 | 2.59 | 0.67 | 0.29 | 294 | 2220 | 288 | 346 | 667 | 111 | 181 |
| 401 | 3.87 | 2.62 | 0.66 | 0.29 | 299 | 2207 | 280 | 348 | 668 | 112 | 181 |
| 402 | 3.80 | 2.64 | 0.65 | 0.30 | 304 | 2195 | 273 | 351 | 668 | 113 | 180 |
| 403 | 3.74 | 2.66 | 0.63 | 0.30 | 309 | 2182 | 265 | 353 | 669 | 114 | 180 |
| 404 | 3.68 | 2.69 | 0.62 | 0.31 | 314 | 2170 | 258 | 355 | 669 | 115 | 180 |
| 405 | 3.62 | 2.72 | 0.60 | 0.31 | 320 | 2157 | 251 | 358 | 669 | 116 | 179 |
| 406 | 3.55 | 2.75 | 0.59 | 0.31 | 325 | 2144 | 244 | 361 | 669 | 116 | 179 |
| 407 | 3.49 | 2.78 | 0.57 | 0.32 | 331 | 2131 | 237 | 363 | 670 | 117 | 179 |
| 408 | 3.43 | 2.81 | 0.56 | 0.33 | 337 | 2119 | 230 | 365 | 670 | 118 | 178 |
| 409 | 3.36 | 2.84 | 0.55 | 0.33 | 342 | 2106 | 224 | 368 | 670 | 119 | 178 |
| 410 | 3.30 | 2.88 | 0.53 | 0.34 | 348 | 2093 | 217 | 371 | 670 | 120 | 178 |
| 411 | 3.24 | 2.92 | 0.52 | 0.35 | 354 | 2080 | 211 | 373 | 670 | 121 | 177 |
| 412 | 3.17 | 2.95 | 0.50 | 0.36 | 359 | 2066 | 205 | 376 | 670 | 122 | 177 |
| 413 | 3.11 | 2.99 | 0.49 | 0.36 | 365 | 2053 | 199 | 378 | 671 | 122 | 176 |
| 414 | 3.04 | 3.03 | 0.48 | 0.37 | 371 | 2039 | 193 | 381 | 671 | 123 | 176 |
| 415 | 2.98 | 3.07 | 0.46 | 0.38 | 377 | 2026 | 187 | 382 | 671 | 124 | 175 |
| 416 | 2.92 | 3.11 | 0.45 | 0.38 | 383 | 2012 | 181 | 386 | 671 | 125 | 174 |
| 417 | 2.85 | 3.15 | 0.44 | 0.39 | 389 | 1998 | 176 | 388 | 671 | 126 | 174 |
| 418 | 2.79 | 3.20 | 0.42 | 0.40 | 395 | 1985 | 170 | 391 | 671 | 126 | 173 |
| 419 | 2.73 | 3.24 | 0.41 | 0.41 | 401 | 1971 | 165 | 393 | 672 | 127 | 173 |
| 420 | 2.67 | 3.29 | 0.40 | 0.42 | 407 | 1957 | 160 | 396 | 672 | 128 | 172 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | Sec.var. | (v.c.1) | Diff. | Sec.var. | (v.s.2) | Diff. | Sec.var. | (v.c.2) | Diff. | Sec.var. |
|------|---------|-------|---------|-------|----------|---------|-------|----------|---------|-------|----------|---------|-------|----------|
| | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 420 | 4.88 | | 14.47 | | 2.07 | 94.71 | | 2.77 | 1.37 | | 2.73 | 144.35 | | 2.19 |
| 421 | 4.95 | +0.07 | 13.90 | -0.57 | 2.08 | 96.16 | +1.45 | 2.76 | 1.54 | +0.17 | 2.74 | 145.72 | +1.37 | 2.18 |
| 422 | 5.02 | 0.07 | 13.35 | 0.55 | 2.10 | 97.62 | 1.46 | 2.76 | 1.72 | 0.18 | 2.74 | 147.09 | 1.37 | 2.16 |
| 423 | 5.10 | 0.08 | 12.81 | 0.54 | 2.11 | 99.09 | 1.47 | 2.75 | 1.92 | 0.20 | 2.75 | 148.45 | 1.36 | 2.15 |
| 424 | 5.18 | 0.08 | 12.28 | 0.53 | 2.13 | 100.56 | 1.47 | 2.75 | 2.13 | 0.21 | 2.75 | 149.80 | 1.35 | 2.13 |
| | | 0.08 | | 0.51 | | | 1.48 | | | 0.22 | | | 1.35 | |
| 425 | 5.26 | | 11.77 | | 2.14 | 102.04 | | 2.74 | 2.35 | | 2.76 | 151.15 | | 2.12 |
| 426 | 5.35 | +0.09 | 11.28 | -0.49 | 2.15 | 103.52 | +1.48 | 2.73 | 2.59 | +0.24 | 2.77 | 152.50 | +1.35 | 2.11 |
| 427 | 5.44 | 0.09 | 10.80 | 0.48 | 2.17 | 105.01 | 1.49 | 2.73 | 2.84 | 0.25 | 2.77 | 153.84 | 1.34 | 2.09 |
| 428 | 5.53 | 0.09 | 10.34 | 0.46 | 2.18 | 106.50 | 1.49 | 2.72 | 3.11 | 0.27 | 2.78 | 155.19 | 1.35 | 2.08 |
| 429 | 5.63 | 0.10 | 9.89 | 0.45 | 2.20 | 108.00 | 1.50 | 2.72 | 3.39 | 0.28 | 2.78 | 156.54 | 1.35 | 2.06 |
| | | 0.10 | | 0.43 | | | 1.51 | | | 0.30 | | | 1.34 | |
| 430 | 5.73 | | 9.46 | | 2.21 | 109.51 | | 2.71 | 3.69 | | 2.79 | 157.88 | | 2.05 |
| 431 | 5.83 | +0.10 | 9.05 | -0.41 | 2.22 | 111.02 | +1.51 | 2.70 | 4.00 | +0.31 | 2.79 | 159.22 | +1.34 | 2.04 |
| 432 | 5.94 | 0.11 | 8.66 | 0.39 | 2.23 | 112.53 | 1.51 | 2.69 | 4.33 | 0.33 | 2.80 | 160.55 | 1.33 | 2.02 |
| 433 | 6.05 | 0.11 | 8.28 | 0.38 | 2.24 | 114.04 | 1.51 | 2.68 | 4.67 | 0.34 | 2.80 | 161.88 | 1.33 | 2.01 |
| 434 | 6.17 | 0.12 | 7.91 | 0.37 | 2.25 | 115.56 | 1.52 | 2.67 | 5.02 | 0.35 | 2.81 | 163.20 | 1.32 | 1.99 |
| | | 0.12 | | 0.35 | | | 1.53 | | | 0.37 | | | 1.32 | |
| 435 | 6.29 | | 7.56 | | 2.27 | 117.09 | | 2.66 | 5.39 | | 2.81 | 164.52 | | 1.98 |
| 436 | 6.41 | +0.12 | 7.23 | -0.33 | 2.28 | 118.62 | +1.53 | 2.66 | 5.77 | +0.38 | 2.82 | 165.84 | +1.32 | 1.97 |
| 437 | 6.54 | 0.13 | 6.92 | 0.31 | 2.29 | 120.16 | 1.54 | 2.65 | 6.17 | 0.40 | 2.82 | 167.15 | 1.31 | 1.95 |
| 438 | 6.67 | 0.13 | 6.62 | 0.30 | 2.30 | 121.70 | 1.54 | 2.64 | 6.58 | 0.41 | 2.83 | 168.46 | 1.31 | 1.94 |
| 439 | 6.80 | 0.13 | 6.34 | 0.28 | 2.31 | 123.24 | 1.54 | 2.63 | 7.00 | 0.42 | 2.83 | 169.76 | 1.30 | 1.92 |
| | | 0.14 | | 0.27 | | | 1.54 | | | 0.44 | | | 1.30 | |
| 440 | 6.94 | | 6.07 | | 2.32 | 124.78 | | 2.62 | 7.44 | | 2.84 | 171.06 | | 1.91 |
| 441 | 7.08 | +0.14 | 5.82 | -0.25 | 2.33 | 126.33 | +1.55 | 2.61 | 7.89 | +0.45 | 2.84 | 172.35 | +1.29 | 1.90 |
| 442 | 7.22 | 0.14 | 5.59 | 0.23 | 2.34 | 127.88 | 1.55 | 2.60 | 8.36 | 0.47 | 2.85 | 173.64 | 1.29 | 1.88 |
| 443 | 7.37 | 0.15 | 5.38 | 0.21 | 2.36 | 129.43 | 1.55 | 2.59 | 8.84 | 0.48 | 2.85 | 174.93 | 1.29 | 1.87 |
| 444 | 7.52 | 0.15 | 5.18 | 0.20 | 2.37 | 130.98 | 1.55 | 2.58 | 9.33 | 0.49 | 2.86 | 176.21 | 1.28 | 1.85 |
| | | 0.15 | | 0.18 | | | 1.56 | | | 0.51 | | | 1.27 | |
| 445 | 7.67 | | 5.00 | | 2.38 | 132.54 | | 2.57 | 9.84 | | 2.86 | 177.48 | | 1.84 |
| 446 | 7.83 | +0.16 | 4.83 | -0.17 | 2.39 | 134.10 | +1.56 | 2.57 | 10.36 | +0.52 | 2.86 | 178.75 | +1.27 | 1.83 |
| 447 | 7.99 | 0.16 | 4.68 | 0.15 | 2.40 | 135.66 | 1.56 | 2.56 | 10.89 | 0.53 | 2.87 | 180.01 | 1.26 | 1.81 |
| 448 | 8.15 | 0.16 | 4.55 | 0.13 | 2.42 | 137.22 | 1.56 | 2.55 | 11.44 | 0.55 | 2.87 | 181.26 | 1.25 | 1.80 |
| 449 | 8.31 | 0.16 | 4.43 | 0.12 | 2.43 | 138.78 | 1.56 | 2.54 | 12.00 | 0.56 | 2.88 | 182.51 | 1.25 | 1.78 |
| | | 0.17 | | 0.10 | | | 1.57 | | | 0.57 | | | 1.24 | |
| 450 | 8.48 | | 4.33 | | 2.44 | 140.35 | | 2.53 | 12.57 | | 2.88 | 183.75 | | 1.77 |
| 451 | 8.65 | +0.17 | 4.25 | -0.08 | 2.45 | 141.92 | +1.57 | 2.52 | 13.16 | +0.59 | 2.88 | 184.99 | +1.24 | 1.76 |
| 452 | 8.82 | 0.17 | 4.19 | 0.06 | 2.46 | 143.49 | 1.57 | 2.51 | 13.76 | 0.60 | 2.88 | 186.22 | 1.23 | 1.74 |
| 453 | 9.00 | 0.18 | 4.14 | 0.05 | 2.47 | 145.06 | 1.57 | 2.50 | 14.37 | 0.61 | 2.89 | 187.44 | 1.22 | 1.73 |
| 454 | 9.18 | 0.18 | 4.10 | 0.04 | 2.48 | 146.63 | 1.57 | 2.49 | 14.99 | 0.62 | 2.89 | 188.66 | 1.22 | 1.71 |
| | | 0.18 | | 0.01 | | | 1.57 | | | 0.64 | | | 1.21 | |
| 455 | 9.36 | | 4.09 | | 2.49 | 148.20 | | 2.47 | 15.63 | | 2.89 | 189.87 | | 1.70 |
| 456 | 9.54 | +0.18 | 4.09 | 0.00 | 2.50 | 149.77 | +1.57 | 2.46 | 16.28 | +0.65 | 2.89 | 191.07 | +1.20 | 1.69 |
| 457 | 9.73 | 0.19 | 4.11 | +0.02 | 2.51 | 151.34 | 1.57 | 2.45 | 16.95 | 0.67 | 2.89 | 192.27 | 1.20 | 1.67 |
| 458 | 9.92 | 0.19 | 4.14 | 0.03 | 2.52 | 152.91 | 1.57 | 2.44 | 17.63 | 0.68 | 2.90 | 193.46 | 1.19 | 1.66 |
| 459 | 10.11 | 0.19 | 4.18 | 0.04 | 2.53 | 154.48 | 1.57 | 2.43 | 18.32 | 0.69 | 2.90 | 194.64 | 1.18 | 1.64 |
| | | 0.19 | | 0.07 | | | 1.57 | | | 0.70 | | | 1.17 | |
| 460 | 10.30 | | 4.25 | | 2.54 | 156.05 | | 2.42 | 19.02 | | 2.90 | 195.81 | | 1.63 |
| 461 | 10.50 | +0.20 | 4.33 | +0.08 | 2.55 | 157.62 | +1.57 | 2.41 | 19.73 | +0.71 | 2.90 | 196.98 | +1.17 | 1.61 |
| 462 | 10.70 | 0.20 | 4.43 | 0.10 | 2.56 | 159.19 | 1.57 | 2.40 | 20.45 | 0.72 | 2.90 | 198.14 | 1.16 | 1.60 |
| 463 | 10.90 | 0.20 | 4.55 | 0.12 | 2.57 | 160.76 | 1.57 | 2.39 | 21.19 | 0.74 | 2.90 | 199.29 | 1.15 | 1.58 |
| 464 | 11.10 | 0.20 | 4.68 | 0.13 | 2.58 | 162.33 | 1.57 | 2.38 | 21.94 | 0.75 | 2.90 | 200.43 | 1.14 | 1.57 |
| | | 0.21 | | 0.15 | | | 1.56 | | | 0.77 | | | 1.13 | |
| 465 | 11.31 | | 4.83 | | 2.59 | 163.89 | | 2.36 | 22.71 | | 2.91 | 201.56 | | 1.55 |
| 466 | 11.52 | +0.21 | 4.99 | +0.16 | 2.59 | 165.45 | +1.56 | 2.35 | 23.48 | +0.77 | 2.91 | 202.68 | +1.12 | 1.53 |
| 467 | 11.73 | 0.21 | 5.17 | 0.18 | 2.60 | 167.01 | 1.56 | 2.34 | 24.27 | 0.79 | 2.91 | 203.79 | 1.11 | 1.52 |
| 468 | 11.94 | 0.21 | 5.37 | 0.20 | 2.61 | 168.57 | 1.56 | 2.33 | 25.07 | 0.80 | 2.91 | 204.90 | 1.11 | 1.50 |
| 469 | 12.15 | 0.21 | 5.59 | 0.22 | 2.62 | 170.13 | 1.56 | 2.32 | 25.88 | 0.81 | 2.91 | 206.00 | 1.10 | 1.49 |
| | | 0.22 | | 0.23 | | | 1.55 | | | 0.82 | | | 1.09 | |
| 470 | 12.37 | | 5.82 | | 2.63 | 171.68 | | 2.31 | 26.70 | | 2.91 | 207.09 | | 1.47 |
| 471 | 12.59 | +0.22 | 6.07 | +0.25 | 2.64 | 173.23 | +1.55 | 2.30 | 27.53 | +0.83 | 2.91 | 208.17 | +1.08 | 1.46 |
| 472 | 12.81 | 0.22 | 6.34 | 0.27 | 2.65 | 174.78 | 1.55 | 2.29 | 28.37 | 0.84 | 2.91 | 209.24 | 1.07 | 1.44 |
| 473 | 13.03 | 0.22 | 6.62 | 0.28 | 2.65 | 176.33 | 1.55 | 2.27 | 29.22 | 0.85 | 2.90 | 210.30 | 1.06 | 1.43 |
| 474 | 13.26 | 0.23 | 6.91 | 0.29 | 2.66 | 177.87 | 1.54 | 2.26 | 30.09 | 0.87 | 2.90 | 211.36 | 1.06 | 1.41 |
| | | 0.23 | | 0.31 | | | 1.54 | | | 0.88 | | | 1.04 | |
| 475 | 13.49 | | 7.22 | | 2.67 | 179.41 | | 2.25 | 30.97 | | 2.90 | 212.40 | | 1.40 |
| 476 | 13.72 | +0.23 | 7.55 | +0.33 | 2.68 | 180.95 | +1.54 | 2.24 | 31.86 | +0.89 | 2.90 | 213.43 | +1.03 | 1.39 |
| 477 | 13.95 | 0.23 | 7.90 | 0.35 | 2.69 | 182.48 | 1.53 | 2.23 | 32.76 | 0.90 | 2.90 | 214.45 | 1.02 | 1.37 |
| 478 | 14.18 | 0.23 | 8.26 | 0.36 | 2.69 | 184.01 | 1.53 | 2.21 | 33.67 | 0.91 | 2.89 | 215.47 | 1.02 | 1.36 |
| 479 | 14.41 | 0.23 | 8.63 | 0.37 | 2.70 | 185.54 | 1.53 | 2.20 | 34.59 | 0.92 | 2.89 | 216.48 | 1.01 | 1.34 |
| | | 0.24 | | 0.39 | | | 1.52 | | | 0.93 | | | 0.99 | |
| 480 | 14.65 | | 9.02 | | 2.71 | 187.06 | | 2.19 | 35.52 | | 2.89 | 217.47 | | 1.33 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | (p.s.3) | (p.c.3) |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| " | " | " | " | " | " | " | " | " | " | " | " |
| 420 | 2.67 | 3.29 | 0.40 | 0.42 | 407 | 1957 | 160 | 396 | 672 | 128 | 172 |
| 421 | 2.61 | 3.34 | 0.39 | 0.43 | 413 | 1943 | 155 | 399 | 672 | 129 | 172 |
| 422 | 2.55 | 3.39 | 0.38 | 0.44 | 419 | 1930 | 150 | 402 | 673 | 129 | 171 |
| 423 | 2.49 | 3.44 | 0.36 | 0.45 | 425 | 1916 | 146 | 405 | 673 | 130 | 171 |
| 424 | 2.43 | 3.50 | 0.35 | 0.46 | 431 | 1903 | 141 | 407 | 673 | 131 | 171 |
| 425 | 2.37 | 3.55 | 0.34 | 0.47 | 437 | 1889 | 136 | 410 | 673 | 132 | 170 |
| 426 | 2.31 | 3.61 | 0.33 | 0.49 | 443 | 1875 | 132 | 413 | 673 | 133 | 170 |
| 427 | 2.25 | 3.66 | 0.32 | 0.50 | 450 | 1862 | 128 | 416 | 673 | 133 | 169 |
| 428 | 2.20 | 3.72 | 0.31 | 0.51 | 456 | 1848 | 124 | 418 | 673 | 134 | 169 |
| 429 | 2.14 | 3.78 | 0.30 | 0.52 | 463 | 1834 | 120 | 421 | 673 | 135 | 168 |
| 430 | 2.08 | 3.84 | 0.29 | 0.53 | 469 | 1820 | 117 | 424 | 673 | 136 | 167 |
| 431 | 2.03 | 3.90 | 0.28 | 0.54 | 476 | 1806 | 114 | 426 | 673 | 136 | 167 |
| 432 | 1.97 | 3.96 | 0.27 | 0.55 | 482 | 1791 | 111 | 429 | 673 | 137 | 166 |
| 433 | 1.92 | 4.03 | 0.26 | 0.57 | 489 | 1777 | 108 | 432 | 673 | 138 | 165 |
| 434 | 1.86 | 4.10 | 0.25 | 0.58 | 496 | 1762 | 105 | 434 | 673 | 139 | 165 |
| 435 | 1.81 | 4.16 | 0.24 | 0.59 | 503 | 1748 | 102 | 437 | 672 | 139 | 164 |
| 436 | 1.76 | 4.23 | 0.23 | 0.60 | 510 | 1733 | 100 | 440 | 672 | 140 | 163 |
| 437 | 1.70 | 4.30 | 0.22 | 0.62 | 517 | 1719 | 98 | 442 | 672 | 141 | 163 |
| 438 | 1.65 | 4.37 | 0.22 | 0.63 | 524 | 1704 | 96 | 445 | 671 | 142 | 162 |
| 439 | 1.60 | 4.45 | 0.21 | 0.64 | 531 | 1690 | 94 | 447 | 671 | 142 | 161 |
| 440 | 1.55 | 4.52 | 0.20 | 0.66 | 539 | 1675 | 92 | 450 | 671 | 143 | 160 |
| 441 | 1.50 | 4.60 | 0.19 | 0.67 | 546 | 1660 | 91 | 453 | 670 | 144 | 159 |
| 442 | 1.46 | 4.67 | 0.18 | 0.68 | 553 | 1645 | 90 | 455 | 670 | 144 | 159 |
| 443 | 1.41 | 4.75 | 0.18 | 0.70 | 560 | 1631 | 89 | 458 | 669 | 145 | 158 |
| 444 | 1.37 | 4.83 | 0.17 | 0.71 | 567 | 1616 | 88 | 460 | 669 | 145 | 157 |
| 445 | 1.32 | 4.91 | 0.17 | 0.73 | 574 | 1601 | 87 | 463 | 669 | 146 | 156 |
| 446 | 1.28 | 4.99 | 0.16 | 0.74 | 582 | 1586 | 87 | 466 | 668 | 146 | 156 |
| 447 | 1.24 | 5.07 | 0.16 | 0.75 | 589 | 1571 | 86 | 468 | 668 | 147 | 155 |
| 448 | 1.20 | 5.15 | 0.15 | 0.77 | 596 | 1557 | 86 | 471 | 667 | 147 | 154 |
| 449 | 1.16 | 5.24 | 0.14 | 0.79 | 603 | 1542 | 86 | 473 | 667 | 147 | 153 |
| 450 | 1.12 | 5.32 | 0.14 | 0.80 | 610 | 1527 | 86 | 476 | 667 | 148 | 152 |
| 451 | 1.08 | 5.40 | 0.13 | 0.82 | 618 | 1512 | 87 | 479 | 666 | 148 | 152 |
| 452 | 1.04 | 5.49 | 0.13 | 0.83 | 625 | 1497 | 87 | 481 | 666 | 149 | 151 |
| 453 | 1.01 | 5.57 | 0.12 | 0.85 | 633 | 1483 | 88 | 484 | 665 | 149 | 150 |
| 454 | 0.97 | 5.66 | 0.12 | 0.86 | 640 | 1468 | 88 | 486 | 665 | 149 | 149 |
| 455 | 0.94 | 5.75 | 0.12 | 0.88 | 648 | 1453 | 89 | 489 | 664 | 150 | 148 |
| 456 | 0.91 | 5.84 | 0.11 | 0.89 | 656 | 1438 | 90 | 492 | 664 | 150 | 148 |
| 457 | 0.88 | 5.94 | 0.11 | 0.90 | 664 | 1423 | 92 | 494 | 663 | 150 | 147 |
| 458 | 0.85 | 6.03 | 0.11 | 0.92 | 671 | 1409 | 93 | 497 | 662 | 151 | 146 |
| 459 | 0.82 | 6.13 | 0.10 | 0.94 | 679 | 1394 | 95 | 499 | 661 | 151 | 145 |
| 460 | 0.80 | 6.23 | 0.10 | 0.95 | 687 | 1379 | 96 | 502 | 661 | 151 | 144 |
| 461 | 0.77 | 6.32 | 0.10 | 0.97 | 695 | 1364 | 98 | 505 | 660 | 151 | 144 |
| 462 | 0.75 | 6.42 | 0.10 | 0.99 | 703 | 1349 | 100 | 507 | 660 | 152 | 143 |
| 463 | 0.73 | 6.51 | 0.10 | 1.00 | 711 | 1335 | 102 | 510 | 659 | 152 | 142 |
| 464 | 0.71 | 6.61 | 0.10 | 1.02 | 719 | 1320 | 105 | 512 | 658 | 152 | 142 |
| 465 | 0.69 | 6.71 | 0.09 | 1.03 | 727 | 1305 | 108 | 515 | 657 | 152 | 141 |
| 466 | 0.68 | 6.81 | 0.09 | 1.05 | 735 | 1290 | 110 | 517 | 657 | 153 | 140 |
| 467 | 0.66 | 6.91 | 0.09 | 1.06 | 743 | 1275 | 113 | 520 | 656 | 153 | 139 |
| 468 | 0.64 | 7.01 | 0.09 | 1.08 | 751 | 1261 | 116 | 523 | 655 | 153 | 139 |
| 469 | 0.63 | 7.11 | 0.09 | 1.10 | 759 | 1246 | 119 | 525 | 654 | 154 | 138 |
| 470 | 0.62 | 7.21 | 0.09 | 1.11 | 767 | 1232 | 123 | 528 | 653 | 154 | 137 |
| 471 | 0.62 | 7.31 | 0.10 | 1.13 | 775 | 1217 | 127 | 531 | 652 | 154 | 136 |
| 472 | 0.61 | 7.42 | 0.10 | 1.15 | 784 | 1203 | 131 | 533 | 651 | 155 | 136 |
| 473 | 0.60 | 7.52 | 0.10 | 1.16 | 792 | 1189 | 135 | 536 | 649 | 155 | 135 |
| 474 | 0.60 | 7.62 | 0.10 | 1.18 | 800 | 1175 | 139 | 539 | 648 | 155 | 134 |
| 475 | 0.60 | 7.72 | 0.10 | 1.19 | 809 | 1161 | 143 | 542 | 647 | 156 | 133 |
| 476 | 0.60 | 7.83 | 0.11 | 1.21 | 817 | 1147 | 148 | 545 | 646 | 156 | 133 |
| 477 | 0.60 | 7.94 | 0.11 | 1.23 | 826 | 1133 | 153 | 548 | 644 | 156 | 132 |
| 478 | 0.60 | 8.04 | 0.11 | 1.24 | 834 | 1119 | 158 | 551 | 643 | 156 | 131 |
| 479 | 0.61 | 8.15 | 0.12 | 1.25 | 843 | 1105 | 163 | 554 | 642 | 156 | 130 |
| 480 | 0.61 | 8.26 | 0.12 | 1.27 | 851 | 1091 | 168 | 556 | 640 | 156 | 130 |

TABLE IX, ARG. 2.—Continued.

| Arg. | (c.c.0) Diff. | v.s.1) Diff. Sec.var. | v.c.1) Diff. Sec.var. | (v.s.2) Diff. Sec.var. | (v.c.2) Diff. Sec.var. | | | | | | | | | |
|------|---------------|-----------------------|-----------------------|------------------------|------------------------|--------------|-------|--------------|--------|------|--------|--------|------|------|
| " | " | " | " | " | " | | | | | | | | | |
| 480 | 14.65 | 9.02 | 2.71 | 187.06 | 2.19 | 35.52 | 2.89 | 217.47 | 1.33 | | | | | |
| 481 | 14.89 +0.24 | 9.43 +0.41 | 2.72 | 188.57 +1.51 | 2.18 | 36.46 +0.94 | 2.89 | 218.46 +0.99 | 1.32 | | | | | |
| 482 | 15.13 | 9.85 | 0.42 | 2.72 | 190.08 | 1.51 | 2.16 | 37.41 | 0.95 | 2.89 | 219.44 | 0.98 | 1.30 | |
| 483 | 15.37 | 0.24 | 10.29 | 0.44 | 2.73 | 191.59 | 1.51 | 2.15 | 38.37 | 0.96 | 2.88 | 220.40 | 0.96 | 1.29 |
| 484 | 15.61 | 0.24 | 10.74 | 0.45 | 2.74 | 193.09 | 1.50 | 2.14 | 39.35 | 0.98 | 2.88 | 221.35 | 0.95 | 1.27 |
| | | 0.24 | | 0.47 | | | | | | 0.98 | | | 0.94 | |
| 485 | 15.85 | 11.21 | 0.48 | 2.74 | 194.59 | 2.12 | 40.33 | 2.88 | 222.29 | 1.26 | | | | |
| 486 | 16.09 +0.24 | 11.69 +0.48 | 2.75 | 196.08 +1.49 | 2.11 | 41.32 +0.99 | 2.88 | 223.22 +0.93 | 1.25 | | | | | |
| 487 | 16.33 | 12.19 | 0.50 | 2.76 | 197.57 | 1.49 | 2.10 | 42.32 | 1.00 | 2.88 | 224.13 | 0.91 | 1.23 | |
| 488 | 16.58 | 0.25 | 12.71 | 0.52 | 2.77 | 199.05 | 1.48 | 2.09 | 43.33 | 1.01 | 2.87 | 225.04 | 0.91 | 1.22 |
| 489 | 16.83 | 0.25 | 13.24 | 0.53 | 2.77 | 200.53 | 1.48 | 2.07 | 44.35 | 1.02 | 2.87 | 225.94 | 0.90 | 1.20 |
| | | 0.25 | | 0.55 | | | | | | 1.03 | | | 0.89 | |
| 490 | 17.08 | 13.79 | 2.78 | 202.00 | 2.06 | 45.38 | 2.87 | 226.83 | 1.19 | | | | | |
| 491 | 17.33 +0.25 | 14.35 +0.56 | 2.79 | 203.46 +1.46 | 2.05 | 46.42 +1.04 | 2.87 | 227.71 +0.88 | 1.18 | | | | | |
| 492 | 17.58 | 14.93 | 0.58 | 2.79 | 204.92 | 1.46 | 2.03 | 47.46 | 1.04 | 2.86 | 228.58 | 0.87 | 1.16 | |
| 493 | 17.83 | 0.25 | 15.52 | 0.59 | 2.80 | 206.37 | 1.45 | 2.02 | 48.51 | 1.05 | 2.86 | 229.43 | 0.85 | 1.15 |
| 494 | 18.09 | 0.26 | 16.13 | 0.61 | 2.80 | 207.81 | 1.44 | 2.00 | 49.58 | 1.07 | 2.86 | 230.27 | 0.84 | 1.13 |
| | | 0.25 | | 0.62 | | | | | | 1.07 | | | 0.83 | |
| 495 | 18.34 | 16.75 | 2.81 | 209.24 | 1.99 | 50.65 | 2.85 | 231.10 | 1.12 | | | | | |
| 496 | 18.60 +0.26 | 17.39 +0.64 | 2.81 | 210.67 +1.43 | 1.98 | 51.73 +1.08 | 2.85 | 231.92 +0.82 | 1.10 | | | | | |
| 497 | 18.85 | 18.04 | 0.65 | 2.82 | 212.09 | 1.42 | 1.96 | 52.82 | 1.09 | 2.85 | 232.73 | 0.81 | 1.09 | |
| 498 | 19.11 | 0.26 | 18.71 | 0.67 | 2.82 | 213.50 | 1.41 | 1.95 | 53.92 | 1.10 | 2.85 | 233.52 | 0.79 | 1.07 |
| 499 | 19.36 | 0.25 | 19.39 | 0.68 | 2.83 | 214.90 | 1.40 | 1.93 | 55.03 | 1.11 | 2.84 | 234.30 | 0.78 | 1.06 |
| | | 0.26 | | 0.70 | | | | | | 1.11 | | | 0.77 | |
| 500 | 19.62 | 20.09 | 2.83 | 216.30 | 1.92 | 56.14 | 2.84 | 235.07 | 1.04 | | | | | |
| 501 | 19.88 +0.26 | 20.80 +0.71 | 2.83 | 217.69 +1.39 | 1.91 | 57.26 +1.12 | 2.83 | 235.83 +0.76 | 1.03 | | | | | |
| 502 | 20.14 | 21.52 | 0.72 | 2.84 | 219.07 | 1.38 | 1.89 | 58.38 | 1.12 | 2.83 | 236.58 | 0.75 | 1.01 | |
| 503 | 20.40 | 0.26 | 22.26 | 0.74 | 2.84 | 220.44 | 1.37 | 1.88 | 59.52 | 1.14 | 2.82 | 237.31 | 0.73 | 1.00 |
| 504 | 20.66 | 0.26 | 23.01 | 0.75 | 2.84 | 221.81 | 1.37 | 1.86 | 60.67 | 1.15 | 2.82 | 238.03 | 0.72 | 0.99 |
| | | 0.26 | | 0.76 | | | | | | 1.15 | | | 0.71 | |
| 505 | 20.92 | 23.77 | 2.85 | 223.17 | 1.85 | 61.82 | 2.81 | 238.74 | 0.97 | | | | | |
| 506 | 21.18 +0.26 | 24.55 +0.78 | 2.85 | 224.52 +1.35 | 1.84 | 62.98 +1.16 | 2.80 | 239.44 +0.70 | 0.96 | | | | | |
| 507 | 21.44 | 25.34 | 0.79 | 2.85 | 225.85 | 1.33 | 1.82 | 64.14 | 1.16 | 2.80 | 240.12 | 0.68 | 0.95 | |
| 508 | 21.69 | 0.25 | 26.15 | 0.81 | 2.85 | 227.18 | 1.33 | 1.81 | 65.31 | 1.17 | 2.79 | 240.79 | 0.67 | 0.94 |
| 509 | 21.95 | 0.26 | 26.97 | 0.82 | 2.86 | 228.50 | 1.32 | 1.79 | 66.49 | 1.18 | 2.79 | 241.45 | 0.66 | 0.92 |
| | | 0.26 | | 0.84 | | | | | | 1.19 | | | 0.65 | |
| 510 | 22.21 | 27.81 | 2.86 | 229.81 | 1.78 | 67.68 | 2.78 | 242.10 | 0.91 | | | | | |
| 511 | 22.47 +0.26 | 28.66 +0.85 | 2.86 | 231.11 +1.30 | 1.77 | 68.87 +1.19 | 2.77 | 242.73 +0.63 | 0.90 | | | | | |
| 512 | 22.72 | 29.52 | 0.86 | 2.87 | 232.39 | 1.28 | 1.75 | 70.07 | 1.20 | 2.77 | 243.36 | 0.63 | 0.88 | |
| 513 | 22.98 | 0.26 | 30.40 | 0.88 | 2.87 | 233.67 | 1.28 | 1.74 | 71.28 | 1.21 | 2.76 | 243.98 | 0.62 | 0.87 |
| 514 | 23.24 | 0.26 | 31.28 | 0.88 | 2.87 | 234.94 | 1.27 | 1.72 | 72.49 | 1.21 | 2.75 | 244.58 | 0.60 | 0.86 |
| | | 0.25 | | 0.90 | | | | | | 1.22 | | | 0.58 | |
| 515 | 23.49 | 32.18 | 2.88 | 236.20 | 1.71 | 73.71 | 2.74 | 245.16 | 0.84 | | | | | |
| 516 | 23.75 +0.26 | 33.09 +0.91 | 2.88 | 237.45 +1.25 | 1.69 | 74.93 +1.22 | 2.74 | 245.71 +0.55 | 0.83 | | | | | |
| 517 | 24.00 | 34.01 | 0.92 | 2.88 | 238.69 | 1.24 | 1.68 | 76.16 | 1.23 | 2.73 | 246.26 | 0.55 | 0.82 | |
| 518 | 24.26 | 0.26 | 34.95 | 0.94 | 2.88 | 239.92 | 1.23 | 1.66 | 77.39 | 1.23 | 2.72 | 246.80 | 0.54 | 0.81 |
| 519 | 24.51 | 0.25 | 35.90 | 0.95 | 2.89 | 241.13 | 1.21 | 1.65 | 78.63 | 1.24 | 2.72 | 247.33 | 0.53 | 0.79 |
| | | 0.25 | | 0.96 | | | | | | 1.24 | | | 0.51 | |
| 520 | 24.76 | 36.86 | 2.89 | 242.33 | 1.63 | 79.87 | 2.71 | 247.84 | 0.78 | | | | | |
| 521 | 25.02 +0.26 | 37.83 +0.97 | 2.89 | 243.52 +1.19 | 1.62 | 81.12 +1.25 | 2.70 | 248.34 +0.50 | 0.77 | | | | | |
| 522 | 25.27 | 38.82 | 0.99 | 2.89 | 244.70 | 1.18 | 1.60 | 82.37 | 1.25 | 2.69 | 248.83 | 0.49 | 0.75 | |
| 523 | 25.52 | 0.25 | 39.82 | 1.00 | 2.89 | 245.87 | 1.17 | 1.59 | 83.62 | 1.25 | 2.68 | 249.30 | 0.47 | 0.74 |
| 524 | 25.77 | 0.25 | 40.83 | 1.01 | 2.89 | 247.03 | 1.16 | 1.57 | 84.88 | 1.26 | 2.67 | 249.76 | 0.46 | 0.73 |
| | | 0.25 | | 1.02 | | | | | | 1.26 | | | 0.45 | |
| 525 | 26.02 | 41.85 | 2.89 | 248.18 | 1.56 | 86.14 | 2.66 | 250.21 | 0.71 | | | | | |
| 526 | 26.27 +0.25 | 42.88 +1.03 | 2.89 | 249.31 +1.13 | 1.55 | 87.41 +1.27 | 2.66 | 250.64 +0.43 | 0.70 | | | | | |
| 527 | 26.52 | 43.92 | 1.04 | 2.89 | 250.43 | 1.12 | 1.53 | 88.69 | 1.28 | 2.65 | 251.06 | 0.42 | 0.69 | |
| 528 | 26.76 | 0.24 | 44.97 | 1.05 | 2.89 | 251.53 | 1.10 | 1.52 | 89.97 | 1.28 | 2.64 | 251.47 | 0.41 | 0.68 |
| 529 | 27.00 | 0.24 | 46.04 | 1.07 | 2.89 | 252.63 | 1.10 | 1.50 | 91.25 | 1.28 | 2.63 | 251.86 | 0.39 | 0.66 |
| | | 0.24 | | 1.08 | | | | | | 1.29 | | | 0.38 | |
| 530 | 27.24 | 47.12 | 2.89 | 253.72 | 1.49 | 92.54 | 2.62 | 252.24 | 0.65 | | | | | |
| 531 | 27.48 +0.24 | 48.21 +1.09 | 2.89 | 254.79 +1.07 | 1.48 | 93.83 +1.29 | 2.61 | 252.60 +0.36 | 0.64 | | | | | |
| 532 | 27.72 | 49.31 | 1.10 | 2.89 | 255.85 | 1.06 | 1.46 | 95.12 | 1.29 | 2.60 | 252.95 | 0.35 | 0.63 | |
| 533 | 27.96 | 0.24 | 50.42 | 1.11 | 2.89 | 256.90 | 1.05 | 1.45 | 96.42 | 1.30 | 2.59 | 253.29 | 0.34 | 0.62 |
| 534 | 28.19 | 0.23 | 51.53 | 1.11 | 2.89 | 257.93 | 1.03 | 1.43 | 97.72 | 1.30 | 2.58 | 253.61 | 0.32 | 0.61 |
| | | 0.23 | | 1.13 | | | | | | 1.30 | | | 0.31 | |
| 535 | 28.42 | 52.66 | 2.88 | 258.95 | 1.42 | 99.02 | 2.58 | 253.92 | 0.60 | | | | | |
| 536 | 28.65 +0.23 | 53.80 +1.14 | 2.88 | 259.96 +1.01 | 1.40 | 100.32 +1.30 | 2.57 | 254.22 +0.30 | 0.58 | | | | | |
| 537 | 28.88 | 54.95 | 1.15 | 2.88 | 260.95 | 1.15 | 1.39 | 101.63 | 1.31 | 2.56 | 254.50 | 0.28 | 0.57 | |
| 538 | 29.11 | 0.23 | 56.11 | 1.16 | 2.88 | 261.93 | 1.16 | 1.37 | 102.94 | 1.31 | 2.55 | 254.77 | 0.27 | 0.56 |
| 539 | 29.33 | 0.22 | 57.27 | 1.16 | 2.88 | 262.90 | 1.16 | 1.36 | 104.25 | 1.31 | 2.54 | 255.02 | 0.25 | 0.55 |
| | | 0.22 | | 1.18 | | | | | | 1.31 | | | 0.24 | |
| 540 | 29.55 | 58.45 | 2.88 | 263.85 | 1.34 | 105.56 | 2.53 | 255.26 | 0.54 | | | | | |

TABLE IX, ARG. 2.—Continued.

| Arg. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | (p.s.3) | (p.c.3) |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| | " | " | " | " | | | | | | | |
| 480 | 0.61 | 8.26 | 0.11 | 1.27 | 851 | 1091 | 168 | 556 | 640 | 156 | 130 |
| 481 | 0.62 | 8.36 | 0.12 | 1.28 | 860 | 1076 | 173 | 559 | 639 | 156 | 129 |
| 482 | 0.64 | 8.47 | 0.13 | 1.30 | 868 | 1063 | 179 | 561 | 637 | 156 | 128 |
| 483 | 0.65 | 8.58 | 0.13 | 1.31 | 877 | 1049 | 184 | 564 | 636 | 156 | 128 |
| 484 | 0.66 | 8.69 | 0.14 | 1.33 | 885 | 1035 | 190 | 566 | 634 | 156 | 127 |
| 485 | 0.67 | 8.79 | 0.14 | 1.35 | 894 | 1021 | 196 | 569 | 633 | 156 | 126 |
| 486 | 0.69 | 8.90 | 0.15 | 1.36 | 902 | 1007 | 202 | 571 | 632 | 156 | 125 |
| 487 | 0.71 | 9.01 | 0.15 | 1.37 | 911 | 994 | 208 | 574 | 630 | 156 | 124 |
| 488 | 0.73 | 9.11 | 0.16 | 1.39 | 919 | 980 | 214 | 576 | 629 | 156 | 124 |
| 489 | 0.75 | 9.22 | 0.17 | 1.40 | 927 | 966 | 220 | 579 | 627 | 156 | 123 |
| 490 | 0.77 | 9.33 | 0.17 | 1.42 | 936 | 953 | 227 | 581 | 626 | 156 | 122 |
| 491 | 0.80 | 9.43 | 0.18 | 1.43 | 945 | 940 | 234 | 584 | 624 | 156 | 121 |
| 492 | 0.82 | 9.54 | 0.19 | 1.44 | 954 | 926 | 241 | 586 | 623 | 156 | 120 |
| 493 | 0.85 | 9.65 | 0.19 | 1.46 | 963 | 913 | 249 | 589 | 621 | 156 | 120 |
| 494 | 0.88 | 9.75 | 0.20 | 1.47 | 972 | 900 | 256 | 591 | 619 | 156 | 119 |
| 495 | 0.91 | 9.86 | 0.21 | 1.49 | 981 | 887 | 264 | 594 | 618 | 156 | 118 |
| 496 | 0.94 | 9.97 | 0.22 | 1.50 | 990 | 874 | 272 | 596 | 616 | 156 | 117 |
| 497 | 0.98 | 10.07 | 0.23 | 1.51 | 999 | 861 | 280 | 599 | 615 | 156 | 117 |
| 498 | 1.02 | 10.18 | 0.23 | 1.53 | 1008 | 848 | 288 | 601 | 613 | 155 | 116 |
| 499 | 1.06 | 10.29 | 0.24 | 1.54 | 1017 | 835 | 296 | 604 | 611 | 155 | 115 |
| 500 | 1.10 | 10.39 | 0.25 | 1.55 | 1026 | 822 | 304 | 606 | 610 | 155 | 115 |
| 501 | 1.14 | 10.50 | 0.26 | 1.57 | 1035 | 810 | 312 | 608 | 608 | 155 | 114 |
| 502 | 1.19 | 10.60 | 0.27 | 1.58 | 1044 | 797 | 321 | 611 | 606 | 155 | 114 |
| 503 | 1.24 | 10.70 | 0.28 | 1.59 | 1053 | 785 | 329 | 613 | 604 | 154 | 113 |
| 504 | 1.29 | 10.81 | 0.29 | 1.60 | 1063 | 772 | 338 | 615 | 602 | 154 | 113 |
| 505 | 1.34 | 10.91 | 0.30 | 1.62 | 1072 | 760 | 346 | 617 | 600 | 154 | 112 |
| 506 | 1.39 | 11.01 | 0.31 | 1.63 | 1081 | 748 | 355 | 620 | 598 | 154 | 112 |
| 507 | 1.44 | 11.12 | 0.32 | 1.64 | 1090 | 736 | 364 | 622 | 596 | 154 | 111 |
| 508 | 1.50 | 11.22 | 0.33 | 1.65 | 1099 | 724 | 373 | 624 | 594 | 154 | 110 |
| 509 | 1.56 | 11.32 | 0.34 | 1.66 | 1108 | 712 | 382 | 626 | 592 | 153 | 110 |
| 510 | 1.62 | 11.42 | 0.35 | 1.67 | 1117 | 700 | 391 | 628 | 590 | 153 | 109 |
| 511 | 1.68 | 11.52 | 0.37 | 1.68 | 1126 | 688 | 401 | 631 | 588 | 153 | 109 |
| 512 | 1.74 | 11.62 | 0.38 | 1.69 | 1135 | 677 | 410 | 633 | 585 | 153 | 108 |
| 513 | 1.80 | 11.71 | 0.39 | 1.70 | 1144 | 666 | 420 | 635 | 583 | 153 | 108 |
| 514 | 1.87 | 11.81 | 0.40 | 1.71 | 1153 | 654 | 430 | 637 | 581 | 153 | 107 |
| 515 | 1.94 | 11.91 | 0.41 | 1.72 | 1162 | 643 | 440 | 639 | 578 | 153 | 106 |
| 516 | 2.01 | 12.00 | 0.42 | 1.73 | 1171 | 632 | 450 | 642 | 576 | 153 | 106 |
| 517 | 2.08 | 12.09 | 0.43 | 1.74 | 1180 | 621 | 460 | 644 | 573 | 152 | 106 |
| 518 | 2.16 | 12.19 | 0.45 | 1.75 | 1189 | 610 | 471 | 646 | 571 | 152 | 105 |
| 519 | 2.23 | 12.28 | 0.46 | 1.76 | 1197 | 599 | 481 | 648 | 569 | 152 | 105 |
| 520 | 2.31 | 12.37 | 0.47 | 1.77 | 1206 | 588 | 492 | 650 | 566 | 152 | 104 |
| 521 | 2.39 | 12.46 | 0.49 | 1.78 | 1215 | 577 | 503 | 652 | 564 | 152 | 104 |
| 522 | 2.47 | 12.55 | 0.50 | 1.79 | 1224 | 567 | 514 | 654 | 561 | 151 | 103 |
| 523 | 2.55 | 12.63 | 0.51 | 1.79 | 1233 | 557 | 525 | 656 | 558 | 151 | 103 |
| 524 | 2.63 | 12.72 | 0.52 | 1.80 | 1242 | 546 | 536 | 658 | 556 | 151 | 103 |
| 525 | 2.71 | 12.81 | 0.54 | 1.81 | 1250 | 536 | 548 | 660 | 553 | 151 | 102 |
| 526 | 2.80 | 12.89 | 0.55 | 1.82 | 1259 | 526 | 559 | 662 | 550 | 151 | 102 |
| 527 | 2.88 | 12.98 | 0.57 | 1.83 | 1267 | 517 | 571 | 664 | 548 | 150 | 101 |
| 528 | 2.97 | 13.06 | 0.58 | 1.83 | 1276 | 507 | 583 | 666 | 545 | 150 | 101 |
| 529 | 3.06 | 13.14 | 0.60 | 1.84 | 1284 | 498 | 595 | 668 | 542 | 150 | 100 |
| 530 | 3.15 | 13.22 | 0.61 | 1.85 | 1293 | 488 | 607 | 670 | 540 | 150 | 100 |
| 531 | 3.24 | 13.30 | 0.62 | 1.85 | 1301 | 479 | 618 | 672 | 537 | 150 | 100 |
| 532 | 3.34 | 13.37 | 0.64 | 1.86 | 1310 | 470 | 630 | 673 | 535 | 149 | 99 |
| 533 | 3.43 | 13.45 | 0.65 | 1.86 | 1318 | 461 | 642 | 675 | 532 | 149 | 99 |
| 534 | 3.53 | 13.52 | 0.67 | 1.87 | 1327 | 452 | 654 | 677 | 529 | 149 | 98 |
| 535 | 3.63 | 13.59 | 0.68 | 1.87 | 1335 | 443 | 666 | 678 | 527 | 149 | 98 |
| 536 | 3.73 | 13.66 | 0.69 | 1.88 | 1343 | 434 | 678 | 680 | 524 | 148 | 98 |
| 537 | 3.83 | 13.73 | 0.71 | 1.88 | 1352 | 426 | 690 | 682 | 521 | 148 | 98 |
| 538 | 3.93 | 13.80 | 0.72 | 1.89 | 1360 | 417 | 702 | 683 | 519 | 148 | 97 |
| 539 | 4.03 | 13.87 | 0.74 | 1.89 | 1368 | 409 | 714 | 685 | 516 | 148 | 97 |
| 540 | 4.13 | 13.93 | 0.75 | 1.90 | 1376 | 401 | 726 | 686 | 513 | 147 | 97 |

TABLE IX, ARG. 2.—*Concluded.*

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | Sec.var. | (v.c.1) | Diff. | Sec.var. | (v.s.2) | Diff. | Sec.var. | (v.c.2) | Diff. | Sec.var. |
|------|---------|-------|---------|-------|----------|---------|-------|----------|---------|-------|----------|---------|-------|----------|
| | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 540 | 29.55 | | 58.45 | | 2.88 | 263.85 | | 1.34 | 105.56 | | 2.53 | 255.26 | | 0.54 |
| 541 | 29.77 | +0.22 | 59.63 | +1.18 | 2.88 | 264.79 | +0.94 | 1.33 | 106.88 | +1.32 | 2.52 | 255.49 | +0.23 | 0.53 |
| 542 | 29.99 | 0.22 | 60.82 | 1.19 | 2.88 | 265.71 | 0.92 | 1.32 | 108.19 | 1.31 | 2.51 | 255.70 | 0.21 | 0.52 |
| 543 | 30.21 | 0.22 | 62.02 | 1.20 | 2.87 | 266.62 | 0.91 | 1.30 | 109.51 | 1.32 | 2.50 | 255.90 | 0.20 | 0.51 |
| 544 | 30.43 | 0.22 | 63.24 | 1.22 | 2.87 | 267.52 | 0.90 | 1.29 | 110.83 | 1.32 | 2.49 | 256.08 | 0.18 | 0.50 |
| | | 0.21 | | 1.22 | | | 0.88 | | | 1.32 | | | 0.17 | |
| 545 | 30.64 | | 64.46 | | 2.87 | 268.40 | | 1.27 | 112.15 | | 2.47 | 256.25 | | 0.49 |
| 546 | 30.85 | +0.21 | 65.69 | +1.23 | 2.87 | 269.27 | +0.87 | 1.26 | 113.47 | +1.32 | 2.46 | 256.41 | +0.16 | 0.48 |
| 547 | 31.06 | 0.21 | 66.93 | 1.24 | 2.87 | 270.12 | 0.85 | 1.25 | 114.80 | 1.33 | 2.45 | 256.55 | 0.14 | 0.47 |
| 548 | 31.27 | 0.21 | 68.18 | 1.25 | 2.86 | 270.96 | 0.84 | 1.23 | 116.12 | 1.32 | 2.44 | 256.68 | 0.13 | 0.46 |
| 549 | 31.47 | 0.20 | 69.43 | 1.25 | 2.86 | 271.79 | 0.83 | 1.22 | 117.45 | 1.33 | 2.43 | 256.79 | 0.11 | 0.45 |
| | | 0.20 | | 1.26 | | | 0.81 | | | 1.32 | | | 0.10 | |
| 550 | 31.67 | +0.20 | 70.69 | +1.27 | 2.86 | 272.60 | +0.80 | 1.21 | 118.77 | +1.32 | 2.42 | 256.89 | +0.09 | 0.44 |
| 551 | 31.87 | 0.19 | 71.96 | 1.27 | 2.86 | 273.40 | 0.80 | 1.20 | 120.09 | 1.32 | 2.41 | 256.98 | 0.09 | 0.43 |
| 552 | 32.06 | 0.19 | 73.23 | 1.27 | 2.85 | 274.18 | 0.78 | 1.18 | 121.42 | 1.33 | 2.40 | 257.05 | 0.07 | 0.42 |
| 553 | 32.25 | 0.19 | 74.51 | 1.28 | 2.85 | 274.95 | 0.77 | 1.17 | 122.74 | 1.32 | 2.39 | 257.11 | 0.06 | 0.41 |
| 554 | 32.44 | 0.19 | 75.80 | 1.29 | 2.84 | 275.70 | 0.75 | 1.15 | 124.07 | 1.33 | 2.38 | 257.15 | 0.04 | 0.40 |
| | | 0.19 | | 1.30 | | | 0.74 | | | 1.32 | | | 0.03 | |
| 555 | 32.63 | | 77.10 | | 2.84 | 276.44 | | 1.14 | 125.39 | | 2.36 | 257.18 | | 0.40 |
| 556 | 32.81 | +0.18 | 78.40 | +1.30 | 2.84 | 277.16 | +0.72 | 1.12 | 126.71 | +1.32 | 2.35 | 257.20 | +0.02 | 0.39 |
| 557 | 32.99 | 0.18 | 79.71 | 1.31 | 2.83 | 277.86 | 0.70 | 1.11 | 128.03 | 1.32 | 2.34 | 257.20 | 0.00 | 0.38 |
| 558 | 33.17 | 0.18 | 81.03 | 1.32 | 2.83 | 278.55 | 0.69 | 1.09 | 129.35 | 1.32 | 2.33 | 257.19 | -0.01 | 0.37 |
| 559 | 33.35 | 0.18 | 82.35 | 1.32 | 2.82 | 279.23 | 0.68 | 1.08 | 130.67 | 1.32 | 2.32 | 257.16 | 0.03 | 0.36 |
| | | 0.17 | | 1.33 | | | 0.66 | | | 1.34 | | | 0.04 | |
| 560 | 33.52 | | 83.68 | | 2.82 | 279.89 | | 1.06 | 132.01 | | 2.31 | 257.12 | | 0.35 |
| 561 | 33.69 | +0.17 | 85.01 | +1.33 | 2.82 | 280.54 | +0.65 | 1.05 | 133.33 | +1.32 | 2.30 | 257.07 | -0.05 | 0.34 |
| 562 | 33.86 | 0.17 | 86.35 | 1.34 | 2.81 | 281.17 | 0.63 | 1.03 | 134.65 | 1.32 | 2.29 | 257.00 | 0.07 | 0.33 |
| 563 | 34.03 | 0.17 | 87.69 | 1.34 | 2.81 | 281.78 | 0.61 | 1.02 | 135.97 | 1.32 | 2.27 | 256.92 | 0.08 | 0.33 |
| 564 | 34.19 | 0.16 | 89.04 | 1.35 | 2.80 | 282.38 | 0.60 | 1.01 | 137.28 | 1.31 | 2.26 | 256.82 | 0.10 | 0.32 |
| | | 0.16 | | 1.36 | | | 0.59 | | | 1.32 | | | 0.11 | |
| 565 | 34.35 | | 90.40 | | 2.80 | 282.97 | | 0.99 | 138.60 | | 2.25 | 256.71 | | 0.31 |
| 566 | 34.51 | +0.16 | 91.76 | +1.36 | 2.79 | 283.54 | +0.57 | 0.98 | 139.91 | +1.31 | 2.24 | 256.59 | -0.12 | 0.30 |
| 567 | 34.67 | 0.16 | 93.13 | 1.37 | 2.79 | 284.09 | 0.55 | 0.97 | 141.22 | 1.31 | 2.23 | 256.45 | 0.14 | 0.29 |
| 568 | 34.82 | 0.15 | 94.50 | 1.37 | 2.78 | 284.63 | 0.54 | 0.96 | 142.54 | 1.32 | 2.21 | 256.30 | 0.15 | 0.29 |
| 569 | 34.97 | 0.15 | 95.88 | 1.38 | 2.78 | 285.16 | 0.53 | 0.94 | 143.85 | 1.31 | 2.20 | 256.14 | 0.16 | 0.28 |
| | | 0.15 | | 1.38 | | | 0.51 | | | 1.31 | | | 0.18 | |
| 570 | 35.12 | | 97.26 | | 2.77 | 285.67 | | 0.93 | 145.16 | | 2.19 | 255.96 | | 0.27 |
| 571 | 35.27 | +0.15 | 98.64 | +1.38 | 2.76 | 286.16 | +0.49 | 0.92 | 146.46 | +1.30 | 2.18 | 255.77 | -0.19 | 0.26 |
| 572 | 35.41 | 0.14 | 100.03 | 1.39 | 2.76 | 286.64 | 0.48 | 0.90 | 147.76 | 1.30 | 2.16 | 255.57 | 0.20 | 0.26 |
| 573 | 35.55 | 0.14 | 101.42 | 1.39 | 2.75 | 287.10 | 0.46 | 0.89 | 149.05 | 1.29 | 2.15 | 255.36 | 0.21 | 0.25 |
| 574 | 35.69 | 0.14 | 102.82 | 1.40 | 2.75 | 287.55 | 0.45 | 0.87 | 150.35 | 1.30 | 2.13 | 255.13 | 0.23 | 0.25 |
| | | 0.13 | | 1.40 | | | 0.43 | | | 1.29 | | | 0.24 | |
| 575 | 35.82 | | 104.22 | | 2.74 | 287.98 | | 0.86 | 151.64 | | 2.12 | 254.89 | | 0.24 |
| 576 | 35.95 | +0.13 | 105.62 | +1.40 | 2.73 | 288.40 | +0.42 | 0.85 | 152.93 | +1.29 | 2.11 | 254.63 | -0.26 | 0.23 |
| 577 | 36.08 | 0.13 | 107.03 | 1.41 | 2.73 | 288.80 | 0.40 | 0.83 | 154.22 | 1.29 | 2.09 | 254.36 | 0.27 | 0.23 |
| 578 | 36.21 | 0.13 | 108.44 | 1.41 | 2.72 | 289.18 | 0.38 | 0.82 | 155.50 | 1.28 | 2.08 | 254.08 | 0.28 | 0.22 |
| 579 | 36.33 | 0.12 | 109.85 | 1.41 | 2.72 | 289.55 | 0.37 | 0.80 | 156.77 | 1.27 | 2.06 | 253.79 | 0.29 | 0.22 |
| | | 0.12 | | 1.41 | | | 0.36 | | | 1.28 | | | 0.31 | |
| 580 | 36.45 | | 111.26 | | 2.71 | 289.91 | | 0.79 | 158.05 | | 2.05 | 253.48 | | 0.21 |
| 581 | 36.57 | +0.12 | 112.68 | +1.42 | 2.70 | 290.25 | +0.34 | 0.78 | 159.32 | +1.27 | 2.04 | 253.16 | -0.32 | 0.21 |
| 582 | 36.69 | 0.12 | 114.10 | 1.42 | 2.69 | 290.57 | 0.32 | 0.77 | 160.59 | 1.27 | 2.02 | 252.83 | 0.33 | 0.20 |
| 583 | 36.81 | 0.12 | 115.52 | 1.42 | 2.68 | 290.88 | 0.31 | 0.76 | 161.85 | 1.26 | 2.01 | 252.48 | 0.35 | 0.20 |
| 584 | 36.93 | 0.11 | 116.95 | 1.43 | 2.67 | 291.18 | 0.30 | 0.75 | 163.11 | 1.26 | 1.99 | 252.12 | 0.36 | 0.19 |
| | | 0.11 | | 1.43 | | | 0.28 | | | 1.26 | | | 0.37 | |
| 585 | 37.04 | | 118.38 | | 2.67 | 291.46 | | 0.73 | 164.37 | | 1.98 | 251.75 | | 0.19 |
| 586 | 37.15 | +0.11 | 119.81 | +1.43 | 2.66 | 291.72 | +0.26 | 0.72 | 165.62 | +1.25 | 1.97 | 251.36 | -0.39 | 0.18 |
| 587 | 37.26 | 0.11 | 121.24 | 1.43 | 2.65 | 291.96 | 0.24 | 0.71 | 166.87 | 1.25 | 1.95 | 250.96 | 0.40 | 0.18 |
| 588 | 37.37 | 0.11 | 122.67 | 1.43 | 2.64 | 292.19 | 0.23 | 0.70 | 168.11 | 1.24 | 1.94 | 250.55 | 0.41 | 0.17 |
| 589 | 37.48 | 0.11 | 124.11 | 1.44 | 2.63 | 292.41 | 0.22 | 0.69 | 169.35 | 1.24 | 1.92 | 250.13 | 0.42 | 0.17 |
| | | 0.10 | | 1.43 | | | 0.20 | | | 1.23 | | | 0.44 | |
| 590 | 37.58 | | 125.54 | | 2.62 | 292.61 | | 0.68 | 170.58 | | 1.91 | 249.69 | | 0.16 |
| 591 | 37.68 | +0.10 | 126.98 | +1.44 | 2.61 | 292.79 | +0.18 | 0.67 | 171.81 | +1.23 | 1.90 | 249.24 | -0.45 | 0.16 |
| 592 | 37.78 | 0.10 | 128.41 | 1.43 | 2.60 | 292.96 | 0.17 | 0.66 | 173.03 | 1.22 | 1.88 | 248.78 | 0.46 | 0.15 |
| 593 | 37.88 | 0.10 | 129.85 | 1.44 | 2.59 | 293.12 | 0.16 | 0.64 | 174.25 | 1.22 | 1.87 | 248.31 | 0.47 | 0.15 |
| 594 | 37.98 | 0.10 | 131.29 | 1.44 | 2.58 | 293.26 | 0.14 | 0.63 | 175.46 | 1.21 | 1.85 | 247.83 | 0.48 | 0.14 |
| | | 0.10 | | 1.44 | | | 0.12 | | | 1.21 | | | 0.50 | |
| 595 | 38.08 | | 132.73 | | 2.58 | 293.38 | | 0.62 | 176.67 | | 1.84 | 247.33 | | 0.14 |
| 596 | 38.18 | +0.10 | 134.17 | +1.44 | 2.57 | 293.49 | +0.11 | 0.61 | 177.87 | +1.20 | 1.83 | 246.82 | -0.51 | 0.14 |
| 597 | 38.27 | 0.09 | 135.61 | 1.44 | 2.56 | 293.58 | 0.09 | 0.60 | 179.07 | 1.20 | 1.81 | 246.30 | 0.52 | 0.13 |
| 598 | 38.36 | 0.09 | 137.05 | 1.44 | 2.55 | 293.66 | 0.08 | 0.58 | 180.26 | 1.19 | 1.80 | 245.77 | 0.53 | 0.13 |
| 599 | 38.45 | 0.09 | 138.50 | 1.45 | 2.54 | 293.73 | 0.07 | 0.57 | 181.44 | 1.18 | 1.78 | 245.23 | 0.54 | 0.12 |
| | | 0.09 | | 1.44 | | | 0.05 | | | 1.18 | | | 0.56 | |
| 600 | 38.54 | | 139.94 | | 2.53 | 293.78 | | 0.56 | 182.62 | | 1.77 | 244.67 | | 0.12 |

TABLE IX, ARG. 2.—*Concluded.*

| Arg. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | (p.s.3) | p.c.3 |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|-------|
| " | " | " | " | " | " | " | " | " | " | " | " |
| 540 | 4.13 | 13.93 | 0.75 | 1.90 | 1376 | 401 | 726 | 686 | 513 | 147 | 97 |
| 541 | 4.23 | 13.99 | 0.76 | 1.90 | 1384 | 393 | 738 | 688 | 510 | 147 | 97 |
| 542 | 4.34 | 14.05 | 0.78 | 1.90 | 1392 | 386 | 751 | 689 | 507 | 147 | 96 |
| 543 | 4.45 | 14.11 | 0.79 | 1.90 | 1400 | 378 | 763 | 691 | 503 | 146 | 96 |
| 544 | 4.56 | 14.17 | 0.81 | 1.91 | 1408 | 371 | 776 | 692 | 500 | 146 | 95 |
| 545 | 4.67 | 14.23 | 0.83 | 1.91 | 1416 | 363 | 788 | 693 | 497 | 146 | 95 |
| 546 | 4.79 | 14.28 | 0.84 | 1.91 | 1424 | 356 | 801 | 695 | 494 | 146 | 95 |
| 547 | 4.90 | 14.34 | 0.85 | 1.91 | 1431 | 349 | 813 | 696 | 491 | 146 | 94 |
| 548 | 5.01 | 14.39 | 0.87 | 1.92 | 1439 | 343 | 826 | 698 | 487 | 145 | 94 |
| 549 | 5.12 | 14.44 | 0.89 | 1.92 | 1446 | 336 | 838 | 699 | 484 | 145 | 94 |
| 550 | 5.24 | 14.49 | 0.90 | 1.92 | 1454 | 329 | 851 | 700 | 481 | 145 | 93 |
| 551 | 5.35 | 14.54 | 0.92 | 1.92 | 1461 | 323 | 864 | 702 | 478 | 145 | 93 |
| 552 | 5.47 | 14.58 | 0.93 | 1.92 | 1468 | 316 | 877 | 703 | 475 | 145 | 93 |
| 553 | 5.58 | 14.63 | 0.95 | 1.92 | 1475 | 310 | 890 | 704 | 471 | 144 | 93 |
| 554 | 5.70 | 14.67 | 0.96 | 1.92 | 1482 | 304 | 903 | 705 | 468 | 144 | 92 |
| 555 | 5.82 | 14.71 | 0.98 | 1.92 | 1489 | 298 | 916 | 706 | 465 | 144 | 92 |
| 556 | 5.94 | 14.75 | 0.99 | 1.92 | 1496 | 292 | 929 | 708 | 462 | 144 | 92 |
| 557 | 6.06 | 14.79 | 1.01 | 1.92 | 1502 | 287 | 943 | 709 | 458 | 144 | 92 |
| 558 | 6.18 | 14.82 | 1.02 | 1.92 | 1509 | 281 | 956 | 710 | 455 | 143 | 92 |
| 559 | 6.30 | 14.85 | 1.04 | 1.92 | 1515 | 276 | 970 | 711 | 452 | 143 | 91 |
| 560 | 6.42 | 14.88 | 1.05 | 1.92 | 1522 | 270 | 983 | 712 | 449 | 143 | 91 |
| 561 | 6.54 | 14.91 | 1.07 | 1.92 | 1528 | 265 | 996 | 713 | 445 | 143 | 91 |
| 562 | 6.67 | 14.93 | 1.08 | 1.92 | 1534 | 260 | 1010 | 714 | 442 | 143 | 91 |
| 563 | 6.79 | 14.96 | 1.10 | 1.91 | 1541 | 255 | 1023 | 715 | 438 | 142 | 91 |
| 564 | 6.91 | 14.99 | 1.11 | 1.91 | 1547 | 250 | 1036 | 716 | 435 | 142 | 90 |
| 565 | 7.04 | 15.01 | 1.13 | 1.91 | 1553 | 246 | 1049 | 716 | 431 | 142 | 90 |
| 566 | 7.16 | 15.03 | 1.14 | 1.91 | 1559 | 242 | 1063 | 717 | 428 | 142 | 90 |
| 567 | 7.29 | 15.05 | 1.16 | 1.90 | 1564 | 238 | 1076 | 717 | 424 | 142 | 90 |
| 568 | 7.41 | 15.06 | 1.17 | 1.90 | 1570 | 234 | 1089 | 717 | 421 | 141 | 90 |
| 569 | 7.54 | 15.08 | 1.18 | 1.90 | 1575 | 230 | 1102 | 718 | 417 | 141 | 89 |
| 570 | 7.66 | 15.09 | 1.20 | 1.89 | 1581 | 226 | 1116 | 718 | 413 | 141 | 89 |
| 571 | 7.79 | 15.09 | 1.21 | 1.89 | 1586 | 223 | 1129 | 718 | 410 | 141 | 89 |
| 572 | 7.91 | 15.10 | 1.23 | 1.89 | 1591 | 219 | 1142 | 718 | 406 | 141 | 89 |
| 573 | 8.04 | 15.10 | 1.24 | 1.88 | 1596 | 216 | 1155 | 719 | 403 | 140 | 89 |
| 574 | 8.16 | 15.11 | 1.26 | 1.88 | 1601 | 213 | 1169 | 719 | 399 | 140 | 89 |
| 575 | 8.29 | 15.11 | 1.27 | 1.87 | 1605 | 210 | 1182 | 719 | 395 | 140 | 88 |
| 576 | 8.42 | 15.12 | 1.29 | 1.86 | 1610 | 207 | 1195 | 719 | 392 | 140 | 88 |
| 577 | 8.54 | 15.12 | 1.30 | 1.86 | 1614 | 205 | 1208 | 719 | 388 | 140 | 88 |
| 578 | 8.67 | 15.11 | 1.32 | 1.85 | 1618 | 202 | 1221 | 719 | 385 | 140 | 88 |
| 579 | 8.80 | 15.10 | 1.33 | 1.84 | 1622 | 200 | 1234 | 719 | 381 | 139 | 88 |
| 580 | 8.93 | 15.09 | 1.34 | 1.84 | 1626 | 198 | 1248 | 719 | 378 | 139 | 88 |
| 581 | 9.05 | 15.08 | 1.35 | 1.83 | 1630 | 196 | 1261 | 719 | 374 | 139 | 87 |
| 582 | 9.18 | 15.07 | 1.36 | 1.83 | 1634 | 194 | 1274 | 719 | 371 | 139 | 87 |
| 583 | 9.31 | 15.05 | 1.38 | 1.82 | 1637 | 192 | 1288 | 719 | 367 | 139 | 87 |
| 584 | 9.43 | 15.04 | 1.39 | 1.81 | 1641 | 191 | 1301 | 719 | 364 | 139 | 87 |
| 585 | 9.56 | 15.02 | 1.40 | 1.81 | 1645 | 189 | 1315 | 718 | 360 | 139 | 87 |
| 586 | 9.68 | 15.00 | 1.42 | 1.80 | 1648 | 188 | 1328 | 718 | 357 | 139 | 87 |
| 587 | 9.81 | 14.98 | 1.43 | 1.79 | 1651 | 186 | 1342 | 718 | 353 | 139 | 87 |
| 588 | 9.93 | 14.96 | 1.44 | 1.79 | 1654 | 185 | 1356 | 717 | 350 | 138 | 86 |
| 589 | 10.06 | 14.93 | 1.45 | 1.78 | 1657 | 184 | 1369 | 717 | 346 | 138 | 86 |
| 590 | 10.18 | 14.91 | 1.46 | 1.77 | 1660 | 183 | 1382 | 717 | 343 | 138 | 86 |
| 591 | 10.31 | 14.88 | 1.48 | 1.76 | 1662 | 182 | 1396 | 716 | 339 | 138 | 86 |
| 592 | 10.43 | 14.85 | 1.49 | 1.75 | 1665 | 182 | 1409 | 716 | 335 | 138 | 86 |
| 593 | 10.55 | 14.81 | 1.50 | 1.74 | 1667 | 181 | 1422 | 716 | 332 | 138 | 86 |
| 594 | 10.68 | 14.78 | 1.51 | 1.73 | 1669 | 181 | 1436 | 715 | 328 | 138 | 85 |
| 595 | 10.80 | 14.74 | 1.52 | 1.72 | 1671 | 181 | 1448 | 715 | 324 | 138 | 85 |
| 596 | 10.92 | 14.71 | 1.53 | 1.72 | 1673 | 181 | 1461 | 714 | 321 | 138 | 85 |
| 597 | 11.04 | 14.67 | 1.55 | 1.71 | 1674 | 181 | 1474 | 714 | 317 | 137 | 85 |
| 598 | 11.16 | 14.63 | 1.56 | 1.70 | 1676 | 181 | 1487 | 713 | 313 | 137 | 85 |
| 599 | 11.28 | 14.58 | 1.57 | 1.69 | 1677 | 182 | 1500 | 712 | 310 | 137 | 85 |
| 600 | 11.40 | 14.53 | 1.58 | 1.68 | 1678 | 182 | 1513 | 711 | 306 | 137 | 85 |

TABLE X, ARG. 3.—ACTION OF NEPTUNE.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | (v.c.1) | Diff. | (v.s.2) | Diff. | (v.c.2) | Diff. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) |
|------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|---------|---------|---------|
| | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 0 | 92.96 | | 0.87 | | 34.22 | | 7.42 | | 3.95 | | 0.80 | 1.15 | -0.85 | -1.13 |
| 1 | 92.67 | -0.29 | 0.98 | +0.11 | 34.77 | +0.55 | 7.36 | -0.06 | 3.82 | -0.13 | 0.81 | 1.16 | 0.86 | 1.14 |
| 2 | 92.38 | 0.29 | 1.10 | 0.12 | 35.31 | 0.54 | 7.30 | 0.06 | 3.70 | 0.12 | 0.83 | 1.18 | 0.87 | 1.15 |
| 3 | 92.07 | 0.31 | 1.23 | 0.13 | 35.85 | 0.54 | 7.23 | 0.07 | 3.58 | 0.12 | 0.84 | 1.20 | 0.89 | 1.16 |
| 4 | 91.77 | 0.30 | 1.38 | 0.15 | 36.39 | 0.54 | 7.16 | 0.07 | 3.47 | 0.11 | 0.86 | 1.22 | 0.91 | 1.17 |
| | | 0.31 | | 0.15 | | 0.53 | | 0.07 | | 0.11 | | | | |
| 5 | 91.46 | | 1.53 | +0.17 | 36.92 | +0.52 | 7.09 | | 3.36 | | 0.88 | 1.23 | -0.93 | -1.18 |
| 6 | 91.15 | -0.31 | 1.70 | 0.18 | 37.44 | +0.52 | 7.01 | -0.08 | 3.25 | -0.11 | 0.90 | 1.24 | 0.95 | 1.18 |
| 7 | 90.84 | 0.31 | 1.88 | 0.20 | 37.96 | 0.52 | 6.92 | 0.09 | 3.15 | 0.10 | 0.91 | 1.25 | 0.97 | 1.19 |
| 8 | 90.52 | 0.32 | 2.08 | 0.21 | 38.48 | 0.52 | 6.82 | 0.10 | 3.05 | 0.10 | 0.93 | 1.26 | 1.00 | 1.19 |
| 9 | 90.19 | 0.33 | 2.29 | 0.22 | 39.00 | 0.52 | 6.72 | 0.10 | 2.95 | 0.10 | 0.95 | 1.27 | 1.02 | 1.20 |
| | | 0.33 | | 0.22 | | 0.51 | | 0.11 | | 0.09 | | | | |
| 10 | 89.86 | | 2.51 | | 39.51 | | 6.61 | | 2.86 | | 0.97 | 1.28 | -1.04 | -1.20 |
| 11 | 89.53 | -0.33 | 2.75 | +0.24 | 40.02 | +0.51 | 6.51 | -0.10 | 2.77 | -0.09 | 1.00 | 1.28 | 1.06 | 1.19 |
| 12 | 89.18 | 0.35 | 2.99 | 0.24 | 40.52 | 0.50 | 6.40 | 0.11 | 2.69 | 0.08 | 1.02 | 1.29 | 1.08 | 1.19 |
| 13 | 88.83 | 0.35 | 3.26 | 0.27 | 41.01 | 0.49 | 6.28 | 0.12 | 2.61 | 0.08 | 1.05 | 1.29 | 1.09 | 1.18 |
| 14 | 88.47 | 0.36 | 3.54 | 0.28 | 41.50 | 0.49 | 6.16 | 0.12 | 2.54 | 0.07 | 1.07 | 1.29 | 1.11 | 1.18 |
| | | 0.38 | | 0.29 | | 0.49 | | 0.12 | | 0.07 | | | | |
| 15 | 88.09 | | 3.83 | | 41.99 | | 6.04 | | 2.47 | | 1.09 | 1.29 | -1.12 | -1.17 |
| 16 | 87.71 | -0.38 | 4.13 | +0.30 | 42.47 | +0.48 | 5.91 | -0.13 | 2.41 | -0.06 | 1.12 | 1.29 | 1.13 | 1.15 |
| 17 | 87.31 | 0.40 | 4.45 | 0.32 | 42.94 | 0.47 | 5.78 | 0.13 | 2.36 | 0.05 | 1.14 | 1.28 | 1.15 | 1.13 |
| 18 | 86.90 | 0.41 | 4.78 | 0.33 | 43.40 | 0.46 | 5.64 | 0.14 | 2.31 | 0.05 | 1.17 | 1.27 | 1.16 | 1.12 |
| 19 | 86.49 | 0.41 | 5.13 | 0.35 | 43.86 | 0.46 | 5.50 | 0.14 | 2.27 | 0.04 | 1.19 | 1.26 | 1.17 | 1.10 |
| | | 0.43 | | 0.36 | | 0.45 | | 0.14 | | 0.04 | | | | |
| 20 | 86.06 | | 5.49 | | 44.31 | | 5.36 | | 2.23 | | 1.21 | 1.25 | -1.18 | -1.08 |
| 21 | 85.62 | -0.44 | 5.87 | +0.38 | 44.74 | +0.43 | 5.22 | -0.14 | 2.20 | -0.03 | 1.23 | 1.23 | 1.19 | 1.06 |
| 22 | 85.16 | 0.46 | 6.26 | 0.39 | 45.16 | 0.42 | 5.08 | 0.14 | 2.18 | 0.02 | 1.25 | 1.22 | 1.19 | 1.03 |
| 23 | 84.69 | 0.47 | 6.67 | 0.41 | 45.58 | 0.42 | 4.93 | 0.15 | 2.16 | -0.02 | 1.27 | 1.20 | 1.20 | 1.00 |
| 24 | 84.22 | 0.47 | 7.09 | 0.42 | 45.98 | 0.40 | 4.78 | 0.15 | 2.16 | 0.00 | 1.29 | 1.19 | 1.20 | 0.99 |
| | | 0.50 | | 0.44 | | 0.40 | | 0.15 | | 0.00 | | | | |
| 25 | 83.72 | | 7.53 | | 46.38 | | 4.63 | | 2.16 | | 1.31 | 1.16 | -1.20 | -0.97 |
| 26 | 83.22 | -0.50 | 7.98 | +0.45 | 46.77 | +0.39 | 4.48 | -0.15 | 2.16 | 0.00 | 1.33 | 1.14 | 1.19 | 0.95 |
| 27 | 82.70 | 0.52 | 8.45 | 0.47 | 47.14 | 0.37 | 4.33 | 0.15 | 2.17 | +0.01 | 1.35 | 1.12 | 1.19 | 0.93 |
| 28 | 82.17 | 0.53 | 8.94 | 0.49 | 47.49 | 0.35 | 4.18 | 0.15 | 2.19 | 0.02 | 1.36 | 1.10 | 1.18 | 0.91 |
| 29 | 81.62 | 0.55 | 9.44 | 0.50 | 47.83 | 0.34 | 4.02 | 0.16 | 2.22 | 0.03 | 1.37 | 1.07 | 1.17 | 0.90 |
| | | 0.57 | | 0.51 | | 0.33 | | 0.15 | | 0.04 | | | | |
| 30 | 81.05 | | 9.95 | | 48.16 | | 3.87 | | 2.26 | | 1.38 | 1.05 | -1.17 | -0.89 |
| 31 | 80.48 | -0.57 | 10.48 | +0.53 | 48.47 | +0.31 | 3.72 | -0.15 | 2.30 | +0.04 | 1.38 | 1.02 | 1.15 | 0.88 |
| 32 | 79.89 | 0.59 | 11.02 | 0.54 | 48.76 | 0.29 | 3.58 | 0.14 | 2.35 | 0.05 | 1.39 | 1.00 | 1.13 | 0.86 |
| 33 | 79.29 | 0.60 | 11.58 | 0.56 | 49.04 | 0.28 | 3.43 | 0.15 | 2.41 | 0.06 | 1.40 | 0.97 | 1.11 | 0.85 |
| 34 | 78.68 | 0.61 | 12.15 | 0.57 | 49.30 | 0.26 | 3.28 | 0.15 | 2.48 | 0.07 | 1.40 | 0.94 | 1.09 | 0.84 |
| | | 0.63 | | 0.58 | | 0.25 | | 0.14 | | 0.07 | | | | |
| 35 | 78.05 | | 12.73 | | 49.55 | | 3.14 | | 2.55 | | 1.40 | 0.91 | -1.09 | -0.83 |
| 36 | 77.41 | -0.64 | 13.33 | +0.60 | 49.77 | +0.22 | 3.01 | -0.13 | 2.64 | +0.09 | 1.40 | 0.88 | 1.07 | 0.83 |
| 37 | 76.76 | 0.65 | 13.94 | 0.61 | 49.98 | 0.21 | 2.87 | 0.14 | 2.73 | 0.09 | 1.40 | 0.85 | 1.05 | 0.82 |
| 38 | 76.10 | 0.66 | 14.57 | 0.63 | 50.16 | 0.18 | 2.74 | 0.13 | 2.83 | 0.10 | 1.39 | 0.82 | 1.03 | 0.82 |
| 39 | 75.42 | 0.68 | 15.21 | 0.64 | 50.32 | 0.16 | 2.61 | 0.13 | 2.94 | 0.11 | 1.38 | 0.79 | 0.99 | 0.81 |
| | | 0.69 | | 0.65 | | 0.14 | | 0.12 | | 0.11 | | | | |
| 40 | 74.73 | | 15.86 | | 50.46 | | 2.49 | | 3.05 | | 1.37 | 0.76 | -0.99 | -0.81 |
| 41 | 74.04 | -0.69 | 16.52 | +0.66 | 50.58 | +0.12 | 2.38 | -0.11 | 3.17 | +0.12 | 1.35 | 0.73 | 0.97 | 0.81 |
| 42 | 73.33 | 0.71 | 17.19 | 0.67 | 50.68 | 0.10 | 2.27 | 0.11 | 3.30 | 0.13 | 1.34 | 0.70 | 0.95 | 0.82 |
| 43 | 72.61 | 0.72 | 17.87 | 0.68 | 50.76 | 0.08 | 2.17 | 0.10 | 3.44 | 0.14 | 1.32 | 0.67 | 0.93 | 0.82 |
| 44 | 71.88 | 0.73 | 18.56 | 0.69 | 50.81 | 0.05 | 2.07 | 0.10 | 3.58 | 0.14 | 1.30 | 0.65 | 0.91 | 0.83 |
| | | 0.74 | | 0.70 | | +0.02 | | 0.10 | | 0.14 | | | | |
| 45 | 71.14 | | 19.26 | | 50.83 | | 1.97 | | 3.72 | | 1.28 | 0.62 | -0.91 | -0.83 |
| 46 | 70.39 | -0.75 | 19.97 | +0.71 | 50.83 | 0.00 | 1.88 | -0.09 | 3.87 | +0.15 | 1.25 | 0.60 | 0.91 | 0.84 |
| 47 | 69.63 | 0.76 | 20.68 | 0.71 | 50.81 | -0.02 | 1.81 | 0.07 | 4.03 | 0.16 | 1.23 | 0.58 | 0.87 | 0.86 |
| 48 | 68.87 | 0.76 | 21.40 | 0.72 | 50.75 | 0.06 | 1.75 | 0.06 | 4.19 | 0.16 | 1.20 | 0.56 | 0.86 | 0.87 |
| 49 | 68.10 | 0.77 | 22.12 | 0.72 | 50.67 | 0.08 | 1.69 | 0.06 | 4.36 | 0.17 | 1.17 | 0.54 | 0.85 | 0.88 |
| | | 0.78 | | 0.73 | | 0.10 | | 0.05 | | 0.17 | | | | |
| 50 | 67.32 | | 22.85 | | 50.57 | | 1.64 | | 4.53 | | 1.14 | 0.52 | -0.84 | -0.90 |
| 51 | 66.54 | -0.78 | 23.58 | +0.73 | 50.44 | -0.13 | 1.61 | -0.03 | 4.71 | +0.18 | 1.10 | 0.51 | 0.84 | 0.92 |
| 52 | 65.75 | 0.79 | 24.32 | 0.74 | 50.29 | 0.15 | 1.58 | 0.03 | 4.89 | 0.18 | 1.07 | 0.49 | 0.83 | 0.94 |
| 53 | 64.96 | 0.79 | 25.06 | 0.74 | 50.11 | 0.18 | 1.56 | 0.02 | 5.07 | 0.18 | 1.03 | 0.48 | 0.82 | 0.96 |
| 54 | 64.16 | 0.80 | 25.79 | 0.73 | 49.90 | 0.21 | 1.55 | 0.01 | 5.25 | 0.18 | 1.00 | 0.47 | 0.81 | 0.98 |
| | | 0.81 | | 0.74 | | 0.24 | | 0.00 | | 0.18 | | | | |
| 55 | 63.35 | | 26.53 | | 49.66 | | 1.55 | | 5.43 | | 0.96 | 0.47 | -0.81 | -1.00 |
| 56 | 62.54 | -0.81 | 27.27 | +0.74 | 49.39 | -0.27 | 1.57 | +0.02 | 5.62 | +0.19 | 0.92 | 0.47 | 0.81 | 1.01 |
| 57 | 61.73 | 0.81 | 27.99 | 0.72 | 49.11 | 0.28 | 1.59 | 0.02 | 5.81 | 0.19 | 0.89 | 0.47 | 0.82 | 1.03 |
| 58 | 60.92 | 0.81 | 28.72 | 0.73 | 48.79 | 0.32 | 1.62 | 0.03 | 5.99 | 0.18 | 0.85 | 0.48 | 0.82 | 1.05 |
| 59 | 60.11 | 0.81 | 29.45 | 0.73 | 48.44 | 0.35 | 1.66 | 0.04 | 6.17 | 0.18 | 0.81 | 0.48 | 0.83 | 1.07 |
| | | 0.81 | | 0.72 | | 0.37 | | 0.06 | | 0.19 | | | | |
| 60 | 59.30 | | 30.17 | | 48.07 | | 1.72 | | 6.36 | | 0.77 | 0.49 | -0.83 | -1.09 |

TABLE X, ARG. 3.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | (v.c.1) | Diff. | (v.s.2) | Diff. | (v.c.2) | Diff. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) |
|------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|---------|---------|---------|
| " | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 60 | 59.30 | | 30.17 | | 48.07 | | 1.72 | | 6.36 | | 0.77 | 0.49 | -0.83 | -1.09 |
| 61 | 58.49 | -0.81 | 30.88 | +0.71 | 47.66 | -0.41 | 1.78 | +0.06 | 6.54 | +0.18 | 0.73 | 0.50 | 0.84 | 1.10 |
| 62 | 57.67 | 0.82 | 31.59 | 0.71 | 47.24 | 0.42 | 1.86 | 0.08 | 6.71 | 0.17 | 0.69 | 0.52 | 0.85 | 1.11 |
| 63 | 56.86 | 0.81 | 32.29 | 0.70 | 46.79 | 0.45 | 1.95 | 0.09 | 6.89 | 0.18 | 0.66 | 0.54 | 0.87 | 1.13 |
| 64 | 56.05 | 0.81 | 32.97 | 0.67 | 46.31 | 0.50 | 2.05 | 0.10 | 7.06 | 0.17 | 0.62 | 0.56 | 0.88 | 1.14 |
| 65 | 55.24 | | 33.64 | | 45.81 | | 2.16 | | 7.23 | | 0.59 | 0.59 | -0.89 | -1.15 |
| 66 | 54.44 | -0.80 | 34.30 | +0.66 | 45.28 | -0.53 | 2.28 | +0.12 | 7.39 | +0.16 | 0.55 | 0.62 | 0.91 | 1.16 |
| 67 | 53.64 | 0.80 | 34.95 | 0.65 | 44.73 | 0.55 | 2.41 | 0.13 | 7.54 | 0.15 | 0.53 | 0.65 | 0.93 | 1.16 |
| 68 | 52.84 | 0.80 | 35.58 | 0.63 | 44.16 | 0.57 | 2.55 | 0.14 | 7.69 | 0.15 | 0.50 | 0.68 | 0.95 | 1.17 |
| 69 | 52.05 | 0.79 | 36.21 | 0.63 | 43.56 | 0.60 | 2.70 | 0.15 | 7.83 | 0.14 | 0.47 | 0.71 | 0.97 | 1.18 |
| 70 | 51.26 | | 36.82 | | 42.93 | | 2.85 | | 7.96 | | 0.45 | 0.75 | -0.99 | -1.19 |
| 71 | 50.47 | -0.79 | 37.41 | +0.59 | 42.28 | -0.65 | 3.02 | +0.17 | 8.08 | +0.12 | 0.43 | 0.78 | 1.00 | 1.19 |
| 72 | 49.70 | 0.77 | 37.98 | 0.57 | 41.62 | 0.66 | 3.19 | 0.17 | 8.20 | 0.12 | 0.41 | 0.82 | 1.02 | 1.19 |
| 73 | 48.93 | 0.77 | 38.53 | 0.55 | 40.94 | 0.63 | 3.37 | 0.18 | 8.31 | 0.11 | 0.39 | 0.87 | 1.04 | 1.18 |
| 74 | 48.17 | 0.76 | 39.06 | 0.53 | 40.23 | 0.71 | 3.56 | 0.19 | 8.40 | 0.09 | 0.38 | 0.91 | 1.06 | 1.18 |
| 75 | 47.43 | | 39.56 | | 39.51 | | 3.75 | | 8.49 | | 0.37 | 0.96 | -1.08 | -1.18 |
| 76 | 46.69 | -0.74 | 40.05 | +0.49 | 38.77 | -0.74 | 3.96 | +0.21 | 8.57 | +0.08 | 0.36 | 1.00 | 1.10 | 1.17 |
| 77 | 45.96 | 0.73 | 40.52 | 0.47 | 38.01 | 0.76 | 4.16 | 0.20 | 8.64 | 0.07 | 0.36 | 1.05 | 1.11 | 1.16 |
| 78 | 45.24 | 0.72 | 40.96 | 0.44 | 37.23 | 0.78 | 4.37 | 0.21 | 8.69 | 0.05 | 0.36 | 1.10 | 1.12 | 1.15 |
| 79 | 44.52 | 0.72 | 41.38 | 0.42 | 36.44 | 0.79 | 4.59 | 0.22 | 8.73 | 0.04 | 0.36 | 1.14 | 1.13 | 1.13 |
| 80 | 43.82 | | 41.78 | | 35.63 | | 4.80 | | 8.76 | | 0.37 | 1.18 | -1.14 | -1.12 |
| 81 | 43.14 | -0.68 | 42.16 | +0.38 | 34.81 | -0.82 | 5.02 | +0.22 | 8.77 | +0.01 | 0.38 | 1.23 | 1.15 | 1.10 |
| 82 | 42.46 | 0.68 | 42.51 | 0.35 | 33.98 | 0.83 | 5.24 | 0.22 | 8.78 | +0.01 | 0.40 | 1.28 | 1.16 | 1.08 |
| 83 | 41.80 | 0.66 | 42.83 | 0.32 | 33.13 | 0.85 | 5.47 | 0.23 | 8.77 | -0.01 | 0.41 | 1.32 | 1.17 | 1.06 |
| 84 | 41.15 | 0.65 | 43.12 | 0.29 | 32.28 | 0.85 | 5.69 | 0.22 | 8.75 | 0.02 | 0.43 | 1.36 | 1.17 | 1.04 |
| 85 | 40.52 | 0.63 | 43.38 | 0.26 | 31.42 | 0.86 | 5.91 | 0.22 | 8.72 | 0.03 | 0.46 | 1.41 | -1.18 | -1.02 |
| 86 | 39.90 | -0.62 | 43.61 | +0.23 | 30.55 | -0.87 | 6.13 | +0.22 | 8.68 | -0.04 | 0.49 | 1.45 | 1.18 | 1.01 |
| 87 | 39.30 | 0.60 | 43.82 | 0.21 | 29.68 | 0.87 | 6.35 | 0.22 | 8.62 | 0.06 | 0.52 | 1.49 | 1.18 | 0.99 |
| 88 | 38.71 | 0.59 | 44.00 | 0.18 | 28.80 | 0.88 | 6.57 | 0.22 | 8.55 | 0.07 | 0.55 | 1.52 | 1.17 | 0.97 |
| 89 | 38.14 | 0.57 | 44.16 | 0.16 | 27.91 | 0.89 | 6.79 | 0.22 | 8.47 | 0.08 | 0.59 | 1.56 | 1.16 | 0.95 |
| 90 | 37.58 | | 44.28 | | 27.01 | | 7.00 | | 8.38 | | 0.62 | 1.59 | -1.16 | -0.94 |
| 91 | 37.04 | -0.54 | 44.38 | +0.10 | 26.12 | -0.89 | 7.21 | +0.21 | 8.27 | -0.11 | 0.66 | 1.61 | 1.15 | 0.92 |
| 92 | 36.52 | 0.52 | 44.44 | 0.06 | 25.23 | 0.89 | 7.41 | 0.20 | 8.15 | 0.12 | 0.71 | 1.64 | 1.14 | 0.91 |
| 93 | 36.02 | 0.50 | 44.48 | +0.04 | 24.33 | 0.90 | 7.60 | 0.19 | 8.02 | 0.13 | 0.76 | 1.66 | 1.13 | 0.90 |
| 94 | 35.53 | 0.49 | 44.48 | 0.00 | 23.43 | 0.90 | 7.79 | 0.19 | 7.89 | 0.13 | 0.80 | 1.68 | 1.12 | 0.89 |
| 95 | 35.07 | 0.46 | | -0.02 | | 0.89 | | 0.18 | | 0.15 | | | | |
| 95 | 35.07 | | 44.46 | | 22.54 | | 7.97 | | 7.74 | | 0.85 | 1.70 | -1.11 | -0.88 |
| 96 | 34.62 | -0.45 | 44.40 | -0.06 | 21.65 | -0.89 | 8.14 | +0.17 | 7.58 | -0.16 | 0.90 | 1.71 | 1.10 | 0.87 |
| 97 | 34.20 | 0.42 | 44.32 | 0.08 | 20.77 | 0.88 | 8.30 | 0.16 | 7.41 | 0.17 | 0.95 | 1.72 | 1.08 | 0.86 |
| 98 | 33.79 | 0.41 | 44.20 | 0.12 | 19.90 | 0.87 | 8.45 | 0.15 | 7.24 | 0.17 | 1.00 | 1.72 | 1.06 | 0.85 |
| 99 | 33.41 | 0.38 | 44.07 | 0.13 | 19.03 | 0.87 | 8.60 | 0.15 | 7.05 | 0.19 | 1.05 | 1.73 | 1.04 | 0.84 |
| 100 | 33.04 | | 43.90 | | 18.16 | | 8.73 | | 6.85 | | 1.10 | 1.72 | -1.03 | -0.83 |
| 101 | 32.70 | -0.34 | 43.70 | -0.20 | 17.31 | -0.85 | 8.86 | +0.13 | 6.65 | -0.20 | 1.15 | 1.72 | 1.02 | 0.83 |
| 102 | 32.38 | 0.32 | 43.47 | 0.23 | 16.46 | 0.85 | 8.97 | 0.11 | 6.44 | 0.21 | 1.21 | 1.70 | 1.00 | 0.83 |
| 103 | 32.08 | 0.30 | 43.21 | 0.26 | 15.62 | 0.84 | 9.06 | 0.09 | 6.23 | 0.21 | 1.26 | 1.69 | 0.98 | 0.83 |
| 104 | 31.80 | 0.28 | 42.92 | 0.29 | 14.80 | 0.82 | 9.15 | 0.09 | 6.01 | 0.22 | 1.31 | 1.67 | 0.96 | 0.83 |
| 105 | 31.54 | | 42.61 | | 13.99 | | 9.23 | | 5.79 | | 1.35 | 1.65 | -0.95 | -0.83 |
| 106 | 31.31 | -0.23 | 42.27 | -0.34 | 13.20 | -0.79 | 9.30 | +0.07 | 5.57 | -0.22 | 1.40 | 1.63 | 0.94 | 0.84 |
| 107 | 31.11 | 0.20 | 41.89 | 0.38 | 12.42 | 0.78 | 9.35 | 0.05 | 5.34 | 0.23 | 1.44 | 1.60 | 0.92 | 0.85 |
| 108 | 30.92 | 0.19 | 41.49 | 0.40 | 11.66 | 0.76 | 9.39 | 0.04 | 5.10 | 0.24 | 1.48 | 1.56 | 0.90 | 0.86 |
| 109 | 30.76 | 0.16 | 41.08 | 0.41 | 10.92 | 0.74 | 9.42 | 0.03 | 4.87 | 0.23 | 1.53 | 1.53 | 0.89 | 0.87 |
| 110 | 30.62 | | 40.64 | | 10.19 | | 9.43 | | 4.64 | | 1.56 | 1.49 | -0.88 | -0.88 |
| 111 | 30.51 | -0.11 | 40.17 | -0.47 | 9.49 | -0.70 | 9.44 | +0.01 | 4.40 | -0.24 | 1.59 | 1.45 | 0.87 | 0.89 |
| 112 | 30.42 | 0.09 | 39.68 | 0.49 | 8.80 | 0.69 | 9.42 | -0.02 | 4.17 | 0.23 | 1.62 | 1.41 | 0.86 | 0.91 |
| 113 | 30.35 | 0.07 | 39.16 | 0.52 | 8.14 | 0.66 | 9.40 | 0.02 | 3.94 | 0.23 | 1.65 | 1.37 | 0.86 | 0.93 |
| 114 | 30.30 | 0.05 | 38.61 | 0.55 | 7.50 | 0.64 | 9.36 | 0.04 | 3.71 | 0.23 | 1.67 | 1.32 | 0.85 | 0.95 |
| 115 | 30.28 | -0.02 | | -0.56 | | 0.68 | | 0.31 | | 0.23 | | | | |
| 115 | 30.28 | | 38.05 | | 6.88 | | 9.31 | | 3.48 | | 1.70 | 1.27 | -0.84 | -0.96 |
| 116 | 30.29 | +0.01 | 37.47 | -0.58 | 6.28 | -0.60 | 9.24 | -0.07 | 3.26 | -0.22 | 1.72 | 1.22 | 0.84 | 0.97 |
| 117 | 30.32 | 0.03 | 36.86 | 0.61 | 5.71 | 0.57 | 9.17 | 0.07 | 3.04 | 0.22 | 1.73 | 1.17 | 0.84 | 0.99 |
| 118 | 30.37 | 0.05 | 36.24 | 0.62 | 5.17 | 0.54 | 9.08 | 0.09 | 2.82 | 0.22 | 1.74 | 1.12 | 0.84 | 1.01 |
| 119 | 30.46 | 0.09 | 35.60 | 0.64 | 4.65 | 0.52 | 8.99 | 0.09 | 2.61 | 0.21 | 1.74 | 1.07 | 0.84 | 1.02 |
| 120 | 30.56 | 0.10 | 34.95 | 0.65 | 4.16 | 0.49 | 8.88 | 0.11 | 2.41 | 0.20 | 1.74 | 1.02 | -0.84 | -1.04 |

TABLE X, ARG. 3.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | (v.c.1) | Diff. | (v.s.2) | Diff. | (v.c.2) | Diff. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) |
|------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|---------|---------|---------|
| " | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 120 | 30.56 | | 34.95 | | 4.16 | | 8.88 | | 2.41 | | 1.74 | 1.02 | -0.84 | -1.04 |
| 121 | 30.70 | +0.14 | 34.27 | -0.68 | 3.69 | -0.47 | 8.76 | -0.12 | 2.21 | -0.20 | 1.74 | 0.97 | 0.84 | 1.06 |
| 122 | 30.85 | 0.15 | 33.58 | 0.69 | 3.26 | 0.43 | 8.63 | 0.13 | 2.02 | 0.19 | 1.73 | 0.91 | 0.85 | 1.08 |
| 123 | 31.03 | 0.18 | 32.88 | 0.70 | 2.85 | 0.41 | 8.49 | 0.14 | 1.84 | 0.18 | 1.72 | 0.86 | 0.86 | 1.09 |
| 124 | 31.23 | 0.20 | 32.15 | 0.73 | 2.47 | 0.38 | 8.34 | 0.15 | 1.67 | 0.17 | 1.71 | 0.81 | 0.87 | 1.10 |
| 125 | 31.46 | 0.23 | 31.42 | 0.73 | 2.13 | 0.34 | 8.18 | 0.16 | 1.51 | 0.16 | 1.70 | 0.77 | -0.88 | -1.11 |
| 126 | 31.71 | +0.25 | 30.67 | -0.75 | 1.81 | -0.32 | 8.02 | -0.16 | 1.36 | -0.15 | 1.67 | 0.72 | 0.90 | 1.12 |
| 127 | 31.98 | 0.27 | 29.92 | 0.75 | 1.52 | 0.29 | 7.85 | 0.17 | 1.21 | 0.15 | 1.65 | 0.68 | 0.92 | 1.13 |
| 128 | 32.28 | 0.30 | 29.16 | 0.76 | 1.26 | 0.26 | 7.67 | 0.18 | 1.08 | 0.13 | 1.62 | 0.63 | 0.93 | 1.13 |
| 129 | 32.61 | 0.33 | 28.39 | 0.77 | 1.03 | 0.23 | 7.48 | 0.19 | 0.96 | 0.12 | 1.59 | 0.59 | 0.94 | 1.14 |
| 130 | 32.96 | 0.35 | 27.60 | 0.78 | 0.83 | 0.20 | 7.29 | 0.19 | 0.84 | 0.12 | 1.56 | 0.55 | -0.95 | -1.15 |
| 131 | 33.34 | +0.38 | 26.82 | -0.79 | 0.67 | -0.16 | 7.09 | -0.20 | 0.74 | -0.10 | 1.53 | 0.51 | 0.96 | 1.15 |
| 132 | 33.74 | 0.40 | 26.03 | 0.79 | 0.54 | 0.13 | 6.88 | 0.21 | 0.65 | 0.09 | 1.49 | 0.48 | 0.98 | 1.15 |
| 133 | 34.16 | 0.42 | 25.24 | 0.79 | 0.44 | 0.10 | 6.67 | 0.21 | 0.57 | 0.08 | 1.45 | 0.45 | 0.99 | 1.15 |
| 134 | 34.60 | 0.44 | 24.44 | 0.80 | 0.37 | 0.07 | 6.46 | 0.21 | 0.51 | 0.06 | 1.41 | 0.42 | 1.01 | 1.16 |
| 135 | 35.06 | 0.46 | 23.65 | 0.79 | 0.33 | 0.04 | 6.25 | 0.21 | 0.46 | 0.05 | 1.37 | 0.40 | -1.03 | -1.16 |
| 136 | 35.54 | +0.48 | 22.86 | -0.79 | 0.32 | -0.01 | 6.03 | -0.22 | 0.41 | -0.05 | 1.32 | 0.38 | 1.05 | 1.15 |
| 137 | 36.05 | 0.51 | 22.06 | 0.80 | 0.35 | +0.03 | 5.81 | 0.22 | 0.38 | 0.03 | 1.28 | 0.37 | 1.06 | 1.14 |
| 138 | 36.58 | 0.53 | 21.27 | 0.79 | 0.40 | 0.05 | 5.59 | 0.22 | 0.36 | -0.02 | 1.23 | 0.35 | 1.07 | 1.13 |
| 139 | 37.13 | 0.55 | 20.48 | 0.79 | 0.49 | 0.09 | 5.38 | 0.21 | 0.36 | 0.00 | 1.18 | 0.34 | 1.08 | 1.12 |
| 140 | 37.70 | 0.57 | 19.69 | 0.79 | 0.61 | 0.12 | 5.16 | 0.22 | 0.36 | 0.00 | 1.13 | 0.34 | -1.09 | -1.11 |
| 141 | 38.30 | +0.60 | 18.90 | -0.79 | 0.76 | +0.15 | 4.95 | -0.21 | 0.38 | +0.02 | 1.09 | 0.33 | 1.09 | 1.09 |
| 142 | 38.92 | 0.62 | 18.13 | 0.77 | 0.94 | 0.18 | 4.73 | 0.22 | 0.41 | 0.03 | 1.04 | 0.34 | 1.10 | 1.07 |
| 143 | 39.55 | 0.63 | 17.36 | 0.77 | 1.15 | 0.21 | 4.52 | 0.21 | 0.45 | 0.04 | 1.00 | 0.34 | 1.11 | 1.06 |
| 144 | 40.21 | 0.66 | 16.60 | 0.76 | 1.39 | 0.24 | 4.31 | 0.21 | 0.50 | 0.05 | 0.95 | 0.35 | 1.12 | 1.05 |
| 145 | 40.88 | 0.67 | 15.85 | 0.75 | 1.66 | 0.27 | 4.10 | 0.21 | 0.57 | 0.07 | 0.91 | 0.36 | -1.13 | -1.04 |
| 146 | 41.56 | +0.68 | 15.12 | -0.73 | 1.95 | +0.29 | 3.90 | -0.20 | 0.65 | +0.08 | 0.86 | 0.38 | 1.13 | 1.03 |
| 147 | 42.27 | 0.71 | 14.39 | 0.73 | 2.28 | 0.33 | 3.71 | 0.19 | 0.73 | 0.08 | 0.82 | 0.40 | 1.13 | 1.02 |
| 148 | 43.00 | 0.73 | 13.67 | 0.72 | 2.63 | 0.35 | 3.52 | 0.19 | 0.82 | 0.09 | 0.78 | 0.41 | 1.13 | 1.01 |
| 149 | 43.75 | 0.75 | 12.97 | 0.70 | 3.01 | 0.38 | 3.33 | 0.19 | 0.93 | 0.11 | 0.74 | 0.44 | 1.14 | 0.99 |
| 150 | 44.51 | 0.76 | 12.28 | 0.69 | 3.42 | 0.41 | 3.15 | 0.18 | 1.04 | 0.11 | 0.71 | 0.46 | -1.14 | -0.98 |
| 151 | 45.30 | +0.79 | 11.60 | -0.68 | 3.86 | +0.44 | 2.98 | -0.17 | 1.16 | +0.12 | 0.67 | 0.50 | 1.14 | 0.96 |
| 152 | 46.09 | 0.79 | 10.94 | 0.66 | 4.32 | 0.46 | 2.82 | 0.16 | 1.29 | 0.13 | 0.64 | 0.53 | 1.13 | 0.95 |
| 153 | 46.90 | 0.81 | 10.30 | 0.64 | 4.80 | 0.48 | 2.66 | 0.16 | 1.43 | 0.14 | 0.62 | 0.56 | 1.13 | 0.94 |
| 154 | 47.73 | 0.83 | 9.68 | 0.62 | 5.29 | 0.49 | 2.51 | 0.15 | 1.58 | 0.15 | 0.59 | 0.59 | 1.12 | 0.93 |
| 155 | 48.57 | 0.84 | 9.08 | 0.60 | 5.82 | 0.53 | 2.37 | 0.14 | 1.74 | 0.16 | 0.57 | 0.63 | -1.11 | -0.92 |
| 156 | 49.42 | +0.85 | 8.49 | -0.59 | 6.36 | +0.54 | 2.25 | -0.12 | 1.90 | +0.16 | 0.55 | 0.66 | 1.10 | 0.91 |
| 157 | 50.29 | 0.87 | 7.92 | 0.57 | 6.92 | 0.56 | 2.13 | 0.12 | 2.07 | 0.17 | 0.53 | 0.70 | 1.09 | 0.90 |
| 158 | 51.17 | 0.88 | 7.37 | 0.55 | 7.51 | 0.59 | 2.01 | 0.12 | 2.24 | 0.17 | 0.52 | 0.74 | 1.08 | 0.89 |
| 159 | 52.07 | 0.90 | 6.84 | 0.53 | 8.12 | 0.61 | 1.91 | 0.10 | 2.42 | 0.18 | 0.51 | 0.78 | 1.06 | 0.88 |
| 160 | 52.97 | 0.90 | 6.34 | 0.50 | 8.74 | 0.62 | 1.82 | 0.09 | 2.60 | 0.18 | 0.50 | 0.82 | -1.05 | -0.87 |
| 161 | 53.89 | +0.92 | 5.85 | -0.49 | 9.39 | +0.65 | 1.74 | -0.08 | 2.78 | +0.18 | 0.50 | 0.86 | 1.04 | 0.87 |
| 162 | 54.82 | 0.93 | 5.39 | 0.46 | 10.05 | 0.66 | 1.66 | 0.08 | 2.97 | 0.19 | 0.50 | 0.90 | 1.03 | 0.87 |
| 163 | 55.75 | 0.93 | 4.95 | 0.44 | 10.72 | 0.67 | 1.60 | 0.06 | 3.16 | 0.19 | 0.50 | 0.94 | 1.02 | 0.86 |
| 164 | 56.70 | 0.95 | 4.54 | 0.41 | 11.40 | 0.68 | 1.55 | 0.05 | 3.35 | 0.19 | 0.51 | 0.97 | 1.00 | 0.86 |
| 165 | 57.66 | 0.96 | 4.14 | 0.40 | 12.10 | 0.70 | 1.51 | 0.04 | 3.55 | 0.20 | 0.52 | 1.01 | -0.99 | -0.86 |
| 166 | 58.63 | +0.97 | 3.77 | -0.37 | 12.80 | +0.70 | 1.48 | -0.03 | 3.75 | +0.20 | 0.53 | 1.05 | 0.97 | 0.87 |
| 167 | 59.60 | 0.97 | 3.42 | 0.35 | 13.52 | 0.72 | 1.46 | 0.02 | 3.94 | 0.19 | 0.54 | 1.08 | 0.96 | 0.87 |
| 168 | 60.58 | 0.98 | 3.10 | 0.32 | 14.25 | 0.73 | 1.45 | -0.01 | 4.14 | 0.20 | 0.56 | 1.11 | 0.95 | 0.88 |
| 169 | 61.57 | 0.99 | 2.80 | 0.30 | 15.00 | 0.75 | 1.45 | 0.00 | 4.34 | 0.20 | 0.57 | 1.15 | 0.94 | 0.88 |
| 170 | 62.56 | 0.99 | 2.52 | 0.28 | 15.74 | 0.74 | 1.46 | +0.01 | 4.53 | 0.19 | 0.59 | 1.18 | -0.93 | -0.89 |
| 171 | 63.56 | +1.00 | 2.27 | -0.25 | 16.49 | +0.75 | 1.48 | +0.02 | 4.72 | +0.19 | 0.62 | 1.21 | 0.92 | 0.91 |
| 172 | 64.56 | 1.00 | 2.05 | 0.22 | 17.25 | 0.76 | 1.51 | 0.03 | 4.91 | 0.19 | 0.65 | 1.23 | 0.91 | 0.92 |
| 173 | 65.57 | 1.01 | 1.84 | 0.21 | 18.02 | 0.77 | 1.55 | 0.04 | 5.09 | 0.18 | 0.67 | 1.25 | 0.90 | 0.93 |
| 174 | 66.58 | 1.01 | 1.67 | 0.17 | 18.78 | 0.76 | 1.60 | 0.05 | 5.28 | 0.19 | 0.70 | 1.27 | 0.89 | 0.94 |
| 175 | 67.60 | 1.02 | 1.51 | 0.16 | 19.55 | 0.77 | 1.66 | 0.06 | 5.46 | 0.18 | 0.72 | 1.29 | -0.89 | -0.95 |
| 176 | 68.61 | +1.01 | 1.38 | -0.13 | 20.33 | +0.78 | 1.73 | +0.07 | 5.64 | +0.18 | 0.75 | 1.30 | 0.88 | 0.96 |
| 177 | 69.63 | 1.02 | 1.27 | 0.11 | 21.09 | 0.76 | 1.81 | 0.08 | 5.81 | 0.17 | 0.78 | 1.31 | 0.88 | 0.97 |
| 178 | 70.65 | 1.02 | 1.19 | 0.08 | 21.86 | 0.77 | 1.89 | 0.08 | 5.97 | 0.16 | 0.81 | 1.32 | 0.88 | 0.98 |
| 179 | 71.67 | 1.02 | 1.13 | 0.06 | 22.63 | 0.77 | 1.93 | 0.09 | 6.12 | 0.15 | 0.84 | 1.33 | 0.88 | 0.99 |
| 180 | 72.69 | 1.02 | 1.08 | 0.05 | 23.40 | 0.77 | 2.08 | 0.10 | 6.27 | 0.15 | 0.88 | 1.34 | -0.88 | -1.00 |

TABLE X, ARG. 3.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | (v.c.1) | Diff. | (v.s.2) | Diff. | (v.c.2) | Diff. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) |
|------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|---------|---------|---------|
| | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 180 | 72.69 | | 1.08 | -0.01 | 23.40 | +0.77 | 2.08 | +0.12 | 6.27 | +0.15 | 0.88 | 1.34 | -0.88 | -1.00 |
| 181 | 73.71 | +1.02 | 1.07 | +0.01 | 24.17 | 0.75 | 2.20 | 0.11 | 6.42 | 0.13 | 0.91 | 1.34 | 0.88 | 1.02 |
| 182 | 74.73 | 1.02 | 1.08 | 0.03 | 24.92 | 0.76 | 2.31 | 0.12 | 6.55 | 0.13 | 0.94 | 1.34 | 0.89 | 1.04 |
| 183 | 75.75 | 1.01 | 1.11 | 0.05 | 25.68 | 0.75 | 2.43 | 0.13 | 6.68 | 0.13 | 0.97 | 1.34 | 0.89 | 1.05 |
| 184 | 76.76 | 1.01 | 1.16 | 0.08 | 26.43 | 0.74 | 2.56 | 0.13 | 6.81 | 0.12 | 1.00 | 1.33 | -0.90 | -1.06 |
| 185 | 77.77 | | 1.24 | | 27.17 | | 2.69 | | 1.93 | | 1.03 | 1.33 | 0.92 | 1.06 |
| 186 | 78.78 | +1.01 | 1.34 | +0.10 | 27.90 | +0.73 | 2.83 | +0.14 | 7.04 | +0.11 | 1.06 | 1.32 | 0.93 | 1.07 |
| 187 | 79.78 | 1.00 | 1.45 | 0.11 | 28.63 | 0.73 | 2.96 | 0.13 | 7.13 | 0.09 | 1.09 | 1.31 | 0.94 | 1.08 |
| 188 | 80.77 | 0.99 | 1.59 | 0.14 | 29.34 | 0.71 | 3.10 | 0.14 | 7.22 | 0.08 | 1.11 | 1.29 | 0.95 | 1.09 |
| 189 | 81.76 | 0.99 | 1.74 | 0.15 | 30.05 | 0.70 | 3.25 | 0.15 | 7.30 | 0.07 | 1.13 | 1.28 | -0.96 | -1.10 |
| 190 | 82.75 | | 1.91 | | 30.75 | | 3.40 | | 7.37 | | 1.16 | 1.26 | 0.97 | 1.10 |
| 191 | 83.74 | +0.99 | 2.10 | +0.19 | 31.43 | +0.68 | 3.55 | +0.15 | 7.43 | +0.06 | 1.18 | 1.24 | 0.98 | 1.10 |
| 192 | 84.71 | 0.97 | 2.32 | 0.22 | 32.10 | 0.67 | 3.71 | 0.16 | 7.48 | 0.05 | 1.20 | 1.22 | 0.99 | 1.10 |
| 193 | 85.65 | 0.95 | 2.55 | 0.23 | 32.76 | 0.66 | 3.86 | 0.15 | 7.53 | 0.04 | 1.21 | 1.19 | 1.00 | 1.10 |
| 194 | 86.61 | 0.95 | 2.79 | 0.24 | 33.41 | 0.65 | 4.02 | 0.16 | 7.57 | 0.03 | 1.23 | 1.17 | 1.00 | 1.10 |
| 195 | 87.56 | | 3.05 | | 34.05 | | 4.17 | | 7.60 | | 1.24 | 1.15 | -1.01 | -1.11 |
| 196 | 88.50 | +0.94 | 3.33 | +0.28 | 34.67 | +0.62 | 4.33 | +0.16 | 7.62 | +0.02 | 1.25 | 1.12 | 1.01 | 1.11 |
| 197 | 89.42 | 0.92 | 3.62 | 0.29 | 35.27 | 0.60 | 4.49 | 0.16 | 7.63 | +0.01 | 1.26 | 1.09 | 1.02 | 1.10 |
| 198 | 90.34 | 0.92 | 3.93 | 0.31 | 35.86 | 0.59 | 4.64 | 0.15 | 7.63 | 0.00 | 1.27 | 1.07 | 1.03 | 1.10 |
| 199 | 91.24 | 0.90 | 4.25 | 0.32 | 36.43 | 0.57 | 4.79 | 0.15 | 7.62 | -0.01 | 1.27 | 1.04 | 1.04 | 1.10 |
| 200 | 92.14 | | 4.58 | | 36.99 | | 4.94 | | 7.60 | | 1.28 | 1.02 | -1.05 | -1.09 |
| 201 | 93.02 | +0.88 | 4.92 | +0.34 | 37.53 | +0.54 | 5.09 | +0.15 | 7.57 | -0.03 | 1.28 | 1.00 | 1.05 | 1.08 |
| 202 | 93.89 | 0.87 | 5.28 | 0.36 | 38.05 | 0.52 | 5.23 | 0.14 | 7.54 | 0.03 | 1.28 | 0.97 | 1.06 | 1.07 |
| 203 | 94.74 | 0.85 | 5.65 | 0.37 | 38.56 | 0.51 | 5.37 | 0.14 | 7.50 | 0.04 | 1.27 | 0.94 | 1.07 | 1.06 |
| 204 | 95.59 | 0.85 | 6.03 | 0.38 | 39.04 | 0.48 | 5.51 | 0.14 | 7.46 | 0.04 | 1.26 | 0.92 | 1.08 | 1.05 |
| 205 | 96.41 | | 6.42 | | 39.51 | | 5.64 | | 7.41 | | 1.25 | 0.89 | -1.09 | -1.04 |
| 206 | 97.23 | +0.82 | 6.81 | +0.39 | 39.96 | +0.45 | 5.77 | +0.13 | 7.35 | -0.06 | 1.24 | 0.87 | 1.09 | 1.04 |
| 207 | 98.02 | 0.79 | 7.22 | 0.41 | 40.40 | 0.44 | 5.89 | 0.12 | 7.28 | 0.07 | 1.23 | 0.85 | 1.09 | 1.03 |
| 208 | 98.81 | 0.79 | 7.64 | 0.42 | 40.81 | 0.41 | 6.01 | 0.12 | 7.21 | 0.07 | 1.22 | 0.83 | 1.09 | 1.02 |
| 209 | 99.57 | 0.76 | 8.06 | 0.42 | 41.21 | 0.40 | 6.12 | 0.11 | 7.13 | 0.08 | 1.21 | 0.81 | 1.09 | 1.01 |
| 210 | 100.32 | | 8.49 | | 41.58 | | 6.22 | | 7.05 | | 1.19 | 0.79 | -1.10 | -1.00 |
| 211 | 101.06 | +0.74 | 8.92 | +0.43 | 41.94 | +0.36 | 6.32 | +0.10 | 6.96 | -0.09 | 1.17 | 0.77 | 1.10 | 0.99 |
| 212 | 101.78 | 0.72 | 9.36 | 0.44 | 42.28 | 0.34 | 6.41 | 0.09 | 6.85 | 0.10 | 1.15 | 0.76 | 1.09 | 0.98 |
| 213 | 102.48 | 0.70 | 9.80 | 0.44 | 42.59 | 0.31 | 6.50 | 0.09 | 6.76 | 0.10 | 1.13 | 0.75 | 1.09 | 0.97 |
| 214 | 103.17 | 0.69 | 10.24 | 0.44 | 42.89 | 0.30 | 6.58 | 0.08 | 6.66 | 0.10 | 1.11 | 0.73 | 1.09 | 0.97 |
| 215 | 103.84 | | 10.69 | | 43.18 | | 6.65 | | 6.56 | | 1.09 | 0.72 | -1.08 | -0.96 |
| 216 | 104.49 | +0.65 | 11.14 | +0.45 | 43.44 | +0.26 | 6.71 | +0.06 | 6.45 | -0.11 | 1.06 | 0.71 | 1.08 | 0.96 |
| 217 | 105.12 | 0.63 | 11.59 | 0.45 | 43.68 | 0.24 | 6.77 | 0.06 | 6.33 | 0.12 | 1.04 | 0.70 | 1.07 | 0.95 |
| 218 | 105.73 | 0.61 | 12.04 | 0.45 | 43.90 | 0.22 | 6.82 | 0.05 | 6.22 | 0.11 | 1.02 | 0.70 | 1.06 | 0.94 |
| 219 | 106.33 | 0.60 | 12.50 | 0.46 | 44.11 | 0.21 | 6.87 | 0.05 | 6.11 | 0.11 | 1.00 | 0.70 | 1.05 | 0.93 |
| 220 | 106.90 | 0.57 | 12.95 | 0.45 | 44.29 | 0.18 | 6.91 | 0.04 | 6.00 | 0.12 | 0.98 | 0.70 | -1.04 | -0.92 |
| 221 | 107.45 | +0.55 | 13.41 | +0.46 | 44.45 | +0.16 | 6.94 | +0.03 | 5.87 | -0.12 | 0.95 | 0.70 | 1.04 | 0.92 |
| 222 | 107.99 | 0.54 | 13.86 | 0.45 | 44.55 | 0.15 | 6.96 | 0.02 | 5.76 | 0.11 | 0.92 | 0.70 | 1.03 | 0.92 |
| 223 | 108.50 | 0.51 | 14.30 | 0.44 | 44.60 | 0.13 | 6.98 | 0.02 | 5.65 | 0.11 | 0.90 | 0.70 | 1.02 | 0.92 |
| 224 | 109.00 | 0.50 | 14.75 | 0.45 | 44.73 | 0.11 | 6.99 | +0.01 | 5.53 | 0.12 | 0.88 | 0.71 | 1.01 | 0.92 |
| 225 | 109.48 | 0.48 | 15.19 | 0.44 | 44.84 | 0.10 | 6.99 | 0.00 | 5.42 | 0.11 | 0.86 | 0.72 | -1.00 | -0.92 |
| 226 | 109.94 | +0.46 | 15.63 | +0.44 | 44.94 | +0.08 | 6.98 | -0.01 | 5.31 | -0.11 | 0.84 | 0.72 | 1.00 | 0.92 |
| 227 | 110.37 | 0.43 | 16.07 | 0.44 | 45.02 | 0.06 | 6.97 | 0.01 | 5.20 | 0.11 | 0.81 | 0.73 | 1.00 | 0.92 |
| 228 | 110.78 | 0.41 | 16.50 | 0.43 | 45.08 | 0.04 | 6.96 | 0.01 | 5.09 | 0.11 | 0.79 | 0.74 | 0.99 | 0.92 |
| 229 | 111.18 | 0.40 | 16.93 | 0.43 | 45.12 | 0.02 | 6.94 | 0.02 | 4.98 | 0.11 | 0.77 | 0.76 | 0.98 | 0.92 |
| 230 | 111.55 | | 17.35 | | 45.14 | | 6.91 | | 4.88 | | 0.76 | 0.78 | -0.97 | -0.93 |
| 231 | 111.89 | +0.37 | 17.76 | +0.41 | 45.15 | +0.01 | 6.88 | -0.03 | 4.78 | -0.10 | 0.74 | 0.79 | 0.97 | 0.93 |
| 232 | 112.23 | 0.34 | 18.17 | 0.41 | 45.15 | 0.00 | 6.85 | 0.03 | 4.68 | 0.10 | 0.72 | 0.81 | 0.96 | 0.94 |
| 233 | 112.54 | 0.31 | 18.57 | 0.40 | 45.13 | -0.02 | 6.80 | 0.05 | 4.58 | 0.10 | 0.71 | 0.83 | 0.95 | 0.95 |
| 234 | 112.83 | 0.29 | 18.96 | 0.39 | 45.09 | 0.04 | 6.80 | 0.05 | 4.49 | 0.09 | 0.69 | 0.85 | 0.94 | 0.96 |
| 235 | 113.10 | 0.27 | 19.34 | 0.38 | 45.04 | 0.05 | 6.75 | 0.05 | 4.40 | 0.09 | 0.68 | 0.87 | -0.93 | -0.97 |
| 236 | 113.35 | +0.25 | 19.72 | +0.38 | 44.98 | +0.07 | 6.70 | +0.06 | 4.31 | -0.09 | 0.67 | 0.89 | 0.93 | 0.97 |
| 237 | 113.57 | 0.22 | 20.09 | 0.37 | 44.91 | 0.09 | 6.64 | 0.06 | 4.23 | 0.08 | 0.66 | 0.91 | 0.93 | 0.98 |
| 238 | 113.77 | 0.20 | 20.45 | 0.36 | 44.82 | 0.11 | 6.58 | 0.07 | 4.16 | 0.07 | 0.65 | 0.93 | 0.93 | 0.99 |
| 239 | 113.95 | 0.18 | 20.80 | 0.35 | 44.71 | 0.12 | 6.51 | 0.06 | 4.09 | 0.07 | 0.65 | 0.96 | 0.93 | 1.00 |
| 240 | 114.11 | 0.16 | 21.15 | 0.35 | 44.59 | 0.12 | 6.45 | 0.07 | 4.02 | 0.07 | 0.65 | 0.98 | -0.93 | -1.01 |

TABLE X, ARG. 3.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | (v.c.1) | Diff. | (v.s.2) | Diff. | (v.c.2) | Diff. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) |
|------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|---------|---------|---------|
| | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 240 | 114.11 | | 21.15 | | 44.47 | | 6.38 | | 4.02 | | 0.65 | 0.98 | -0.98 | -1.01 |
| 241 | 114.24 | +0.13 | 21.48 | +0.33 | 44.33 | -0.14 | 6.30 | -0.08 | 3.96 | -0.06 | 0.65 | 1.00 | 0.93 | 1.01 |
| 242 | 114.36 | 0.12 | 21.80 | 0.32 | 44.18 | 0.15 | 6.23 | 0.07 | 3.90 | 0.06 | 0.65 | 1.03 | 0.93 | 1.02 |
| 243 | 114.46 | 0.10 | 22.12 | 0.32 | 44.02 | 0.16 | 6.15 | 0.08 | 3.85 | 0.05 | 0.65 | 1.05 | 0.94 | 1.03 |
| 244 | 114.54 | 0.08 | 22.43 | 0.31 | 43.86 | 0.16 | 6.07 | 0.08 | 3.80 | 0.05 | 0.65 | 1.07 | 0.95 | 1.04 |
| | | 0.06 | | 0.30 | | 0.18 | | 0.08 | | 0.05 | | | | |
| 245 | 114.60 | | 22.73 | | 43.68 | | 5.99 | | 3.75 | | 0.65 | 1.10 | -0.95 | -1.04 |
| 246 | 114.63 | +0.03 | 23.01 | +0.28 | 43.49 | -0.19 | 5.91 | -0.08 | 3.71 | -0.04 | 0.65 | 1.12 | 0.96 | 1.04 |
| 247 | 114.65 | +0.02 | 23.29 | 0.28 | 43.30 | 0.19 | 5.83 | 0.08 | 3.68 | 0.03 | 0.66 | 1.14 | 0.97 | 1.04 |
| 248 | 114.64 | -0.01 | 23.56 | 0.27 | 43.10 | 0.20 | 5.74 | 0.09 | 3.65 | 0.03 | 0.66 | 1.16 | 0.97 | 1.05 |
| 249 | 114.61 | 0.03 | 23.82 | 0.26 | 42.88 | 0.22 | 5.66 | 0.08 | 3.62 | 0.03 | 0.67 | 1.19 | 0.98 | 1.06 |
| | | 0.06 | | 0.24 | | 0.21 | | 0.08 | | 0.02 | | | | |
| 250 | 114.55 | | 24.06 | | 42.67 | | 5.58 | | 3.60 | | 0.69 | 1.21 | -0.98 | -1.06 |
| 251 | 114.48 | -0.07 | 24.30 | +0.24 | 42.45 | -0.22 | 5.49 | -0.09 | 3.59 | -0.01 | 0.70 | 1.23 | 0.98 | 1.06 |
| 252 | 114.39 | 0.09 | 24.53 | 0.23 | 42.22 | 0.23 | 5.41 | 0.08 | 3.58 | 0.01 | 0.72 | 1.24 | 0.99 | 1.06 |
| 253 | 114.29 | 0.10 | 24.75 | 0.22 | 41.98 | 0.24 | 5.33 | 0.08 | 3.57 | -0.01 | 0.73 | 1.26 | 1.00 | 1.07 |
| 254 | 114.16 | 0.13 | 24.95 | 0.20 | 41.74 | 0.24 | 5.25 | 0.08 | 3.57 | 0.00 | 0.75 | 1.28 | 1.01 | 1.07 |
| | | 0.15 | | 0.20 | | 0.25 | | 0.09 | | 0.00 | | | | |
| 255 | 114.01 | | 25.15 | | 41.49 | | 5.16 | | 3.57 | | 0.77 | 1.30 | -1.01 | -1.07 |
| 256 | 113.84 | -0.17 | 25.34 | +0.19 | 41.24 | -0.25 | 5.08 | -0.08 | 3.57 | 0.00 | 0.78 | 1.31 | 1.01 | 1.07 |
| 257 | 113.66 | 0.18 | 25.52 | 0.18 | 40.98 | 0.26 | 5.01 | 0.07 | 3.57 | 0.00 | 0.80 | 1.32 | 1.01 | 1.07 |
| 258 | 113.45 | 0.21 | 25.70 | 0.18 | 40.73 | 0.25 | 4.93 | 0.08 | 3.58 | +0.01 | 0.82 | 1.34 | 1.02 | 1.06 |
| 259 | 113.22 | 0.23 | 25.86 | 0.16 | 40.47 | 0.26 | 4.85 | 0.08 | 3.60 | 0.02 | 0.84 | 1.35 | 1.03 | 1.05 |
| | | 0.24 | | 0.16 | | 0.26 | | 0.07 | | 0.02 | | | | |
| 260 | 112.98 | | 26.02 | | 40.21 | | 4.78 | | 3.62 | | 0.87 | 1.36 | -1.03 | -1.05 |
| 261 | 112.71 | -0.27 | 26.16 | +0.14 | 39.95 | -0.26 | 4.71 | -0.07 | 3.64 | +0.02 | 0.89 | 1.37 | 1.03 | 1.04 |
| 262 | 112.43 | 0.28 | 26.30 | 0.14 | 39.68 | 0.27 | 4.64 | 0.07 | 3.66 | 0.02 | 0.91 | 1.38 | 1.03 | 1.03 |
| 263 | 112.14 | 0.29 | 26.43 | 0.13 | 39.41 | 0.27 | 4.57 | 0.07 | 3.69 | 0.03 | 0.93 | 1.38 | 1.04 | 1.02 |
| 264 | 111.84 | 0.30 | 26.55 | 0.12 | 39.13 | 0.28 | 4.51 | 0.06 | 3.72 | 0.03 | 0.96 | 1.39 | 1.05 | 1.01 |
| | | 0.33 | | 0.11 | | 0.27 | | 0.06 | | 0.03 | | | | |
| 265 | 111.51 | | 26.66 | | 38.86 | | 4.45 | | 3.75 | | 0.98 | 1.39 | -1.05 | -1.01 |
| 266 | 111.16 | -0.35 | 26.77 | +0.11 | 38.59 | -0.27 | 4.39 | -0.06 | 3.79 | +0.04 | 1.00 | 1.39 | 1.05 | 1.01 |
| 267 | 110.80 | 0.36 | 26.87 | 0.10 | 38.31 | 0.28 | 4.33 | 0.06 | 3.83 | 0.04 | 1.02 | 1.39 | 1.05 | 1.01 |
| 268 | 110.43 | 0.37 | 26.96 | 0.09 | 38.03 | 0.28 | 4.28 | 0.05 | 3.86 | 0.03 | 1.05 | 1.38 | 1.05 | 1.00 |
| 269 | 110.04 | 0.39 | 27.04 | 0.08 | 37.76 | 0.27 | 4.22 | 0.06 | 3.90 | 0.04 | 1.07 | 1.38 | 1.05 | 0.99 |
| | | 0.41 | | 0.08 | | 0.27 | | 0.05 | | 0.04 | | | | |
| 270 | 109.63 | | 27.12 | | 37.49 | | 4.17 | | 3.94 | | 1.09 | 1.38 | -1.05 | -0.99 |
| 271 | 109.21 | -0.42 | 27.19 | +0.07 | 37.21 | -0.28 | 4.12 | -0.05 | 3.99 | +0.05 | 1.12 | 1.37 | 1.05 | 0.99 |
| 272 | 108.77 | 0.44 | 27.26 | 0.07 | 36.93 | 0.28 | 4.07 | 0.05 | 4.03 | 0.04 | 1.14 | 1.37 | 1.05 | 0.98 |
| 273 | 108.32 | 0.45 | 27.32 | 0.06 | 36.65 | 0.28 | 4.02 | 0.05 | 4.08 | 0.05 | 1.16 | 1.36 | 1.04 | 0.98 |
| 274 | 107.86 | 0.46 | 27.37 | 0.05 | 36.37 | 0.28 | 3.98 | 0.04 | 4.12 | 0.04 | 1.18 | 1.35 | 1.04 | 0.97 |
| | | 0.48 | | 0.05 | | 0.27 | | 0.03 | | 0.05 | | | | |
| 275 | 107.38 | | 27.42 | | 36.10 | | 3.95 | | 4.17 | | 1.20 | 1.34 | -1.04 | -0.97 |
| 276 | 106.90 | -0.48 | 27.46 | +0.04 | 35.82 | -0.28 | 3.92 | -0.03 | 4.22 | +0.05 | 1.22 | 1.32 | 1.03 | 0.97 |
| 277 | 106.40 | 0.50 | 27.50 | 0.04 | 35.54 | 0.28 | 3.89 | 0.03 | 4.27 | 0.05 | 1.24 | 1.31 | 1.02 | 0.97 |
| 278 | 105.89 | 0.51 | 27.53 | 0.03 | 35.27 | 0.27 | 3.86 | 0.03 | 4.32 | 0.05 | 1.26 | 1.30 | 1.01 | 0.97 |
| 279 | 105.37 | 0.52 | 27.56 | 0.03 | 35.00 | 0.27 | 3.83 | 0.03 | 4.37 | 0.05 | 1.28 | 1.28 | 1.00 | 0.96 |
| | | 0.53 | | 0.03 | | 0.28 | | 0.03 | | 0.05 | | | | |
| 280 | 104.84 | | 27.59 | | 34.72 | | 3.80 | | 4.42 | | 1.29 | 1.26 | -1.00 | -0.96 |
| 281 | 104.30 | -0.54 | 27.61 | +0.02 | 34.44 | -0.28 | 3.77 | -0.03 | 4.47 | +0.05 | 1.31 | 1.24 | 1.00 | 0.96 |
| 282 | 103.75 | 0.55 | 27.63 | 0.02 | 34.17 | 0.27 | 3.75 | 0.02 | 4.53 | 0.06 | 1.32 | 1.22 | 1.00 | 0.96 |
| 283 | 103.19 | 0.56 | 27.64 | 0.01 | 33.90 | 0.27 | 3.74 | 0.01 | 4.58 | 0.05 | 1.34 | 1.20 | 0.99 | 0.96 |
| 284 | 102.62 | 0.57 | 27.65 | +0.01 | 33.63 | 0.27 | 3.72 | 0.02 | 4.64 | 0.06 | 1.35 | 1.18 | 0.98 | 0.95 |
| | | 0.58 | | 0.00 | | 0.27 | | 0.01 | | 0.05 | | | | |
| 285 | 102.04 | | 27.65 | | 33.36 | | 3.71 | | 4.69 | | 1.36 | 1.16 | -0.98 | -0.95 |
| 286 | 101.46 | -0.58 | 27.65 | 0.00 | 33.09 | -0.27 | 3.70 | -0.01 | 4.74 | +0.05 | 1.37 | 1.14 | 0.98 | 0.95 |
| 287 | 100.87 | 0.59 | 27.65 | 0.00 | 32.82 | 0.27 | 3.69 | 0.01 | 4.79 | 0.05 | 1.38 | 1.12 | 0.98 | 0.96 |
| 288 | 100.27 | 0.60 | 27.64 | -0.01 | 32.55 | 0.27 | 3.68 | 0.01 | 4.85 | 0.06 | 1.38 | 1.09 | 0.97 | 0.97 |
| 289 | 99.67 | 0.60 | 27.63 | 0.01 | 32.29 | 0.26 | 3.67 | -0.01 | 4.90 | 0.05 | 1.39 | 1.07 | 0.96 | 0.97 |
| | | 0.61 | | 0.01 | | 0.27 | | 0.00 | | 0.05 | | | | |
| 290 | 99.06 | | 27.62 | | 32.02 | | 3.67 | | 4.95 | | 1.39 | 1.05 | -0.96 | -0.98 |
| 291 | 98.45 | -0.61 | 27.60 | -0.02 | 31.76 | -0.26 | 3.67 | 0.00 | 5.00 | +0.05 | 1.40 | 1.02 | 0.96 | 0.98 |
| 292 | 97.83 | 0.62 | 27.58 | 0.02 | 31.50 | 0.26 | 3.67 | 0.00 | 5.06 | 0.06 | 1.40 | 1.00 | 0.96 | 0.99 |
| 293 | 97.21 | 0.62 | 27.56 | 0.02 | 31.23 | 0.27 | 3.67 | 0.00 | 5.11 | 0.05 | 1.40 | 0.98 | 0.95 | 1.00 |
| 294 | 96.58 | 0.63 | 27.54 | 0.02 | 30.97 | 0.26 | 3.68 | +0.01 | 5.16 | 0.05 | 1.40 | 0.95 | 0.95 | 1.01 |
| | | 0.63 | | 0.02 | | 0.26 | | 0.01 | | 0.05 | | | | |
| 295 | 95.95 | | 27.52 | | 30.71 | | 3.69 | | 5.21 | | 1.39 | 0.93 | -0.95 | -1.02 |
| 296 | 95.31 | -0.64 | 27.48 | -0.04 | 30.45 | -0.26 | 3.70 | +0.01 | 5.26 | +0.05 | 1.39 | 0.91 | 0.95 | 1.02 |
| 297 | 94.67 | 0.64 | 27.45 | 0.03 | 30.19 | 0.26 | 3.71 | 0.01 | 5.32 | 0.06 | 1.38 | 0.88 | 0.96 | 1.02 |
| 298 | 94.04 | 0.63 | 27.42 | 0.03 | 29.93 | 0.26 | 3.72 | 0.01 | 5.37 | 0.05 | 1.38 | 0.86 | 0.96 | 1.03 |
| 299 | 93.40 | 0.64 | 27.38 | 0.04 | 29.67 | 0.26 | 3.74 | 0.02 | 5.42 | 0.05 | 1.37 | 0.84 | 0.97 | 1.03 |
| | | 0.63 | | 0.04 | | 0.27 | | 0.02 | | 0.05 | | | | |
| 300 | 92.77 | | 27.34 | | 29.40 | | 3.76 | | 5.47 | | 1.35 | 0.82 | -0.97 | -1.03 |

TABLE X, ARG. 3.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | (v.c.1) | Diff. | (v.s.2) | Diff. | (v.c.2) | Diff. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) |
|------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|---------|---------|---------|
| " | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 300 | 92.77 | | 27.34 | | 29.40 | | 3.76 | | 5.47 | | 1.35 | 0.82 | -0.97 | -1.03 |
| 301 | 92.13 | -0.64 | 27.30 | -0.04 | 29.16 | -0.24 | 3.77 | +0.01 | 5.51 | +0.04 | 1.34 | 0.80 | 0.97 | 1.03 |
| 302 | 91.50 | 0.63 | 27.26 | 0.04 | 28.89 | 0.27 | 3.80 | 0.03 | 5.56 | 0.05 | 1.33 | 0.78 | 0.98 | 1.03 |
| 303 | 90.87 | 0.63 | 27.21 | 0.05 | 28.63 | 0.26 | 3.82 | 0.02 | 5.61 | 0.05 | 1.31 | 0.76 | 0.98 | 1.04 |
| 304 | 90.24 | 0.63 | 27.16 | 0.05 | 28.37 | 0.26 | 3.85 | 0.03 | 5.66 | 0.05 | 1.30 | 0.74 | 0.99 | 1.04 |
| | | 0.64 | | 0.06 | | 0.26 | | 0.02 | | 0.05 | | | | |
| 305 | 89.60 | | 27.10 | | 28.11 | | 3.87 | | 5.71 | | 1.28 | 0.72 | -0.99 | -1.04 |
| 306 | 88.97 | -0.63 | 27.05 | -0.05 | 27.86 | -0.25 | 3.91 | +0.04 | 5.75 | +0.04 | 1.27 | 0.71 | 1.00 | 1.04 |
| 307 | 88.35 | 0.62 | 26.99 | 0.06 | 27.60 | 0.26 | 3.94 | 0.03 | 5.80 | 0.05 | 1.25 | 0.69 | 1.00 | 1.04 |
| 308 | 87.72 | 0.62 | 26.93 | 0.06 | 27.35 | 0.25 | 3.97 | 0.03 | 5.84 | 0.04 | 1.23 | 0.68 | 1.01 | 1.04 |
| 309 | 87.10 | 0.62 | 26.86 | 0.07 | 27.10 | 0.25 | 4.01 | 0.04 | 5.88 | 0.04 | 1.21 | 0.66 | 1.01 | 1.04 |
| | | 0.61 | | 0.07 | | 0.25 | | 0.04 | | 0.04 | | | | |
| 310 | 86.49 | | 26.79 | | 26.85 | | 4.05 | | 5.92 | | 1.19 | 0.65 | -1.02 | -1.04 |
| 311 | 85.89 | -0.60 | 26.72 | -0.07 | 26.59 | -0.26 | 4.09 | +0.04 | 5.96 | +0.04 | 1.17 | 0.64 | 1.02 | 1.04 |
| 312 | 85.29 | 0.60 | 26.65 | 0.07 | 26.34 | 0.25 | 4.14 | 0.05 | 6.00 | 0.04 | 1.15 | 0.63 | 1.03 | 1.03 |
| 313 | 84.69 | 0.60 | 26.58 | 0.07 | 26.09 | 0.25 | 4.18 | 0.04 | 6.03 | 0.03 | 1.12 | 0.62 | 1.03 | 1.03 |
| 314 | 84.09 | 0.60 | 26.49 | 0.09 | 25.84 | 0.25 | 4.22 | 0.04 | 6.07 | 0.04 | 1.10 | 0.62 | 1.04 | 1.03 |
| | | 0.58 | | 0.09 | | 0.25 | | 0.05 | | 0.03 | | | | |
| 315 | 83.51 | | 26.40 | | 25.59 | | 4.27 | | 6.10 | | 1.08 | 0.61 | -1.04 | -1.03 |
| 316 | 82.93 | -0.58 | 26.31 | -0.09 | 25.34 | -0.25 | 4.32 | +0.05 | 6.14 | +0.04 | 1.05 | 0.61 | 1.04 | 1.02 |
| 317 | 82.37 | 0.56 | 26.22 | 0.09 | 25.10 | 0.24 | 4.37 | 0.05 | 6.17 | 0.03 | 1.03 | 0.61 | 1.04 | 1.02 |
| 318 | 81.81 | 0.56 | 26.12 | 0.10 | 24.85 | 0.25 | 4.43 | 0.06 | 6.19 | 0.02 | 1.01 | 0.61 | 1.04 | 1.01 |
| 319 | 81.27 | 0.54 | 26.02 | 0.10 | 24.60 | 0.25 | 4.48 | 0.05 | 6.22 | 0.03 | 0.98 | 0.61 | 1.04 | 1.00 |
| | | 0.54 | | 0.11 | | 0.25 | | 0.06 | | 0.03 | | | | |
| 320 | 80.73 | | 25.91 | | 24.35 | | 4.54 | | 6.25 | | 0.96 | 0.61 | -1.04 | -1.00 |
| 321 | 80.20 | -0.53 | 25.80 | -0.11 | 24.11 | -0.24 | 4.60 | +0.06 | 6.27 | +0.02 | 0.94 | 0.61 | 1.04 | 0.99 |
| 322 | 79.68 | 0.52 | 25.68 | 0.12 | 23.87 | 0.24 | 4.66 | 0.06 | 6.29 | 0.02 | 0.91 | 0.62 | 1.03 | 0.99 |
| 323 | 79.17 | 0.51 | 25.56 | 0.12 | 23.63 | 0.24 | 4.72 | 0.06 | 6.30 | 0.01 | 0.89 | 0.62 | 1.03 | 0.98 |
| 324 | 78.68 | 0.49 | 25.44 | 0.12 | 23.39 | 0.24 | 4.78 | 0.06 | 6.32 | 0.02 | 0.87 | 0.63 | 1.03 | 0.98 |
| | | 0.49 | | 0.13 | | 0.23 | | 0.07 | | 0.02 | | | | |
| 325 | 78.19 | | 25.31 | | 23.16 | | 4.85 | | 6.34 | | 0.85 | 0.64 | -1.03 | -0.97 |
| 326 | 77.72 | -0.47 | 25.17 | -0.14 | 22.92 | -0.24 | 4.92 | +0.07 | 6.35 | +0.01 | 0.83 | 0.65 | 1.03 | 0.97 |
| 327 | 77.26 | 0.46 | 25.02 | 0.15 | 22.69 | 0.23 | 4.99 | 0.07 | 6.35 | 0.00 | 0.81 | 0.66 | 1.02 | 0.96 |
| 328 | 76.82 | 0.44 | 24.87 | 0.15 | 22.46 | 0.23 | 5.06 | 0.07 | 6.35 | 0.00 | 0.79 | 0.67 | 1.02 | 0.96 |
| 329 | 76.39 | 0.43 | 24.72 | 0.15 | 22.23 | 0.23 | 5.13 | 0.07 | 6.35 | 0.00 | 0.77 | 0.69 | 1.01 | 0.95 |
| | | 0.42 | | 0.16 | | 0.23 | | 0.07 | | 0.00 | | | | |
| 330 | 75.97 | | 24.56 | | 22.00 | | 5.20 | | 6.35 | | 0.75 | 0.70 | -1.01 | -0.95 |
| 331 | 75.57 | -0.40 | 24.39 | -0.17 | 21.78 | -0.22 | 5.27 | +0.07 | 6.35 | 0.00 | 0.73 | 0.72 | 1.00 | 0.95 |
| 332 | 75.18 | 0.39 | 24.21 | 0.18 | 21.57 | 0.21 | 5.34 | 0.07 | 6.34 | -0.01 | 0.72 | 0.74 | 1.00 | 0.95 |
| 333 | 74.81 | 0.37 | 24.03 | 0.18 | 21.35 | 0.22 | 5.41 | 0.07 | 6.33 | 0.01 | 0.70 | 0.76 | 0.99 | 0.95 |
| 334 | 74.45 | 0.36 | 23.84 | 0.19 | 21.14 | 0.21 | 5.48 | 0.07 | 6.32 | 0.01 | 0.69 | 0.78 | 0.99 | 0.95 |
| | | 0.34 | | 0.19 | | 0.20 | | 0.08 | | 0.02 | | | | |
| 335 | 74.11 | | 23.65 | | 20.94 | | 5.56 | | 6.30 | | 0.68 | 0.80 | -0.99 | -0.95 |
| 336 | 73.79 | -0.32 | 23.45 | -0.20 | 20.73 | -0.21 | 5.63 | +0.07 | 6.28 | -0.02 | 0.67 | 0.82 | 0.98 | 0.95 |
| 337 | 73.48 | 0.31 | 23.24 | 0.21 | 20.53 | 0.20 | 5.70 | 0.07 | 6.25 | 0.03 | 0.66 | 0.84 | 0.97 | 0.95 |
| 338 | 73.18 | 0.30 | 23.02 | 0.22 | 20.34 | 0.19 | 5.78 | 0.08 | 6.22 | 0.03 | 0.65 | 0.86 | 0.96 | 0.95 |
| 339 | 72.91 | 0.27 | 22.80 | 0.22 | 20.15 | 0.19 | 5.85 | 0.07 | 6.19 | 0.03 | 0.64 | 0.88 | 0.95 | 0.95 |
| | | 0.25 | | 0.23 | | 0.19 | | 0.07 | | 0.04 | | | | |
| 340 | 72.66 | | 22.57 | | 19.96 | | 5.92 | | 6.15 | | 0.64 | 0.91 | -0.95 | -0.95 |
| 341 | 72.42 | -0.24 | 22.33 | -0.24 | 19.79 | -0.17 | 5.98 | +0.06 | 6.11 | -0.04 | 0.63 | 0.93 | 0.95 | 0.96 |
| 342 | 72.20 | 0.22 | 22.09 | 0.24 | 19.62 | 0.17 | 6.05 | 0.07 | 6.07 | 0.04 | 0.63 | 0.95 | 0.94 | 0.96 |
| 343 | 72.00 | 0.20 | 21.84 | 0.25 | 19.45 | 0.17 | 6.12 | 0.07 | 6.02 | 0.05 | 0.63 | 0.98 | 0.94 | 0.96 |
| 344 | 71.82 | 0.18 | 21.58 | 0.26 | 19.30 | 0.15 | 6.19 | 0.07 | 5.97 | 0.05 | 0.63 | 1.00 | 0.93 | 0.97 |
| | | 0.16 | | 0.27 | | 0.15 | | 0.06 | | 0.06 | | | | |
| 345 | 71.66 | | 21.31 | | 19.15 | | 6.25 | | 5.91 | | 0.63 | 1.02 | -0.93 | -0.97 |
| 346 | 71.52 | -0.14 | 21.04 | -0.27 | 19.00 | -0.15 | 6.31 | +0.06 | 5.86 | -0.05 | 0.64 | 1.05 | 0.93 | 0.98 |
| 347 | 71.39 | 0.13 | 20.75 | 0.29 | 18.87 | 0.13 | 6.37 | 0.06 | 5.80 | 0.06 | 0.64 | 1.07 | 0.94 | 0.99 |
| 348 | 71.29 | 0.10 | 20.46 | 0.29 | 18.74 | 0.13 | 6.43 | 0.06 | 5.73 | 0.07 | 0.65 | 1.09 | 0.94 | 1.00 |
| 349 | 71.21 | 0.08 | 20.16 | 0.30 | 18.62 | 0.12 | 6.48 | 0.05 | 5.66 | 0.07 | 0.66 | 1.12 | 0.94 | 1.01 |
| | | 0.06 | | 0.31 | | 0.10 | | 0.05 | | 0.07 | | | | |
| 350 | 71.15 | | 19.85 | | 18.52 | | 6.53 | | 5.59 | | 0.67 | 1.14 | -0.94 | -1.02 |
| 351 | 71.10 | -0.05 | 19.54 | -0.31 | 18.43 | -0.09 | 6.58 | +0.05 | 5.51 | -0.08 | 0.68 | 1.16 | 0.94 | 1.02 |
| 352 | 71.08 | -0.02 | 19.22 | 0.32 | 18.34 | 0.09 | 6.62 | 0.04 | 5.43 | 0.08 | 0.69 | 1.18 | 0.95 | 1.03 |
| 353 | 71.08 | 0.00 | 18.90 | 0.32 | 18.26 | 0.08 | 6.66 | 0.04 | 5.35 | 0.08 | 0.71 | 1.20 | 0.95 | 1.04 |
| 354 | 71.10 | +0.02 | 18.56 | 0.34 | 18.19 | 0.07 | 6.70 | 0.04 | 5.27 | 0.08 | 0.72 | 1.22 | 0.95 | 1.05 |
| | | 0.05 | | 0.34 | | 0.05 | | 0.03 | | 0.09 | | | | |
| 355 | 71.15 | | 18.22 | | 18.14 | | 6.73 | | 5.18 | | 0.74 | 1.23 | -0.95 | -1.06 |
| 356 | 71.21 | +0.06 | 17.88 | -0.34 | 18.10 | -0.04 | 6.76 | +0.03 | 5.09 | -0.09 | 0.76 | 1.25 | 0.96 | 1.06 |
| 357 | 71.29 | 0.08 | 17.53 | 0.35 | 18.07 | 0.03 | 6.79 | 0.03 | 5.00 | 0.09 | 0.78 | 1.26 | 0.97 | 1.06 |
| 358 | 71.40 | 0.11 | 17.17 | 0.36 | 18.05 | -0.02 | 6.81 | 0.02 | 4.91 | 0.09 | 0.80 | 1.27 | 0.98 | 1.07 |
| 359 | 71.52 | 0.12 | 16.80 | 0.37 | 18.05 | 0.00 | 6.83 | 0.02 | 4.81 | 0.10 | 0.82 | 1.28 | 0.99 | 1.07 |
| | | 0.15 | | 0.37 | | +0.01 | | 0.01 | | 0.10 | | | | |
| 360 | 71.67 | | 16.43 | | 18.06 | | 6.84 | | 4.71 | | 0.84 | 1.29 | -0.99 | -1.07 |

TABLE X, ARG. 3.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | (v.c.1) | Diff. | (v.s.2) | Diff. | (v.c.2) | Diff. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) |
|------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|---------|---------|---------|
| " | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 360 | 71.67 | | 16.43 | | 18.06 | | 6.84 | | 4.71 | | 0.84 | 1.29 | -0.99 | -1.07 |
| 361 | 71.84 | +0.17 | 16.05 | -0.38 | 18.08 | +0.02 | 6.84 | 0.00 | 4.61 | -0.10 | 0.87 | 1.30 | 1.00 | 1.07 |
| 362 | 72.02 | 0.18 | 15.67 | 0.38 | 18.12 | 0.04 | 6.84 | 0.00 | 4.51 | 0.10 | 0.89 | 1.31 | 1.01 | 1.07 |
| 363 | 72.24 | 0.22 | 15.28 | 0.39 | 18.17 | 0.05 | 6.84 | 0.00 | 4.41 | 0.10 | 0.91 | 1.31 | 1.02 | 1.07 |
| 364 | 72.48 | 0.24 | 14.89 | 0.39 | 18.24 | 0.07 | 6.83 | -0.01 | 4.31 | 0.10 | 0.94 | 1.32 | 1.03 | 1.07 |
| | | 0.25 | | 0.39 | | 0.08 | | 0.02 | | 0.10 | | | | |
| 365 | 72.73 | | 14.50 | | 18.32 | | 6.81 | | 4.21 | | 0.96 | 1.32 | -1.03 | -1.07 |
| 366 | 73.00 | +0.27 | 14.11 | -0.39 | 18.42 | +0.10 | 6.79 | -0.02 | 4.11 | -0.10 | 0.98 | 1.32 | 1.03 | 1.06 |
| 367 | 73.30 | 0.30 | 13.71 | 0.40 | 18.53 | 0.11 | 6.76 | 0.03 | 4.01 | 0.10 | 1.01 | 1.32 | 1.04 | 1.06 |
| 368 | 73.62 | 0.32 | 13.31 | 0.40 | 18.66 | 0.13 | 6.73 | 0.03 | 3.90 | 0.11 | 1.03 | 1.31 | 1.05 | 1.05 |
| 369 | 73.96 | 0.34 | 12.91 | 0.40 | 18.81 | 0.15 | 6.68 | 0.05 | 3.80 | 0.10 | 1.06 | 1.31 | 1.06 | 1.05 |
| | | 0.37 | | 0.40 | | 0.16 | | 0.05 | | 0.10 | | | | |
| 370 | 74.33 | | 12.51 | | 18.97 | | 6.63 | | 3.70 | | 1.08 | 1.30 | -1.07 | -1.05 |
| 371 | 74.71 | +0.38 | 12.10 | -0.41 | 19.15 | +0.18 | 6.58 | -0.05 | 3.60 | -0.10 | 1.10 | 1.29 | 1.07 | 1.05 |
| 372 | 75.12 | 0.41 | 11.70 | 0.40 | 19.35 | 0.20 | 6.52 | 0.06 | 3.51 | 0.09 | 1.12 | 1.28 | 1.07 | 1.04 |
| 373 | 75.54 | 0.42 | 11.30 | 0.40 | 19.57 | 0.22 | 6.46 | 0.06 | 3.41 | 0.10 | 1.14 | 1.27 | 1.08 | 1.04 |
| 374 | 75.99 | 0.45 | 10.90 | 0.40 | 19.80 | 0.23 | 6.39 | 0.07 | 3.32 | 0.09 | 1.16 | 1.25 | 1.08 | 1.03 |
| | | 0.46 | | 0.40 | | 0.24 | | 0.07 | | 0.09 | | | | |
| 375 | 76.45 | | 10.50 | | 20.04 | | 6.32 | | 3.23 | | 1.18 | 1.23 | -1.08 | -1.02 |
| 376 | 76.94 | +0.49 | 10.10 | -0.40 | 20.30 | +0.26 | 6.24 | -0.08 | 3.15 | -0.08 | 1.20 | 1.22 | 1.08 | 1.00 |
| 377 | 77.44 | 0.50 | 9.71 | 0.39 | 20.58 | 0.28 | 6.15 | 0.09 | 3.07 | 0.08 | 1.22 | 1.20 | 1.08 | 0.99 |
| 378 | 77.96 | 0.52 | 9.32 | 0.39 | 20.89 | 0.31 | 6.06 | 0.09 | 2.99 | 0.08 | 1.23 | 1.19 | 1.08 | 0.98 |
| 379 | 78.51 | 0.55 | 8.93 | 0.39 | 21.21 | 0.32 | 5.96 | 0.10 | 2.92 | 0.07 | 1.25 | 1.16 | 1.08 | 0.97 |
| | | 0.57 | | 0.39 | | 0.34 | | 0.11 | | 0.07 | | | | |
| 380 | 79.08 | | 8.54 | | 21.55 | | 5.85 | | 2.85 | | 1.26 | 1.13 | -1.08 | -0.96 |
| 381 | 79.66 | +0.58 | 8.16 | -0.38 | 21.91 | +0.36 | 5.74 | -0.11 | 2.78 | -0.07 | 1.27 | 1.11 | 1.07 | 0.96 |
| 382 | 80.27 | 0.61 | 7.79 | 0.37 | 22.28 | 0.37 | 5.63 | 0.11 | 2.72 | 0.06 | 1.28 | 1.09 | 1.07 | 0.95 |
| 383 | 80.90 | 0.63 | 7.43 | 0.36 | 22.67 | 0.39 | 5.51 | 0.12 | 2.67 | 0.05 | 1.28 | 1.06 | 1.07 | 0.94 |
| 384 | 81.54 | 0.64 | 7.07 | 0.36 | 23.08 | 0.41 | 5.39 | 0.12 | 2.62 | 0.05 | 1.29 | 1.04 | 1.06 | 0.93 |
| | | 0.65 | | 0.34 | | 0.43 | | 0.12 | | 0.04 | | | | |
| 385 | 82.19 | | 6.73 | | 23.51 | | 5.27 | | 2.58 | | 1.29 | 1.01 | -1.06 | -0.92 |
| 386 | 82.87 | +0.68 | 6.39 | -0.34 | 23.95 | +0.44 | 5.14 | -0.13 | 2.55 | -0.03 | 1.29 | 0.98 | 1.06 | 0.92 |
| 387 | 83.56 | 0.69 | 6.06 | 0.33 | 24.41 | 0.46 | 5.01 | 0.13 | 2.53 | 0.02 | 1.29 | 0.96 | 1.05 | 0.92 |
| 388 | 84.27 | 0.71 | 5.74 | 0.32 | 24.89 | 0.48 | 4.87 | 0.14 | 2.51 | 0.02 | 1.28 | 0.93 | 1.04 | 0.91 |
| 389 | 85.01 | 0.74 | 5.42 | 0.32 | 25.38 | 0.49 | 4.73 | 0.14 | 2.49 | -0.02 | 1.28 | 0.91 | 1.03 | 0.91 |
| | | 0.74 | | 0.30 | | 0.51 | | 0.14 | | 0.00 | | | | |
| 390 | 85.75 | | 5.12 | | 25.89 | | 4.59 | | 2.49 | | 1.27 | 0.88 | -1.02 | -0.90 |
| 391 | 86.51 | +0.76 | 4.84 | -0.28 | 26.42 | +0.53 | 4.45 | -0.14 | 2.49 | 0.00 | 1.26 | 0.86 | 1.01 | 0.90 |
| 392 | 87.29 | 0.78 | 4.56 | 0.28 | 26.96 | 0.54 | 4.31 | 0.14 | 2.50 | +0.01 | 1.25 | 0.83 | 1.00 | 0.90 |
| 393 | 88.08 | 0.79 | 4.30 | 0.26 | 27.51 | 0.55 | 4.16 | 0.15 | 2.52 | 0.02 | 1.23 | 0.81 | 0.99 | 0.90 |
| 394 | 88.89 | 0.81 | 4.06 | 0.24 | 28.08 | 0.57 | 4.02 | 0.14 | 2.55 | 0.03 | 1.22 | 0.78 | 0.98 | 0.90 |
| | | 0.82 | | 0.23 | | 0.59 | | 0.15 | | 0.03 | | | | |
| 395 | 89.71 | | 3.83 | | 28.67 | | 3.87 | | 2.58 | | 1.20 | 0.76 | -0.97 | -0.90 |
| 396 | 90.54 | +0.83 | 3.61 | -0.22 | 29.27 | +0.60 | 3.73 | -0.14 | 2.63 | +0.05 | 1.17 | 0.74 | 0.95 | 0.91 |
| 397 | 91.39 | 0.85 | 3.41 | 0.20 | 29.88 | 0.61 | 3.58 | 0.15 | 2.68 | 0.05 | 1.15 | 0.72 | 0.94 | 0.91 |
| 398 | 92.25 | 0.86 | 3.22 | 0.19 | 30.50 | 0.62 | 3.44 | 0.14 | 2.74 | 0.06 | 1.13 | 0.70 | 0.93 | 0.92 |
| 399 | 93.13 | 0.88 | 3.06 | 0.16 | 31.14 | 0.64 | 3.30 | 0.14 | 2.81 | 0.07 | 1.10 | 0.69 | 0.92 | 0.92 |
| | | 0.89 | | 0.15 | | 0.65 | | 0.14 | | 0.07 | | | | |
| 400 | 94.02 | | 2.91 | | 31.79 | | 3.16 | | 2.88 | | 1.08 | 0.68 | -0.91 | -0.93 |
| 401 | 94.92 | +0.90 | 2.77 | -0.14 | 32.45 | +0.66 | 3.02 | -0.14 | 2.96 | +0.03 | 1.05 | 0.67 | 0.91 | 0.93 |
| 402 | 95.83 | 0.91 | 2.65 | 0.12 | 33.11 | 0.66 | 2.89 | 0.13 | 3.06 | 0.10 | 1.02 | 0.66 | 0.90 | 0.94 |
| 403 | 96.75 | 0.92 | 2.56 | 0.09 | 33.79 | 0.68 | 2.76 | 0.13 | 3.16 | 0.10 | 0.99 | 0.65 | 0.90 | 0.95 |
| 404 | 97.68 | 0.93 | 2.48 | 0.08 | 34.48 | 0.69 | 2.64 | 0.12 | 3.27 | 0.11 | 0.96 | 0.65 | 0.89 | 0.96 |
| | | 0.94 | | 0.05 | | 0.69 | | 0.12 | | 0.12 | | | | |
| 405 | 98.62 | | 2.43 | | 35.17 | | 2.52 | | 3.39 | | 0.93 | 0.64 | -0.89 | -0.97 |
| 406 | 99.57 | +0.95 | 2.39 | -0.04 | 35.87 | +0.70 | 2.41 | -0.11 | 3.51 | +0.12 | 0.90 | 0.64 | 0.89 | 0.99 |
| 407 | 100.53 | 0.96 | 2.37 | -0.02 | 36.58 | 0.71 | 2.31 | 0.10 | 3.64 | 0.13 | 0.87 | 0.65 | 0.89 | 1.01 |
| 408 | 101.50 | 0.97 | 2.37 | 0.00 | 37.30 | 0.72 | 2.20 | 0.11 | 3.78 | 0.14 | 0.84 | 0.65 | 0.90 | 1.02 |
| 409 | 102.47 | 0.97 | 2.39 | +0.02 | 38.02 | 0.72 | 2.10 | 0.10 | 3.93 | 0.15 | 0.80 | 0.66 | 0.90 | 1.03 |
| | | 0.99 | | 0.05 | | 0.72 | | 0.09 | | 0.15 | | | | |
| 410 | 103.46 | | 2.44 | | 38.74 | | 2.01 | | 4.08 | | 0.77 | 0.67 | -0.90 | -1.04 |
| 411 | 104.45 | +0.99 | 2.52 | +0.08 | 39.47 | +0.73 | 1.93 | -0.08 | 4.23 | +0.15 | 0.74 | 0.69 | 0.91 | 1.05 |
| 412 | 105.44 | 0.99 | 2.61 | 0.09 | 40.19 | 0.72 | 1.86 | 0.07 | 4.39 | 0.16 | 0.72 | 0.70 | 0.91 | 1.07 |
| 413 | 106.44 | 1.00 | 2.72 | 0.11 | 40.92 | 0.73 | 1.79 | 0.07 | 4.56 | 0.17 | 0.69 | 0.72 | 0.92 | 1.08 |
| 414 | 107.44 | 1.00 | 2.85 | 0.13 | 41.66 | 0.74 | 1.73 | 0.06 | 4.73 | 0.17 | 0.66 | 0.74 | 0.92 | 1.09 |
| | | 1.00 | | 0.15 | | 0.73 | | 0.04 | | 0.18 | | | | |
| 415 | 108.44 | | 3.00 | | 42.39 | | 1.69 | | 4.91 | | 0.63 | 0.77 | -0.92 | -1.10 |
| 416 | 109.45 | +1.01 | 3.18 | +0.18 | 43.12 | +0.73 | 1.65 | -0.04 | 5.09 | +0.18 | 0.61 | 0.79 | 0.93 | 1.10 |
| 417 | 110.46 | 1.01 | 3.38 | 0.20 | 43.84 | 0.72 | 1.62 | 0.03 | 5.27 | 0.18 | 0.59 | 0.82 | 0.94 | 1.10 |
| 418 | 111.47 | 1.01 | 3.60 | 0.22 | 44.57 | 0.73 | 1.60 | 0.02 | 5.46 | 0.19 | 0.56 | 0.85 | 0.96 | 1.11 |
| 419 | 112.49 | 1.02 | 3.85 | 0.25 | 45.29 | 0.72 | 1.59 | 0.01 | 5.65 | 0.19 | 0.55 | 0.88 | 0.97 | 1.12 |
| | | 1.02 | | 0.27 | | 0.72 | | 0.00 | | 0.19 | | | | |
| 420 | 113.51 | | 4.12 | | 46.01 | | 1.59 | | 5.84 | | 0.53 | 0.92 | -0.98 | -1.12 |

TABLE X, ARG. 3.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | (v.c.1) | Diff. | (v.s.2) | Diff. | (v.c.2) | Diff. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) |
|------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|---------|---------|---------|
| | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 420 | 113.51 | | 4.12 | | 46.01 | | 1.59 | | 5.84 | | 0.53 | 0.92 | -0.98 | -1.12 |
| 421 | 114.53 | +1.02 | 4.42 | +0.30 | 46.71 | +0.70 | 1.60 | +0.01 | 6.04 | +0.20 | 0.52 | 0.95 | 0.99 | 1.12 |
| 422 | 115.55 | 1.02 | 4.73 | 0.31 | 47.41 | 0.70 | 1.63 | 0.03 | 6.23 | 0.19 | 0.51 | 0.99 | 1.01 | 1.11 |
| 423 | 116.57 | 1.02 | 5.07 | 0.34 | 48.10 | 0.69 | 1.65 | 0.02 | 6.42 | 0.19 | 0.50 | 1.02 | 1.02 | 1.11 |
| 424 | 117.59 | 1.00 | 5.42 | 0.35 | 48.79 | 0.69 | 1.69 | 0.04 | 6.61 | 0.19 | 0.49 | 1.06 | 1.03 | 1.11 |
| 425 | 118.59 | | 5.80 | | 49.48 | | 1.75 | | 6.80 | | 0.49 | 1.10 | -1.04 | -1.11 |
| 426 | 119.59 | +1.00 | 6.21 | +0.41 | 50.15 | +0.67 | 1.81 | +0.06 | 6.98 | +0.18 | 0.49 | 1.14 | 1.06 | 1.10 |
| 427 | 120.59 | 1.00 | 6.63 | 0.42 | 50.80 | 0.65 | 1.88 | 0.07 | 7.17 | 0.19 | 0.49 | 1.18 | 1.08 | 1.10 |
| 428 | 121.60 | 1.01 | 7.07 | 0.44 | 51.44 | 0.64 | 1.96 | 0.08 | 7.35 | 0.18 | 0.50 | 1.22 | 1.09 | 1.10 |
| 429 | 122.59 | 0.99 | 7.54 | 0.47 | 52.06 | 0.62 | 2.06 | 0.10 | 7.52 | 0.17 | 0.51 | 1.26 | 1.10 | 1.09 |
| 430 | 123.58 | 0.99 | 8.03 | 0.49 | 52.67 | 0.61 | 2.16 | 0.10 | 7.70 | 0.18 | 0.53 | 1.30 | -1.11 | -1.09 |
| 431 | 124.57 | +0.99 | 8.54 | +0.51 | 53.27 | +0.60 | 2.26 | +0.10 | 7.87 | +0.17 | 0.54 | 1.34 | 1.11 | 1.08 |
| 432 | 125.55 | 0.98 | 9.08 | 0.54 | 53.85 | 0.58 | 2.38 | 0.12 | 8.03 | 0.16 | 0.56 | 1.38 | 1.11 | 1.07 |
| 433 | 126.52 | 0.97 | 9.62 | 0.54 | 54.41 | 0.56 | 2.52 | 0.14 | 8.19 | 0.16 | 0.58 | 1.41 | 1.12 | 1.06 |
| 434 | 127.48 | 0.96 | 10.18 | 0.56 | 54.95 | 0.54 | 2.66 | 0.14 | 8.34 | 0.15 | 0.61 | 1.45 | 1.12 | 1.05 |
| 435 | 128.44 | 0.96 | 10.76 | 0.58 | 55.48 | 0.53 | 2.81 | 0.15 | 8.48 | 0.14 | 0.64 | 1.48 | -1.13 | -1.04 |
| 436 | 129.38 | +0.94 | 11.37 | +0.61 | 55.99 | +0.51 | 2.96 | +0.15 | 8.62 | +0.14 | 0.66 | 1.51 | 1.13 | 1.02 |
| 437 | 130.31 | 0.93 | 11.99 | 0.62 | 56.48 | 0.49 | 3.13 | 0.17 | 8.74 | 0.12 | 0.70 | 1.54 | 1.13 | 1.00 |
| 438 | 131.24 | 0.93 | 12.62 | 0.63 | 56.94 | 0.46 | 3.30 | 0.17 | 8.86 | 0.12 | 0.73 | 1.57 | 1.13 | 0.99 |
| 439 | 132.15 | 0.91 | 13.26 | 0.64 | 57.38 | 0.44 | 3.47 | 0.17 | 8.97 | 0.11 | 0.77 | 1.59 | 1.13 | 0.98 |
| 440 | 133.06 | 0.91 | 13.93 | 0.67 | 57.80 | 0.42 | 3.65 | 0.18 | 9.07 | 0.10 | 0.82 | 1.61 | -1.13 | -0.97 |
| 441 | 133.95 | +0.89 | 14.61 | +0.68 | 58.19 | +0.39 | 3.84 | +0.19 | 9.16 | +0.09 | 0.86 | 1.63 | 1.12 | 0.95 |
| 442 | 134.83 | 0.88 | 15.31 | 0.70 | 58.56 | 0.37 | 4.04 | 0.20 | 9.24 | 0.08 | 0.90 | 1.64 | 1.12 | 0.93 |
| 443 | 135.70 | 0.87 | 16.01 | 0.70 | 58.90 | 0.34 | 4.24 | 0.20 | 9.31 | 0.07 | 0.95 | 1.65 | 1.12 | 0.92 |
| 444 | 136.55 | 0.85 | 16.73 | 0.72 | 59.22 | 0.32 | 4.44 | 0.20 | 9.37 | 0.06 | 0.99 | 1.66 | 1.11 | 0.91 |
| 445 | 137.39 | 0.84 | 17.45 | 0.72 | 59.51 | 0.29 | 4.65 | 0.21 | 9.42 | 0.05 | 1.04 | 1.67 | -1.11 | -0.90 |
| 446 | 138.21 | +0.82 | 18.19 | +0.74 | 59.78 | +0.27 | 4.86 | +0.21 | 9.45 | +0.03 | 1.09 | 1.67 | 1.09 | 0.90 |
| 447 | 139.02 | 0.81 | 18.93 | 0.74 | 60.02 | 0.24 | 5.07 | 0.21 | 9.48 | 0.03 | 1.13 | 1.67 | 1.07 | 0.89 |
| 448 | 139.81 | 0.79 | 19.68 | 0.75 | 60.22 | 0.20 | 5.28 | 0.21 | 9.49 | +0.01 | 1.18 | 1.66 | 1.06 | 0.88 |
| 449 | 140.59 | 0.78 | 20.45 | 0.77 | 60.40 | 0.18 | 5.49 | 0.21 | 9.49 | 0.00 | 1.23 | 1.65 | 1.05 | 0.87 |
| 450 | 141.36 | 0.77 | 21.22 | 0.77 | 60.55 | 0.15 | 5.71 | 0.22 | 9.48 | -0.01 | 1.28 | 1.64 | -1.04 | -0.86 |
| 451 | 142.11 | +0.75 | 21.99 | +0.77 | 60.67 | +0.12 | 5.92 | +0.21 | 9.45 | -0.03 | 1.32 | 1.62 | 1.02 | 0.86 |
| 452 | 142.83 | 0.72 | 22.77 | 0.78 | 60.76 | 0.09 | 6.13 | 0.21 | 9.42 | 0.03 | 1.37 | 1.60 | 1.00 | 0.86 |
| 453 | 143.54 | 0.71 | 23.55 | 0.78 | 60.82 | 0.06 | 6.34 | 0.21 | 9.38 | 0.04 | 1.41 | 1.58 | 0.99 | 0.86 |
| 454 | 144.23 | 0.69 | 24.33 | 0.78 | 60.85 | +0.03 | 6.55 | 0.21 | 9.32 | 0.06 | 1.45 | 1.55 | 0.98 | 0.86 |
| 455 | 144.91 | 0.68 | 25.11 | 0.78 | 60.85 | 0.00 | 6.75 | 0.20 | 9.25 | 0.07 | 1.49 | 1.52 | -0.97 | -0.86 |
| 456 | 145.56 | +0.65 | 25.89 | +0.78 | 60.82 | -0.03 | 6.96 | +0.21 | 9.17 | -0.08 | 1.53 | 1.49 | 0.95 | 0.87 |
| 457 | 146.19 | 0.63 | 26.67 | 0.78 | 60.76 | 0.06 | 7.15 | 0.19 | 9.07 | 0.10 | 1.57 | 1.45 | 0.93 | 0.87 |
| 458 | 146.81 | 0.62 | 27.45 | 0.78 | 60.66 | 0.10 | 7.35 | 0.20 | 8.97 | 0.10 | 1.60 | 1.41 | 0.91 | 0.88 |
| 459 | 147.40 | 0.59 | 28.22 | 0.77 | 60.54 | 0.12 | 7.53 | 0.18 | 8.86 | 0.11 | 1.63 | 1.37 | 0.90 | 0.88 |
| 460 | 147.98 | 0.58 | 28.99 | 0.77 | 60.54 | 0.16 | 7.70 | 0.17 | 8.74 | 0.12 | 1.66 | 1.33 | -0.89 | -0.89 |
| 461 | 148.54 | +0.56 | 29.75 | +0.76 | 60.20 | -0.18 | 7.87 | +0.17 | 8.60 | -0.14 | 1.68 | 1.29 | 0.88 | 0.90 |
| 462 | 149.07 | 0.53 | 30.51 | 0.76 | 60.20 | 0.22 | 8.04 | 0.17 | 8.46 | 0.14 | 1.70 | 1.24 | 0.87 | 0.91 |
| 463 | 149.59 | 0.52 | 31.26 | 0.75 | 59.98 | 0.25 | 8.20 | 0.16 | 8.31 | 0.15 | 1.72 | 1.19 | 0.86 | 0.92 |
| 464 | 150.08 | 0.49 | 32.00 | 0.74 | 59.73 | 0.28 | 8.35 | 0.15 | 8.14 | 0.17 | 1.73 | 1.14 | 0.85 | 0.93 |
| 465 | 150.54 | 0.46 | 32.73 | 0.73 | 59.45 | 0.31 | 8.49 | 0.14 | 7.97 | 0.17 | 1.74 | 1.09 | -0.85 | -0.94 |
| 466 | 150.98 | +0.44 | 33.45 | +0.72 | 58.81 | -0.33 | 8.62 | +0.13 | 7.79 | -0.18 | 1.75 | 1.03 | 0.85 | 0.96 |
| 467 | 151.40 | 0.42 | 34.16 | 0.71 | 58.44 | 0.37 | 8.74 | 0.12 | 7.61 | 0.18 | 1.75 | 0.98 | 0.85 | 0.98 |
| 468 | 151.80 | 0.40 | 34.85 | 0.69 | 58.05 | 0.39 | 8.85 | 0.11 | 7.42 | 0.19 | 1.75 | 0.93 | 0.85 | 1.00 |
| 469 | 152.18 | 0.38 | 35.52 | 0.67 | 57.62 | 0.43 | 8.95 | 0.10 | 7.22 | 0.20 | 1.75 | 0.87 | 0.85 | 1.02 |
| 470 | 152.53 | 0.35 | 36.18 | 0.66 | 57.17 | 0.45 | 9.03 | 0.08 | 7.01 | 0.21 | 1.73 | 0.82 | -0.85 | -1.03 |
| 471 | 152.87 | +0.34 | 36.83 | +0.65 | 56.69 | -0.48 | 9.11 | +0.08 | 6.81 | -0.20 | 1.72 | 0.77 | 0.86 | 1.05 |
| 472 | 153.18 | 0.31 | 37.46 | 0.63 | 56.18 | 0.51 | 9.17 | 0.06 | 6.60 | 0.21 | 1.70 | 0.72 | 0.86 | 1.07 |
| 473 | 153.46 | 0.28 | 38.07 | 0.61 | 55.64 | 0.54 | 9.22 | 0.05 | 6.38 | 0.22 | 1.68 | 0.67 | 0.87 | 1.09 |
| 474 | 153.72 | 0.26 | 38.67 | 0.60 | 55.09 | 0.55 | 9.27 | 0.05 | 6.16 | 0.22 | 1.66 | 0.62 | 0.87 | 1.10 |
| 475 | 153.95 | 0.23 | 39.24 | 0.57 | 54.51 | 0.58 | 9.30 | 0.03 | 5.93 | 0.23 | 1.63 | 0.58 | -0.88 | -1.11 |
| 476 | 154.16 | +0.21 | 39.79 | +0.55 | 53.91 | -0.60 | 9.31 | +0.01 | 5.71 | -0.22 | 1.60 | 0.54 | 0.90 | 1.12 |
| 477 | 154.34 | 0.18 | 40.32 | 0.53 | 53.28 | 0.63 | 9.32 | +0.01 | 5.48 | 0.23 | 1.56 | 0.50 | 0.91 | 1.13 |
| 478 | 154.51 | 0.17 | 40.82 | 0.50 | 52.62 | 0.66 | 9.31 | -0.01 | 5.25 | 0.23 | 1.53 | 0.46 | 0.92 | 1.14 |
| 479 | 154.66 | 0.15 | 41.31 | 0.49 | 51.94 | 0.68 | 9.29 | 0.02 | 5.03 | 0.22 | 1.48 | 0.42 | 0.93 | 1.15 |
| 480 | 154.77 | 0.11 | 41.77 | 0.46 | 51.25 | 0.69 | 9.25 | 0.04 | 4.82 | 0.21 | 1.44 | 0.39 | -0.94 | -1.16 |

TABLE X, ARG. 3.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | (v.c.1) | Diff. | (v.s.2) | Diff. | (v.c.2) | Diff. | (v.s.3) | (v.c.3) | (v.s.4) | (v.c.4) |
|------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|---------|---------|---------|
| " | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 480 | 154.77 | | 41.77 | | 51.25 | | 9.25 | | 4.82 | | 1.44 | 0.39 | 0.94 | -1.16 |
| 481 | 154.86 | +0.09 | 42.20 | +0.43 | 50.53 | -0.72 | 9.20 | -0.05 | 4.60 | -0.22 | 1.39 | 0.36 | 0.96 | 1.16 |
| 482 | 154.93 | 0.07 | 42.61 | 0.41 | 49.80 | 0.73 | 9.15 | 0.05 | 4.39 | 0.21 | 1.35 | 0.34 | 0.98 | 1.16 |
| 483 | 154.98 | 0.05 | 42.99 | 0.38 | 49.04 | 0.76 | 9.08 | 0.07 | 4.17 | 0.22 | 1.30 | 0.31 | 1.00 | 1.16 |
| 484 | 154.99 | +0.01 | 43.34 | 0.35 | 48.28 | 0.76 | 9.00 | 0.08 | 3.96 | 0.21 | 1.25 | 0.30 | 1.01 | 1.16 |
| 485 | 154.98 | -0.01 | 43.67 | 0.33 | | 0.79 | | 0.10 | | 0.21 | | | | |
| 486 | 154.95 | -0.03 | 43.98 | +0.31 | 47.49 | -0.80 | 8.90 | -0.10 | 3.75 | -0.21 | 1.20 | 0.28 | -1.02 | -1.16 |
| 487 | 154.90 | 0.05 | 44.25 | 0.27 | 46.69 | 0.81 | 8.80 | 0.11 | 3.54 | 0.20 | 1.15 | 0.27 | 1.04 | 1.15 |
| 488 | 154.82 | 0.08 | 44.49 | 0.24 | 45.88 | 0.83 | 8.69 | 0.12 | 3.34 | 0.19 | 1.10 | 0.26 | 1.06 | 1.15 |
| 489 | 154.72 | 0.10 | 44.71 | 0.22 | 45.05 | 0.84 | 8.57 | 0.13 | 3.15 | 0.17 | 1.04 | 0.26 | 1.08 | 1.15 |
| 490 | 154.59 | 0.13 | 44.90 | 0.19 | 44.21 | 0.84 | 8.44 | 0.14 | 2.98 | 0.17 | 0.99 | 0.26 | 1.09 | 1.14 |
| 491 | 154.45 | -0.14 | 44.90 | +0.15 | 43.37 | -0.86 | 8.30 | -0.16 | 2.81 | -0.17 | 0.94 | 0.27 | -1.10 | -1.14 |
| 492 | 154.45 | -0.14 | 45.05 | +0.15 | 42.51 | -0.86 | 8.14 | -0.16 | 2.64 | -0.17 | 0.88 | 0.28 | 1.11 | 1.13 |
| 493 | 154.28 | 0.17 | 45.18 | 0.13 | 41.64 | 0.87 | 7.98 | 0.16 | 2.49 | 0.15 | 0.83 | 0.29 | 1.12 | 1.12 |
| 494 | 154.08 | 0.20 | 45.28 | 0.10 | 40.77 | 0.87 | 7.81 | 0.17 | 2.34 | 0.15 | 0.78 | 0.30 | 1.13 | 1.11 |
| 495 | 153.87 | 0.21 | 45.35 | 0.07 | 39.90 | 0.87 | 7.64 | 0.17 | 2.20 | 0.14 | 0.74 | 0.32 | 1.14 | 1.10 |
| 496 | 153.63 | 0.24 | 45.38 | 0.03 | | 0.89 | | 0.19 | | 0.13 | | | | |
| 497 | 153.37 | -0.26 | 45.39 | +0.01 | 39.01 | -0.89 | 7.45 | -0.19 | 2.07 | -0.12 | 0.69 | 0.35 | -1.15 | -1.09 |
| 498 | 153.09 | 0.28 | 45.37 | -0.02 | 38.12 | 0.89 | 7.26 | 0.19 | 1.95 | 0.12 | 0.65 | 0.38 | 1.15 | 1.07 |
| 499 | 152.79 | 0.30 | 45.32 | 0.05 | 37.24 | 0.88 | 7.06 | 0.20 | 1.84 | 0.11 | 0.60 | 0.41 | 1.15 | 1.05 |
| 500 | 152.46 | 0.33 | 45.32 | 0.09 | 36.35 | 0.89 | 6.86 | 0.20 | 1.74 | 0.10 | 0.56 | 0.44 | 1.16 | 1.03 |
| 501 | 152.11 | 0.35 | 45.23 | 0.09 | 35.46 | 0.89 | 6.66 | 0.20 | 1.66 | 0.08 | 0.53 | 0.47 | 1.16 | 1.01 |
| 502 | 151.74 | -0.37 | 45.12 | 0.11 | | 0.89 | | 0.21 | | 0.07 | | | | |
| 503 | 151.36 | -0.38 | 45.12 | -0.14 | 34.57 | -0.89 | 6.45 | -0.22 | 1.59 | -0.07 | 0.49 | 0.51 | -1.17 | -0.99 |
| 504 | 150.95 | 0.41 | 44.98 | 0.17 | 33.68 | 0.89 | 6.23 | 0.22 | 1.54 | 0.05 | 0.46 | 0.55 | 1.17 | 0.97 |
| 505 | 150.52 | 0.43 | 44.81 | 0.17 | 32.80 | 0.88 | 6.02 | 0.21 | 1.49 | 0.05 | 0.43 | 0.59 | 1.16 | 0.95 |
| 506 | 150.08 | 0.44 | 44.60 | 0.21 | 31.92 | 0.88 | 5.80 | 0.22 | 1.45 | 0.04 | 0.41 | 0.64 | 1.16 | 0.93 |
| 507 | 149.61 | -0.47 | 44.38 | 0.22 | 31.04 | 0.88 | 5.59 | 0.21 | 1.42 | 0.03 | 0.39 | 0.63 | 1.15 | 0.91 |
| 508 | 149.13 | 0.48 | 44.12 | 0.26 | | 0.86 | | 0.22 | | -0.02 | | | | |
| 509 | 148.63 | 0.50 | 43.84 | -0.28 | 30.18 | -0.86 | 5.37 | -0.22 | 1.40 | 0.00 | 0.37 | 0.73 | -1.15 | -0.90 |
| 510 | 148.11 | 0.52 | 43.52 | 0.32 | 29.32 | 0.86 | 5.15 | 0.22 | 1.40 | 0.00 | 0.36 | 0.78 | 1.13 | 0.89 |
| 511 | 147.58 | 0.53 | 43.18 | 0.34 | 28.46 | 0.86 | 4.93 | 0.22 | 1.40 | 0.00 | 0.35 | 0.83 | 1.11 | 0.87 |
| 512 | 147.03 | -0.55 | 43.18 | 0.34 | 27.62 | 0.84 | 4.72 | 0.21 | 1.43 | +0.03 | 0.34 | 0.87 | 1.10 | 0.86 |
| 513 | 146.46 | -0.57 | 42.82 | 0.36 | 26.80 | 0.82 | 4.51 | 0.21 | 1.46 | 0.03 | 0.34 | 0.92 | 1.09 | 0.85 |
| 514 | 145.87 | 0.59 | 42.42 | 0.40 | | 0.82 | | 0.21 | | 0.05 | | | | |
| 515 | 145.27 | 0.61 | 42.00 | -0.42 | 25.98 | -0.81 | 4.30 | -0.21 | 1.51 | +0.06 | 0.34 | 0.97 | -1.08 | -0.84 |
| 516 | 144.66 | 0.62 | 41.56 | -0.44 | 25.17 | -0.81 | 4.09 | -0.21 | 1.57 | +0.06 | 0.34 | 1.02 | 1.06 | 0.84 |
| 517 | 144.04 | 0.64 | 41.09 | 0.44 | 24.38 | 0.79 | 3.89 | 0.20 | 1.64 | 0.07 | 0.35 | 1.06 | 1.04 | 0.84 |
| 518 | 143.40 | 0.66 | 41.09 | 0.47 | 23.61 | 0.77 | 3.69 | 0.20 | 1.72 | 0.08 | 0.36 | 1.11 | 1.02 | 0.83 |
| 519 | 142.74 | 0.67 | 40.60 | 0.49 | 22.85 | 0.76 | 3.50 | 0.19 | 1.81 | 0.09 | 0.37 | 1.16 | 1.01 | 0.83 |
| 520 | 142.07 | 0.68 | | 0.51 | | 0.74 | | 0.18 | | 0.09 | | | | |
| 521 | 141.39 | -0.69 | 40.09 | 0.51 | 22.11 | 0.74 | 3.32 | 0.18 | 1.90 | 0.09 | 0.39 | 1.20 | -1.00 | -0.82 |
| 522 | 140.70 | -0.70 | 39.55 | -0.54 | 21.38 | -0.73 | 3.14 | -0.18 | 2.01 | +0.11 | 0.41 | 1.24 | 0.98 | 0.82 |
| 523 | 140.00 | 0.71 | 38.99 | 0.56 | 20.68 | 0.70 | 2.98 | 0.16 | 2.12 | 0.11 | 0.43 | 1.28 | 0.96 | 0.82 |
| 524 | 139.29 | 0.72 | 38.41 | 0.58 | 19.99 | 0.69 | 2.82 | 0.16 | 2.25 | 0.13 | 0.46 | 1.32 | 0.94 | 0.83 |
| 525 | 138.57 | 0.73 | 37.81 | 0.60 | 19.32 | 0.67 | 2.66 | 0.16 | 2.39 | 0.14 | 0.49 | 1.35 | 0.92 | 0.83 |
| 526 | 137.84 | 0.76 | 37.19 | 0.62 | | 0.64 | | 0.15 | | 0.14 | | | | |
| 527 | 137.10 | -0.74 | 36.56 | -0.63 | 18.68 | -0.63 | 2.51 | -0.13 | 2.53 | +0.15 | 0.52 | 1.39 | -0.90 | -0.84 |
| 528 | 136.35 | 0.75 | 35.91 | -0.65 | 18.05 | -0.60 | 2.33 | -0.13 | 2.68 | +0.16 | 0.55 | 1.42 | 0.98 | 0.85 |
| 529 | 135.59 | 0.76 | 35.25 | 0.66 | 17.45 | 0.58 | 2.25 | 0.12 | 2.84 | 0.16 | 0.58 | 1.44 | 0.97 | 0.87 |
| 530 | 134.83 | 0.76 | 34.57 | 0.68 | 16.87 | 0.55 | 2.13 | 0.12 | 3.00 | 0.16 | 0.62 | 1.47 | 0.96 | 0.88 |
| 531 | 134.07 | 0.77 | | 0.70 | 16.32 | 0.53 | 2.03 | 0.10 | 3.16 | 0.16 | 0.66 | 1.49 | 0.95 | 0.89 |
| 532 | 133.30 | -0.77 | 33.87 | 0.70 | | 0.53 | | 0.09 | | 0.17 | | | | |
| 533 | 132.52 | 0.78 | 33.15 | -0.72 | 15.79 | -0.51 | 1.94 | -0.08 | 3.33 | 0.17 | 0.70 | 1.51 | -0.94 | -0.90 |
| 534 | 131.73 | 0.79 | 32.43 | -0.72 | 15.28 | -0.51 | 1.86 | -0.08 | 3.51 | +0.18 | 0.74 | 1.53 | 0.94 | 0.91 |
| 535 | 130.94 | 0.79 | 31.70 | 0.73 | 14.79 | 0.49 | 1.78 | 0.08 | 3.69 | 0.18 | 0.78 | 1.54 | 0.93 | 0.93 |
| 536 | 130.15 | 0.79 | 31.06 | 0.73 | 14.34 | 0.45 | 1.72 | 0.06 | 3.87 | 0.18 | 0.82 | 1.55 | 0.93 | 0.95 |
| 537 | 129.35 | -0.80 | 30.36 | 0.74 | 13.91 | 0.43 | 1.67 | 0.05 | 4.06 | 0.19 | 0.86 | 1.55 | 0.92 | 0.97 |
| 538 | 128.55 | 0.80 | 30.02 | 0.74 | | 0.40 | | 0.04 | | 0.19 | | | | |
| 539 | 127.75 | 0.80 | 30.22 | -0.76 | 13.51 | -0.38 | 1.63 | -0.03 | 4.25 | +0.19 | 0.90 | 1.56 | -0.91 | -0.99 |
| 540 | 126.95 | 0.80 | 29.46 | -0.76 | 13.13 | -0.35 | 1.60 | -0.02 | 4.44 | +0.19 | 0.94 | 1.55 | 0.91 | 1.01 |
| 541 | 126.15 | 0.80 | 28.69 | 0.77 | 12.78 | 0.35 | 1.58 | 0.02 | 4.63 | 0.19 | 0.98 | 1.55 | 0.92 | 1.03 |
| 542 | | 0.80 | 27.92 | 0.77 | 12.46 | 0.32 | 1.57 | -0.01 | 4.82 | 0.19 | 1.02 | 1.54 | 0.92 | 1.05 |
| 543 | | 0.80 | 27.15 | 0.77 | 12.16 | 0.30 | 1.58 | +0.01 | 5.00 | 0.18 | 1.06 | 1.54 | 0.93 | 1.07 |
| 544 | | 0.80 | | 0.78 | | 0.26 | | 0.02 | | 0.18 | | | | |
| 545 | 130.15 | 0.79 | 26.37 | 0.78 | 11.90 | 0.26 | 1.60 | 0.02 | 5.18 | 0.18 | 1.09 | 1.53 | -0.93 | -1.09 |
| 546 | 129.35 | -0.80 | 25.58 | -0.79 | 11.66 | -0.24 | 1.62 | +0.02 | 5.36 | +0.18 | 1.13 | 1.51 | 0.95 | 1.10 |
| 547 | 128.55 | 0.80 | 24.81 | 0.77 | 11.45 | 0.21 | 1.65 | 0.03 | 5.54 | 0.18 | 1.17 | 1.50 | 0.96 | 1.12 |
| 548 | 127.75 | 0.80 | 24.03 | 0.78 | 11.26 | 0.19 | 1.70 | 0.05 | 5.72 | 0.18 | 1.20 | 1.48 | 0.97 | 1.13 |
| 549 | 126.95 | 0.80 | 23.25 | 0.78 | 11.10 | 0.16 | 1.76 | 0.06 | 5.90 | 0.18 | 1.23 | 1.46 | 0.98 | 1.14 |
| 550 | 126.15 | 0.80 | 22.48 | 0.77 | 10.98 | 0.12 | 1.82 | 0.06 | 6.07 | 0.17 | 1.26 | 1.43 | -0.99 | -1.15 |

TABLE X, Arg. 3.—Continued.

| Arg. | (v.c.0) | Diff. | (v.s.1) | Diff. | (v.c.1) | Diff. | (v.s.2) | Diff. | (v.c.2) | Diff. | (v.s.3) | v.c.3 | (v.s.4) | (v.c.4) | |
|------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|---------|-------|
| | " | " | " | " | " | " | " | " | " | " | " | " | " | " | |
| 540 | 126.15 | | 22.48 | | 10.98 | | 1.82 | | 6.07 | | | 1.26 | 1.43 | -0.89 | -1.15 |
| 541 | 125.36 | -0.79 | 21.70 | -0.78 | 10.88 | -0.10 | 1.90 | +0.08 | 6.23 | +0.16 | 1.28 | 1.41 | 0.91 | 1.15 | |
| 542 | 124.56 | 0.80 | 20.93 | 0.77 | 10.80 | 0.08 | 1.98 | 0.08 | 6.39 | 0.16 | 1.30 | 1.38 | 0.93 | 1.16 | |
| 543 | 123.76 | 0.80 | 20.16 | 0.77 | 10.75 | 0.05 | 2.08 | 0.10 | 6.54 | 0.15 | 1.33 | 1.36 | 0.95 | 1.16 | |
| 544 | 122.96 | 0.80 | 19.40 | 0.76 | 10.73 | -0.02 | 2.18 | 0.10 | 6.69 | 0.15 | 1.35 | 1.33 | 0.96 | 1.17 | |
| 545 | 122.17 | 0.79 | 18.65 | 0.75 | 10.74 | +0.01 | 2.29 | 0.11 | 6.83 | 0.14 | 1.37 | 1.30 | -0.97 | -1.18 | |
| 546 | 121.37 | -0.80 | 17.90 | -0.75 | 10.77 | +0.03 | 2.40 | +0.11 | 6.96 | +0.13 | 1.38 | 1.26 | 0.99 | 1.18 | |
| 547 | 120.58 | 0.79 | 17.17 | 0.73 | 10.83 | 0.06 | 2.52 | 0.12 | 7.08 | 0.12 | 1.40 | 1.23 | 1.02 | 1.18 | |
| 548 | 119.80 | 0.78 | 16.44 | 0.73 | 10.92 | 0.09 | 2.65 | 0.13 | 7.20 | 0.12 | 1.41 | 1.20 | 1.04 | 1.18 | |
| 549 | 119.03 | 0.77 | 15.73 | 0.71 | 11.02 | 0.10 | 2.79 | 0.14 | 7.31 | 0.11 | 1.41 | 1.17 | 1.06 | 1.18 | |
| 550 | 118.26 | 0.77 | 15.03 | 0.70 | 11.15 | 0.13 | 2.93 | 0.14 | 7.41 | 0.10 | 1.42 | 1.14 | -1.08 | -1.18 | |
| 551 | 117.50 | -0.76 | 14.34 | -0.69 | 11.31 | +0.16 | 3.08 | +0.15 | 7.51 | +0.10 | 1.42 | 1.11 | 1.10 | 1.17 | |
| 552 | 116.74 | 0.76 | 13.66 | 0.68 | 11.49 | 0.18 | 3.23 | 0.15 | 7.59 | 0.08 | 1.42 | 1.08 | 1.12 | 1.16 | |
| 553 | 115.99 | 0.75 | 12.99 | 0.67 | 11.69 | 0.20 | 3.38 | 0.15 | 7.67 | 0.08 | 1.41 | 1.04 | 1.13 | 1.15 | |
| 554 | 115.24 | 0.75 | 12.35 | 0.64 | 11.91 | 0.22 | 3.54 | 0.16 | 7.73 | 0.06 | 1.41 | 1.01 | 1.14 | 1.14 | |
| 555 | 114.50 | 0.74 | 11.71 | 0.64 | 12.16 | 0.25 | 3.70 | 0.16 | 7.79 | 0.06 | 1.41 | 0.98 | -1.15 | -1.13 | |
| 556 | 113.77 | -0.73 | 11.09 | -0.62 | 12.43 | +0.27 | 3.86 | +0.16 | 7.83 | +0.04 | 1.40 | 0.95 | 1.15 | 1.11 | |
| 557 | 113.05 | 0.72 | 10.48 | 0.61 | 12.71 | 0.28 | 4.02 | 0.16 | 7.87 | 0.04 | 1.39 | 0.92 | 1.16 | 1.09 | |
| 558 | 112.35 | 0.70 | 9.89 | 0.59 | 13.02 | 0.31 | 4.19 | 0.17 | 7.90 | 0.03 | 1.38 | 0.90 | 1.17 | 1.07 | |
| 559 | 111.65 | 0.70 | 9.32 | 0.57 | 13.35 | 0.33 | 4.35 | 0.16 | 7.92 | 0.02 | 1.36 | 0.87 | 1.18 | 1.05 | |
| 560 | 110.96 | 0.69 | 8.76 | 0.56 | 13.69 | 0.34 | 4.51 | 0.16 | 7.93 | 0.01 | 1.34 | 0.85 | -1.19 | -1.03 | |
| 561 | 110.29 | -0.67 | 8.23 | -0.53 | 14.04 | +0.35 | 4.67 | +0.16 | 7.94 | +0.01 | 1.32 | 0.83 | 1.19 | 1.01 | |
| 562 | 109.63 | 0.66 | 7.71 | 0.52 | 14.42 | 0.38 | 4.84 | 0.17 | 7.93 | -0.01 | 1.30 | 0.81 | 1.19 | 0.99 | |
| 563 | 108.98 | 0.65 | 7.21 | 0.50 | 14.81 | 0.39 | 5.00 | 0.16 | 7.92 | 0.01 | 1.28 | 0.79 | 1.19 | 0.97 | |
| 564 | 108.33 | 0.65 | 6.73 | 0.48 | 15.22 | 0.41 | 5.16 | 0.16 | 7.90 | 0.02 | 1.26 | 0.77 | 1.19 | 0.95 | |
| 565 | 107.70 | 0.63 | 6.26 | 0.47 | 15.64 | 0.42 | 5.32 | 0.16 | 7.87 | 0.03 | 1.24 | 0.76 | -1.19 | -0.93 | |
| 566 | 107.08 | -0.62 | 5.82 | -0.44 | 16.07 | +0.43 | 5.47 | +0.15 | 7.83 | -0.04 | 1.22 | 0.74 | 1.17 | 0.91 | |
| 567 | 106.48 | 0.60 | 5.39 | 0.43 | 16.52 | 0.45 | 5.62 | 0.15 | 7.78 | 0.05 | 1.20 | 0.73 | 1.16 | 0.89 | |
| 568 | 105.88 | 0.60 | 4.98 | 0.41 | 16.98 | 0.46 | 5.77 | 0.15 | 7.73 | 0.05 | 1.17 | 0.72 | 1.15 | 0.87 | |
| 569 | 105.30 | 0.58 | 4.60 | 0.38 | 17.45 | 0.47 | 5.91 | 0.14 | 7.67 | 0.06 | 1.15 | 0.72 | 1.14 | 0.86 | |
| 570 | 104.74 | 0.56 | 4.23 | 0.37 | 17.93 | 0.48 | 6.05 | 0.14 | 7.60 | 0.07 | 1.12 | 0.71 | -1.13 | -0.85 | |
| 571 | 104.19 | -0.55 | 3.87 | -0.36 | 18.42 | +0.49 | 6.19 | +0.14 | 7.52 | -0.08 | 1.09 | 0.71 | 1.11 | 0.85 | |
| 572 | 103.65 | 0.54 | 3.54 | 0.33 | 18.92 | 0.50 | 6.32 | 0.13 | 7.44 | 0.08 | 1.07 | 0.71 | 1.09 | 0.84 | |
| 573 | 103.13 | 0.52 | 3.22 | 0.32 | 19.42 | 0.50 | 6.44 | 0.12 | 7.35 | 0.09 | 1.04 | 0.71 | 1.07 | 0.83 | |
| 574 | 102.62 | 0.51 | 2.93 | 0.29 | 19.93 | 0.51 | 6.56 | 0.12 | 7.26 | 0.09 | 1.02 | 0.71 | 1.05 | 0.82 | |
| 575 | 102.12 | 0.50 | 2.65 | 0.28 | 20.45 | 0.52 | 6.68 | 0.12 | 7.17 | 0.09 | 1.00 | 0.71 | -1.04 | -0.81 | |
| 576 | 101.63 | -0.49 | 2.39 | -0.26 | 20.97 | +0.52 | 6.79 | +0.11 | 7.06 | -0.11 | 0.98 | 0.72 | 1.02 | 0.81 | |
| 577 | 101.15 | 0.48 | 2.15 | 0.24 | 21.49 | 0.52 | 6.89 | 0.10 | 6.95 | 0.11 | 0.95 | 0.72 | 1.00 | 0.81 | |
| 578 | 100.69 | 0.46 | 1.92 | 0.23 | 22.03 | 0.54 | 6.98 | 0.09 | 6.84 | 0.11 | 0.93 | 0.73 | 0.98 | 0.80 | |
| 579 | 100.25 | 0.43 | 1.71 | 0.21 | 22.58 | 0.55 | 7.07 | 0.09 | 6.73 | 0.11 | 0.91 | 0.74 | 0.96 | 0.80 | |
| 580 | 99.81 | 0.44 | 1.53 | 0.18 | 23.12 | 0.54 | 7.16 | 0.09 | 6.61 | 0.12 | 0.89 | 0.76 | -0.94 | -0.80 | |
| 581 | 99.39 | -0.42 | 1.35 | -0.18 | 23.66 | +0.54 | 7.24 | +0.08 | 6.48 | -0.13 | 0.87 | 0.77 | 0.92 | 0.81 | |
| 582 | 98.98 | 0.41 | 1.19 | 0.16 | 24.21 | 0.55 | 7.30 | 0.06 | 6.35 | 0.13 | 0.85 | 0.78 | 0.90 | 0.82 | |
| 583 | 98.57 | 0.41 | 1.06 | 0.13 | 24.77 | 0.56 | 7.36 | 0.06 | 6.23 | 0.12 | 0.83 | 0.80 | 0.88 | 0.83 | |
| 584 | 98.18 | 0.39 | 0.93 | 0.13 | 25.33 | 0.56 | 7.42 | 0.06 | 6.10 | 0.13 | 0.82 | 0.82 | 0.87 | 0.84 | |
| 585 | 97.80 | 0.38 | 0.82 | 0.11 | 25.88 | 0.55 | 7.47 | 0.05 | 5.97 | 0.13 | 0.81 | 0.84 | -0.86 | -0.85 | |
| 586 | 97.43 | -0.37 | 0.73 | -0.09 | 26.44 | +0.56 | 7.52 | +0.05 | 5.83 | -0.14 | 0.80 | 0.86 | 0.86 | 0.87 | |
| 587 | 97.07 | 0.36 | 0.65 | 0.08 | 27.00 | 0.56 | 7.55 | 0.03 | 5.69 | 0.14 | 0.79 | 0.87 | 0.85 | 0.89 | |
| 588 | 96.72 | 0.35 | 0.58 | 0.07 | 27.56 | 0.56 | 7.58 | 0.03 | 5.56 | 0.13 | 0.78 | 0.90 | 0.84 | 0.91 | |
| 589 | 96.38 | 0.34 | 0.53 | 0.05 | 28.12 | 0.56 | 7.61 | 0.03 | 5.42 | 0.14 | 0.77 | 0.92 | 0.83 | 0.93 | |
| 590 | 96.04 | 0.34 | 0.50 | 0.03 | 28.68 | 0.56 | 7.62 | 0.01 | 5.28 | 0.14 | 0.76 | 0.94 | -0.82 | -0.94 | |
| 591 | 95.72 | -0.32 | 0.47 | -0.03 | 29.24 | +0.56 | 7.63 | +0.01 | 5.14 | -0.14 | 0.76 | 0.96 | 0.82 | 0.96 | |
| 592 | 95.40 | 0.32 | 0.46 | 0.01 | 29.79 | 0.55 | 7.63 | 0.00 | 5.01 | 0.13 | 0.76 | 0.98 | 0.82 | 0.98 | |
| 593 | 95.08 | 0.32 | 0.47 | +0.01 | 30.35 | 0.56 | 7.63 | 0.00 | 4.87 | 0.14 | 0.76 | 1.00 | 0.81 | 1.00 | |
| 594 | 94.77 | 0.31 | 0.49 | 0.02 | 30.91 | 0.56 | 7.62 | -0.01 | 4.73 | 0.14 | 0.76 | 1.02 | 0.81 | 1.02 | |
| 595 | 94.46 | 0.31 | 0.52 | 0.03 | 31.46 | 0.55 | 7.60 | 0.02 | 4.60 | 0.13 | 0.76 | 1.04 | -0.81 | -1.04 | |
| 596 | 94.15 | -0.31 | 0.57 | +0.05 | 32.01 | +0.55 | 7.58 | -0.02 | 4.47 | -0.13 | 0.77 | 1.06 | 0.82 | 1.06 | |
| 597 | 93.85 | 0.30 | 0.63 | 0.06 | 32.56 | 0.55 | 7.55 | 0.03 | 4.34 | 0.13 | 0.78 | 1.08 | 0.83 | 1.08 | |
| 598 | 93.55 | 0.30 | 0.70 | 0.07 | 33.11 | 0.55 | 7.51 | 0.04 | 4.21 | 0.13 | 0.78 | 1.10 | 0.84 | 1.10 | |
| 599 | 93.26 | 0.29 | 0.78 | 0.08 | 33.67 | 0.56 | 7.46 | 0.05 | 4.08 | 0.13 | 0.79 | 1.13 | 0.85 | 1.12 | |
| 600 | 92.96 | 0.30 | 0.87 | 0.09 | 34.22 | 0.55 | 7.42 | 0.04 | 3.95 | 0.13 | 0.80 | 1.15 | -0.85 | -1.13 | |

TABLE X, ARG. 3.—Continued.

| Arg. | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | Arg. | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) |
|------|---------|---------|---------|---------|---------|------|---------|---------|---------|---------|---------|
| 0 | 524 | 180 | 89 | 49 | 58 | 60 | 746 | 17 | 244 | 33 | 10 |
| 1 | 525 | 178 | 90 | 50 | 58 | 61 | 747 | 17 | 249 | 32 | 11 |
| 2 | 525 | 176 | 90 | 50 | 57 | 62 | 748 | 16 | 255 | 30 | 11 |
| 3 | 526 | 174 | 91 | 51 | 57 | 63 | 749 | 16 | 260 | 29 | 11 |
| 4 | 527 | 172 | 91 | 52 | 56 | 64 | 749 | 16 | 266 | 27 | 11 |
| 5 | 528 | 170 | 92 | 53 | 56 | 65 | 750 | 16 | 271 | 26 | 12 |
| 6 | 529 | 168 | 92 | 54 | 55 | 66 | 750 | 16 | 277 | 24 | 13 |
| 7 | 531 | 165 | 92 | 55 | 55 | 67 | 750 | 17 | 282 | 23 | 13 |
| 8 | 533 | 163 | 93 | 55 | 54 | 68 | 750 | 17 | 287 | 22 | 14 |
| 9 | 535 | 161 | 93 | 56 | 53 | 69 | 749 | 18 | 293 | 20 | 14 |
| 10 | 538 | 158 | 94 | 57 | 53 | 70 | 748 | 19 | 298 | 19 | 15 |
| 11 | 540 | 156 | 94 | 57 | 52 | 71 | 747 | 21 | 303 | 18 | 16 |
| 12 | 543 | 154 | 95 | 58 | 51 | 72 | 746 | 22 | 308 | 17 | 17 |
| 13 | 546 | 151 | 96 | 59 | 50 | 73 | 744 | 24 | 313 | 16 | 18 |
| 14 | 550 | 149 | 96 | 59 | 49 | 74 | 742 | 26 | 318 | 14 | 19 |
| 15 | 553 | 146 | 97 | 60 | 48 | 75 | 740 | 29 | 323 | 13 | 20 |
| 16 | 557 | 144 | 98 | 60 | 47 | 76 | 737 | 31 | 327 | 12 | 21 |
| 17 | 560 | 141 | 99 | 61 | 46 | 77 | 734 | 34 | 332 | 11 | 22 |
| 18 | 564 | 138 | 100 | 61 | 45 | 78 | 731 | 37 | 336 | 10 | 23 |
| 19 | 568 | 135 | 101 | 62 | 44 | 79 | 728 | 40 | 340 | 09 | 24 |
| 20 | 573 | 132 | 102 | 62 | 44 | 80 | 724 | 43 | 344 | 09 | 26 |
| 21 | 577 | 129 | 103 | 62 | 42 | 81 | 721 | 47 | 348 | 08 | 27 |
| 22 | 582 | 126 | 105 | 62 | 41 | 82 | 717 | 51 | 352 | 07 | 28 |
| 23 | 586 | 123 | 106 | 63 | 40 | 83 | 713 | 55 | 355 | 07 | 30 |
| 24 | 591 | 120 | 108 | 63 | 39 | 84 | 708 | 59 | 359 | 06 | 31 |
| 25 | 596 | 117 | 109 | 63 | 38 | 85 | 703 | 63 | 362 | 06 | 33 |
| 26 | 601 | 113 | 111 | 63 | 37 | 86 | 698 | 68 | 364 | 05 | 34 |
| 27 | 606 | 110 | 113 | 63 | 36 | 87 | 693 | 72 | 367 | 05 | 36 |
| 28 | 611 | 107 | 115 | 63 | 34 | 88 | 688 | 77 | 369 | 05 | 37 |
| 29 | 617 | 103 | 117 | 63 | 33 | 89 | 682 | 82 | 372 | 05 | 39 |
| 30 | 622 | 100 | 119 | 62 | 32 | 90 | 676 | 87 | 373 | 05 | 41 |
| 31 | 627 | 96 | 121 | 62 | 31 | 91 | 670 | 92 | 375 | 05 | 42 |
| 32 | 633 | 93 | 123 | 62 | 30 | 92 | 663 | 97 | 376 | 05 | 44 |
| 33 | 638 | 90 | 126 | 62 | 29 | 93 | 657 | 102 | 378 | 05 | 45 |
| 34 | 643 | 86 | 129 | 61 | 28 | 94 | 650 | 108 | 378 | 06 | 47 |
| 35 | 648 | 82 | 132 | 61 | 27 | 95 | 643 | 114 | 379 | 06 | 48 |
| 36 | 654 | 79 | 135 | 60 | 26 | 96 | 636 | 119 | 379 | 06 | 50 |
| 37 | 659 | 76 | 139 | 60 | 24 | 97 | 629 | 125 | 379 | 07 | 51 |
| 38 | 664 | 72 | 142 | 59 | 23 | 98 | 622 | 130 | 379 | 08 | 52 |
| 39 | 669 | 69 | 145 | 58 | 22 | 99 | 614 | 136 | 379 | 08 | 54 |
| 40 | 674 | 65 | 149 | 58 | 21 | 100 | 606 | 141 | 378 | 09 | 55 |
| 41 | 680 | 62 | 152 | 57 | 20 | 101 | 598 | 147 | 377 | 10 | 56 |
| 42 | 685 | 58 | 156 | 56 | 19 | 102 | 590 | 153 | 376 | 11 | 57 |
| 43 | 689 | 55 | 160 | 55 | 18 | 103 | 582 | 159 | 374 | 12 | 58 |
| 44 | 694 | 52 | 164 | 54 | 18 | 104 | 574 | 164 | 373 | 13 | 60 |
| 45 | 698 | 49 | 168 | 53 | 17 | 105 | 565 | 170 | 371 | 14 | 61 |
| 46 | 703 | 46 | 173 | 52 | 16 | 106 | 557 | 176 | 368 | 15 | 62 |
| 47 | 707 | 43 | 177 | 51 | 15 | 107 | 548 | 181 | 366 | 16 | 62 |
| 48 | 711 | 40 | 182 | 50 | 14 | 108 | 539 | 186 | 363 | 17 | 63 |
| 49 | 715 | 37 | 186 | 49 | 14 | 109 | 530 | 192 | 360 | 19 | 64 |
| 50 | 719 | 35 | 191 | 47 | 13 | 110 | 521 | 197 | 357 | 20 | 64 |
| 51 | 722 | 32 | 196 | 46 | 13 | 111 | 512 | 202 | 353 | 21 | 65 |
| 52 | 726 | 30 | 201 | 45 | 12 | 112 | 503 | 207 | 350 | 23 | 65 |
| 53 | 729 | 28 | 206 | 43 | 12 | 113 | 494 | 212 | 346 | 24 | 66 |
| 54 | 732 | 26 | 211 | 42 | 11 | 114 | 484 | 217 | 342 | 25 | 66 |
| 55 | 735 | 24 | 216 | 40 | 11 | 115 | 475 | 222 | 338 | 27 | 66 |
| 56 | 737 | 22 | 222 | 39 | 11 | 116 | 465 | 227 | 333 | 28 | 67 |
| 57 | 740 | 21 | 227 | 38 | 11 | 117 | 456 | 231 | 329 | 30 | 67 |
| 58 | 742 | 19 | 233 | 36 | 11 | 118 | 446 | 235 | 324 | 31 | 67 |
| 59 | 744 | 18 | 238 | 35 | 10 | 119 | 436 | 239 | 319 | 32 | 67 |
| 60 | 746 | 17 | 244 | 33 | 10 | 120 | 427 | 243 | 314 | 34 | 66 |

TABLE X, ARG. 3.—Continued.

| Arg. | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | Arg. | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) |
|------|---------|---------|---------|---------|---------|------|---------|---------|---------|---------|---------|
| 120 | 427 | 243 | 314 | 34 | 66 | 180 | 11 | 172 | 66 | 26 | 33 |
| 121 | 417 | 247 | 309 | 35 | 66 | 181 | 10 | 168 | 66 | 25 | 33 |
| 122 | 407 | 250 | 303 | 37 | 66 | 182 | 9 | 165 | 67 | 25 | 34 |
| 123 | 397 | 254 | 298 | 38 | 65 | 183 | 9 | 162 | 67 | 24 | 35 |
| 124 | 387 | 257 | 292 | 39 | 65 | 184 | 8 | 159 | 68 | 24 | 36 |
| 125 | 378 | 260 | 286 | 41 | 64 | 185 | 8 | 156 | 69 | 24 | 37 |
| 126 | 368 | 263 | 280 | 42 | 63 | 186 | 8 | 153 | 71 | 24 | 38 |
| 127 | 359 | 266 | 274 | 43 | 63 | 187 | 8 | 150 | 72 | 23 | 39 |
| 128 | 349 | 268 | 268 | 44 | 62 | 188 | 9 | 147 | 74 | 23 | 40 |
| 129 | 339 | 270 | 262 | 45 | 61 | 189 | 9 | 144 | 75 | 23 | 41 |
| 130 | 330 | 272 | 256 | 46 | 60 | 190 | 10 | 141 | 77 | 23 | 42 |
| 131 | 320 | 274 | 250 | 47 | 59 | 191 | 11 | 138 | 79 | 24 | 43 |
| 132 | 310 | 275 | 244 | 48 | 58 | 192 | 13 | 136 | 81 | 24 | 44 |
| 133 | 301 | 277 | 238 | 49 | 57 | 193 | 14 | 133 | 84 | 24 | 45 |
| 134 | 292 | 278 | 232 | 50 | 56 | 194 | 16 | 131 | 86 | 24 | 46 |
| 135 | 282 | 279 | 225 | 50 | 55 | 195 | 18 | 129 | 89 | 25 | 47 |
| 136 | 273 | 279 | 219 | 51 | 54 | 196 | 21 | 127 | 91 | 25 | 48 |
| 137 | 263 | 280 | 213 | 52 | 52 | 197 | 23 | 125 | 94 | 26 | 49 |
| 138 | 254 | 280 | 207 | 52 | 51 | 198 | 26 | 123 | 97 | 26 | 50 |
| 139 | 245 | 280 | 201 | 52 | 50 | 199 | 29 | 121 | 99 | 27 | 51 |
| 140 | 236 | 280 | 195 | 53 | 49 | 200 | 32 | 120 | 102 | 28 | 52 |
| 141 | 227 | 280 | 189 | 53 | 47 | 201 | 36 | 118 | 105 | 28 | 52 |
| 142 | 218 | 280 | 183 | 53 | 46 | 202 | 39 | 117 | 108 | 29 | 53 |
| 143 | 209 | 279 | 177 | 53 | 45 | 203 | 43 | 116 | 112 | 30 | 54 |
| 144 | 201 | 278 | 171 | 53 | 44 | 204 | 47 | 114 | 115 | 31 | 54 |
| 145 | 193 | 277 | 165 | 53 | 42 | 205 | 51 | 113 | 118 | 32 | 55 |
| 146 | 184 | 275 | 160 | 53 | 41 | 206 | 55 | 112 | 122 | 33 | 55 |
| 147 | 176 | 274 | 154 | 52 | 40 | 207 | 60 | 112 | 125 | 34 | 56 |
| 148 | 168 | 272 | 149 | 52 | 39 | 208 | 65 | 111 | 129 | 35 | 56 |
| 149 | 160 | 271 | 144 | 52 | 38 | 209 | 69 | 110 | 132 | 36 | 57 |
| 150 | 152 | 269 | 139 | 51 | 36 | 210 | 75 | 110 | 135 | 37 | 57 |
| 151 | 145 | 267 | 134 | 51 | 35 | 211 | 80 | 110 | 139 | 38 | 57 |
| 152 | 137 | 265 | 129 | 50 | 35 | 212 | 85 | 110 | 143 | 39 | 57 |
| 153 | 130 | 263 | 124 | 49 | 34 | 213 | 91 | 110 | 146 | 40 | 57 |
| 154 | 123 | 260 | 120 | 49 | 33 | 214 | 96 | 110 | 150 | 41 | 57 |
| 155 | 116 | 257 | 115 | 48 | 32 | 215 | 102 | 110 | 153 | 42 | 57 |
| 156 | 109 | 255 | 111 | 47 | 31 | 216 | 108 | 110 | 157 | 43 | 57 |
| 157 | 103 | 252 | 107 | 46 | 30 | 217 | 114 | 110 | 160 | 44 | 57 |
| 158 | 96 | 249 | 103 | 45 | 30 | 218 | 120 | 111 | 164 | 45 | 57 |
| 159 | 90 | 246 | 100 | 45 | 29 | 219 | 126 | 112 | 168 | 46 | 57 |
| 160 | 84 | 243 | 96 | 44 | 29 | 220 | 133 | 113 | 171 | 47 | 57 |
| 161 | 78 | 240 | 93 | 43 | 28 | 221 | 139 | 114 | 175 | 48 | 56 |
| 162 | 73 | 236 | 90 | 42 | 28 | 222 | 145 | 115 | 178 | 49 | 56 |
| 163 | 67 | 233 | 87 | 41 | 27 | 223 | 152 | 116 | 182 | 50 | 55 |
| 164 | 62 | 229 | 84 | 40 | 27 | 224 | 159 | 117 | 185 | 50 | 55 |
| 165 | 57 | 226 | 81 | 39 | 27 | 225 | 166 | 118 | 188 | 51 | 54 |
| 166 | 53 | 222 | 79 | 38 | 27 | 226 | 173 | 119 | 192 | 52 | 53 |
| 167 | 48 | 219 | 77 | 36 | 27 | 227 | 180 | 120 | 195 | 53 | 52 |
| 168 | 44 | 215 | 75 | 35 | 27 | 228 | 187 | 122 | 198 | 54 | 52 |
| 169 | 40 | 212 | 73 | 34 | 27 | 229 | 194 | 123 | 201 | 54 | 51 |
| 170 | 36 | 208 | 71 | 33 | 27 | 230 | 201 | 125 | 204 | 55 | 51 |
| 171 | 32 | 204 | 70 | 32 | 27 | 231 | 208 | 127 | 207 | 55 | 50 |
| 172 | 29 | 201 | 68 | 31 | 28 | 232 | 216 | 128 | 210 | 56 | 49 |
| 173 | 26 | 197 | 67 | 30 | 28 | 233 | 223 | 130 | 213 | 56 | 49 |
| 174 | 23 | 193 | 67 | 29 | 28 | 234 | 230 | 132 | 216 | 57 | 48 |
| 175 | 21 | 190 | 66 | 28 | 29 | 235 | 238 | 134 | 218 | 57 | 47 |
| 176 | 18 | 186 | 66 | 28 | 30 | 236 | 245 | 136 | 221 | 57 | 46 |
| 177 | 16 | 182 | 65 | 27 | 30 | 237 | 253 | 138 | 224 | 58 | 45 |
| 178 | 14 | 179 | 65 | 27 | 31 | 238 | 260 | 140 | 226 | 58 | 45 |
| 179 | 12 | 175 | 65 | 26 | 32 | 239 | 268 | 142 | 228 | 58 | 44 |
| 180 | 11 | 172 | 66 | 26 | 33 | 240 | 275 | 144 | 231 | 58 | 43 |

TABLE X, ARG. 3.—Continued.

| Arg. | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | Arg. | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) |
|------|---------|---------|---------|---------|---------|------|---------|---------|---------|---------|---------|
| 240 | 275 | 144 | 231 | 58 | 43 | 300 | 562 | 208 | 250 | 42 | 32 |
| 241 | 283 | 146 | 233 | 58 | 42 | 301 | 562 | 208 | 250 | 42 | 32 |
| 242 | 290 | 148 | 235 | 58 | 41 | 302 | 562 | 208 | 250 | 42 | 32 |
| 243 | 298 | 150 | 237 | 58 | 40 | 303 | 561 | 208 | 250 | 42 | 32 |
| 244 | 305 | 152 | 239 | 58 | 40 | 304 | 560 | 208 | 250 | 41 | 32 |
| 245 | 313 | 154 | 240 | 57 | 39 | 305 | 559 | 208 | 250 | 41 | 31 |
| 246 | 320 | 156 | 242 | 57 | 38 | 306 | 558 | 208 | 250 | 41 | 31 |
| 247 | 328 | 159 | 244 | 57 | 38 | 307 | 557 | 208 | 250 | 41 | 31 |
| 248 | 335 | 161 | 245 | 57 | 37 | 308 | 555 | 208 | 251 | 41 | 31 |
| 249 | 342 | 163 | 246 | 57 | 36 | 309 | 553 | 208 | 251 | 40 | 30 |
| 250 | 350 | 165 | 248 | 56 | 36 | 310 | 551 | 208 | 251 | 40 | 30 |
| 251 | 357 | 167 | 249 | 56 | 35 | 311 | 549 | 209 | 251 | 40 | 30 |
| 252 | 364 | 169 | 250 | 55 | 34 | 312 | 547 | 209 | 251 | 39 | 30 |
| 253 | 371 | 171 | 251 | 55 | 34 | 313 | 544 | 209 | 251 | 39 | 29 |
| 254 | 378 | 173 | 252 | 54 | 33 | 314 | 542 | 210 | 251 | 39 | 29 |
| 255 | 385 | 175 | 253 | 54 | 33 | 315 | 539 | 210 | 251 | 38 | 29 |
| 256 | 392 | 177 | 254 | 54 | 33 | 316 | 536 | 210 | 252 | 38 | 29 |
| 257 | 399 | 179 | 254 | 53 | 32 | 317 | 532 | 211 | 252 | 38 | 28 |
| 258 | 406 | 180 | 255 | 52 | 32 | 318 | 529 | 211 | 252 | 37 | 28 |
| 259 | 412 | 182 | 256 | 52 | 32 | 319 | 525 | 212 | 252 | 37 | 28 |
| 260 | 419 | 184 | 256 | 51 | 31 | 320 | 521 | 213 | 252 | 36 | 28 |
| 261 | 426 | 185 | 256 | 51 | 21 | 321 | 517 | 213 | 252 | 36 | 28 |
| 262 | 432 | 187 | 257 | 50 | 31 | 322 | 513 | 214 | 252 | 35 | 28 |
| 263 | 438 | 189 | 257 | 50 | 31 | 323 | 509 | 215 | 252 | 34 | 28 |
| 264 | 444 | 190 | 257 | 50 | 31 | 324 | 504 | 216 | 252 | 34 | 28 |
| 265 | 450 | 192 | 257 | 49 | 31 | 325 | 500 | 217 | 252 | 33 | 28 |
| 266 | 456 | 193 | 257 | 49 | 31 | 326 | 495 | 218 | 252 | 33 | 28 |
| 267 | 462 | 194 | 257 | 48 | 31 | 327 | 490 | 219 | 252 | 32 | 28 |
| 268 | 468 | 196 | 257 | 48 | 31 | 328 | 485 | 221 | 251 | 31 | 28 |
| 269 | 473 | 197 | 257 | 47 | 31 | 329 | 480 | 222 | 251 | 31 | 28 |
| 270 | 478 | 198 | 257 | 47 | 31 | 330 | 474 | 223 | 251 | 30 | 28 |
| 271 | 484 | 199 | 257 | 46 | 31 | 331 | 469 | 224 | 250 | 30 | 28 |
| 272 | 489 | 200 | 257 | 46 | 31 | 332 | 463 | 225 | 249 | 29 | 29 |
| 273 | 494 | 201 | 257 | 45 | 31 | 333 | 457 | 227 | 249 | 29 | 29 |
| 274 | 499 | 202 | 256 | 45 | 31 | 334 | 451 | 228 | 248 | 28 | 29 |
| 275 | 503 | 203 | 256 | 45 | 31 | 335 | 446 | 229 | 248 | 27 | 30 |
| 276 | 509 | 203 | 256 | 45 | 31 | 336 | 439 | 231 | 247 | 27 | 30 |
| 277 | 512 | 204 | 256 | 44 | 31 | 337 | 433 | 233 | 246 | 26 | 31 |
| 278 | 517 | 205 | 255 | 44 | 31 | 338 | 427 | 234 | 245 | 25 | 31 |
| 279 | 521 | 205 | 255 | 44 | 32 | 339 | 421 | 236 | 244 | 25 | 31 |
| 280 | 524 | 206 | 255 | 44 | 32 | 340 | 414 | 238 | 243 | 24 | 32 |
| 281 | 528 | 206 | 254 | 43 | 32 | 341 | 407 | 239 | 242 | 24 | 33 |
| 282 | 531 | 207 | 254 | 43 | 32 | 342 | 401 | 241 | 241 | 24 | 33 |
| 283 | 535 | 207 | 253 | 43 | 32 | 343 | 394 | 243 | 240 | 23 | 34 |
| 284 | 538 | 207 | 253 | 43 | 32 | 344 | 387 | 244 | 238 | 23 | 35 |
| 285 | 541 | 208 | 253 | 43 | 32 | 345 | 380 | 246 | 236 | 23 | 35 |
| 286 | 544 | 208 | 252 | 43 | 33 | 346 | 373 | 248 | 235 | 22 | 36 |
| 287 | 546 | 208 | 252 | 43 | 33 | 347 | 366 | 249 | 233 | 22 | 37 |
| 288 | 549 | 208 | 252 | 43 | 33 | 348 | 359 | 251 | 231 | 22 | 38 |
| 289 | 551 | 208 | 251 | 42 | 33 | 349 | 352 | 253 | 229 | 22 | 38 |
| 290 | 553 | 208 | 251 | 42 | 33 | 350 | 344 | 255 | 227 | 21 | 39 |
| 291 | 555 | 208 | 251 | 42 | 33 | 351 | 337 | 256 | 225 | 21 | 40 |
| 292 | 556 | 208 | 251 | 42 | 33 | 352 | 330 | 258 | 223 | 21 | 41 |
| 293 | 558 | 208 | 251 | 42 | 33 | 353 | 323 | 260 | 221 | 21 | 42 |
| 294 | 559 | 208 | 251 | 42 | 33 | 354 | 315 | 261 | 218 | 21 | 42 |
| 295 | 560 | 208 | 250 | 42 | 33 | 355 | 308 | 263 | 216 | 22 | 43 |
| 296 | 561 | 208 | 250 | 42 | 33 | 356 | 300 | 264 | 213 | 22 | 44 |
| 297 | 562 | 208 | 250 | 42 | 32 | 357 | 293 | 266 | 211 | 22 | 45 |
| 298 | 562 | 208 | 250 | 42 | 32 | 358 | 285 | 267 | 208 | 22 | 46 |
| 299 | 562 | 208 | 250 | 42 | 32 | 359 | 278 | 269 | 205 | 23 | 47 |
| 300 | 562 | 208 | 250 | 42 | 32 | 360 | 270 | 270 | 203 | 23 | 47 |

TABLE X, ARG. 3.—Continued.

| Arg. | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | Arg. | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) |
|------|---------|---------|---------|---------|---------|------|---------|---------|---------|---------|---------|
| 360 | 270 | 270 | 203 | 23 | 47 | 420 | 14 | 183 | 69 | 44 | 32 |
| 361 | 263 | 272 | 200 | 23 | 48 | 421 | 16 | 180 | 70 | 44 | 32 |
| 362 | 255 | 273 | 197 | 24 | 49 | 422 | 18 | 176 | 72 | 43 | 32 |
| 363 | 248 | 274 | 194 | 25 | 50 | 423 | 20 | 173 | 74 | 42 | 32 |
| 364 | 241 | 275 | 190 | 25 | 50 | 424 | 22 | 170 | 76 | 41 | 32 |
| 365 | 233 | 276 | 187 | 26 | 51 | 425 | 24 | 166 | 78 | 41 | 32 |
| 366 | 226 | 277 | 184 | 26 | 52 | 426 | 27 | 163 | 81 | 40 | 32 |
| 367 | 219 | 278 | 181 | 27 | 52 | 427 | 30 | 160 | 83 | 39 | 32 |
| 368 | 211 | 278 | 177 | 28 | 53 | 428 | 33 | 157 | 86 | 38 | 33 |
| 369 | 204 | 279 | 174 | 29 | 53 | 429 | 36 | 153 | 89 | 38 | 33 |
| 370 | 197 | 280 | 171 | 29 | 54 | 430 | 40 | 150 | 92 | 37 | 33 |
| 371 | 190 | 280 | 167 | 30 | 54 | 431 | 44 | 147 | 95 | 36 | 34 |
| 372 | 183 | 281 | 164 | 31 | 54 | 432 | 48 | 144 | 99 | 35 | 34 |
| 373 | 176 | 281 | 160 | 32 | 55 | 433 | 53 | 142 | 103 | 35 | 35 |
| 374 | 169 | 281 | 157 | 33 | 55 | 434 | 57 | 139 | 106 | 34 | 36 |
| 375 | 162 | 281 | 153 | 34 | 55 | 435 | 62 | 136 | 111 | 34 | 36 |
| 376 | 155 | 281 | 150 | 35 | 55 | 436 | 67 | 134 | 115 | 33 | 37 |
| 377 | 149 | 281 | 146 | 36 | 55 | 437 | 72 | 132 | 119 | 33 | 38 |
| 378 | 142 | 281 | 142 | 37 | 55 | 438 | 77 | 130 | 124 | 32 | 39 |
| 379 | 136 | 281 | 139 | 38 | 55 | 439 | 83 | 128 | 128 | 32 | 40 |
| 380 | 129 | 280 | 135 | 39 | 55 | 440 | 89 | 126 | 133 | 32 | 40 |
| 381 | 123 | 280 | 132 | 40 | 55 | 441 | 95 | 124 | 138 | 32 | 41 |
| 382 | 117 | 279 | 128 | 41 | 55 | 442 | 102 | 122 | 143 | 31 | 42 |
| 383 | 111 | 278 | 125 | 42 | 55 | 443 | 108 | 121 | 148 | 31 | 43 |
| 384 | 105 | 278 | 122 | 42 | 54 | 444 | 115 | 120 | 153 | 31 | 44 |
| 385 | 99 | 277 | 118 | 43 | 54 | 445 | 121 | 118 | 159 | 31 | 45 |
| 386 | 94 | 276 | 115 | 44 | 54 | 446 | 128 | 118 | 164 | 31 | 46 |
| 387 | 89 | 274 | 111 | 45 | 53 | 447 | 135 | 117 | 170 | 31 | 47 |
| 388 | 83 | 273 | 108 | 46 | 53 | 448 | 142 | 116 | 176 | 32 | 48 |
| 389 | 78 | 272 | 105 | 47 | 52 | 449 | 150 | 116 | 181 | 32 | 49 |
| 390 | 73 | 270 | 102 | 47 | 52 | 450 | 157 | 115 | 187 | 32 | 50 |
| 391 | 68 | 268 | 99 | 48 | 51 | 451 | 165 | 115 | 193 | 33 | 51 |
| 392 | 63 | 266 | 97 | 48 | 50 | 452 | 173 | 115 | 199 | 33 | 52 |
| 393 | 59 | 265 | 94 | 49 | 50 | 453 | 181 | 116 | 205 | 34 | 53 |
| 394 | 54 | 263 | 91 | 49 | 49 | 454 | 189 | 117 | 211 | 34 | 54 |
| 395 | 50 | 261 | 88 | 50 | 48 | 455 | 197 | 117 | 217 | 35 | 55 |
| 396 | 46 | 258 | 86 | 50 | 47 | 456 | 206 | 118 | 223 | 36 | 56 |
| 397 | 42 | 256 | 83 | 51 | 47 | 457 | 215 | 120 | 229 | 36 | 57 |
| 398 | 39 | 254 | 81 | 51 | 46 | 458 | 223 | 121 | 236 | 37 | 58 |
| 399 | 35 | 251 | 79 | 51 | 45 | 459 | 232 | 123 | 242 | 38 | 58 |
| 400 | 32 | 248 | 77 | 52 | 45 | 460 | 241 | 125 | 248 | 39 | 59 |
| 401 | 29 | 246 | 75 | 52 | 44 | 461 | 250 | 127 | 254 | 40 | 60 |
| 402 | 27 | 243 | 73 | 52 | 43 | 462 | 259 | 129 | 260 | 41 | 60 |
| 403 | 24 | 240 | 72 | 52 | 42 | 463 | 268 | 131 | 265 | 42 | 61 |
| 404 | 22 | 237 | 70 | 52 | 41 | 464 | 278 | 134 | 271 | 43 | 61 |
| 405 | 19 | 234 | 69 | 52 | 40 | 465 | 287 | 137 | 277 | 44 | 62 |
| 406 | 17 | 231 | 68 | 52 | 39 | 466 | 296 | 140 | 283 | 45 | 62 |
| 407 | 15 | 228 | 67 | 51 | 38 | 467 | 306 | 143 | 288 | 47 | 62 |
| 408 | 14 | 225 | 66 | 51 | 38 | 468 | 315 | 147 | 294 | 48 | 62 |
| 409 | 13 | 221 | 65 | 51 | 37 | 469 | 324 | 150 | 299 | 49 | 62 |
| 410 | 12 | 218 | 64 | 50 | 36 | 470 | 334 | 154 | 305 | 51 | 62 |
| 411 | 11 | 215 | 64 | 50 | 36 | 471 | 343 | 158 | 310 | 52 | 62 |
| 412 | 11 | 211 | 64 | 49 | 35 | 472 | 353 | 162 | 315 | 53 | 62 |
| 413 | 10 | 208 | 64 | 49 | 34 | 473 | 362 | 167 | 320 | 54 | 62 |
| 414 | 10 | 204 | 64 | 48 | 34 | 474 | 372 | 171 | 325 | 55 | 62 |
| 415 | 10 | 201 | 64 | 48 | 33 | 475 | 382 | 176 | 329 | 57 | 62 |
| 416 | 10 | 197 | 65 | 47 | 33 | 476 | 391 | 181 | 334 | 58 | 61 |
| 417 | 11 | 194 | 66 | 46 | 33 | 477 | 401 | 186 | 338 | 59 | 61 |
| 418 | 12 | 190 | 67 | 46 | 32 | 478 | 411 | 191 | 342 | 60 | 60 |
| 419 | 13 | 187 | 68 | 45 | 32 | 479 | 420 | 196 | 346 | 61 | 60 |
| 420 | 14 | 183 | 69 | 44 | 32 | 480 | 430 | 202 | 349 | 62 | 59 |

TABLE X, ARG. 3.—*Concluded.*

| Arg. | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | Arg. | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) |
|------|---------|---------|---------|---------|---------|------|---------|---------|---------|---------|---------|
| 480 | 430 | 202 | 349 | 62 | 59 | 540 | 741 | 381 | 186 | 29 | 19 |
| 481 | 439 | 207 | 353 | 63 | 58 | 541 | 739 | 378 | 181 | 28 | 20 |
| 482 | 449 | 213 | 356 | 64 | 57 | 542 | 737 | 375 | 176 | 27 | 21 |
| 483 | 458 | 218 | 359 | 65 | 56 | 543 | 735 | 372 | 171 | 26 | 22 |
| 484 | 468 | 223 | 361 | 66 | 55 | 544 | 732 | 368 | 167 | 25 | 23 |
| 485 | 477 | 230 | 364 | 67 | 54 | 545 | 730 | 365 | 162 | 24 | 24 |
| 486 | 486 | 236 | 366 | 68 | 53 | 546 | 727 | 361 | 158 | 24 | 25 |
| 487 | 496 | 242 | 368 | 69 | 52 | 547 | 724 | 357 | 153 | 23 | 26 |
| 488 | 505 | 248 | 370 | 69 | 51 | 548 | 721 | 353 | 149 | 22 | 27 |
| 489 | 514 | 254 | 371 | 70 | 50 | 549 | 717 | 350 | 145 | 22 | 28 |
| 490 | 523 | 260 | 373 | 70 | 49 | 550 | 714 | 346 | 141 | 21 | 29 |
| 491 | 532 | 266 | 374 | 71 | 48 | 551 | 710 | 342 | 137 | 21 | 30 |
| 492 | 540 | 272 | 374 | 71 | 46 | 552 | 706 | 338 | 133 | 21 | 31 |
| 493 | 549 | 278 | 375 | 72 | 45 | 553 | 702 | 333 | 130 | 20 | 32 |
| 494 | 558 | 284 | 375 | 72 | 44 | 554 | 698 | 329 | 126 | 20 | 33 |
| 495 | 566 | 289 | 375 | 72 | 42 | 555 | 693 | 325 | 123 | 20 | 34 |
| 496 | 574 | 295 | 374 | 72 | 41 | 556 | 689 | 321 | 120 | 20 | 35 |
| 497 | 583 | 301 | 374 | 72 | 39 | 557 | 684 | 316 | 117 | 19 | 36 |
| 498 | 591 | 307 | 373 | 72 | 38 | 558 | 679 | 312 | 114 | 19 | 38 |
| 499 | 599 | 313 | 872 | 72 | 37 | 559 | 674 | 308 | 112 | 19 | 39 |
| 500 | 606 | 318 | 371 | 72 | 35 | 560 | 669 | 303 | 109 | 20 | 40 |
| 501 | 614 | 324 | 369 | 71 | 34 | 561 | 664 | 299 | 107 | 20 | 41 |
| 502 | 621 | 329 | 367 | 71 | 33 | 562 | 659 | 295 | 105 | 20 | 42 |
| 503 | 629 | 334 | 365 | 71 | 31 | 563 | 654 | 290 | 103 | 20 | 43 |
| 504 | 636 | 339 | 363 | 70 | 30 | 564 | 649 | 286 | 101 | 20 | 44 |
| 505 | 643 | 344 | 360 | 70 | 29 | 565 | 644 | 282 | 99 | 20 | 45 |
| 506 | 649 | 349 | 357 | 69 | 28 | 566 | 638 | 278 | 97 | 21 | 46 |
| 507 | 656 | 353 | 354 | 68 | 27 | 567 | 633 | 274 | 96 | 21 | 47 |
| 508 | 662 | 358 | 351 | 68 | 25 | 568 | 628 | 270 | 94 | 22 | 48 |
| 509 | 668 | 362 | 347 | 67 | 24 | 569 | 623 | 266 | 93 | 22 | 49 |
| 510 | 674 | 366 | 344 | 66 | 23 | 570 | 618 | 262 | 92 | 23 | 50 |
| 511 | 680 | 370 | 340 | 65 | 22 | 571 | 613 | 258 | 91 | 23 | 50 |
| 512 | 686 | 374 | 336 | 64 | 21 | 572 | 608 | 255 | 90 | 24 | 51 |
| 513 | 691 | 377 | 332 | 63 | 20 | 573 | 602 | 251 | 90 | 25 | 52 |
| 514 | 696 | 380 | 327 | 62 | 19 | 574 | 598 | 247 | 89 | 25 | 53 |
| 515 | 701 | 383 | 323 | 61 | 18 | 575 | 593 | 244 | 88 | 26 | 54 |
| 516 | 706 | 386 | 318 | 59 | 18 | 576 | 588 | 240 | 88 | 27 | 54 |
| 517 | 710 | 388 | 313 | 58 | 17 | 577 | 583 | 237 | 87 | 28 | 55 |
| 518 | 714 | 390 | 308 | 57 | 16 | 578 | 578 | 234 | 87 | 28 | 56 |
| 519 | 718 | 392 | 303 | 56 | 16 | 579 | 574 | 231 | 87 | 29 | 56 |
| 520 | 721 | 394 | 298 | 54 | 15 | 580 | 570 | 228 | 86 | 30 | 57 |
| 521 | 725 | 396 | 293 | 53 | 15 | 581 | 565 | 225 | 86 | 31 | 57 |
| 522 | 728 | 397 | 287 | 52 | 14 | 582 | 561 | 222 | 86 | 32 | 58 |
| 523 | 731 | 398 | 282 | 50 | 14 | 583 | 557 | 219 | 86 | 32 | 58 |
| 524 | 733 | 399 | 276 | 49 | 14 | 584 | 554 | 216 | 86 | 33 | 58 |
| 525 | 736 | 400 | 270 | 48 | 14 | 585 | 551 | 214 | 86 | 34 | 58 |
| 526 | 738 | 400 | 265 | 46 | 14 | 586 | 547 | 211 | 86 | 35 | 59 |
| 527 | 740 | 400 | 259 | 45 | 14 | 587 | 544 | 209 | 86 | 36 | 59 |
| 528 | 742 | 400 | 253 | 44 | 14 | 588 | 541 | 206 | 86 | 37 | 59 |
| 529 | 743 | 399 | 247 | 42 | 14 | 589 | 539 | 204 | 86 | 38 | 59 |
| 530 | 744 | 399 | 242 | 41 | 14 | 590 | 536 | 202 | 87 | 39 | 60 |
| 531 | 745 | 398 | 236 | 40 | 14 | 591 | 534 | 199 | 87 | 40 | 60 |
| 532 | 745 | 397 | 230 | 38 | 14 | 592 | 532 | 197 | 87 | 41 | 60 |
| 533 | 746 | 396 | 225 | 37 | 15 | 593 | 530 | 195 | 88 | 42 | 60 |
| 534 | 746 | 394 | 219 | 36 | 15 | 594 | 528 | 193 | 88 | 43 | 60 |
| 535 | 745 | 392 | 213 | 34 | 16 | 595 | 527 | 190 | 88 | 44 | 59 |
| 536 | 745 | 390 | 208 | 33 | 17 | 596 | 526 | 188 | 88 | 45 | 59 |
| 537 | 744 | 388 | 202 | 32 | 17 | 597 | 525 | 186 | 88 | 46 | 59 |
| 538 | 743 | 386 | 197 | 31 | 18 | 598 | 525 | 184 | 89 | 47 | 59 |
| 539 | 742 | 383 | 192 | 30 | 19 | 599 | 524 | 182 | 89 | 48 | 58 |
| 540 | 741 | 381 | 186 | 29 | 19 | 600 | 524 | 180 | 89 | 49 | 58 |

| TAB. | XI. | | XII. | | | XIII. | | | XIV. | | | XV. | | | XVI. | | |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| ARG. | 4. | | 5. | | | 6. | | | 7. | | | 8. | | | 9. | | |
| | (v.s.1) | (v.e.1) | (v.e.0) | (v.s.1) | (v.e.1) | (v.e.0) | (v.s.1) | (v.e.1) | (v.e.0) | (v.s.1) | (v.e.1) | (v.e.0) | (v.s.1) | (v.e.1) | (v.e.0) | (v.s.1) | (v.e.1) |
| | " | " | " | " | " | " | " | " | " | " | " | " | " | " | " | " | " |
| 0 | 0.67 | 0.13 | 1.11 | 0.03 | 0.21 | 0.06 | 0.13 | 0.18 | 0.10 | 0.18 | 0.19 | 0.05 | 0.17 | 0.14 | 0.13 | 0.10 | 0.13 |
| 10 | 0.64 | 0.13 | 1.12 | 0.03 | 0.22 | 0.06 | 0.14 | 0.18 | 0.16 | 0.18 | 0.19 | 0.09 | 0.15 | 0.19 | 0.12 | 0.10 | 0.12 |
| 20 | 0.61 | 0.14 | 1.12 | 0.03 | 0.24 | 0.07 | 0.15 | 0.18 | 0.10 | 0.18 | 0.19 | 0.15 | 0.13 | 0.25 | 0.12 | 0.09 | 0.11 |
| 30 | 0.58 | 0.16 | 1.11 | 0.03 | 0.26 | 0.07 | 0.16 | 0.17 | 0.10 | 0.18 | 0.19 | 0.22 | 0.11 | 0.31 | 0.11 | 0.09 | 0.10 |
| 40 | 0.54 | 0.17 | 1.10 | 0.03 | 0.28 | 0.08 | 0.16 | 0.16 | 0.10 | 0.18 | 0.19 | 0.30 | 0.10 | 0.37 | 0.11 | 0.09 | 0.09 |
| 50 | 0.50 | 0.19 | 1.08 | 0.04 | 0.30 | 0.08 | 0.17 | 0.16 | 0.10 | 0.18 | 0.19 | 0.39 | 0.10 | 0.44 | 0.10 | 0.08 | 0.08 |
| 60 | 0.47 | 0.21 | 1.06 | 0.04 | 0.31 | 0.08 | 0.17 | 0.15 | 0.10 | 0.18 | 0.19 | 0.50 | 0.11 | 0.52 | 0.09 | 0.08 | 0.07 |
| 70 | 0.43 | 0.24 | 1.04 | 0.05 | 0.32 | 0.09 | 0.18 | 0.14 | 0.09 | 0.17 | 0.18 | 0.61 | 0.13 | 0.59 | 0.09 | 0.08 | 0.06 |
| 80 | 0.39 | 0.26 | 1.00 | 0.06 | 0.34 | 0.09 | 0.18 | 0.13 | 0.09 | 0.17 | 0.18 | 0.74 | 0.15 | 0.67 | 0.08 | 0.07 | 0.05 |
| 90 | 0.35 | 0.29 | 0.97 | 0.07 | 0.35 | 0.09 | 0.18 | 0.12 | 0.09 | 0.16 | 0.17 | 0.87 | 0.17 | 0.75 | 0.08 | 0.07 | 0.04 |
| 100 | 0.31 | 0.31 | 0.93 | 0.08 | 0.36 | 0.09 | 0.18 | 0.12 | 0.08 | 0.16 | 0.17 | 1.00 | 0.21 | 0.83 | 0.07 | 0.07 | 0.03 |
| 110 | 0.27 | 0.34 | 0.88 | 0.09 | 0.37 | 0.09 | 0.19 | 0.11 | 0.08 | 0.15 | 0.16 | 1.14 | 0.25 | 0.91 | 0.06 | 0.07 | 0.02 |
| 120 | 0.23 | 0.37 | 0.84 | 0.11 | 0.37 | 0.09 | 0.19 | 0.10 | 0.07 | 0.14 | 0.15 | 1.28 | 0.29 | 0.96 | 0.06 | 0.06 | 0.02 |
| 130 | 0.20 | 0.40 | 0.79 | 0.12 | 0.38 | 0.09 | 0.18 | 0.09 | 0.07 | 0.14 | 0.14 | 1.42 | 0.34 | 1.06 | 0.06 | 0.06 | 0.01 |
| 140 | 0.17 | 0.43 | 0.74 | 0.14 | 0.38 | 0.09 | 0.18 | 0.08 | 0.06 | 0.13 | 0.13 | 1.56 | 0.40 | 1.13 | 0.05 | 0.06 | 0.01 |
| 150 | 0.14 | 0.46 | 0.68 | 0.16 | 0.38 | 0.09 | 0.18 | 0.07 | 0.06 | 0.12 | 0.12 | 1.70 | 0.45 | 1.19 | 0.05 | 0.06 | 0.01 |
| 160 | 0.11 | 0.49 | 0.63 | 0.18 | 0.38 | 0.09 | 0.18 | 0.06 | 0.05 | 0.11 | 0.11 | 1.83 | 0.52 | 1.25 | 0.04 | 0.06 | 0.00 |
| 170 | 0.09 | 0.51 | 0.57 | 0.19 | 0.37 | 0.08 | 0.17 | 0.05 | 0.05 | 0.10 | 0.10 | 1.96 | 0.58 | 1.31 | 0.04 | 0.06 | 0.00 |
| 180 | 0.07 | 0.54 | 0.52 | 0.21 | 0.37 | 0.08 | 0.17 | 0.04 | 0.04 | 0.10 | 0.03 | 2.09 | 0.65 | 1.36 | 0.04 | 0.06 | 0.00 |
| 190 | 0.05 | 0.56 | 0.46 | 0.23 | 0.36 | 0.08 | 0.16 | 0.04 | 0.04 | 0.09 | 0.08 | 2.20 | 0.72 | 1.40 | 0.04 | 0.07 | 0.00 |
| 200 | 0.04 | 0.59 | 0.41 | 0.25 | 0.35 | 0.08 | 0.16 | 0.04 | 0.03 | 0.08 | 0.07 | 2.31 | 0.78 | 1.43 | 0.04 | 0.07 | 0.00 |
| 210 | 0.03 | 0.61 | 0.36 | 0.26 | 0.34 | 0.07 | 0.15 | 0.02 | 0.03 | 0.07 | 0.06 | 2.40 | 0.85 | 1.46 | 0.04 | 0.07 | 0.00 |
| 220 | 0.02 | 0.63 | 0.32 | 0.28 | 0.32 | 0.07 | 0.14 | 0.02 | 0.02 | 0.07 | 0.05 | 2.48 | 0.92 | 1.48 | 0.04 | 0.07 | 0.01 |
| 230 | 0.02 | 0.64 | 0.27 | 0.30 | 0.31 | 0.07 | 0.13 | 0.02 | 0.02 | 0.06 | 0.05 | 2.55 | 0.98 | 1.50 | 0.04 | 0.08 | 0.01 |
| 240 | 0.02 | 0.66 | 0.23 | 0.31 | 0.30 | 0.06 | 0.12 | 0.01 | 0.02 | 0.05 | 0.04 | 2.61 | 1.05 | 1.50 | 0.04 | 0.08 | 0.02 |
| 250 | 0.03 | 0.67 | 0.20 | 0.32 | 0.28 | 0.06 | 0.11 | 0.01 | 0.01 | 0.04 | 0.03 | 2.65 | 1.10 | 1.50 | 0.05 | 0.08 | 0.02 |
| 260 | 0.04 | 0.67 | 0.16 | 0.34 | 0.26 | 0.06 | 0.11 | 0.01 | 0.01 | 0.04 | 0.03 | 2.68 | 1.16 | 1.48 | 0.05 | 0.09 | 0.03 |
| 270 | 0.06 | 0.68 | 0.14 | 0.35 | 0.24 | 0.05 | 0.10 | 0.01 | 0.01 | 0.03 | 0.02 | 2.70 | 1.21 | 1.46 | 0.06 | 0.09 | 0.04 |
| 280 | 0.08 | 0.68 | 0.12 | 0.35 | 0.22 | 0.05 | 0.09 | 0.01 | 0.00 | 0.03 | 0.02 | 2.70 | 1.25 | 1.43 | 0.06 | 0.09 | 0.05 |
| 290 | 0.10 | 0.68 | 0.10 | 0.36 | 0.21 | 0.04 | 0.08 | 0.01 | 0.00 | 0.02 | 0.01 | 2.68 | 1.29 | 1.40 | 0.06 | 0.10 | 0.06 |
| 300 | 0.13 | 0.67 | 0.08 | 0.37 | 0.19 | 0.04 | 0.07 | 0.02 | 0.00 | 0.02 | 0.01 | 2.65 | 1.33 | 1.36 | 0.07 | 0.10 | 0.07 |
| 310 | 0.16 | 0.67 | 0.08 | 0.37 | 0.18 | 0.04 | 0.06 | 0.02 | 0.00 | 0.02 | 0.01 | 2.61 | 1.35 | 1.31 | 0.08 | 0.10 | 0.08 |
| 320 | 0.19 | 0.66 | 0.08 | 0.37 | 0.16 | 0.03 | 0.05 | 0.02 | 0.00 | 0.02 | 0.01 | 2.55 | 1.37 | 1.25 | 0.08 | 0.11 | 0.09 |
| 330 | 0.22 | 0.64 | 0.09 | 0.37 | 0.14 | 0.03 | 0.04 | 0.03 | 0.00 | 0.02 | 0.01 | 2.48 | 1.39 | 1.19 | 0.09 | 0.11 | 0.10 |
| 340 | 0.26 | 0.63 | 0.10 | 0.37 | 0.12 | 0.02 | 0.04 | 0.04 | 0.00 | 0.02 | 0.01 | 2.40 | 1.40 | 1.13 | 0.09 | 0.11 | 0.11 |
| 350 | 0.30 | 0.61 | 0.12 | 0.36 | 0.10 | 0.02 | 0.03 | 0.04 | 0.00 | 0.02 | 0.01 | 2.31 | 1.40 | 1.06 | 0.10 | 0.12 | 0.12 |
| 360 | 0.33 | 0.59 | 0.14 | 0.36 | 0.09 | 0.01 | 0.03 | 0.05 | 0.00 | 0.02 | 0.02 | 2.20 | 1.39 | 0.98 | 0.11 | 0.12 | 0.13 |
| 370 | 0.37 | 0.56 | 0.16 | 0.35 | 0.08 | 0.01 | 0.02 | 0.06 | 0.01 | 0.03 | 0.02 | 2.09 | 1.37 | 0.91 | 0.11 | 0.12 | 0.14 |
| 380 | 0.41 | 0.54 | 0.20 | 0.34 | 0.06 | 0.01 | 0.02 | 0.07 | 0.01 | 0.03 | 0.03 | 1.96 | 1.35 | 0.83 | 0.12 | 0.13 | 0.15 |
| 390 | 0.45 | 0.51 | 0.23 | 0.33 | 0.05 | 0.01 | 0.02 | 0.08 | 0.01 | 0.04 | 0.03 | 1.83 | 1.32 | 0.75 | 0.12 | 0.13 | 0.16 |
| 400 | 0.49 | 0.49 | 0.27 | 0.32 | 0.04 | 0.01 | 0.02 | 0.08 | 0.02 | 0.04 | 0.04 | 1.70 | 1.29 | 0.67 | 0.13 | 0.13 | 0.17 |
| 410 | 0.53 | 0.46 | 0.32 | 0.31 | 0.03 | 0.01 | 0.01 | 0.09 | 0.02 | 0.05 | 0.05 | 1.56 | 1.25 | 0.59 | 0.13 | 0.13 | 0.18 |
| 420 | 0.57 | 0.43 | 0.36 | 0.29 | 0.03 | 0.01 | 0.01 | 0.10 | 0.03 | 0.06 | 0.06 | 1.42 | 1.21 | 0.52 | 0.14 | 0.14 | 0.18 |
| 430 | 0.60 | 0.40 | 0.41 | 0.28 | 0.02 | 0.01 | 0.02 | 0.11 | 0.03 | 0.06 | 0.06 | 1.28 | 1.16 | 0.44 | 0.14 | 0.14 | 0.19 |
| 440 | 0.63 | 0.37 | 0.46 | 0.26 | 0.02 | 0.01 | 0.02 | 0.12 | 0.04 | 0.07 | 0.07 | 1.14 | 1.10 | 0.37 | 0.15 | 0.14 | 0.19 |
| 450 | 0.66 | 0.34 | 0.52 | 0.24 | 0.02 | 0.01 | 0.02 | 0.13 | 0.04 | 0.08 | 0.08 | 1.00 | 1.05 | 0.31 | 0.15 | 0.14 | 0.19 |
| 460 | 0.69 | 0.31 | 0.57 | 0.22 | 0.02 | 0.01 | 0.02 | 0.14 | 0.05 | 0.09 | 0.09 | 0.87 | 0.98 | 0.25 | 0.16 | 0.14 | 0.20 |
| 470 | 0.71 | 0.29 | 0.63 | 0.21 | 0.03 | 0.02 | 0.03 | 0.15 | 0.05 | 0.10 | 0.10 | 0.74 | 0.92 | 0.19 | 0.16 | 0.14 | 0.20 |
| 480 | 0.73 | 0.26 | 0.68 | 0.19 | 0.03 | 0.02 | 0.03 | 0.16 | 0.06 | 0.10 | 0.11 | 0.61 | 0.85 | 0.14 | 0.16 | 0.14 | 0.20 |
| 490 | 0.75 | 0.24 | 0.74 | 0.17 | 0.04 | 0.02 | 0.04 | 0.16 | 0.07 | 0.11 | 0.12 | 0.50 | 0.78 | 0.10 | 0.16 | 0.13 | 0.20 |
| 500 | 0.76 | 0.21 | 0.79 | 0.15 | 0.05 | 0.02 | 0.04 | 0.17 | 0.07 | 0.12 | 0.13 | 0.39 | 0.72 | 0.07 | 0.16 | 0.13 | 0.20 |
| 510 | 0.77 | 0.19 | 0.84 | 0.14 | 0.06 | 0.03 | 0.05 | 0.18 | 0.07 | 0.13 | 0.14 | 0.30 | 0.65 | 0.04 | 0.16 | 0.13 | 0.20 |
| 520 | 0.78 | 0.17 | 0.88 | 0.12 | 0.08 | 0.03 | 0.06 | 0.18 | 0.08 | 0.14 | 0.14 | 0.22 | 0.58 | 0.02 | 0.16 | 0.13 | 0.19 |
| 530 | 0.78 | 0.16 | 0.93 | 0.10 | 0.09 | 0.03 | 0.07 | 0.18 | 0.08 | 0.14 | 0.15 | 0.15 | 0.52 | 0.00 | 0.16 | 0.12 | 0.19 |
| 540 | 0.78 | 0.14 | 0.97 | 0.09 | 0.10 | 0.04 | 0.08 | 0.19 | 0.08 | 0.15 | 0.16 | 0.09 | 0.45 | 0.00 | 0.16 | 0.12 | 0.18 |
| 550 | 0.77 | 0.13 | 1.00 | 0.08 | 0.12 | 0.04 | 0.09 | 0.19 | 0.09 | 0.16 | 0.17 | 0.05 | 0.40 | 0.00 | 0.15 | 0.12 | 0.18 |
| 560 | 0.76 | 0.13 | 1.04 | 0.07 | 0.14 | 0.04 | 0.09 | 0.19 | 0.09 | 0.16 | 0.17 | 0.02 | 0.34 | 0.02 | 0.15 | 0.11 | 0.17 |
| 570 | 0.74 | 0.12 | 1.06 | 0.06 | 0.16 | 0.05 | 0.10 | 0.19 | 0.09 | 0.17 | 0.18 | 0.00 | 0.29 | 0.04 | 0.15 | 0.11 | 0.16 |
| 580 | 0.72 | 0.12 | 1.08 | 0.05 | 0.18 | 0.05 | 0.11 | 0.19 | 0.10 | 0.17 | 0.18 | 0.00 | 0.25 | 0.07 | 0.14 | 0.11 | 0.15 |
| 590 | 0.70 | 0.12 | 1.10 | 0.04 | 0.19 | 0.06 | 0.12 | 0.19 | 0.10 | 0.18 | 0.19 | 0.02 | 0.21 | 0.10 | 0.14 | 0.10 | 0.14 |
| 600 | 0.67 | 0.13 | 1.11 | 0.03 | 0.21 | 0.06 | 0.13 | 0.18 | 0.10 | 0.18 | 0.19 | 0.05 | 0.17 | 0.14 | 0.13 | 0.10 | 0.13 |

| TABLE XVII a. | | | | | TABLE XVII b. | | | |
|---------------|---------|---------|-------|---------|---------------|-------|----------------|---------------|
| Year. | (v.s.1) | (v.c.1) | Year. | (v.s.1) | (v.c.1) | Year. | (v.s.1) | (v.c.1) |
| | " | " | | " | " | | " | " |
| 1800 | -240.33 | -162.11 | 1850 | -183.57 | -189.69 | 1500 | -560.44 | -96.24 |
| 1801 | 239.19 | 162.61 | 1851 | 182.44 | 190.29 | 1510 | 550.85 + 9.59 | 95.81 + 0.43 |
| 1802 | 238.06 | 163.12 | 1852 | 181.30 | 190.89 | 1520 | 541.17 9.68 | 95.54 0.27 |
| 1803 | 236.92 | 163.63 | 1853 | 180.17 | 191.50 | 1530 | 531.40 9.77 | 95.44 + 0.10 |
| 1804 | 235.79 | 164.14 | 1854 | 179.04 | 192.10 | 1540 | 521.54 9.86 | 95.51 - 0.07 |
| 1805 | -234.65 | -164.65 | 1855 | -177.91 | -192.71 | 1550 | -511.59 9.95 | -95.75 0.24 |
| 1806 | 233.51 | 165.16 | 1856 | 176.78 | 193.32 | 1560 | 501.55 + 10.04 | 96.16 - 0.41 |
| 1807 | 232.38 | 165.68 | 1857 | 175.65 | 193.93 | 1570 | 491.42 10.13 | 96.75 0.59 |
| 1808 | 231.24 | 166.20 | 1858 | 174.51 | 194.54 | 1580 | 481.20 10.22 | 97.52 0.77 |
| 1809 | 230.11 | 166.72 | 1859 | 173.38 | 195.15 | 1590 | 470.90 10.30 | 98.47 0.95 |
| 1810 | -228.97 | -167.24 | 1860 | -172.25 | -195.77 | 1600 | -460.52 10.38 | -99.60 1.13 |
| 1811 | 227.83 | 167.76 | 1861 | 171.12 | 196.39 | 1610 | 450.06 + 10.46 | 100.93 - 1.33 |
| 1812 | 226.70 | 168.29 | 1862 | 169.99 | 197.01 | 1620 | 439.53 10.53 | 102.44 1.51 |
| 1813 | 225.56 | 168.82 | 1863 | 168.86 | 197.63 | 1630 | 428.92 10.61 | 104.14 1.70 |
| 1814 | 224.43 | 169.35 | 1864 | 167.72 | 198.25 | 1640 | 418.24 10.68 | 106.02 1.88 |
| 1815 | -223.29 | -169.88 | 1865 | -166.59 | -198.88 | 1650 | -407.49 10.75 | -108.09 2.07 |
| 1816 | 222.15 | 170.41 | 1866 | 165.46 | 199.51 | 1660 | 396.67 + 10.82 | 110.35 - 2.26 |
| 1817 | 221.02 | 170.95 | 1867 | 164.33 | 200.14 | 1670 | 385.79 10.88 | 112.80 2.45 |
| 1818 | 219.88 | 171.49 | 1868 | 163.20 | 200.77 | 1680 | 374.86 10.93 | 115.45 2.65 |
| 1819 | 218.75 | 172.03 | 1869 | 162.07 | 201.40 | 1690 | 363.87 10.99 | 118.28 2.83 |
| 1820 | -217.61 | -172.57 | 1870 | -160.94 | -202.04 | 1700 | -352.83 11.04 | -121.30 3.02 |
| 1821 | 216.47 | 173.11 | 1871 | 159.81 | 202.68 | 1710 | 341.74 + 11.09 | 124.52 - 3.22 |
| 1822 | 215.34 | 173.66 | 1872 | 158.68 | 203.32 | 1720 | 330.61 11.13 | 127.93 3.41 |
| 1823 | 214.20 | 174.21 | 1873 | 157.55 | 203.96 | 1730 | 319.43 11.18 | 131.53 3.60 |
| 1824 | 213.07 | 174.76 | 1874 | 156.42 | 204.60 | 1740 | 308.22 11.21 | 135.32 3.79 |
| 1825 | -211.93 | -175.31 | 1875 | -155.29 | -205.24 | 1750 | -296.97 11.25 | -139.30 3.98 |
| 1826 | 210.80 | 175.86 | 1876 | 154.16 | 205.89 | 1760 | 285.69 + 11.28 | 143.48 - 4.18 |
| 1827 | 209.66 | 176.42 | 1877 | 153.03 | 206.54 | 1770 | 274.38 11.31 | 147.85 4.37 |
| 1828 | 208.53 | 176.97 | 1878 | 151.91 | 207.19 | 1780 | 263.04 11.34 | 152.42 4.57 |
| 1829 | 207.39 | 177.53 | 1879 | 150.78 | 207.84 | 1790 | 251.69 11.35 | 157.17 4.75 |
| 1830 | -206.26 | -178.09 | 1880 | -149.65 | -208.49 | 1800 | -240.33 11.36 | -162.11 4.94 |
| 1831 | 205.13 | 178.65 | 1881 | 148.52 | 209.15 | 1810 | 228.97 + 11.36 | 167.24 - 5.13 |
| 1832 | 203.99 | 179.22 | 1882 | 147.39 | 209.81 | 1820 | 217.61 11.36 | 172.57 5.33 |
| 1833 | 202.86 | 179.78 | 1883 | 146.27 | 210.47 | 1830 | 206.26 11.35 | 178.09 5.52 |
| 1834 | 201.72 | 180.35 | 1884 | 145.14 | 211.13 | 1840 | 194.91 11.35 | 183.80 5.71 |
| 1835 | -200.59 | -180.92 | 1885 | -144.01 | -211.79 | 1850 | -183.57 11.34 | -189.69 5.89 |
| 1836 | 199.45 | 181.49 | 1886 | 142.89 | 212.45 | 1860 | 172.25 + 11.32 | 195.77 - 6.08 |
| 1837 | 198.32 | 182.06 | 1887 | 141.76 | 213.12 | 1870 | 160.94 11.31 | 202.04 6.27 |
| 1838 | 197.18 | 182.64 | 1888 | 140.64 | 213.79 | 1880 | 149.65 11.29 | 208.49 6.45 |
| 1839 | 196.05 | 183.22 | 1889 | 139.51 | 214.46 | 1890 | 138.39 11.26 | 215.13 6.64 |
| 1840 | -194.91 | -183.80 | 1890 | -138.39 | -215.13 | 1900 | -127.16 11.23 | -221.95 6.82 |
| 1841 | 193.78 | 184.38 | 1891 | 137.27 | 215.80 | 1910 | 115.98 + 11.18 | 228.94 - 6.99 |
| 1842 | 192.64 | 184.96 | 1892 | 136.14 | 216.48 | 1920 | 104.84 11.14 | 236.11 7.17 |
| 1843 | 191.51 | 185.55 | 1893 | 135.02 | 217.17 | 1930 | 93.75 11.09 | 243.46 7.35 |
| 1844 | 190.37 | 186.13 | 1894 | 133.90 | 217.84 | 1940 | 82.72 11.03 | 251.00 7.54 |
| 1845 | -189.24 | -186.72 | 1895 | -132.77 | -218.52 | 1950 | -71.74 10.98 | -258.72 7.72 |
| 1846 | 188.10 | 187.31 | 1896 | 131.65 | 219.20 | 1960 | 60.82 + 10.92 | 266.61 - 7.89 |
| 1847 | 186.97 | 187.90 | 1897 | 130.53 | 219.88 | 1970 | 49.97 10.85 | 274.68 8.07 |
| 1848 | 185.84 | 188.50 | 1898 | 129.40 | 220.57 | 1980 | 39.19 10.78 | 282.93 8.25 |
| 1849 | 184.70 | 189.09 | 1899 | 128.28 | 221.26 | 1990 | 28.49 10.70 | 291.35 8.42 |
| 1850 | -183.57 | -189.69 | 1900 | -127.16 | -221.95 | 2000 | -17.88 10.61 | -299.95 8.60 |

NOTE.—The values of (v.s.1) and (v.c.1) must be taken from only one of the two tables XVII a and XVII b.

TABLE XVIIb.—*Concluded.*

| Year. | (v.s.2) | (v.c.2) | (v.s.3) | (v.c.3) | (p.c.0) | (p.s.1) | (p.c.1) | (p.s.2) | (p.c.2) | (p.s.3) | (p.c.3) |
|-------|--------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| " | " | " | " | " | " | " | " | " | " | " | " |
| 1500 | -152.85 | -124.83 | -10.39 | -6.70 | +1379 | -891 | +2361 | -382 | -158 | 96 | 80 |
| 1510 | 152.43 +0.42 | 124.94 -0.11 | 10.35 | 6.70 | 1369 | 885 | 2258 | 382 | 165 | 96 | 80 |
| 1520 | 152.00 0.43 | 125.06 0.12 | 10.31 | 6.70 | 1359 | 881 | 2154 | 382 | 172 | 96 | 81 |
| 1530 | 151.57 0.43 | 125.19 0.13 | 10.28 | 6.69 | 1349 | 878 | 2047 | 381 | 179 | 96 | 81 |
| 1540 | 151.13 0.44 | 125.34 0.15 | 10.24 | 6.69 | 1338 | 877 | 1944 | 381 | 187 | 96 | 82 |
| 1550 | -150.69 | -125.50 | -10.20 | -6.69 | +1328 | -878 | +1838 | -381 | -195 | 96 | 82 |
| 1560 | 150.24 +0.45 | 125.67 -0.17 | 10.16 | 6.69 | 1318 | 881 | 1731 | 381 | 203 | 96 | 83 |
| 1570 | 149.78 0.46 | 125.85 0.18 | 10.13 | 6.69 | 1307 | 886 | 1623 | 381 | 210 | 96 | 83 |
| 1580 | 149.32 0.46 | 126.04 0.19 | 10.09 | 6.69 | 1297 | 893 | 1513 | 382 | 218 | 96 | 84 |
| 1590 | 148.86 0.46 | 126.24 0.20 | 10.05 | 6.70 | 1286 | 901 | 1403 | 382 | 226 | 96 | 84 |
| 1600 | -148.39 | -126.44 | -10.02 | -6.70 | +1275 | -911 | +1292 | -383 | -234 | 96 | 85 |
| 1610 | 147.92 +0.47 | 126.66 -0.22 | 9.98 | 6.70 | 1264 | 923 | 1180 | 384 | 242 | 96 | 86 |
| 1620 | 147.44 0.48 | 126.89 0.23 | 9.94 | 6.71 | 1253 | 937 | 1067 | 385 | 250 | 96 | 86 |
| 1630 | 146.96 0.48 | 127.13 0.24 | 9.91 | 6.71 | 1241 | 954 | 954 | 386 | 258 | 96 | 87 |
| 1640 | 146.48 0.48 | 127.38 0.25 | 9.87 | 6.72 | 1230 | 972 | 840 | 388 | 266 | 96 | 87 |
| 1650 | -145.99 | -127.64 | -9.83 | -6.73 | +1219 | -992 | +725 | -389 | -274 | 96 | 88 |
| 1660 | 145.50 +0.49 | 127.91 -0.27 | 9.79 | 6.74 | 1208 | 1014 | 610 | 391 | 282 | 96 | 89 |
| 1670 | 145.00 0.50 | 128.19 0.28 | 9.75 | 6.75 | 1197 | 1038 | 495 | 392 | 290 | 96 | 89 |
| 1680 | 144.50 0.50 | 128.49 0.30 | 9.71 | 6.76 | 1185 | 1064 | 379 | 394 | 298 | 96 | 90 |
| 1690 | 144.00 0.50 | 128.80 0.31 | 9.67 | 6.77 | 1174 | 1092 | 262 | 396 | 306 | 96 | 90 |
| 1700 | -143.49 | -129.12 | -9.63 | -6.78 | +1163 | -1123 | +145 | -398 | -314 | 97 | 91 |
| 1710 | 142.98 +0.51 | 129.45 -0.33 | 9.59 | 6.79 | 1152 | 1155 | 27 | 400 | 323 | 97 | 92 |
| 1720 | 142.47 0.51 | 129.79 0.34 | 9.55 | 6.80 | 1141 | 1189 | 92 | 403 | 331 | 97 | 92 |
| 1730 | 141.96 0.51 | 130.14 0.35 | 9.51 | 6.81 | 1130 | 1226 | 212 | 405 | 339 | 97 | 93 |
| 1740 | 141.44 0.52 | 130.51 0.37 | 9.47 | 6.83 | 1119 | 1264 | 333 | 408 | 348 | 98 | 93 |
| 1750 | -140.93 | -130.88 | -9.43 | -6.84 | +1108 | -1304 | -454 | -411 | -356 | 98 | 94 |
| 1760 | 140.41 +0.52 | 131.27 -0.39 | 9.39 | 6.85 | 1097 | 1346 | 575 | 414 | 365 | 98 | 95 |
| 1770 | 139.88 0.53 | 131.67 0.40 | 9.35 | 6.87 | 1086 | 1391 | 696 | 417 | 373 | 98 | 95 |
| 1780 | 139.36 0.52 | 132.08 0.41 | 9.31 | 6.89 | 1074 | 1437 | 817 | 420 | 382 | 99 | 96 |
| 1790 | 138.83 0.53 | 132.50 0.42 | 9.27 | 6.90 | 1063 | 1486 | 938 | 424 | 390 | 99 | 96 |
| 1800 | -138.31 | -132.93 | -9.23 | -6.92 | +1052 | -1536 | -1059 | -427 | -399 | 99 | 97 |
| 1810 | 137.79 +0.52 | 133.37 -0.44 | 9.19 | 6.93 | 1041 | 1588 | 1180 | 431 | 408 | 99 | 98 |
| 1820 | 137.26 0.53 | 133.82 0.45 | 9.14 | 6.95 | 1030 | 1642 | 1301 | 435 | 416 | 99 | 98 |
| 1830 | 136.74 0.52 | 134.28 0.46 | 9.10 | 6.96 | 1019 | 1699 | 1422 | 439 | 425 | 100 | 99 |
| 1840 | 136.21 0.53 | 134.76 0.48 | 9.06 | 6.98 | 1008 | 1758 | 1544 | 443 | 433 | 100 | 99 |
| 1850 | -135.69 | -135.25 | -9.02 | -7.00 | +997 | -1819 | -1665 | -447 | -442 | 100 | 100 |
| 1860 | 135.17 +0.52 | 135.75 -0.50 | 8.98 | 7.02 | 986 | 1881 | 1786 | 451 | 450 | 100 | 101 |
| 1870 | 134.65 0.52 | 136.26 0.51 | 8.94 | 7.04 | 975 | 1945 | 1906 | 456 | 459 | 101 | 101 |
| 1880 | 134.13 0.52 | 136.78 0.52 | 8.90 | 7.07 | 964 | 2012 | 2027 | 461 | 467 | 101 | 102 |
| 1890 | 133.61 0.52 | 137.31 0.53 | 8.86 | 7.09 | 953 | 2081 | 2147 | 466 | 476 | 101 | 102 |
| 1900 | -133.09 | -137.85 | -8.82 | -7.12 | +942 | -2151 | -2267 | -471 | -484 | 102 | 103 |
| 1910 | 132.57 +0.52 | 138.40 -0.55 | 8.78 | 7.15 | 931 | 2223 | 2386 | 476 | 492 | 102 | 104 |
| 1920 | 132.06 0.51 | 138.96 0.56 | 8.74 | 7.17 | 921 | 2297 | 2505 | 481 | 500 | 103 | 104 |
| 1930 | 131.55 0.51 | 139.53 0.57 | 8.70 | 7.20 | 910 | 2372 | 2623 | 486 | 509 | 103 | 105 |
| 1940 | 131.04 0.51 | 140.11 0.58 | 8.66 | 7.22 | 900 | 2449 | 2741 | 492 | 517 | 103 | 105 |
| 1950 | -130.54 | -140.70 | -8.62 | -7.25 | +889 | -2529 | -2859 | -497 | -525 | 104 | 106 |
| 1960 | 130.04 +0.50 | 141.31 -0.61 | 8.58 | 7.28 | 879 | 2611 | 2976 | 503 | 533 | 104 | 106 |
| 1970 | 129.55 0.49 | 141.93 0.62 | 8.54 | 7.31 | 868 | 2695 | 3092 | 509 | 541 | 105 | 107 |
| 1980 | 129.06 0.49 | 142.56 0.63 | 8.50 | 7.34 | 858 | 2780 | 3207 | 515 | 550 | 105 | 107 |
| 1990 | 128.57 0.49 | 143.20 0.64 | 8.46 | 7.37 | 848 | 2867 | 3321 | 521 | 558 | 106 | 108 |
| 2000 | -128.09 | -143.85 | -8.42 | -7.40 | +838 | -2956 | -3434 | -527 | -566 | 106 | 108 |

TABLE XVIII.—REDUCTION TO THE ECLIPTIC. ARGUMENT *u*.

| <i>u</i> | <i>u</i> | <i>R</i> | <i>u</i> | <i>u</i> | <i>R</i> | <i>u</i> | <i>u</i> | <i>R</i> | <i>u</i> | <i>u</i> | <i>R</i> |
|----------|----------|------------|----------|----------|------------|----------|----------|-------------|----------|----------|-------------|
| ° | ° | " | ° | ° | " | ° | ° | " | ° | ° | " |
| 0 | 180 | 10.00 | 45 | 225 | 0.63 | 90 | 270 | 10.00 | 135 | 315 | 19.37 |
| 1 | 181 | 9.67 -0.33 | 46 | 226 | 0.64 +0.01 | 91 | 271 | 10.33 +0.33 | 136 | 316 | 19.36 -0.01 |
| 2 | 182 | 9.35 0.32 | 47 | 227 | 0.65 0.01 | 92 | 272 | 10.65 0.32 | 137 | 317 | 19.35 0.01 |
| 3 | 183 | 9.02 0.33 | 48 | 228 | 0.68 0.03 | 93 | 273 | 10.98 0.33 | 138 | 318 | 19.32 0.03 |
| 4 | 184 | 8.70 0.32 | 49 | 229 | 0.72 0.04 | 94 | 274 | 11.30 0.32 | 139 | 319 | 19.28 0.04 |
| | | 0.33 | | | 0.05 | | | 0.33 | | | 0.05 |
| 5 | 185 | 8.37 | 50 | 230 | 0.77 +0.07 | 95 | 275 | 11.63 | 140 | 320 | 19.23 |
| 6 | 186 | 8.05 -0.32 | 51 | 231 | 0.84 +0.07 | 96 | 276 | 11.95 +0.32 | 141 | 321 | 19.16 -0.07 |
| 7 | 187 | 7.73 0.32 | 52 | 232 | 0.91 0.07 | 97 | 277 | 12.27 0.32 | 142 | 322 | 19.09 0.07 |
| 8 | 188 | 7.42 0.31 | 53 | 233 | 0.99 0.08 | 98 | 278 | 12.58 0.31 | 143 | 323 | 19.01 0.08 |
| 9 | 189 | 7.10 0.32 | 54 | 234 | 1.09 0.10 | 99 | 279 | 12.90 0.32 | 144 | 324 | 18.91 0.10 |
| | | 0.30 | | | 0.11 | | | 0.30 | | | 0.11 |
| 10 | 190 | 6.80 | 55 | 235 | 1.20 | 100 | 280 | 13.20 | 145 | 325 | 18.80 |
| 11 | 191 | 6.49 -0.31 | 56 | 236 | 1.31 +0.11 | 101 | 281 | 13.51 +0.31 | 146 | 326 | 18.69 -0.11 |
| 12 | 192 | 6.19 0.30 | 57 | 237 | 1.44 0.13 | 102 | 282 | 13.81 0.30 | 147 | 327 | 18.56 0.13 |
| 13 | 193 | 5.89 0.30 | 58 | 238 | 1.58 0.14 | 103 | 283 | 14.11 0.30 | 148 | 328 | 18.42 0.14 |
| 14 | 194 | 5.60 0.29 | 59 | 239 | 1.73 0.15 | 104 | 284 | 14.40 0.29 | 149 | 329 | 18.27 0.15 |
| | | 0.28 | | | 0.15 | | | 0.28 | | | 0.15 |
| 15 | 195 | 5.32 | 60 | 240 | 1.88 | 105 | 285 | 14.68 | 150 | 330 | 18.12 |
| 16 | 196 | 5.04 -0.28 | 61 | 241 | 2.05 +0.17 | 106 | 286 | 14.96 +0.28 | 151 | 331 | 17.95 -0.17 |
| 17 | 197 | 4.76 0.28 | 62 | 242 | 2.23 0.18 | 107 | 287 | 15.24 0.28 | 152 | 332 | 17.77 0.18 |
| 18 | 198 | 4.49 0.27 | 63 | 243 | 2.42 0.19 | 108 | 288 | 15.51 0.27 | 153 | 333 | 17.58 0.19 |
| 19 | 199 | 4.23 0.26 | 64 | 244 | 2.62 0.20 | 109 | 289 | 15.77 0.26 | 154 | 334 | 17.38 0.20 |
| | | 0.25 | | | 0.20 | | | 0.25 | | | 0.20 |
| 20 | 200 | 3.98 | 65 | 245 | 2.82 | 110 | 290 | 16.02 | 155 | 335 | 17.18 |
| 21 | 201 | 3.73 -0.25 | 66 | 246 | 3.04 +0.22 | 111 | 291 | 16.27 +0.25 | 156 | 336 | 16.96 -0.22 |
| 22 | 202 | 3.49 0.24 | 67 | 247 | 3.26 0.22 | 112 | 292 | 16.51 0.24 | 157 | 337 | 16.74 0.22 |
| 23 | 203 | 3.26 0.23 | 68 | 248 | 3.49 0.23 | 113 | 293 | 16.74 0.23 | 158 | 338 | 16.51 0.23 |
| 24 | 204 | 3.04 0.22 | 69 | 249 | 3.73 0.24 | 114 | 294 | 16.96 0.22 | 159 | 339 | 16.27 0.24 |
| | | 0.22 | | | 0.25 | | | 0.22 | | | 0.25 |
| 25 | 205 | 2.82 | 70 | 250 | 3.98 | 115 | 295 | 17.18 | 160 | 340 | 16.02 |
| 26 | 206 | 2.62 -0.20 | 71 | 251 | 4.23 +0.25 | 116 | 296 | 17.38 +0.20 | 161 | 341 | 15.77 -0.25 |
| 27 | 207 | 2.42 0.20 | 72 | 252 | 4.49 0.26 | 117 | 297 | 17.58 0.20 | 162 | 342 | 15.51 0.26 |
| 28 | 208 | 2.23 0.19 | 73 | 253 | 4.76 0.27 | 118 | 298 | 17.77 0.19 | 163 | 343 | 15.24 0.27 |
| 29 | 209 | 2.05 0.18 | 74 | 254 | 5.04 0.28 | 119 | 299 | 17.95 0.18 | 164 | 344 | 14.96 0.28 |
| | | 0.17 | | | 0.28 | | | 0.17 | | | 0.28 |
| 30 | 210 | 1.88 | 75 | 255 | 5.32 | 120 | 300 | 18.12 | 165 | 345 | 14.68 |
| 31 | 211 | 1.73 -0.15 | 76 | 256 | 5.60 +0.28 | 121 | 301 | 18.27 +0.15 | 166 | 346 | 14.40 -0.28 |
| 32 | 212 | 1.58 0.15 | 77 | 257 | 5.89 0.29 | 122 | 302 | 19.42 0.15 | 167 | 347 | 14.11 0.29 |
| 33 | 213 | 1.44 0.14 | 78 | 258 | 6.19 0.30 | 123 | 303 | 18.56 0.14 | 168 | 348 | 13.81 0.30 |
| 34 | 214 | 1.31 0.13 | 79 | 259 | 6.49 0.30 | 124 | 304 | 18.69 0.13 | 169 | 349 | 13.51 0.30 |
| | | 0.11 | | | 0.31 | | | 0.11 | | | 0.31 |
| 35 | 215 | 1.20 | 80 | 260 | 6.80 | 125 | 305 | 18.80 | 170 | 350 | 13.20 |
| 36 | 216 | 1.09 -0.11 | 81 | 261 | 7.10 +0.30 | 126 | 306 | 18.91 +0.11 | 171 | 351 | 12.90 -0.30 |
| 37 | 217 | 0.99 0.10 | 82 | 262 | 7.42 0.32 | 127 | 307 | 19.01 0.10 | 172 | 352 | 12.58 0.32 |
| 38 | 218 | 0.91 0.08 | 83 | 263 | 7.73 0.31 | 128 | 308 | 19.09 0.08 | 173 | 353 | 12.27 0.31 |
| 39 | 219 | 0.84 0.07 | 84 | 264 | 8.05 0.32 | 129 | 309 | 19.16 0.07 | 174 | 354 | 11.95 0.32 |
| | | 0.07 | | | 0.32 | | | 0.07 | | | 0.32 |
| 40 | 220 | 0.77 | 85 | 265 | 8.37 | 130 | 310 | 19.23 | 175 | 355 | 11.63 |
| 41 | 221 | 0.72 -0.05 | 86 | 266 | 8.70 +0.33 | 131 | 311 | 19.28 +0.05 | 176 | 356 | 11.30 -0.33 |
| 42 | 222 | 0.68 0.04 | 87 | 267 | 9.02 0.32 | 132 | 312 | 19.32 0.04 | 177 | 357 | 10.98 0.32 |
| 43 | 223 | 0.65 0.03 | 88 | 268 | 9.35 0.33 | 133 | 313 | 19.35 0.03 | 178 | 358 | 10.65 0.33 |
| 44 | 224 | 0.64 0.01 | 89 | 269 | 9.67 0.32 | 134 | 314 | 19.36 0.01 | 179 | 359 | 10.33 0.32 |
| | | 0.01 | | | 0.33 | | | 0.01 | | | 0.33 |
| 45 | 225 | 0.63 | 90 | 270 | 10.00 | 135 | 315 | 19.37 | 180 | 360 | 10.00 |

TABLE XIX, ARG. *u*.—Continued.

| <i>u</i> | β | | <i>u</i> | β | | <i>u</i> | β | | <i>u</i> | β | |
|----------|----------|------|----------|---------|----------|----------|---------|------|----------|---------|------|
| 210° | " | 330° | 220° | " | 320° | 230° | " | 310° | | | |
| 30° | 23 10.24 | 7.00 | 150° | 40° | 29 47.27 | 6.19 | 140° | 50° | 35 29.99 | 5.19 | 130° |
| 10' | 23 17.24 | 6.99 | 50' | 10' | 29 53.46 | 6.17 | 50' | 10' | 35 35.18 | 5.17 | 50' |
| 20 | 23 24.23 | 6.97 | 40 | 20 | 29 59.63 | 6.16 | 40 | 20 | 35 40.35 | 5.15 | 40 |
| 30 | 23 31.20 | 6.97 | 30 | 30 | 30 5.79 | 6.14 | 30 | 30 | 35 45.50 | 5.13 | 30 |
| 40 | 23 38.17 | 6.95 | 20 | 40 | 30 11.93 | 6.13 | 20 | 40 | 35 50.63 | 5.12 | 20 |
| 50 | 23 45.12 | 6.94 | 10 | 50 | 30 18.06 | 6.11 | 10 | 50 | 35 55.75 | 5.10 | 10 |
| 31° | 23 52.06 | 6.93 | 149° | 41° | 30 24.17 | 6.10 | 139° | 51° | 36 0.85 | 5.08 | 129° |
| 10' | 23 58.99 | 6.91 | 50' | 10' | 30 30.27 | 6.08 | 50' | 10' | 36 5.93 | 5.07 | 50' |
| 20 | 24 5.90 | 6.90 | 40 | 20 | 30 36.35 | 6.06 | 40 | 20 | 36 11.00 | 5.05 | 40 |
| 30 | 24 12.80 | 6.89 | 30 | 30 | 30 42.41 | 6.05 | 30 | 30 | 36 16.05 | 5.02 | 30 |
| 40 | 24 19.69 | 6.88 | 20 | 40 | 30 48.46 | 6.03 | 20 | 40 | 36 21.07 | 5.01 | 20 |
| 50 | 24 26.57 | 6.86 | 10 | 50 | 30 54.49 | 6.02 | 10 | 50 | 36 26.08 | 4.99 | 10 |
| 32° | 24 33.43 | 6.85 | 148° | 42° | 31 0.51 | 6.00 | 138° | 52° | 36 31.07 | 4.97 | 128° |
| 10' | 24 40.28 | 6.84 | 50' | 10' | 31 6.51 | 5.99 | 50' | 10' | 36 36.04 | 4.95 | 50' |
| 20 | 24 47.12 | 6.83 | 40 | 20 | 31 12.50 | 5.97 | 40 | 20 | 36 40.99 | 4.94 | 40 |
| 30 | 24 53.95 | 6.82 | 30 | 30 | 31 18.47 | 5.96 | 30 | 30 | 36 45.93 | 4.91 | 30 |
| 40 | 25 0.77 | 6.80 | 20 | 40 | 31 24.43 | 5.94 | 20 | 40 | 36 50.84 | 4.89 | 20 |
| 50 | 25 7.57 | 6.79 | 10 | 50 | 31 30.37 | 5.92 | 10 | 50 | 36 55.73 | 4.88 | 10 |
| 33° | 25 14.36 | 6.78 | 147° | 43° | 31 36.29 | 5.91 | 137° | 53° | 37 0.61 | 4.86 | 127° |
| 10' | 25 21.14 | 6.76 | 50' | 10' | 31 42.20 | 5.89 | 50' | 10' | 37 5.47 | 4.84 | 50' |
| 20 | 25 27.90 | 6.75 | 40 | 20 | 31 48.09 | 5.87 | 40 | 20 | 37 10.31 | 4.82 | 40 |
| 30 | 25 34.65 | 6.74 | 30 | 30 | 31 53.96 | 5.86 | 30 | 30 | 37 15.13 | 4.80 | 30 |
| 40 | 25 41.39 | 6.72 | 20 | 40 | 31 59.82 | 5.84 | 20 | 40 | 37 19.93 | 4.79 | 20 |
| 50 | 25 48.11 | 6.71 | 10 | 50 | 32 5.66 | 5.83 | 10 | 50 | 37 24.72 | 4.76 | 10 |
| 34° | 25 54.82 | 6.70 | 146° | 44° | 32 11.49 | 5.81 | 136° | 54° | 37 29.48 | 4.74 | 126° |
| 10' | 26 1.52 | 6.68 | 50' | 10' | 32 17.30 | 5.79 | 50' | 10' | 37 34.22 | 4.73 | 50' |
| 20 | 26 8.20 | 6.68 | 40 | 20 | 32 23.09 | 5.78 | 40 | 20 | 37 38.95 | 4.71 | 40 |
| 30 | 26 14.88 | 6.66 | 30 | 30 | 32 28.87 | 5.76 | 30 | 30 | 37 43.66 | 4.68 | 30 |
| 40 | 26 21.54 | 6.65 | 20 | 40 | 32 34.63 | 5.75 | 20 | 40 | 37 48.34 | 4.67 | 20 |
| 50 | 26 28.19 | 6.63 | 10 | 50 | 32 40.38 | 5.73 | 10 | 50 | 37 53.01 | 4.65 | 10 |
| 35° | 26 34.82 | 6.62 | 145° | 45° | 32 46.11 | 5.71 | 135° | 55° | 37 57.66 | 4.63 | 125° |
| 10' | 26 41.44 | 6.60 | 50' | 10' | 32 51.82 | 5.69 | 50' | 10' | 38 2.29 | 4.61 | 50' |
| 20 | 26 48.04 | 6.59 | 40 | 20 | 32 57.51 | 5.68 | 40 | 20 | 38 6.90 | 4.59 | 40 |
| 30 | 26 54.63 | 6.58 | 30 | 30 | 33 3.19 | 5.66 | 30 | 30 | 38 11.49 | 4.58 | 30 |
| 40 | 27 1.21 | 6.56 | 20 | 40 | 33 8.85 | 5.64 | 20 | 40 | 38 16.07 | 4.55 | 20 |
| 50 | 27 7.77 | 6.55 | 10 | 50 | 33 14.49 | 5.63 | 10 | 50 | 38 20.62 | 4.53 | 10 |
| 36° | 27 14.32 | 6.54 | 144° | 46° | 33 20.12 | 5.61 | 134° | 56° | 38 25.15 | 4.51 | 124° |
| 10' | 27 20.86 | 6.52 | 50' | 10' | 33 25.73 | 5.59 | 50' | 10' | 38 29.66 | 4.50 | 50' |
| 20 | 27 27.38 | 6.51 | 40 | 20 | 33 31.32 | 5.58 | 40 | 20 | 38 34.16 | 4.47 | 40 |
| 30 | 27 33.89 | 6.50 | 30 | 30 | 33 36.90 | 5.56 | 30 | 30 | 38 38.63 | 4.45 | 30 |
| 40 | 27 40.39 | 6.48 | 20 | 40 | 33 42.46 | 5.54 | 20 | 40 | 38 43.08 | 4.44 | 20 |
| 50 | 27 46.87 | 6.47 | 10 | 50 | 33 48.00 | 5.53 | 10 | 50 | 38 47.52 | 4.41 | 10 |
| 37° | 27 53.34 | 6.45 | 143° | 47° | 33 53.53 | 5.51 | 133° | 57° | 38 51.93 | 4.40 | 123° |
| 10' | 27 59.79 | 6.44 | 50' | 10' | 33 59.04 | 5.49 | 50' | 10' | 38 56.33 | 4.37 | 50' |
| 20 | 28 6.23 | 6.42 | 40 | 20 | 34 4.53 | 5.47 | 40 | 20 | 39 0.70 | 4.36 | 40 |
| 30 | 28 12.65 | 6.41 | 30 | 30 | 34 10.00 | 5.46 | 30 | 30 | 39 5.06 | 4.34 | 30 |
| 40 | 28 19.06 | 6.40 | 20 | 40 | 34 15.46 | 5.44 | 20 | 40 | 39 9.40 | 4.31 | 20 |
| 50 | 28 25.46 | 6.38 | 10 | 50 | 34 20.90 | 5.42 | 10 | 50 | 39 13.71 | 4.30 | 10 |
| 38° | 28 31.84 | 6.37 | 142° | 48° | 34 26.32 | 5.40 | 132° | 58° | 39 18.01 | 4.28 | 122° |
| 10' | 28 38.21 | 6.35 | 50' | 10' | 34 31.72 | 5.39 | 50' | 10' | 39 22.29 | 4.25 | 50' |
| 20 | 28 44.56 | 6.33 | 40 | 20 | 34 37.11 | 5.37 | 40 | 20 | 39 26.54 | 4.24 | 40 |
| 30 | 28 50.89 | 6.33 | 30 | 30 | 34 42.48 | 5.35 | 30 | 30 | 39 30.78 | 4.21 | 30 |
| 40 | 28 57.22 | 6.31 | 20 | 40 | 34 47.83 | 5.33 | 20 | 40 | 39 34.99 | 4.20 | 20 |
| 50 | 29 3.53 | 6.29 | 10 | 50 | 34 53.16 | 5.31 | 10 | 50 | 39 39.19 | 4.17 | 10 |
| 39° | 29 9.82 | 6.28 | 141° | 49° | 34 58.47 | 5.30 | 131° | 59° | 39 43.36 | 4.16 | 121° |
| 10' | 29 16.10 | 6.26 | 50' | 10' | 35 3.77 | 5.28 | 50' | 10' | 39 47.52 | 4.13 | 50' |
| 20 | 29 22.36 | 6.25 | 40 | 20 | 35 9.05 | 5.26 | 40 | 20 | 39 51.65 | 4.12 | 40 |
| 30 | 29 28.61 | 6.24 | 30 | 30 | 35 14.31 | 5.24 | 30 | 30 | 39 55.77 | 4.10 | 30 |
| 40 | 29 34.85 | 6.22 | 20 | 40 | 35 19.55 | 5.23 | 20 | 40 | 39 59.87 | 4.07 | 20 |
| 50 | 29 41.07 | 6.20 | 10 | 50 | 35 24.78 | 5.21 | 10 | 50 | 40 3.94 | 4.06 | 10 |
| 40° | 29 47.27 | | 140° | 50° | 35 29.99 | | 130° | 60° | 40 8.00 | | 120° |
| 220° | β | 320° | 230° | β | 310° | 240° | β | 300° | <i>u</i> | | |

TABLE XIX, ARG. u .—Continued.

| u | β | | u | β | | u | β | | u | β | |
|-------------|----------|------|-------------|---------|----------|-------------|----------|----------|-------------|---------|----------|
| 240° | | | 300° | | | 250° | | | 290° | | |
| | β | | | β | | | β | | | β | |
| 60° | 40 8.00 | | 120° | 50' | | 70° | 43 32.84 | | 110° | 80° | 45 38.29 |
| 10' | 40 12.03 | 4.03 | 50' | 10' | 43 35.60 | 2.76 | 10' | 43 35.60 | 50' | 10' | 45 39.68 |
| 20 | 40 16.04 | 4.01 | 40 | 20 | 43 38.33 | 2.73 | 20 | 43 38.33 | 40 | 20 | 45 41.05 |
| 30 | 40 20.04 | 4.00 | 30 | 30 | 43 41.04 | 2.71 | 30 | 43 41.04 | 30 | 30 | 45 42.40 |
| 40 | 40 24.01 | 3.97 | 20 | 40 | 43 43.73 | 2.69 | 40 | 43 43.73 | 20 | 40 | 45 43.73 |
| 50 | 40 27.96 | 3.95 | 10 | 50 | 43 46.40 | 2.67 | 50 | 43 46.40 | 10 | 50 | 45 45.03 |
| | | 3.93 | | | | | | | | | 1.30 |
| 61° | 40 31.89 | | 119° | 50' | | 71° | 43 49.04 | | 109° | 81° | 45 46.30 |
| 10' | 40 35.80 | 3.91 | 50' | 10' | 43 51.66 | 2.62 | 10' | 43 51.66 | 50' | 10' | 45 47.56 |
| 20 | 40 39.69 | 3.89 | 40 | 20 | 43 54.26 | 2.60 | 20 | 43 54.26 | 40 | 20 | 45 48.79 |
| 30 | 40 43.56 | 3.87 | 30 | 30 | 43 56.84 | 2.58 | 30 | 43 56.84 | 30 | 30 | 45 50.00 |
| 40 | 40 47.41 | 3.85 | 20 | 40 | 43 59.40 | 2.56 | 40 | 43 59.40 | 20 | 40 | 45 51.18 |
| 50 | 40 51.24 | 3.83 | 10 | 50 | 44 1.93 | 2.53 | 50 | 44 1.93 | 10 | 50 | 45 52.34 |
| | | 3.81 | | | | | | | | | 1.27 |
| 62° | 40 55.05 | | 118° | 50' | | 72° | 44 4.44 | | 108° | 82° | 45 53.48 |
| 10' | 40 58.84 | 3.79 | 50' | 10' | 44 6.93 | 2.49 | 10' | 44 6.93 | 50' | 10' | 45 54.59 |
| 20 | 41 2.60 | 3.76 | 40 | 20 | 44 9.39 | 2.46 | 20 | 44 9.39 | 40 | 20 | 45 55.68 |
| 30 | 41 6.35 | 3.75 | 30 | 30 | 44 11.83 | 2.44 | 30 | 44 11.83 | 30 | 30 | 45 56.75 |
| 40 | 41 10.07 | 3.72 | 20 | 40 | 44 14.25 | 2.42 | 40 | 44 14.25 | 20 | 40 | 45 57.79 |
| 50 | 41 13.78 | 3.71 | 10 | 50 | 44 16.65 | 2.40 | 50 | 44 16.65 | 10 | 50 | 45 58.81 |
| | | 3.68 | | | | | | | | | 1.14 |
| 63° | 41 17.46 | | 117° | 50' | | 73° | 44 19.03 | | 107° | 83° | 45 59.80 |
| 10' | 41 21.13 | 3.67 | 50' | 10' | 44 21.38 | 2.35 | 10' | 44 21.38 | 50' | 10' | 46 0.78 |
| 20 | 41 24.77 | 3.64 | 40 | 20 | 44 23.71 | 2.33 | 20 | 44 23.71 | 40 | 20 | 46 1.73 |
| 30 | 41 28.39 | 3.62 | 30 | 30 | 44 26.02 | 2.31 | 30 | 44 26.02 | 30 | 30 | 46 2.66 |
| 40 | 41 31.99 | 3.60 | 20 | 40 | 44 28.30 | 2.28 | 40 | 44 28.30 | 20 | 40 | 46 3.56 |
| 50 | 41 35.56 | 3.57 | 10 | 50 | 44 30.57 | 2.27 | 50 | 44 30.57 | 10 | 50 | 46 4.44 |
| | | 3.56 | | | | | | | | | 0.99 |
| 64° | 41 39.12 | | 116° | 50' | | 74° | 44 32.82 | | 106° | 84° | 46 5.30 |
| 10' | 41 42.65 | 3.53 | 50' | 10' | 44 35.04 | 2.22 | 10' | 44 35.04 | 50' | 10' | 46 6.14 |
| 20 | 41 46.17 | 3.52 | 40 | 20 | 44 37.23 | 2.19 | 20 | 44 37.23 | 40 | 20 | 46 6.95 |
| 30 | 41 49.66 | 3.49 | 30 | 30 | 44 39.40 | 2.17 | 30 | 44 39.40 | 30 | 30 | 46 7.74 |
| 40 | 41 53.13 | 3.47 | 20 | 40 | 44 41.55 | 2.15 | 40 | 44 41.55 | 20 | 40 | 46 8.50 |
| 50 | 41 56.58 | 3.45 | 10 | 50 | 44 43.68 | 2.13 | 50 | 44 43.68 | 10 | 50 | 46 9.24 |
| | | 3.43 | | | | | | | | | 0.86 |
| 65° | 42 0.01 | | 115° | 50' | | 75° | 44 45.79 | | 105° | 85° | 46 9.96 |
| 10' | 42 3.42 | 3.41 | 50' | 10' | 44 47.87 | 2.08 | 10' | 44 47.87 | 50' | 10' | 46 10.65 |
| 20 | 42 6.80 | 3.38 | 40 | 20 | 44 49.93 | 2.06 | 20 | 44 49.93 | 40 | 20 | 46 11.32 |
| 30 | 42 10.17 | 3.37 | 30 | 30 | 44 51.97 | 2.04 | 30 | 44 51.97 | 30 | 30 | 46 11.97 |
| 40 | 42 13.51 | 3.34 | 20 | 40 | 44 53.98 | 2.01 | 40 | 44 53.98 | 20 | 40 | 46 12.59 |
| 50 | 42 16.83 | 3.32 | 10 | 50 | 44 55.97 | 1.99 | 50 | 44 55.97 | 10 | 50 | 46 13.19 |
| | | 3.30 | | | | | | | | | 0.88 |
| 66° | 42 20.13 | | 114° | 50' | | 76° | 44 57.94 | | 104° | 86° | 46 13.76 |
| 10' | 42 23.41 | 3.28 | 50' | 10' | 44 59.89 | 1.95 | 10' | 44 59.89 | 50' | 10' | 46 14.31 |
| 20 | 42 26.67 | 3.26 | 40 | 20 | 45 1.81 | 1.92 | 20 | 45 1.81 | 40 | 20 | 46 14.84 |
| 30 | 42 29.91 | 3.24 | 30 | 30 | 45 3.71 | 1.90 | 30 | 45 3.71 | 30 | 30 | 46 15.35 |
| 40 | 42 33.12 | 3.21 | 20 | 40 | 45 5.59 | 1.88 | 40 | 45 5.59 | 20 | 40 | 46 15.83 |
| 50 | 42 36.32 | 3.20 | 10 | 50 | 45 7.44 | 1.85 | 50 | 45 7.44 | 10 | 50 | 46 16.29 |
| | | 3.17 | | | | | | | | | 0.81 |
| 67° | 42 39.49 | | 113° | 50' | | 77° | 45 9.27 | | 103° | 87° | 46 16.72 |
| 10' | 42 42.64 | 3.15 | 50' | 10' | 45 11.08 | 1.81 | 10' | 45 11.08 | 50' | 10' | 46 17.13 |
| 20 | 42 45.77 | 3.13 | 40 | 20 | 45 12.86 | 1.78 | 20 | 45 12.86 | 40 | 20 | 46 17.52 |
| 30 | 42 48.87 | 3.10 | 30 | 30 | 45 14.62 | 1.76 | 30 | 45 14.62 | 30 | 30 | 46 17.89 |
| 40 | 42 51.96 | 3.09 | 20 | 40 | 45 16.36 | 1.74 | 40 | 45 16.36 | 20 | 40 | 46 18.23 |
| 50 | 42 55.02 | 3.06 | 10 | 50 | 45 18.08 | 1.72 | 50 | 45 18.08 | 10 | 50 | 46 18.55 |
| | | 3.04 | | | | | | | | | 0.74 |
| 68° | 42 58.06 | | 112° | 50' | | 78° | 45 19.77 | | 102° | 88° | 46 18.84 |
| 10' | 43 1.08 | 3.02 | 50' | 10' | 45 21.44 | 1.67 | 10' | 45 21.44 | 50' | 10' | 46 19.11 |
| 20 | 43 4.08 | 3.00 | 40 | 20 | 45 23.09 | 1.65 | 20 | 45 23.09 | 40 | 20 | 46 19.36 |
| 30 | 43 7.05 | 2.97 | 30 | 30 | 45 24.71 | 1.62 | 30 | 45 24.71 | 30 | 30 | 46 19.59 |
| 40 | 43 10.01 | 2.96 | 20 | 40 | 45 26.31 | 1.60 | 40 | 45 26.31 | 20 | 40 | 46 19.79 |
| 50 | 43 12.94 | 2.93 | 10 | 50 | 45 27.89 | 1.58 | 50 | 45 27.89 | 10 | 50 | 46 19.97 |
| | | 2.91 | | | | | | | | | 0.84 |
| 69° | 43 15.85 | | 111° | 50' | | 79° | 45 29.45 | | 101° | 89° | 46 20.12 |
| 10' | 43 18.74 | 2.89 | 50' | 10' | 45 30.98 | 1.53 | 10' | 45 30.98 | 50' | 10' | 46 20.25 |
| 20 | 43 21.60 | 2.86 | 40 | 20 | 45 32.49 | 1.51 | 20 | 45 32.49 | 40 | 20 | 46 20.36 |
| 30 | 43 24.45 | 2.85 | 30 | 30 | 45 33.97 | 1.48 | 30 | 45 33.97 | 30 | 30 | 46 20.44 |
| 40 | 43 27.27 | 2.82 | 20 | 40 | 45 35.43 | 1.46 | 40 | 45 35.43 | 20 | 40 | 46 20.50 |
| 50 | 43 30.06 | 2.79 | 10 | 50 | 45 36.87 | 1.44 | 50 | 45 36.87 | 10 | 50 | 46 20.53 |
| | | 2.78 | | | | | | | | | 0.88 |
| 70° | 43 32.84 | | 110° | 50' | | 80° | 45 38.29 | | 100° | 90° | 46 20.54 |
| | | | | | | | | | | | 0.95 |
| 250° | | | 290° | | | 260° | | | 270° | | |
| | β | | | β | | | β | | | β | |
| | | | u | | | u | | | u | | |

| TABLE | XX. | | XXI. | | | | | XXII. | | | | |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| ARG. | 1 | | 2 | | | | | 3 | | | | |
| | (b.s.1) | (b.c.1) | (b.c.0) | (b.s.1) | (b.c.1) | (b.s.2) | (b.c.2) | (b.c.0) | (b.s.1) | (b.c.1) | (b.s.2) | (b.c.2) |
| | " | " | " | " | " | " | " | " | " | " | " | " |
| 0 | 1.20 | 1.12 | 0.04 | 5.99 | 5.42 | 0.21 | 0.17 | 0.06 | 1.58 | 1.34 | 0.07 | 0.17 |
| 10 | 1.15 | 1.17 | 0.03 | 6.13 | 5.23 | 0.19 | 0.18 | 0.06 | 1.63 | 1.14 | 0.08 | 0.23 |
| 20 | 1.10 | 1.21 | 0.03 | 6.25 | 5.04 | 0.18 | 0.18 | 0.05 | 1.67 | 0.93 | 0.11 | 0.29 |
| 30 | 1.04 | 1.25 | 0.02 | 6.34 | 4.84 | 0.17 | 0.18 | 0.04 | 1.68 | 0.72 | 0.15 | 0.32 |
| 40 | 0.98 | 1.28 | 0.02 | 6.41 | 4.62 | 0.16 | 0.18 | 0.03 | 1.67 | 0.51 | 0.21 | 0.33 |
| 50 | 0.92 | 1.31 | 0.02 | 6.45 | 4.40 | 0.15 | 0.18 | 0.03 | 1.60 | 0.33 | 0.27 | 0.32 |
| 60 | 0.86 | 1.33 | 0.02 | 6.45 | 4.18 | 0.15 | 0.18 | 0.02 | 1.48 | 0.18 | 0.32 | 0.28 |
| 70 | 0.79 | 1.34 | 0.02 | 6.40 | 3.94 | 0.16 | 0.18 | 0.01 | 1.32 | 0.09 | 0.35 | 0.23 |
| 80 | 0.72 | 1.35 | 0.02 | 6.32 | 3.69 | 0.16 | 0.17 | 0.00 | 1.13 | 0.07 | 0.36 | 0.18 |
| 90 | 0.66 | 1.35 | 0.03 | 6.19 | 3.43 | 0.17 | 0.16 | 0.00 | 0.94 | 0.13 | 0.35 | 0.13 |
| 100 | 0.59 | 1.34 | 0.03 | 6.02 | 3.15 | 0.17 | 0.15 | 0.01 | 0.77 | 0.25 | 0.32 | 0.09 |
| 110 | 0.52 | 1.32 | 0.04 | 5.81 | 2.87 | 0.18 | 0.14 | 0.02 | 0.66 | 0.41 | 0.29 | 0.07 |
| 120 | 0.46 | 1.30 | 0.04 | 5.56 | 2.57 | 0.19 | 0.12 | 0.04 | 0.62 | 0.60 | 0.25 | 0.06 |
| 130 | 0.40 | 1.27 | 0.04 | 5.29 | 2.26 | 0.20 | 0.11 | 0.06 | 0.64 | 0.77 | 0.22 | 0.06 |
| 140 | 0.34 | 1.24 | 0.05 | 4.99 | 1.96 | 0.20 | 0.10 | 0.08 | 0.72 | 0.91 | 0.20 | 0.07 |
| 150 | 0.28 | 1.20 | 0.05 | 4.68 | 1.66 | 0.20 | 0.09 | 0.09 | 0.83 | 1.00 | 0.18 | 0.08 |
| 160 | 0.23 | 1.15 | 0.05 | 4.36 | 1.38 | 0.20 | 0.08 | 0.10 | 0.95 | 1.04 | 0.17 | 0.09 |
| 170 | 0.19 | 1.10 | 0.06 | 4.03 | 1.11 | 0.20 | 0.08 | 0.11 | 1.04 | 1.04 | 0.17 | 0.10 |
| 180 | 0.15 | 1.04 | 0.06 | 3.69 | 0.86 | 0.19 | 0.07 | 0.11 | 1.11 | 1.01 | 0.16 | 0.10 |
| 190 | 0.12 | 0.98 | 0.06 | 3.36 | 0.65 | 0.17 | 0.07 | 0.11 | 1.14 | 0.97 | 0.14 | 0.11 |
| 200 | 0.09 | 0.92 | 0.06 | 3.03 | 0.56 | 0.16 | 0.08 | 0.11 | 1.14 | 0.93 | 0.13 | 0.12 |
| 210 | 0.07 | 0.86 | 0.06 | 2.71 | 0.32 | 0.14 | 0.08 | 0.11 | 1.11 | 0.91 | 0.11 | 0.13 |
| 220 | 0.06 | 0.79 | 0.06 | 2.39 | 0.20 | 0.12 | 0.09 | 0.10 | 1.07 | 0.91 | 0.10 | 0.15 |
| 230 | 0.05 | 0.72 | 0.06 | 2.08 | 0.12 | 0.11 | 0.11 | 0.10 | 1.02 | 0.92 | 0.10 | 0.18 |
| 240 | 0.05 | 0.66 | 0.07 | 1.77 | 0.08 | 0.09 | 0.13 | 0.09 | 0.98 | 0.96 | 0.10 | 0.21 |
| 250 | 0.06 | 0.59 | 0.07 | 1.48 | 0.08 | 0.07 | 0.15 | 0.09 | 0.96 | 1.00 | 0.12 | 0.23 |
| 260 | 0.08 | 0.52 | 0.07 | 1.21 | 0.10 | 0.05 | 0.17 | 0.08 | 0.95 | 1.05 | 0.14 | 0.25 |
| 270 | 0.10 | 0.46 | 0.08 | 1.05 | 0.16 | 0.03 | 0.20 | 0.08 | 0.94 | 1.09 | 0.16 | 0.26 |
| 280 | 0.13 | 0.40 | 0.08 | 0.72 | 0.26 | 0.02 | 0.22 | 0.07 | 0.95 | 1.13 | 0.19 | 0.26 |
| 290 | 0.16 | 0.34 | 0.09 | 0.52 | 0.38 | 0.01 | 0.25 | 0.06 | 0.95 | 1.16 | 0.22 | 0.25 |
| 300 | 0.20 | 0.28 | 0.10 | 0.35 | 0.54 | 0.01 | 0.27 | 0.06 | 0.94 | 1.18 | 0.23 | 0.23 |
| 310 | 0.25 | 0.23 | 0.11 | 0.22 | 0.73 | 0.01 | 0.29 | 0.06 | 0.93 | 1.20 | 0.24 | 0.21 |
| 320 | 0.30 | 0.19 | 0.12 | 0.12 | 0.94 | 0.02 | 0.31 | 0.07 | 0.91 | 1.22 | 0.23 | 0.19 |
| 330 | 0.36 | 0.15 | 0.13 | 0.06 | 1.18 | 0.03 | 0.33 | 0.08 | 0.88 | 1.25 | 0.22 | 0.18 |
| 340 | 0.42 | 0.12 | 0.14 | 0.04 | 1.44 | 0.05 | 0.35 | 0.09 | 0.86 | 1.28 | 0.20 | 0.17 |
| 350 | 0.48 | 0.09 | 0.15 | 0.06 | 1.72 | 0.07 | 0.36 | 0.09 | 0.84 | 1.34 | 0.18 | 0.17 |
| 360 | 0.54 | 0.07 | 0.16 | 0.12 | 2.01 | 0.10 | 0.37 | 0.10 | 0.85 | 1.40 | 0.16 | 0.18 |
| 370 | 0.61 | 0.06 | 0.17 | 0.21 | 2.31 | 0.13 | 0.37 | 0.11 | 0.88 | 1.46 | 0.15 | 0.19 |
| 380 | 0.68 | 0.05 | 0.18 | 0.34 | 2.62 | 0.16 | 0.37 | 0.12 | 0.92 | 1.51 | 0.14 | 0.20 |
| 390 | 0.74 | 0.05 | 0.19 | 0.50 | 2.92 | 0.19 | 0.36 | 0.12 | 0.98 | 1.54 | 0.13 | 0.21 |
| 400 | 0.81 | 0.06 | 0.20 | 0.70 | 3.24 | 0.23 | 0.35 | 0.11 | 1.03 | 1.54 | 0.13 | 0.22 |
| 410 | 0.88 | 0.08 | 0.20 | 0.92 | 3.54 | 0.26 | 0.34 | 0.10 | 1.07 | 1.50 | 0.12 | 0.23 |
| 420 | 0.94 | 0.10 | 0.21 | 1.17 | 3.84 | 0.30 | 0.33 | 0.08 | 1.06 | 1.44 | 0.12 | 0.25 |
| 430 | 1.00 | 0.13 | 0.21 | 1.45 | 4.14 | 0.33 | 0.31 | 0.06 | 1.01 | 1.37 | 0.12 | 0.27 |
| 440 | 1.06 | 0.16 | 0.21 | 1.75 | 4.42 | 0.36 | 0.29 | 0.04 | 0.91 | 1.30 | 0.12 | 0.29 |
| 450 | 1.12 | 0.20 | 0.21 | 2.06 | 4.70 | 0.38 | 0.27 | 0.03 | 0.77 | 1.28 | 0.12 | 0.32 |
| 460 | 1.17 | 0.25 | 0.21 | 2.39 | 4.96 | 0.40 | 0.25 | 0.02 | 0.61 | 1.31 | 0.14 | 0.35 |
| 470 | 1.21 | 0.30 | 0.20 | 2.72 | 5.20 | 0.41 | 0.23 | 0.01 | 0.47 | 1.40 | 0.17 | 0.38 |
| 480 | 1.25 | 0.36 | 0.19 | 3.06 | 5.42 | 0.42 | 0.21 | 0.01 | 0.36 | 1.56 | 0.21 | 0.39 |
| 490 | 1.28 | 0.42 | 0.18 | 3.39 | 5.62 | 0.42 | 0.20 | 0.01 | 0.32 | 1.76 | 0.27 | 0.39 |
| 500 | 1.31 | 0.48 | 0.17 | 3.71 | 5.79 | 0.42 | 0.18 | 0.01 | 0.36 | 1.96 | 0.32 | 0.37 |
| 510 | 1.33 | 0.54 | 0.16 | 4.02 | 5.92 | 0.41 | 0.17 | 0.01 | 0.46 | 2.15 | 0.37 | 0.33 |
| 520 | 1.34 | 0.61 | 0.14 | 4.31 | 6.01 | 0.39 | 0.16 | 0.02 | 0.62 | 2.21 | 0.40 | 0.27 |
| 530 | 1.35 | 0.68 | 0.13 | 4.58 | 6.06 | 0.38 | 0.16 | 0.02 | 0.81 | 2.35 | 0.40 | 0.20 |
| 540 | 1.35 | 0.74 | 0.11 | 4.84 | 6.08 | 0.35 | 0.15 | 0.03 | 1.00 | 2.34 | 0.38 | 0.14 |
| 550 | 1.34 | 0.81 | 0.10 | 5.07 | 6.04 | 0.33 | 0.15 | 0.03 | 1.18 | 2.26 | 0.33 | 0.08 |
| 560 | 1.32 | 0.88 | 0.09 | 5.28 | 5.98 | 0.30 | 0.15 | 0.04 | 1.31 | 2.11 | 0.27 | 0.05 |
| 570 | 1.30 | 0.94 | 0.07 | 5.48 | 5.88 | 0.28 | 0.16 | 0.04 | 1.41 | 1.94 | 0.20 | 0.04 |
| 580 | 1.27 | 1.00 | 0.06 | 5.67 | 5.75 | 0.26 | 0.16 | 0.05 | 1.48 | 1.74 | 0.14 | 0.07 |
| 590 | 1.24 | 1.06 | 0.05 | 5.84 | 5.59 | 0.23 | 0.17 | 0.06 | 1.53 | 1.54 | 0.10 | 0.11 |
| 600 | 1.20 | 1.12 | 0.04 | 5.99 | 5.42 | 0.21 | 0.17 | 0.06 | 1.58 | 1.34 | 0.07 | 0.17 |

THE ORBIT OF URANUS.

TABLE XXIII.

| Year. | (b.c.0) | (b.s.1) | (b.c.1) | (b.s.2) | (b.c.2) | Year. | (b.c.0) | (b.s.1) | (b.c.1) | (b.s.2) | (b.c.2) |
|-------|---------|---------|---------|---------|---------|-------|---------|---------|---------|---------|---------|
| | " | " | " | " | " | | " | " | " | " | " |
| 1300 | +0.66 | -3.85 | -12.74 | -0.34 | -0.97 | 1800 | +0.15 | -4.60 | -6.00 | -0.44 | -0.46 |
| 1310 | 0.65 | 3.81 | 12.65 | 0.34 | 0.96 | 1810 | 0.14 | 4.68 | 5.80 | 0.45 | 0.45 |
| 1320 | 0.64 | 3.78 | 12.56 | 0.34 | 0.95 | 1820 | 0.13 | 4.75 | 5.61 | 0.45 | 0.44 |
| 1330 | 0.62 | 3.74 | 12.46 | 0.34 | 0.94 | 1830 | 0.12 | 4.83 | 5.41 | 0.45 | 0.43 |
| 1340 | 0.61 | 3.70 | 12.37 | 0.34 | 0.93 | 1840 | 0.11 | 4.92 | 5.21 | 0.46 | 0.42 |
| 1350 | +0.60 | -3.66 | -12.27 | -0.34 | -0.92 | 1850 | +0.10 | -5.00 | -5.00 | -0.46 | -0.41 |
| 1360 | 0.59 | 3.63 | 12.17 | 0.34 | 0.90 | 1860 | 0.09 | 5.09 | 4.79 | 0.47 | 0.40 |
| 1370 | 0.58 | 3.60 | 12.07 | 0.34 | 0.89 | 1870 | 0.08 | 5.18 | 4.58 | 0.47 | 0.39 |
| 1380 | 0.57 | 3.57 | 11.97 | 0.34 | 0.88 | 1880 | 0.07 | 5.27 | 4.36 | 0.48 | 0.38 |
| 1390 | 0.56 | 3.54 | 11.86 | 0.34 | 0.87 | 1890 | 0.06 | 5.36 | 4.14 | 0.48 | 0.37 |
| 1400 | +0.55 | -3.51 | -11.76 | -0.35 | -0.86 | 1900 | +0.05 | -5.45 | -3.92 | -0.49 | -0.36 |
| 1410 | 0.54 | 3.48 | 11.65 | 0.35 | 0.85 | 1910 | 0.04 | 5.55 | 3.69 | 0.49 | 0.35 |
| 1420 | 0.53 | 3.46 | 11.54 | 0.35 | 0.84 | 1920 | 0.03 | 5.64 | 3.46 | 0.50 | 0.34 |
| 1430 | 0.52 | 3.44 | 11.43 | 0.35 | 0.83 | 1930 | 0.02 | 5.74 | 3.22 | 0.50 | 0.33 |
| 1440 | 0.51 | 3.43 | 11.32 | 0.35 | 0.82 | 1940 | +0.01 | 5.84 | 2.97 | 0.50 | 0.32 |
| 1450 | +0.50 | -3.42 | -11.21 | -0.35 | -0.80 | 1950 | 0.00 | -5.94 | -2.73 | -0.51 | -0.30 |
| 1460 | 0.49 | 3.41 | 11.10 | 0.35 | 0.79 | 1960 | -0.01 | 6.04 | 2.48 | 0.51 | 0.29 |
| 1470 | 0.48 | 3.40 | 10.98 | 0.36 | 0.78 | 1970 | 0.02 | 6.14 | 2.24 | 0.52 | 0.28 |
| 1480 | 0.46 | 3.40 | 10.86 | 0.36 | 0.77 | 1980 | 0.04 | 6.25 | 1.99 | 0.52 | 0.27 |
| 1490 | 0.45 | 3.40 | 10.74 | 0.36 | 0.76 | 1990 | 0.05 | 6.36 | 1.73 | 0.52 | 0.26 |
| 1500 | +0.44 | -3.40 | -10.62 | -0.36 | -0.75 | 2000 | -0.06 | -6.47 | -1.47 | -0.53 | -0.25 |
| 1510 | 0.43 | 3.40 | 10.50 | 0.36 | 0.74 | 2010 | 0.07 | 6.58 | 1.21 | 0.53 | 0.24 |
| 1520 | 0.42 | 3.41 | 10.38 | 0.36 | 0.73 | 2020 | 0.08 | 6.69 | 0.94 | 0.54 | 0.23 |
| 1530 | 0.41 | 3.42 | 10.26 | 0.37 | 0.72 | 2030 | 0.09 | 6.80 | 0.66 | 0.54 | 0.22 |
| 1540 | 0.40 | 3.43 | 10.14 | 0.37 | 0.71 | 2040 | 0.10 | 6.92 | 0.38 | 0.54 | 0.20 |
| 1550 | +0.39 | -3.45 | -10.01 | -0.37 | -0.70 | 2050 | -0.11 | -7.03 | -0.10 | -0.55 | -0.19 |
| 1560 | 0.38 | 3.47 | 9.88 | 0.37 | 0.70 | 2060 | 0.12 | 7.14 | +0.18 | 0.55 | 0.18 |
| 1570 | 0.37 | 3.49 | 9.75 | 0.37 | 0.69 | 2070 | 0.13 | 7.26 | 0.47 | 0.56 | 0.17 |
| 1580 | 0.36 | 3.51 | 9.61 | 0.37 | 0.68 | 2080 | 0.15 | 7.37 | 0.76 | 0.56 | 0.16 |
| 1590 | 0.35 | 3.53 | 9.47 | 0.38 | 0.67 | 2090 | 0.16 | 7.48 | 1.05 | 0.56 | 0.15 |
| 1600 | +0.34 | -3.56 | -9.33 | -0.38 | -0.66 | 2100 | -0.17 | -7.60 | +1.35 | -0.57 | -0.14 |
| 1610 | 0.33 | 3.59 | 9.19 | 0.38 | 0.65 | 2110 | 0.18 | 7.72 | 1.65 | 0.57 | 0.13 |
| 1620 | 0.32 | 3.62 | 9.05 | 0.39 | 0.64 | 2120 | 0.19 | 7.84 | 1.96 | 0.57 | 0.12 |
| 1630 | 0.32 | 3.65 | 8.90 | 0.39 | 0.63 | 2130 | 0.21 | 7.97 | 2.27 | 0.58 | 0.10 |
| 1640 | 0.31 | 3.69 | 8.75 | 0.39 | 0.62 | 2140 | 0.22 | 8.09 | 2.58 | 0.58 | 0.09 |
| 1650 | +0.30 | -3.73 | -8.60 | -0.39 | -0.61 | 2150 | -0.23 | -8.21 | +2.90 | -0.58 | -0.08 |
| 1660 | 0.29 | 3.77 | 8.44 | 0.40 | 0.60 | 2160 | 0.24 | 8.33 | 3.22 | 0.59 | 0.07 |
| 1670 | 0.28 | 3.81 | 8.29 | 0.40 | 0.59 | 2170 | 0.25 | 8.44 | 3.54 | 0.59 | 0.06 |
| 1680 | 0.27 | 3.85 | 8.13 | 0.40 | 0.58 | 2180 | 0.27 | 8.56 | 3.87 | 0.59 | 0.05 |
| 1690 | 0.26 | 3.90 | 7.96 | 0.41 | 0.57 | 2190 | 0.28 | 8.68 | 4.20 | 0.60 | 0.03 |
| 1700 | +0.25 | -3.95 | -7.79 | -0.41 | -0.56 | 2200 | -0.29 | -8.80 | +4.54 | -0.60 | -0.02 |
| 1710 | 0.24 | 4.00 | 7.63 | 0.41 | 0.55 | 2210 | 0.30 | 8.92 | 4.88 | 0.60 | -0.01 |
| 1720 | 0.23 | 4.06 | 7.46 | 0.42 | 0.54 | 2220 | 0.31 | 9.04 | 5.23 | 0.61 | 0.00 |
| 1730 | 0.22 | 4.12 | 7.29 | 0.42 | 0.53 | 2230 | 0.33 | 9.16 | 5.58 | 0.61 | +0.01 |
| 1740 | 0.21 | 4.18 | 7.12 | 0.42 | 0.52 | 2240 | 0.34 | 9.28 | 5.94 | 0.61 | 0.03 |
| 1750 | +0.20 | -4.25 | -6.95 | -0.43 | -0.51 | 2250 | -0.35 | -9.40 | +6.29 | -0.62 | +0.04 |
| 1760 | 0.19 | 4.32 | 6.77 | 0.43 | 0.50 | 2260 | 0.36 | 9.52 | 6.65 | 0.62 | 0.05 |
| 1770 | 0.18 | 4.38 | 6.58 | 0.43 | 0.49 | 2270 | 0.38 | 9.64 | 7.01 | 0.63 | 0.07 |
| 1780 | 0.17 | 4.45 | 6.39 | 0.44 | 0.48 | 2280 | 0.39 | 9.76 | 7.37 | 0.63 | 0.08 |
| 1790 | 0.16 | 4.53 | 6.20 | 0.44 | 0.47 | 2290 | 0.40 | 9.88 | 7.74 | 0.63 | 0.10 |
| 1800 | +0.15 | -4.60 | -6.00 | -0.44 | -0.46 | 2300 | -0.42 | -10.00 | +8.10 | -0.64 | +0.11 |

TABLE FOR FORMING THE PRODUCTS OF GIVEN NUMBERS BY THE SINE OR COSINE OF A GIVEN ANGLE.

THIS table is formed for the especial purpose of facilitating the formation of the products (*v.s.3*) $\sin 3g$, (*v.c.3*) $\cos 3g$, etc., (*p.s.1*) $\sin g$, (*p.c.1*) $\cos g$, for entire degrees of g . It is so arranged that the required products can be taken out at sight. Supposing the number to be given in seconds and decimal fractions of a second, we first seek the given angle at the top or bottom of the page, and then enter one of the first nine lines of the table with the fraction part of the second, interpolating for the hundredths. We then add the result mentally to the number corresponding to the entire seconds. The algebraic signs at the sides of the angles are those of the sines or cosines corresponding to the angle and to the column above or below. If the number does not exceed $3''$ we can enter the table as if it were ten times greater, and remove the decimal point one place to the left in the result.

For example, to find the value of

$$21''.67 \sin 280^\circ + 2''.25 \cos 280^\circ$$

we find the angle 280° at the bottom of a pair of columns, the right hand one being the sine column. Entering this column with 0.67 as the argument, we find 0.66. Entering with 2.1, we find 20.68, to which adding 0.66, we have $21''.34$ as the sine product. Entering the other column with 22.5, and moving the decimal point, we find $0''.39$ for the cosine product. Noticing the algebraic signs on each side of 280° , we find the result to be $-21''.34 + 0''.39 = -20''.95$.

| | + 1° +
+179 -
-181 -
-359 + | | + 2° +
+178 -
-182 -
-358 + | | + 3° +
+177 -
-183 -
-357 + | | + 4° +
+176 -
-184 -
-356 + | | + 5° +
+175 -
-185 -
-355 + | | |
|------|--|-------|---------------------------------------|-------|---------------------------------------|-------|---------------------------------------|-------|---------------------------------------|-------|------|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0.1 | 0.00 | 0.10 | 0.00 | 0.10 | 0.01 | 0.10 | 0.01 | 0.10 | 0.01 | 0.10 | 0.1 |
| 0.2 | 0.00 | 0.20 | 0.01 | 0.20 | 0.01 | 0.20 | 0.01 | 0.20 | 0.02 | 0.20 | 0.2 |
| 0.3 | 0.01 | 0.30 | 0.01 | 0.30 | 0.02 | 0.30 | 0.02 | 0.30 | 0.03 | 0.30 | 0.3 |
| 0.4 | 0.01 | 0.40 | 0.01 | 0.40 | 0.02 | 0.40 | 0.03 | 0.40 | 0.03 | 0.40 | 0.4 |
| 0.5 | 0.01 | 0.50 | 0.02 | 0.50 | 0.03 | 0.50 | 0.03 | 0.50 | 0.04 | 0.50 | 0.5 |
| 0.6 | 0.01 | 0.60 | 0.02 | 0.60 | 0.03 | 0.60 | 0.04 | 0.60 | 0.05 | 0.60 | 0.6 |
| 0.7 | 0.01 | 0.70 | 0.02 | 0.70 | 0.04 | 0.70 | 0.05 | 0.70 | 0.06 | 0.70 | 0.7 |
| 0.8 | 0.01 | 0.80 | 0.03 | 0.80 | 0.04 | 0.80 | 0.06 | 0.80 | 0.07 | 0.80 | 0.8 |
| 0.9 | 0.02 | 0.90 | 0.03 | 0.90 | 0.05 | 0.90 | 0.06 | 0.90 | 0.08 | 0.90 | 0.9 |
| 1.0 | 0.02 | 1.00 | 0.03 | 1.00 | 0.05 | 1.00 | 0.07 | 1.00 | 0.09 | 1.00 | 1.0 |
| 2.0 | 0.03 | 2.00 | 0.07 | 2.00 | 0.10 | 2.00 | 0.14 | 2.00 | 0.17 | 1.99 | 2.0 |
| 3.0 | 0.05 | 3.00 | 0.10 | 3.00 | 0.16 | 3.00 | 0.21 | 2.99 | 0.26 | 2.99 | 3.0 |
| 4.0 | 0.07 | 4.00 | 0.14 | 4.00 | 0.21 | 3.99 | 0.28 | 3.99 | 0.35 | 3.98 | 4.0 |
| 5.0 | 0.09 | 5.00 | 0.17 | 5.00 | 0.26 | 4.99 | 0.35 | 4.99 | 0.44 | 4.98 | 5.0 |
| 6.0 | 0.10 | 6.00 | 0.21 | 6.00 | 0.31 | 5.99 | 0.42 | 5.99 | 0.52 | 5.98 | 6.0 |
| 7.0 | 0.12 | 7.00 | 0.24 | 7.00 | 0.37 | 6.99 | 0.49 | 6.98 | 0.61 | 6.97 | 7.0 |
| 8.0 | 0.14 | 8.00 | 0.28 | 8.00 | 0.42 | 7.99 | 0.56 | 7.98 | 0.70 | 7.97 | 8.0 |
| 9.0 | 0.16 | 9.00 | 0.31 | 8.99 | 0.47 | 8.99 | 0.63 | 8.98 | 0.78 | 8.97 | 9.0 |
| 10.0 | 0.17 | 10.00 | 0.35 | 9.99 | 0.52 | 9.99 | 0.70 | 9.98 | 0.87 | 9.96 | 10.0 |
| 11.0 | 0.19 | 11.00 | 0.38 | 10.99 | 0.58 | 10.98 | 0.77 | 10.97 | 0.96 | 10.96 | 11.0 |
| 12.0 | 0.21 | 12.00 | 0.42 | 11.99 | 0.63 | 11.98 | 0.84 | 11.97 | 1.05 | 11.95 | 12.0 |
| 13.0 | 0.23 | 13.00 | 0.45 | 12.99 | 0.68 | 12.98 | 0.91 | 12.97 | 1.13 | 12.95 | 13.0 |
| 14.0 | 0.24 | 14.00 | 0.49 | 13.99 | 0.73 | 13.98 | 0.98 | 13.97 | 1.22 | 13.95 | 14.0 |
| 15.0 | 0.26 | 15.00 | 0.52 | 14.99 | 0.79 | 14.98 | 1.05 | 14.96 | 1.31 | 14.94 | 15.0 |
| 16.0 | 0.28 | 16.00 | 0.56 | 15.99 | 0.84 | 15.98 | 1.12 | 15.96 | 1.39 | 15.94 | 16.0 |
| 17.0 | 0.30 | 17.00 | 0.59 | 16.99 | 0.89 | 16.98 | 1.19 | 16.96 | 1.48 | 16.94 | 17.0 |
| 18.0 | 0.31 | 18.00 | 0.63 | 17.99 | 0.94 | 17.98 | 1.26 | 17.96 | 1.57 | 17.93 | 18.0 |
| 19.0 | 0.33 | 19.00 | 0.66 | 18.99 | 0.99 | 18.97 | 1.33 | 18.95 | 1.66 | 18.93 | 19.0 |
| 20.0 | 0.35 | 20.00 | 0.70 | 19.99 | 1.05 | 19.97 | 1.40 | 19.95 | 1.74 | 19.92 | 20.0 |
| 21.0 | 0.37 | 21.00 | 0.73 | 20.99 | 1.10 | 20.97 | 1.46 | 20.95 | 1.83 | 20.92 | 21.0 |
| 22.0 | 0.38 | 22.00 | 0.77 | 21.99 | 1.15 | 21.97 | 1.53 | 21.95 | 1.92 | 21.92 | 22.0 |
| 23.0 | 0.40 | 23.00 | 0.80 | 22.99 | 1.20 | 22.97 | 1.60 | 22.94 | 2.00 | 22.91 | 23.0 |
| 24.0 | 0.42 | 24.00 | 0.84 | 23.99 | 1.26 | 23.97 | 1.67 | 23.94 | 2.09 | 23.91 | 24.0 |
| 25.0 | 0.44 | 25.00 | 0.87 | 24.98 | 1.31 | 24.97 | 1.74 | 24.94 | 2.18 | 24.90 | 25.0 |
| 26.0 | 0.45 | 26.00 | 0.91 | 25.98 | 1.36 | 25.96 | 1.81 | 25.94 | 2.27 | 25.90 | 26.0 |
| 27.0 | 0.47 | 27.00 | 0.94 | 26.98 | 1.41 | 26.96 | 1.88 | 26.93 | 2.35 | 26.90 | 27.0 |
| 28.0 | 0.49 | 28.00 | 0.98 | 27.98 | 1.47 | 27.96 | 1.95 | 27.93 | 2.44 | 27.89 | 28.0 |
| 29.0 | 0.51 | 29.00 | 1.01 | 28.98 | 1.52 | 28.96 | 2.02 | 28.93 | 2.53 | 28.89 | 29.0 |
| 30.0 | 0.52 | 30.00 | 1.05 | 29.98 | 1.57 | 29.96 | 2.09 | 29.93 | 2.61 | 29.89 | 20.0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| | + 271° -
-269 -
- 91 +
+ 89 + | | + 272 -
-268 -
- 92 +
+ 88 + | | + 273 -
-267 -
- 93 +
+ 87 + | | + 274 -
-266 -
- 94 +
+ 86 + | | + 275 -
-265 -
- 95 +
+ 85 + | | |

| | + 6° +
+ 174 -
- 186 -
- 354 + | | + 7° +
+ 173 -
- 187 -
- 353 + | | + 8° +
+ 172 -
- 188 -
- 352 + | | + 9° +
+ 171 -
- 189 -
- 351 + | | + 10° +
+ 170 -
- 190 -
- 350 + | | |
|------|---|-------|---|-------|---|-------|---|-------|--|-------|------|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0.1 | 0.01 | 0.10 | 0.01 | 0.10 | 0.01 | 0.10 | 0.02 | 0.10 | 0.02 | 0.10 | 0.1 |
| 0.2 | 0.02 | 0.20 | 0.02 | 0.20 | 0.03 | 0.20 | 0.03 | 0.20 | 0.03 | 0.20 | 0.2 |
| 0.3 | 0.03 | 0.30 | 0.04 | 0.30 | 0.04 | 0.30 | 0.05 | 0.30 | 0.05 | 0.30 | 0.3 |
| 0.4 | 0.04 | 0.40 | 0.05 | 0.40 | 0.06 | 0.40 | 0.06 | 0.40 | 0.07 | 0.39 | 0.4 |
| 0.5 | 0.05 | 0.50 | 0.06 | 0.50 | 0.07 | 0.50 | 0.08 | 0.49 | 0.09 | 0.49 | 0.5 |
| 0.6 | 0.06 | 0.60 | 0.07 | 0.60 | 0.08 | 0.59 | 0.09 | 0.59 | 0.10 | 0.59 | 0.6 |
| 0.7 | 0.07 | 0.70 | 0.09 | 0.69 | 0.10 | 0.69 | 0.11 | 0.69 | 0.12 | 0.69 | 0.7 |
| 0.8 | 0.08 | 0.80 | 0.10 | 0.79 | 0.11 | 0.79 | 0.13 | 0.79 | 0.14 | 0.79 | 0.8 |
| 0.9 | 0.09 | 0.90 | 0.11 | 0.89 | 0.13 | 0.89 | 0.14 | 0.89 | 0.16 | 0.89 | 0.9 |
| 1.0 | 0.10 | 0.99 | 0.12 | 0.99 | 0.14 | 0.99 | 0.16 | 0.99 | 0.17 | 0.98 | 1.0 |
| 2.0 | 0.21 | 1.99 | 0.24 | 1.99 | 0.28 | 1.98 | 0.31 | 1.98 | 0.35 | 1.97 | 2.0 |
| 3.0 | 0.31 | 2.98 | 0.37 | 2.98 | 0.42 | 2.97 | 0.47 | 2.96 | 0.52 | 2.95 | 3.0 |
| 4.0 | 0.42 | 3.98 | 0.49 | 3.97 | 0.56 | 3.96 | 0.63 | 3.95 | 0.69 | 3.94 | 4.0 |
| 5.0 | 0.52 | 4.97 | 0.61 | 4.96 | 0.70 | 4.95 | 0.78 | 4.94 | 0.87 | 4.92 | 5.0 |
| 6.0 | 0.63 | 5.97 | 0.73 | 5.96 | 0.84 | 5.94 | 0.94 | 5.93 | 1.04 | 5.91 | 6.0 |
| 7.0 | 0.73 | 6.96 | 0.85 | 6.95 | 0.97 | 6.93 | 1.10 | 6.91 | 1.22 | 6.89 | 7.0 |
| 8.0 | 0.84 | 7.96 | 0.97 | 7.94 | 1.11 | 7.92 | 1.25 | 7.90 | 1.39 | 7.88 | 8.0 |
| 9.0 | 0.94 | 8.95 | 1.10 | 8.93 | 1.25 | 8.91 | 1.41 | 8.89 | 1.56 | 8.86 | 9.0 |
| 10.0 | 1.05 | 9.95 | 1.22 | 9.93 | 1.39 | 9.90 | 1.56 | 9.88 | 1.74 | 9.85 | 10.0 |
| 11.0 | 1.15 | 10.94 | 1.34 | 10.92 | 1.53 | 10.89 | 1.72 | 10.86 | 1.91 | 10.83 | 11.0 |
| 12.0 | 1.25 | 11.93 | 1.46 | 11.91 | 1.67 | 11.88 | 1.88 | 11.85 | 2.08 | 11.82 | 12.0 |
| 13.0 | 1.36 | 12.93 | 1.58 | 12.90 | 1.81 | 12.87 | 2.03 | 12.84 | 2.26 | 12.80 | 13.0 |
| 14.0 | 1.46 | 13.92 | 1.71 | 13.90 | 1.95 | 13.86 | 2.19 | 13.83 | 2.43 | 13.79 | 14.0 |
| 15.0 | 1.57 | 14.92 | 1.83 | 14.89 | 2.09 | 14.85 | 2.35 | 14.82 | 2.60 | 14.77 | 15.0 |
| 16.0 | 1.67 | 15.91 | 1.95 | 15.88 | 2.23 | 15.84 | 2.50 | 15.80 | 2.78 | 15.76 | 16.0 |
| 17.0 | 1.78 | 16.91 | 2.07 | 16.87 | 2.37 | 16.83 | 2.66 | 16.79 | 2.95 | 16.74 | 17.0 |
| 18.0 | 1.88 | 17.90 | 2.19 | 17.87 | 2.51 | 17.82 | 2.82 | 17.78 | 3.13 | 17.73 | 18.0 |
| 19.0 | 1.99 | 18.90 | 2.32 | 18.86 | 2.64 | 18.82 | 2.97 | 18.77 | 3.30 | 18.71 | 19.0 |
| 20.0 | 2.09 | 19.89 | 2.44 | 19.85 | 2.78 | 19.81 | 3.13 | 19.75 | 3.47 | 19.70 | 20.0 |
| 21.0 | 2.20 | 20.88 | 2.56 | 20.84 | 2.92 | 20.80 | 3.29 | 20.74 | 3.65 | 20.68 | 21.0 |
| 22.0 | 2.30 | 21.88 | 2.68 | 21.84 | 3.06 | 21.79 | 3.44 | 21.73 | 3.82 | 21.67 | 22.0 |
| 23.0 | 2.40 | 22.87 | 2.80 | 22.83 | 3.20 | 22.78 | 3.60 | 22.72 | 3.99 | 22.65 | 23.0 |
| 24.0 | 2.51 | 23.87 | 2.92 | 23.82 | 3.34 | 23.77 | 3.75 | 23.70 | 4.17 | 23.64 | 24.0 |
| 25.0 | 2.61 | 24.86 | 3.05 | 24.81 | 3.48 | 24.76 | 3.91 | 24.69 | 4.34 | 24.62 | 25.0 |
| 26.0 | 2.72 | 25.86 | 3.17 | 25.81 | 3.62 | 25.75 | 4.07 | 25.68 | 4.51 | 25.61 | 26.0 |
| 27.0 | 2.82 | 26.85 | 3.29 | 26.80 | 3.76 | 26.74 | 4.22 | 26.67 | 4.69 | 26.59 | 27.0 |
| 28.0 | 2.93 | 27.85 | 3.41 | 27.79 | 3.90 | 27.73 | 4.38 | 27.66 | 4.86 | 27.57 | 28.0 |
| 29.0 | 3.03 | 28.84 | 3.53 | 28.78 | 4.04 | 28.72 | 4.54 | 28.64 | 5.04 | 28.56 | 29.0 |
| 30.0 | 3.14 | 29.84 | 3.66 | 29.78 | 4.18 | 29.71 | 4.69 | 29.63 | 5.21 | 29.54 | 30.0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| | + 276 -
- 264 -
- 96 +
+ 84 + | | + 277 -
- 263 -
- 97 +
+ 83 + | | + 278 -
- 262 -
- 98 +
+ 82 + | | + 279 -
- 261 -
- 99 +
+ 81 + | | + 280 -
- 260 -
- 100 +
+ 80 + | | |

| | + 11° +
+ 169 -
- 191 -
- 349 + | | + 12° +
+ 168 -
- 192 -
- 348 + | | + 13° +
+ 167 -
- 193 -
- 347 + | | + 14° +
+ 166 -
- 194 -
- 346 + | | + 15° +
+ 165 -
- 195 -
- 345 + | | |
|------|--|-------|--|-------|--|-------|--|-------|--|-------|------|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0.1 | 0.02 | 0.10 | 0.02 | 0.10 | 0.02 | 0.10 | 0.02 | 0.10 | 0.03 | 0.10 | 0.1 |
| 0.2 | 0.04 | 0.20 | 0.04 | 0.20 | 0.04 | 0.19 | 0.05 | 0.19 | 0.05 | 0.19 | 0.2 |
| 0.3 | 0.06 | 0.29 | 0.06 | 0.29 | 0.07 | 0.29 | 0.07 | 0.29 | 0.08 | 0.29 | 0.3 |
| 0.4 | 0.08 | 0.39 | 0.08 | 0.39 | 0.09 | 0.39 | 0.10 | 0.39 | 0.10 | 0.39 | 0.4 |
| 0.5 | 0.10 | 0.49 | 0.10 | 0.49 | 0.11 | 0.49 | 0.12 | 0.49 | 0.13 | 0.48 | 0.5 |
| 0.6 | 0.11 | 0.59 | 0.12 | 0.59 | 0.13 | 0.58 | 0.15 | 0.58 | 0.16 | 0.58 | 0.6 |
| 0.7 | 0.13 | 0.69 | 0.15 | 0.68 | 0.16 | 0.68 | 0.17 | 0.68 | 0.18 | 0.68 | 0.7 |
| 0.8 | 0.15 | 0.79 | 0.17 | 0.78 | 0.18 | 0.78 | 0.19 | 0.78 | 0.21 | 0.77 | 0.8 |
| 0.9 | 0.17 | 0.88 | 0.19 | 0.88 | 0.20 | 0.88 | 0.22 | 0.87 | 0.23 | 0.87 | 0.9 |
| 1.0 | 0.19 | 0.98 | 0.21 | 0.98 | 0.22 | 0.97 | 0.24 | 0.97 | 0.26 | 0.97 | 1.0 |
| 2.0 | 0.38 | 1.96 | 0.42 | 1.96 | 0.45 | 1.95 | 0.48 | 1.94 | 0.52 | 1.93 | 2.0 |
| 3.0 | 0.57 | 2.94 | 0.62 | 2.93 | 0.67 | 2.92 | 0.73 | 2.91 | 0.78 | 2.90 | 3.0 |
| 4.0 | 0.76 | 3.93 | 0.83 | 3.91 | 0.90 | 3.90 | 0.97 | 3.88 | 1.04 | 3.86 | 4.0 |
| 5.0 | 0.95 | 4.91 | 1.04 | 4.89 | 1.12 | 4.87 | 1.21 | 4.85 | 1.29 | 4.83 | 5.0 |
| 6.0 | 1.14 | 5.89 | 1.25 | 5.87 | 1.35 | 5.85 | 1.45 | 5.82 | 1.55 | 5.80 | 6.0 |
| 7.0 | 1.34 | 6.87 | 1.46 | 6.85 | 1.57 | 6.82 | 1.69 | 6.79 | 1.81 | 6.76 | 7.0 |
| 8.0 | 1.53 | 7.85 | 1.66 | 7.83 | 1.80 | 7.79 | 1.94 | 7.76 | 2.07 | 7.73 | 8.0 |
| 9.0 | 1.72 | 8.83 | 1.87 | 8.80 | 2.02 | 8.77 | 2.18 | 8.73 | 2.33 | 8.69 | 9.0 |
| 10.0 | 1.91 | 9.82 | 2.08 | 9.78 | 2.25 | 9.74 | 2.42 | 9.70 | 2.59 | 9.66 | 10.0 |
| 11.0 | 2.10 | 10.80 | 2.29 | 10.76 | 2.47 | 10.72 | 2.66 | 10.67 | 2.85 | 10.63 | 11.0 |
| 12.0 | 2.29 | 11.78 | 2.49 | 11.74 | 2.70 | 11.69 | 2.90 | 11.64 | 3.11 | 11.59 | 12.0 |
| 13.0 | 2.48 | 12.76 | 2.70 | 12.72 | 2.92 | 12.67 | 3.14 | 12.61 | 3.36 | 12.56 | 13.0 |
| 14.0 | 2.67 | 13.74 | 2.91 | 13.69 | 3.15 | 13.64 | 3.39 | 13.58 | 3.62 | 13.52 | 14.0 |
| 15.0 | 2.86 | 14.72 | 3.12 | 14.67 | 3.37 | 14.62 | 3.63 | 14.55 | 3.88 | 14.49 | 15.0 |
| 16.0 | 3.05 | 15.71 | 3.33 | 15.65 | 3.60 | 15.59 | 3.87 | 15.52 | 4.14 | 15.45 | 16.0 |
| 17.0 | 3.24 | 16.69 | 3.53 | 16.63 | 3.82 | 16.56 | 4.11 | 16.50 | 4.40 | 16.42 | 17.0 |
| 18.0 | 3.43 | 17.67 | 3.74 | 17.61 | 4.05 | 17.54 | 4.35 | 17.47 | 4.66 | 17.39 | 18.0 |
| 19.0 | 3.63 | 18.65 | 3.95 | 18.58 | 4.27 | 18.51 | 4.60 | 18.44 | 4.92 | 18.35 | 19.0 |
| 20.0 | 3.82 | 19.63 | 4.16 | 19.56 | 4.50 | 19.49 | 4.84 | 19.41 | 5.18 | 19.32 | 20.0 |
| 21.0 | 4.01 | 20.61 | 4.37 | 20.54 | 4.72 | 20.46 | 5.08 | 20.38 | 5.44 | 20.28 | 21.0 |
| 22.0 | 4.20 | 21.60 | 4.57 | 21.52 | 4.95 | 21.44 | 5.32 | 21.35 | 5.69 | 21.25 | 22.0 |
| 23.0 | 4.39 | 22.58 | 4.78 | 22.50 | 5.17 | 22.41 | 5.56 | 22.32 | 5.95 | 22.22 | 23.0 |
| 24.0 | 4.58 | 23.56 | 4.99 | 23.48 | 5.40 | 23.38 | 5.81 | 23.29 | 6.21 | 23.18 | 24.0 |
| 25.0 | 4.77 | 24.54 | 5.20 | 24.45 | 5.62 | 24.36 | 6.05 | 24.26 | 6.47 | 24.15 | 25.0 |
| 26.0 | 4.96 | 25.52 | 5.41 | 25.43 | 5.85 | 25.33 | 6.29 | 25.23 | 6.73 | 25.11 | 26.0 |
| 27.0 | 5.15 | 26.50 | 5.61 | 26.41 | 6.07 | 26.31 | 6.53 | 26.20 | 6.99 | 26.08 | 27.0 |
| 28.0 | 5.34 | 27.49 | 5.82 | 27.39 | 6.30 | 27.28 | 6.77 | 27.17 | 7.25 | 27.05 | 28.0 |
| 29.0 | 5.53 | 28.47 | 6.03 | 28.37 | 6.52 | 28.26 | 7.02 | 28.14 | 7.51 | 28.01 | 29.0 |
| 30.0 | 5.72 | 29.45 | 6.24 | 29.34 | 6.75 | 29.23 | 7.26 | 29.11 | 7.76 | 28.98 | 30.0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| | + 281 -
- 259 -
- 101 +
+ 79 + | | + 282 -
- 258 -
- 102 +
+ 78 + | | + 283 -
- 257 -
- 103 +
+ 77 + | | + 284 -
- 256 -
- 104 +
+ 76 + | | + 285 -
- 255 -
- 105 +
+ 75 + | | |

| | + 16° +
+ 164 -
- 196 -
- 344 + | | + 17° +
+ 163 -
- 197 -
- 343 + | | + 18° +
+ 162 -
- 198 -
- 342 + | | + 19° +
+ 161 -
- 199 -
- 341 + | | + 20° +
+ 160 -
- 200 -
- 340 + | | |
|------|--|-------|--|-------|--|-------|--|-------|--|-------|------|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0.1 | 0.03 | 0.10 | 0.03 | 0.10 | 0.03 | 0.10 | 0.03 | 0.09 | 0.03 | 0.09 | 0.1 |
| 0.2 | 0.06 | 0.19 | 0.06 | 0.19 | 0.06 | 0.19 | 0.07 | 0.19 | 0.07 | 0.19 | 0.2 |
| 0.3 | 0.08 | 0.29 | 0.09 | 0.29 | 0.09 | 0.29 | 0.10 | 0.28 | 0.10 | 0.28 | 0.3 |
| 0.4 | 0.11 | 0.38 | 0.12 | 0.38 | 0.12 | 0.38 | 0.13 | 0.38 | 0.14 | 0.38 | 0.4 |
| 0.5 | 0.14 | 0.48 | 0.15 | 0.48 | 0.15 | 0.48 | 0.16 | 0.47 | 0.17 | 0.47 | 0.5 |
| 0.6 | 0.17 | 0.58 | 0.18 | 0.57 | 0.19 | 0.57 | 0.20 | 0.57 | 0.21 | 0.56 | 0.6 |
| 0.7 | 0.19 | 0.67 | 0.20 | 0.67 | 0.22 | 0.67 | 0.23 | 0.66 | 0.24 | 0.66 | 0.7 |
| 0.8 | 0.22 | 0.77 | 0.23 | 0.77 | 0.25 | 0.76 | 0.26 | 0.76 | 0.27 | 0.75 | 0.8 |
| 0.9 | 0.25 | 0.87 | 0.26 | 0.86 | 0.28 | 0.86 | 0.29 | 0.85 | 0.31 | 0.85 | 0.9 |
| 1.0 | 0.28 | 0.96 | 0.29 | 0.96 | 0.31 | 0.95 | 0.33 | 0.95 | 0.34 | 0.94 | 1.0 |
| 2.0 | 0.55 | 1.92 | 0.58 | 1.91 | 0.62 | 1.90 | 0.65 | 1.89 | 0.68 | 1.88 | 2.0 |
| 3.0 | 0.83 | 2.88 | 0.88 | 2.87 | 0.93 | 2.85 | 0.98 | 2.84 | 1.03 | 2.82 | 3.0 |
| 4.0 | 1.10 | 3.85 | 1.17 | 3.83 | 1.24 | 3.80 | 1.30 | 3.78 | 1.37 | 3.76 | 4.0 |
| 5.0 | 1.38 | 4.81 | 1.46 | 4.78 | 1.55 | 4.76 | 1.63 | 4.73 | 1.71 | 4.70 | 5.0 |
| 6.0 | 1.65 | 5.77 | 1.75 | 5.74 | 1.85 | 5.71 | 1.95 | 5.67 | 2.05 | 5.64 | 6.0 |
| 7.0 | 1.93 | 6.73 | 2.05 | 6.69 | 2.16 | 6.66 | 2.28 | 6.62 | 2.39 | 6.58 | 7.0 |
| 8.0 | 2.21 | 7.69 | 2.34 | 7.65 | 2.47 | 7.61 | 2.60 | 7.56 | 2.74 | 7.52 | 8.0 |
| 9.0 | 2.48 | 8.65 | 2.63 | 8.61 | 2.78 | 8.56 | 2.93 | 8.51 | 3.08 | 8.46 | 9.0 |
| 10.0 | 2.76 | 9.61 | 2.92 | 9.56 | 3.09 | 9.51 | 3.26 | 9.46 | 3.42 | 9.40 | 10.0 |
| 11.0 | 3.03 | 10.57 | 3.22 | 10.52 | 3.40 | 10.46 | 3.58 | 10.40 | 3.76 | 10.34 | 11.0 |
| 12.0 | 3.31 | 11.54 | 3.51 | 11.48 | 3.71 | 11.41 | 3.91 | 11.35 | 4.10 | 11.28 | 12.0 |
| 13.0 | 3.58 | 12.50 | 3.80 | 12.43 | 4.02 | 12.36 | 4.23 | 12.29 | 4.45 | 12.22 | 13.0 |
| 14.0 | 3.86 | 13.46 | 4.09 | 13.39 | 4.33 | 13.31 | 4.56 | 13.24 | 4.79 | 13.16 | 14.0 |
| 15.0 | 4.13 | 14.42 | 4.39 | 14.34 | 4.64 | 14.27 | 4.88 | 14.18 | 5.13 | 14.10 | 15.0 |
| 16.0 | 4.41 | 15.38 | 4.68 | 15.30 | 4.94 | 15.22 | 5.21 | 15.13 | 5.47 | 15.04 | 16.0 |
| 17.0 | 4.69 | 16.34 | 4.97 | 16.26 | 5.25 | 16.17 | 5.53 | 16.07 | 5.81 | 15.97 | 17.0 |
| 18.0 | 4.96 | 17.30 | 5.26 | 17.21 | 5.56 | 17.12 | 5.86 | 17.02 | 6.16 | 16.91 | 18.0 |
| 19.0 | 5.24 | 18.26 | 5.56 | 18.17 | 5.87 | 18.07 | 6.19 | 17.96 | 6.50 | 17.85 | 19.0 |
| 20.0 | 5.51 | 19.23 | 5.85 | 19.13 | 6.18 | 19.02 | 6.51 | 18.91 | 6.84 | 18.79 | 20.0 |
| 21.0 | 5.79 | 20.19 | 6.14 | 20.08 | 6.49 | 19.97 | 6.84 | 19.86 | 7.18 | 19.73 | 21.0 |
| 22.0 | 6.06 | 21.15 | 6.43 | 21.04 | 6.80 | 20.92 | 7.16 | 20.80 | 7.52 | 20.67 | 22.0 |
| 23.0 | 6.34 | 22.11 | 6.72 | 22.00 | 7.11 | 21.87 | 7.49 | 21.75 | 7.87 | 21.61 | 23.0 |
| 24.0 | 6.62 | 23.07 | 7.02 | 22.95 | 7.42 | 22.83 | 7.81 | 22.69 | 8.21 | 22.55 | 24.0 |
| 25.0 | 6.89 | 24.03 | 7.31 | 23.91 | 7.73 | 23.78 | 8.14 | 23.64 | 8.55 | 23.49 | 25.0 |
| 26.0 | 7.17 | 24.99 | 7.60 | 24.86 | 8.03 | 24.73 | 8.46 | 24.58 | 8.89 | 24.43 | 26.0 |
| 27.0 | 7.44 | 25.95 | 7.89 | 25.82 | 8.34 | 25.68 | 8.79 | 25.53 | 9.23 | 25.37 | 27.0 |
| 28.0 | 7.72 | 26.92 | 8.19 | 26.78 | 8.65 | 26.63 | 9.12 | 26.47 | 9.58 | 26.31 | 28.0 |
| 29.0 | 7.99 | 27.88 | 8.48 | 27.73 | 8.96 | 27.58 | 9.44 | 27.42 | 9.92 | 27.25 | 29.0 |
| 30.0 | 8.27 | 28.84 | 8.77 | 28.69 | 9.27 | 28.53 | 9.77 | 28.37 | 10.26 | 28.19 | 30.0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
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- 106 +
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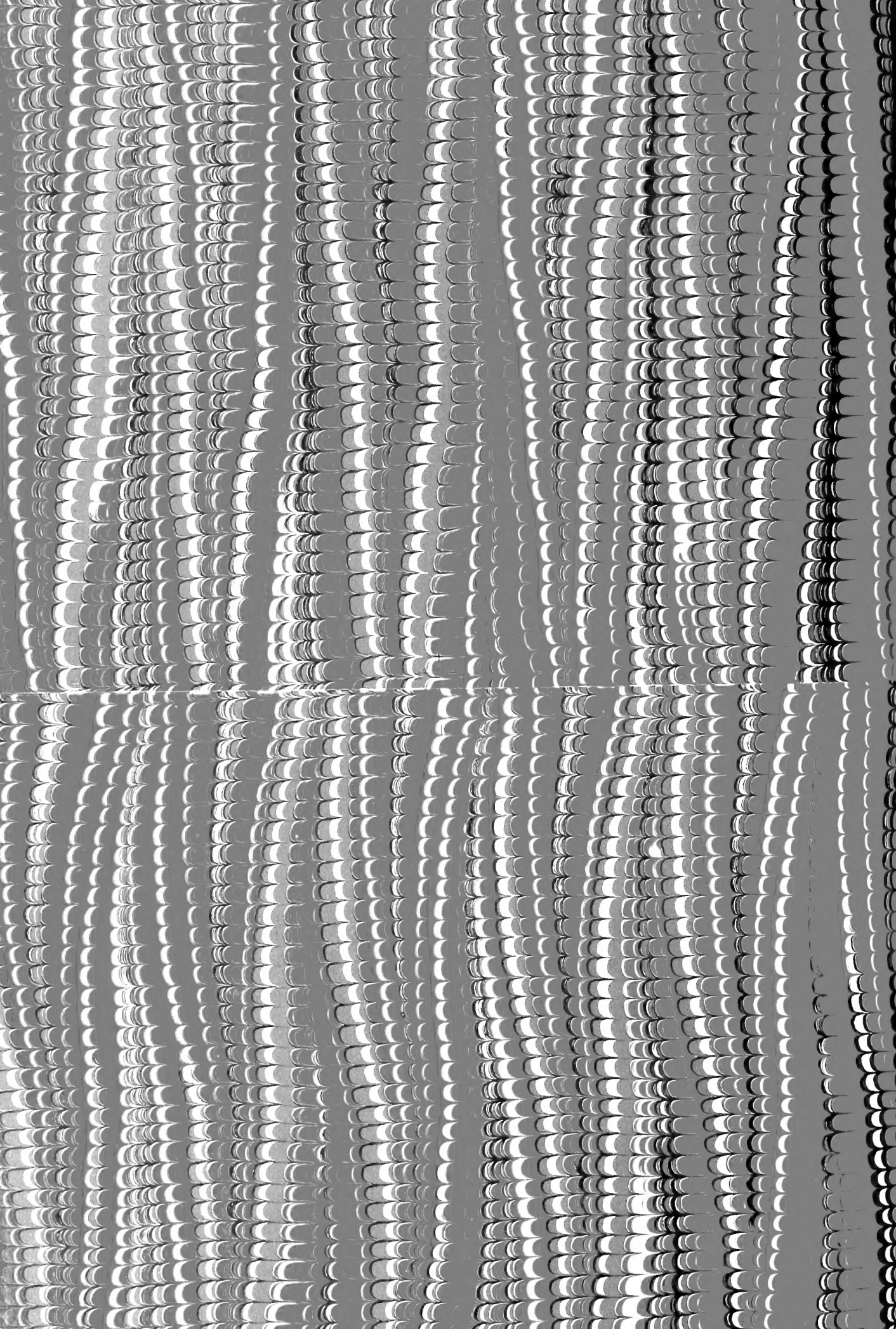
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- 204 -
- 336 + | | + 25° +
+ 155 -
- 205 -
- 335 + | | |
|------|--|-------|--|-------|--|-------|--|-------|--|-------|------|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
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| 0.2 | 0.07 | 0.19 | 0.07 | 0.19 | 0.08 | 0.18 | 0.08 | 0.18 | 0.08 | 0.18 | 0.2 |
| 0.3 | 0.11 | 0.28 | 0.11 | 0.28 | 0.12 | 0.28 | 0.12 | 0.27 | 0.13 | 0.27 | 0.3 |
| 0.4 | 0.14 | 0.37 | 0.15 | 0.37 | 0.16 | 0.37 | 0.16 | 0.37 | 0.17 | 0.36 | 0.4 |
| 0.5 | 0.18 | 0.47 | 0.19 | 0.46 | 0.20 | 0.46 | 0.20 | 0.46 | 0.21 | 0.45 | 0.5 |
| 0.6 | 0.22 | 0.56 | 0.22 | 0.56 | 0.23 | 0.55 | 0.24 | 0.55 | 0.25 | 0.54 | 0.6 |
| 0.7 | 0.25 | 0.65 | 0.26 | 0.65 | 0.27 | 0.64 | 0.28 | 0.64 | 0.30 | 0.63 | 0.7 |
| 0.8 | 0.29 | 0.75 | 0.30 | 0.74 | 0.31 | 0.74 | 0.33 | 0.73 | 0.34 | 0.73 | 0.8 |
| 0.9 | 0.32 | 0.84 | 0.34 | 0.83 | 0.35 | 0.83 | 0.37 | 0.82 | 0.38 | 0.82 | 0.9 |
| 1.0 | 0.36 | 0.93 | 0.37 | 0.93 | 0.39 | 0.92 | 0.41 | 0.91 | 0.42 | 0.91 | 1.0 |
| 2.0 | 0.72 | 1.87 | 0.75 | 1.85 | 0.78 | 1.84 | 0.81 | 1.83 | 0.85 | 1.81 | 2.0 |
| 3.0 | 1.08 | 2.80 | 1.12 | 2.78 | 1.17 | 2.76 | 1.22 | 2.74 | 1.27 | 2.72 | 3.0 |
| 4.0 | 1.43 | 3.73 | 1.50 | 3.71 | 1.56 | 3.68 | 1.63 | 3.65 | 1.69 | 3.63 | 4.0 |
| 5.0 | 1.79 | 4.67 | 1.87 | 4.64 | 1.95 | 4.60 | 2.03 | 4.57 | 2.11 | 4.53 | 5.0 |
| 6.0 | 2.15 | 5.60 | 2.25 | 5.56 | 2.34 | 5.52 | 2.44 | 5.48 | 2.54 | 5.44 | 6.0 |
| 7.0 | 2.51 | 6.54 | 2.62 | 6.49 | 2.74 | 6.44 | 2.85 | 6.39 | 2.96 | 6.34 | 7.0 |
| 8.0 | 2.87 | 7.47 | 3.00 | 7.42 | 3.13 | 7.36 | 3.25 | 7.31 | 3.38 | 7.25 | 8.0 |
| 9.0 | 3.23 | 8.40 | 3.37 | 8.34 | 3.52 | 8.28 | 3.66 | 8.22 | 3.80 | 8.16 | 9.0 |
| 10.0 | 3.58 | 9.34 | 3.75 | 9.27 | 3.91 | 9.21 | 4.07 | 9.14 | 4.23 | 9.06 | 10.0 |
| 11.0 | 3.94 | 10.27 | 4.12 | 10.20 | 4.30 | 10.13 | 4.47 | 10.05 | 4.65 | 9.97 | 11.0 |
| 12.0 | 4.30 | 11.20 | 4.50 | 11.13 | 4.69 | 11.05 | 4.88 | 10.96 | 5.07 | 10.88 | 12.0 |
| 13.0 | 4.66 | 12.14 | 4.87 | 12.05 | 5.08 | 11.97 | 5.29 | 11.88 | 5.49 | 11.78 | 13.0 |
| 14.0 | 5.02 | 13.07 | 5.24 | 12.98 | 5.47 | 12.89 | 5.69 | 12.79 | 5.92 | 12.69 | 14.0 |
| 15.0 | 5.38 | 14.00 | 5.62 | 13.91 | 5.86 | 13.81 | 6.10 | 13.70 | 6.34 | 13.59 | 15.0 |
| 16.0 | 5.73 | 14.94 | 5.99 | 14.83 | 6.25 | 14.73 | 6.51 | 14.62 | 6.76 | 14.50 | 16.0 |
| 17.0 | 6.09 | 15.87 | 6.37 | 15.76 | 6.64 | 15.65 | 6.91 | 15.53 | 7.18 | 15.41 | 17.0 |
| 18.0 | 6.45 | 16.80 | 6.74 | 16.69 | 7.03 | 16.57 | 7.32 | 16.44 | 7.61 | 16.31 | 18.0 |
| 19.0 | 6.81 | 17.74 | 7.12 | 17.62 | 7.42 | 17.49 | 7.73 | 17.36 | 8.03 | 17.22 | 19.0 |
| 20.0 | 7.17 | 18.67 | 7.49 | 18.54 | 7.81 | 18.41 | 8.13 | 18.27 | 8.45 | 18.13 | 20.0 |
| 21.0 | 7.53 | 19.61 | 7.87 | 19.47 | 8.21 | 19.33 | 8.54 | 19.18 | 8.87 | 19.03 | 21.0 |
| 22.0 | 7.88 | 20.54 | 8.24 | 20.40 | 8.60 | 20.25 | 8.95 | 20.10 | 9.30 | 19.94 | 22.0 |
| 23.0 | 8.24 | 21.47 | 8.62 | 21.33 | 8.99 | 21.17 | 9.35 | 21.01 | 9.72 | 20.85 | 23.0 |
| 24.0 | 8.60 | 22.41 | 8.99 | 22.25 | 9.38 | 22.09 | 9.76 | 21.93 | 10.14 | 21.75 | 24.0 |
| 25.0 | 8.96 | 23.34 | 9.37 | 23.18 | 9.77 | 23.01 | 10.17 | 22.84 | 10.57 | 22.66 | 25.0 |
| 26.0 | 9.32 | 24.27 | 9.74 | 24.11 | 10.16 | 23.93 | 10.58 | 23.75 | 10.99 | 23.56 | 26.0 |
| 27.0 | 9.68 | 25.21 | 10.11 | 25.03 | 10.55 | 24.85 | 10.98 | 24.67 | 11.41 | 24.47 | 27.0 |
| 28.0 | 10.03 | 26.14 | 10.49 | 25.96 | 10.94 | 25.77 | 11.39 | 25.58 | 11.83 | 25.38 | 28.0 |
| 29.0 | 10.39 | 27.07 | 10.86 | 26.89 | 11.33 | 26.69 | 11.80 | 26.49 | 12.26 | 26.28 | 29.0 |
| 30.0 | 10.75 | 28.01 | 11.24 | 27.82 | 11.72 | 27.62 | 12.20 | 27.41 | 12.68 | 27.19 | 30.0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| | + 291 -
- 249 -
- 111 +
+ 69 + | | + 292 -
- 248 -
- 112 +
+ 68 + | | + 293 -
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- 114 +
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- 245 -
- 115 +
+ 65 + | | |

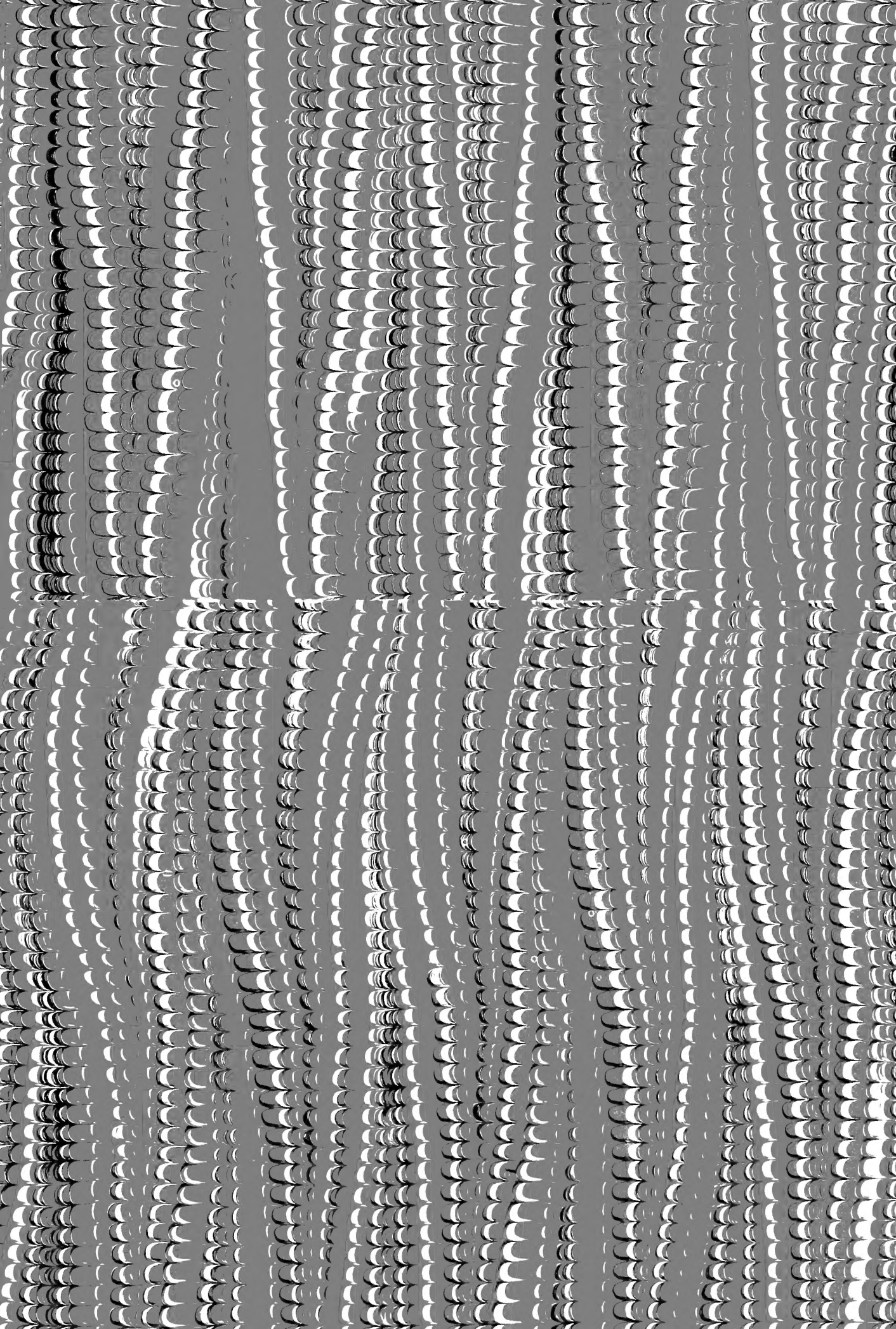
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+ 151 -
- 209 -
- 331 + | | + 30° +
+ 150 -
- 210 -
- 330 + | | |
|------|--|---|--|---|--|-------|--|-------|--|-------|------|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
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| 0.2 | 0.09 | 0.18 | 0.09 | 0.18 | 0.09 | 0.18 | 0.10 | 0.17 | 0.10 | 0.17 | 0.2 |
| 0.3 | 0.13 | 0.27 | 0.14 | 0.27 | 0.14 | 0.26 | 0.15 | 0.26 | 0.15 | 0.26 | 0.3 |
| 0.4 | 0.18 | 0.36 | 0.18 | 0.36 | 0.19 | 0.35 | 0.19 | 0.35 | 0.20 | 0.35 | 0.4 |
| 0.5 | 0.22 | 0.45 | 0.23 | 0.45 | 0.23 | 0.44 | 0.24 | 0.44 | 0.25 | 0.43 | 0.5 |
| 0.6 | 0.26 | 0.54 | 0.27 | 0.53 | 0.28 | 0.53 | 0.29 | 0.52 | 0.30 | 0.52 | 0.6 |
| 0.7 | 0.31 | 0.63 | 0.32 | 0.62 | 0.33 | 0.62 | 0.34 | 0.61 | 0.35 | 0.61 | 0.7 |
| 0.8 | 0.35 | 0.72 | 0.36 | 0.71 | 0.38 | 0.71 | 0.39 | 0.70 | 0.40 | 0.69 | 0.8 |
| 0.9 | 0.39 | 0.81 | 0.41 | 0.80 | 0.42 | 0.79 | 0.44 | 0.79 | 0.45 | 0.78 | 0.9 |
| 1.0 | 0.44 | 0.90 | 0.45 | 0.89 | 0.47 | 0.88 | 0.48 | 0.87 | 0.50 | 0.87 | 1.0 |
| 2.0 | 0.88 | 1.80 | 0.91 | 1.78 | 0.94 | 1.77 | 0.97 | 1.75 | 1.00 | 1.73 | 2.0 |
| 3.0 | 1.32 | 2.70 | 1.36 | 2.67 | 1.41 | 2.65 | 1.45 | 2.62 | 1.50 | 2.60 | 3.0 |
| 4.0 | 1.75 | 3.60 | 1.82 | 3.56 | 1.88 | 3.53 | 1.94 | 3.50 | 2.00 | 3.46 | 4.0 |
| 5.0 | 2.19 | 4.49 | 2.27 | 4.46 | 2.35 | 4.41 | 2.42 | 4.37 | 2.50 | 4.33 | 5.0 |
| 6.0 | 2.63 | 5.39 | 2.72 | 5.35 | 2.82 | 5.30 | 2.91 | 5.25 | 3.00 | 5.20 | 6.0 |
| 7.0 | 3.07 | 6.29 | 3.18 | 6.24 | 3.29 | 6.18 | 3.39 | 6.12 | 3.50 | 6.06 | 7.0 |
| 8.0 | 3.51 | 7.19 | 3.63 | 7.13 | 3.76 | 7.06 | 3.88 | 7.00 | 4.00 | 6.93 | 8.0 |
| 9.0 | 3.95 | 8.09 | 4.09 | 8.02 | 4.23 | 7.95 | 4.36 | 7.87 | 4.50 | 7.79 | 9.0 |
| 10.0 | 4.38 | 8.99 | 4.54 | 8.91 | 4.69 | 8.83 | 4.85 | 8.75 | 5.00 | 8.66 | 10.0 |
| 11.0 | 4.82 | 9.89 | 4.99 | 9.80 | 5.16 | 9.71 | 5.33 | 9.62 | 5.50 | 9.53 | 11.0 |
| 12.0 | 5.26 | 10.79 | 5.45 | 10.69 | 5.63 | 10.60 | 5.82 | 10.50 | 6.00 | 10.39 | 12.0 |
| 13.0 | 5.70 | 11.68 | 5.90 | 11.58 | 6.10 | 11.48 | 6.30 | 11.37 | 6.50 | 11.26 | 13.0 |
| 14.0 | 6.14 | 12.58 | 6.36 | 12.47 | 6.57 | 12.36 | 6.79 | 12.24 | 7.00 | 12.12 | 14.0 |
| 15.0 | 6.58 | 13.48 | 6.81 | 13.37 | 7.04 | 13.24 | 7.27 | 13.12 | 7.50 | 12.99 | 15.0 |
| 16.0 | 7.01 | 14.38 | 7.26 | 14.26 | 7.51 | 14.13 | 7.76 | 13.99 | 8.00 | 13.86 | 16.0 |
| 17.0 | 7.45 | 15.28 | 7.72 | 15.15 | 7.98 | 15.01 | 8.24 | 14.87 | 8.50 | 14.72 | 17.0 |
| 18.0 | 7.89 | 16.18 | 8.17 | 16.04 | 8.45 | 15.89 | 8.73 | 15.74 | 9.00 | 15.59 | 18.0 |
| 19.0 | 8.33 | 17.08 | 8.63 | 16.93 | 8.92 | 16.78 | 9.21 | 16.62 | 9.50 | 16.45 | 19.0 |
| 20.0 | 8.77 | 17.98 | 9.08 | 17.82 | 9.39 | 17.66 | 9.70 | 17.49 | 10.00 | 17.32 | 20.0 |
| 21.0 | 9.21 | 18.87 | 9.53 | 18.71 | 9.86 | 18.54 | 10.18 | 18.37 | 10.50 | 18.19 | 21.0 |
| 22.0 | 9.64 | 19.77 | 9.99 | 19.60 | 10.33 | 19.42 | 10.67 | 19.24 | 11.00 | 19.05 | 22.0 |
| 23.0 | 10.08 | 20.67 | 10.44 | 20.49 | 10.80 | 20.31 | 11.15 | 20.12 | 11.50 | 19.92 | 23.0 |
| 24.0 | 10.52 | 21.57 | 10.90 | 21.38 | 11.27 | 21.19 | 11.64 | 20.99 | 12.00 | 20.78 | 24.0 |
| 25.0 | 10.96 | 22.47 | 11.35 | 22.28 | 11.74 | 22.07 | 12.12 | 21.87 | 12.50 | 21.65 | 25.0 |
| 26.0 | 11.40 | 23.37 | 11.80 | 23.17 | 12.21 | 22.96 | 12.61 | 22.74 | 13.00 | 22.52 | 26.0 |
| 27.0 | 11.84 | 24.27 | 12.26 | 24.06 | 12.68 | 23.84 | 13.09 | 23.61 | 13.50 | 23.38 | 27.0 |
| 28.0 | 12.27 | 25.17 | 12.71 | 24.95 | 13.15 | 24.72 | 13.57 | 24.49 | 14.00 | 24.25 | 28.0 |
| 29.0 | 12.71 | 26.07 | 13.17 | 25.84 | 13.61 | 25.61 | 14.06 | 25.36 | 14.50 | 25.11 | 29.0 |
| 30.0 | 13.15 | 26.96 | 13.62 | 26.73 | 14.08 | 26.49 | 14.54 | 26.24 | 15.00 | 25.98 | 30.0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
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- 244 -
- 116 +
+ 64 + | + 297 -
- 243 -
- 117 +
+ 63 + | + 298 -
- 242 -
- 118 +
+ 62 + | + 299 -
- 241 -
- 119 +
+ 61 + | + 300 -
- 240 -
- 120 +
+ 60 + | | | | | | |

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+149 -
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-212 -
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-327 + | | + 34° +
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|------|--|-------|---------------------------------------|-------|---------------------------------------|-------|---------------------------------------|-------|---------------------------------------|-------|------|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
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| 0.3 | 0.15 | 0.26 | 0.16 | 0.25 | 0.16 | 0.25 | 0.17 | 0.25 | 0.17 | 0.25 | 0.3 |
| 0.4 | 0.21 | 0.34 | 0.21 | 0.34 | 0.22 | 0.34 | 0.22 | 0.33 | 0.23 | 0.33 | 0.4 |
| 0.5 | 0.26 | 0.43 | 0.26 | 0.42 | 0.27 | 0.42 | 0.28 | 0.41 | 0.29 | 0.41 | 0.5 |
| 0.6 | 0.31 | 0.51 | 0.32 | 0.51 | 0.33 | 0.50 | 0.34 | 0.50 | 0.34 | 0.49 | 0.6 |
| 0.7 | 0.36 | 0.60 | 0.37 | 0.59 | 0.38 | 0.59 | 0.39 | 0.58 | 0.40 | 0.57 | 0.7 |
| 0.8 | 0.41 | 0.69 | 0.42 | 0.68 | 0.44 | 0.67 | 0.45 | 0.66 | 0.46 | 0.66 | 0.8 |
| 0.9 | 0.46 | 0.77 | 0.48 | 0.76 | 0.49 | 0.75 | 0.50 | 0.75 | 0.52 | 0.74 | 0.9 |
| 1.0 | 0.52 | 0.86 | 0.53 | 0.85 | 0.54 | 0.84 | 0.56 | 0.83 | 0.57 | 0.82 | 1.0 |
| 2.0 | 1.03 | 1.71 | 1.06 | 1.70 | 1.09 | 1.68 | 1.12 | 1.66 | 1.15 | 1.64 | 2.0 |
| 3.0 | 1.55 | 2.57 | 1.59 | 2.54 | 1.63 | 2.52 | 1.68 | 2.49 | 1.72 | 2.46 | 3.0 |
| 4.0 | 2.06 | 3.43 | 2.12 | 3.39 | 2.18 | 3.35 | 2.24 | 3.32 | 2.29 | 3.28 | 4.0 |
| 5.0 | 2.58 | 4.29 | 2.65 | 4.24 | 2.72 | 4.19 | 2.80 | 4.15 | 2.87 | 4.10 | 5.0 |
| 6.0 | 3.09 | 5.14 | 3.18 | 5.09 | 3.27 | 5.03 | 3.36 | 4.97 | 3.44 | 4.91 | 6.0 |
| 7.0 | 3.61 | 6.00 | 3.71 | 5.94 | 3.81 | 5.87 | 3.91 | 5.80 | 4.02 | 5.73 | 7.0 |
| 8.0 | 4.12 | 6.86 | 4.24 | 6.78 | 4.36 | 6.71 | 4.47 | 6.63 | 4.59 | 6.55 | 8.0 |
| 9.0 | 4.64 | 7.71 | 4.77 | 7.63 | 4.90 | 7.55 | 5.03 | 7.46 | 5.16 | 7.37 | 9.0 |
| 10.0 | 5.15 | 8.57 | 5.30 | 8.48 | 5.45 | 8.39 | 5.59 | 8.29 | 5.74 | 8.19 | 10.0 |
| 11.0 | 5.67 | 9.43 | 5.83 | 9.33 | 5.99 | 9.23 | 6.15 | 9.12 | 6.31 | 9.01 | 11.0 |
| 12.0 | 6.18 | 10.29 | 6.36 | 10.18 | 6.54 | 10.06 | 6.71 | 9.95 | 6.88 | 9.83 | 12.0 |
| 13.0 | 6.70 | 11.14 | 6.89 | 11.02 | 7.08 | 10.90 | 7.27 | 10.78 | 7.46 | 10.65 | 13.0 |
| 14.0 | 7.21 | 12.00 | 7.42 | 11.87 | 7.62 | 11.74 | 7.83 | 11.61 | 8.03 | 11.47 | 14.0 |
| 15.0 | 7.73 | 12.86 | 7.95 | 12.72 | 8.17 | 12.58 | 8.39 | 12.44 | 8.60 | 12.29 | 15.0 |
| 16.0 | 8.24 | 13.71 | 8.48 | 13.57 | 8.71 | 13.42 | 8.95 | 13.26 | 9.18 | 13.11 | 16.0 |
| 17.0 | 8.76 | 14.57 | 9.01 | 14.42 | 9.26 | 14.26 | 9.51 | 14.09 | 9.75 | 13.93 | 17.0 |
| 18.0 | 9.27 | 15.43 | 9.54 | 15.26 | 9.80 | 15.10 | 10.07 | 14.92 | 10.32 | 14.74 | 18.0 |
| 19.0 | 9.79 | 16.29 | 10.07 | 16.11 | 10.35 | 15.93 | 10.62 | 15.75 | 10.90 | 15.56 | 19.0 |
| 20.0 | 10.30 | 17.14 | 10.60 | 16.96 | 10.89 | 16.77 | 11.18 | 16.58 | 11.47 | 16.38 | 20.0 |
| 21.0 | 10.82 | 18.00 | 11.13 | 17.81 | 11.44 | 17.61 | 11.74 | 17.41 | 12.05 | 17.20 | 21.0 |
| 22.0 | 11.33 | 18.86 | 11.66 | 18.66 | 11.98 | 18.45 | 12.30 | 18.24 | 12.62 | 18.02 | 22.0 |
| 23.0 | 11.85 | 19.71 | 12.19 | 19.51 | 12.53 | 19.29 | 12.86 | 19.07 | 13.19 | 18.84 | 23.0 |
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| 25.0 | 12.88 | 21.43 | 13.25 | 21.20 | 13.62 | 20.97 | 13.98 | 20.73 | 14.34 | 20.48 | 25.0 |
| 26.0 | 13.39 | 22.29 | 13.78 | 22.05 | 14.16 | 21.81 | 14.54 | 21.55 | 14.91 | 21.30 | 26.0 |
| 27.0 | 13.91 | 23.14 | 14.31 | 22.90 | 14.71 | 22.64 | 15.10 | 22.38 | 15.49 | 22.12 | 27.0 |
| 28.0 | 14.42 | 24.00 | 14.84 | 23.75 | 15.25 | 23.48 | 15.66 | 23.21 | 16.06 | 22.94 | 28.0 |
| 29.0 | 14.94 | 24.86 | 15.37 | 24.59 | 15.79 | 24.32 | 16.22 | 24.04 | 16.63 | 23.76 | 29.0 |
| 30.0 | 15.45 | 25.71 | 15.90 | 25.44 | 16.34 | 25.16 | 16.78 | 24.87 | 17.21 | 24.57 | 30.0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| | + 301° -
-239 -
-121 +
+ 59 + | | + 302 -
-238 -
-122 +
+ 58 + | | + 303 -
-237 -
-123 +
+ 57 + | | + 304 -
-236 -
-124 +
+ 56 + | | + 305 -
-235 -
-125 +
+ 55 + | | |

| | + 36° +
+ 144 -
- 216 -
- 324 + | | + 37° +
+ 143 -
- 217 -
- 323 + | | + 38° +
+ 142 -
- 218 -
- 322 + | | + 39° +
+ 141 -
- 219 -
- 321 + | | + 40° +
+ 140 -
- 220 -
- 320 + | | |
|------|--|-------|--|-------|--|-------|--|-------|--|-------|------|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0.1 | 0.06 | 0.08 | 0.06 | 0.08 | 0.06 | 0.08 | 0.06 | 0.08 | 0.06 | 0.08 | 0.1 |
| 0.2 | 0.12 | 0.16 | 0.12 | 0.16 | 0.12 | 0.16 | 0.13 | 0.16 | 0.13 | 0.15 | 0.2 |
| 0.3 | 0.18 | 0.24 | 0.18 | 0.24 | 0.18 | 0.24 | 0.19 | 0.23 | 0.19 | 0.23 | 0.3 |
| 0.4 | 0.24 | 0.32 | 0.24 | 0.32 | 0.25 | 0.32 | 0.25 | 0.31 | 0.26 | 0.31 | 0.4 |
| 0.5 | 0.29 | 0.40 | 0.30 | 0.40 | 0.31 | 0.39 | 0.31 | 0.39 | 0.32 | 0.38 | 0.5 |
| 0.6 | 0.35 | 0.49 | 0.36 | 0.48 | 0.37 | 0.47 | 0.38 | 0.47 | 0.39 | 0.46 | 0.6 |
| 0.7 | 0.41 | 0.57 | 0.42 | 0.56 | 0.43 | 0.55 | 0.44 | 0.54 | 0.45 | 0.54 | 0.7 |
| 0.8 | 0.47 | 0.65 | 0.48 | 0.64 | 0.49 | 0.63 | 0.50 | 0.62 | 0.51 | 0.61 | 0.8 |
| 0.9 | 0.53 | 0.73 | 0.54 | 0.72 | 0.55 | 0.71 | 0.57 | 0.70 | 0.58 | 0.69 | 0.9 |
| 1.0 | 0.59 | 0.81 | 0.60 | 0.80 | 0.62 | 0.79 | 0.63 | 0.78 | 0.64 | 0.77 | 1.0 |
| 2.0 | 1.18 | 1.62 | 1.20 | 1.60 | 1.23 | 1.58 | 1.26 | 1.55 | 1.29 | 1.53 | 2.0 |
| 3.0 | 1.76 | 2.43 | 1.81 | 2.40 | 1.85 | 2.36 | 1.89 | 2.33 | 1.93 | 2.30 | 3.0 |
| 4.0 | 2.35 | 3.24 | 2.41 | 3.19 | 2.46 | 3.15 | 2.52 | 3.11 | 2.57 | 3.06 | 4.0 |
| 5.0 | 2.94 | 4.05 | 3.01 | 3.99 | 3.08 | 3.94 | 3.15 | 3.89 | 3.21 | 3.83 | 5.0 |
| 6.0 | 3.53 | 4.85 | 3.61 | 4.79 | 3.69 | 4.73 | 3.78 | 4.66 | 3.86 | 4.60 | 6.0 |
| 7.0 | 4.11 | 5.66 | 4.21 | 5.59 | 4.31 | 5.52 | 4.41 | 5.44 | 4.50 | 5.36 | 7.0 |
| 8.0 | 4.70 | 6.47 | 4.81 | 6.39 | 4.93 | 6.30 | 5.03 | 6.22 | 5.14 | 6.13 | 8.0 |
| 9.0 | 5.29 | 7.28 | 5.42 | 7.19 | 5.54 | 7.09 | 5.66 | 6.99 | 5.79 | 6.89 | 9.0 |
| 10.0 | 5.88 | 8.09 | 6.02 | 7.99 | 6.16 | 7.88 | 6.29 | 7.77 | 6.43 | 7.66 | 10.0 |
| 11.0 | 6.47 | 8.90 | 6.62 | 8.78 | 6.77 | 8.67 | 6.92 | 8.55 | 7.07 | 8.43 | 11.0 |
| 12.0 | 7.05 | 9.71 | 7.22 | 9.58 | 7.39 | 9.46 | 7.55 | 9.33 | 7.71 | 9.19 | 12.0 |
| 13.0 | 7.64 | 10.52 | 7.82 | 10.38 | 8.00 | 10.24 | 8.18 | 10.10 | 8.36 | 9.96 | 13.0 |
| 14.0 | 8.23 | 11.33 | 8.43 | 11.18 | 8.62 | 11.03 | 8.81 | 10.88 | 9.00 | 10.72 | 14.0 |
| 15.0 | 8.82 | 12.14 | 9.03 | 11.98 | 9.23 | 11.82 | 9.44 | 11.66 | 9.64 | 11.49 | 15.0 |
| 16.0 | 9.40 | 12.94 | 9.63 | 12.78 | 9.85 | 12.61 | 10.07 | 12.43 | 10.28 | 12.26 | 16.0 |
| 17.0 | 9.99 | 13.75 | 10.23 | 13.58 | 10.47 | 13.40 | 10.70 | 13.21 | 10.93 | 13.02 | 17.0 |
| 18.0 | 10.58 | 14.56 | 10.83 | 14.38 | 11.08 | 14.18 | 11.33 | 13.99 | 11.57 | 13.79 | 18.0 |
| 19.0 | 11.17 | 15.37 | 11.43 | 15.17 | 11.70 | 14.97 | 11.96 | 14.77 | 12.21 | 14.55 | 19.0 |
| 20.0 | 11.76 | 16.18 | 12.04 | 15.97 | 12.31 | 15.76 | 12.59 | 15.54 | 12.86 | 15.32 | 20.0 |
| 21.0 | 12.34 | 16.99 | 12.64 | 16.77 | 12.93 | 16.55 | 13.22 | 16.32 | 13.50 | 16.09 | 21.0 |
| 22.0 | 12.93 | 17.80 | 13.24 | 17.57 | 13.54 | 17.34 | 13.85 | 17.10 | 14.14 | 16.85 | 22.0 |
| 23.0 | 13.52 | 18.61 | 13.84 | 18.37 | 14.16 | 18.12 | 14.47 | 17.87 | 14.78 | 17.62 | 23.0 |
| 24.0 | 14.11 | 19.42 | 14.44 | 19.17 | 14.78 | 18.91 | 15.10 | 18.65 | 15.43 | 18.39 | 24.0 |
| 25.0 | 14.69 | 20.23 | 15.05 | 19.97 | 15.39 | 19.70 | 15.73 | 19.43 | 16.07 | 19.15 | 25.0 |
| 26.0 | 15.28 | 21.03 | 15.65 | 20.76 | 16.01 | 20.49 | 16.36 | 20.21 | 16.71 | 19.92 | 26.0 |
| 27.0 | 15.87 | 21.84 | 16.25 | 21.56 | 16.62 | 21.28 | 16.99 | 20.98 | 17.36 | 20.68 | 27.0 |
| 28.0 | 16.46 | 22.65 | 16.85 | 22.36 | 17.24 | 22.06 | 17.62 | 21.76 | 18.00 | 21.45 | 28.0 |
| 29.0 | 17.05 | 23.46 | 17.45 | 23.16 | 17.85 | 22.85 | 18.25 | 22.54 | 18.64 | 22.22 | 29.0 |
| 30.0 | 17.63 | 24.27 | 18.05 | 23.96 | 18.47 | 23.64 | 18.88 | 23.31 | 19.28 | 22.98 | 30.0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| | + 306 -
- 234 -
- 126 +
+ 54 + | | + 307 -
- 233 -
- 127 +
+ 53 + | | + 308 -
- 232 -
- 123 +
+ 52 + | | + 309 -
- 231 -
- 129 +
+ 51 + | | + 310 -
- 230 -
- 130 +
+ 50 + | | |

| | + 41° +
+ 139 -
- 221 -
- 319 + | | + 42° +
+ 138 -
- 222 -
- 318 + | | + 43° +
+ 137 -
- 223 -
- 317 + | | + 44° +
+ 136 -
- 224 -
- 316 + | | + 45° +
+ 135 -
- 225 -
- 315 + | | |
|------|--|-------|--|-------|--|-------|--|-------|--|-------|------|
| | sin | cos | sin | cos | sin | cos | sin | cos | sin | cos | |
| 0.1 | 0.07 | 0.08 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.1 |
| 0.2 | 0.13 | 0.15 | 0.13 | 0.15 | 0.14 | 0.15 | 0.14 | 0.14 | 0.14 | 0.14 | 0.2 |
| 0.3 | 0.20 | 0.23 | 0.20 | 0.22 | 0.20 | 0.22 | 0.21 | 0.22 | 0.21 | 0.21 | 0.3 |
| 0.4 | 0.26 | 0.30 | 0.27 | 0.30 | 0.27 | 0.29 | 0.28 | 0.29 | 0.28 | 0.28 | 0.4 |
| 0.5 | 0.33 | 0.38 | 0.33 | 0.37 | 0.34 | 0.37 | 0.35 | 0.36 | 0.35 | 0.35 | 0.5 |
| 0.6 | 0.39 | 0.45 | 0.40 | 0.45 | 0.41 | 0.44 | 0.42 | 0.43 | 0.42 | 0.42 | 0.6 |
| 0.7 | 0.46 | 0.53 | 0.47 | 0.52 | 0.48 | 0.51 | 0.49 | 0.50 | 0.49 | 0.49 | 0.7 |
| 0.8 | 0.52 | 0.60 | 0.54 | 0.59 | 0.55 | 0.59 | 0.56 | 0.58 | 0.57 | 0.57 | 0.8 |
| 0.9 | 0.59 | 0.68 | 0.60 | 0.67 | 0.61 | 0.66 | 0.63 | 0.65 | 0.64 | 0.64 | 0.9 |
| 1.0 | 0.66 | 0.75 | 0.67 | 0.74 | 0.68 | 0.73 | 0.69 | 0.72 | 0.71 | 0.71 | 1.0 |
| 2.0 | 1.31 | 1.51 | 1.34 | 1.49 | 1.36 | 1.46 | 1.39 | 1.44 | 1.41 | 1.41 | 2.0 |
| 3.0 | 1.97 | 2.26 | 2.01 | 2.23 | 2.05 | 2.19 | 2.08 | 2.16 | 2.12 | 2.12 | 3.0 |
| 4.0 | 2.62 | 3.02 | 2.68 | 2.97 | 2.73 | 2.93 | 2.78 | 2.88 | 2.83 | 2.83 | 4.0 |
| 5.0 | 3.28 | 3.77 | 3.35 | 3.72 | 3.41 | 3.66 | 3.47 | 3.60 | 3.54 | 3.54 | 5.0 |
| 6.0 | 3.94 | 4.53 | 4.01 | 4.46 | 4.09 | 4.39 | 4.17 | 4.32 | 4.24 | 4.24 | 6.0 |
| 7.0 | 4.59 | 5.28 | 4.68 | 5.20 | 4.77 | 5.12 | 4.86 | 5.04 | 4.95 | 4.95 | 7.0 |
| 8.0 | 5.25 | 6.04 | 5.35 | 5.95 | 5.46 | 5.85 | 5.56 | 5.75 | 5.66 | 5.66 | 8.0 |
| 9.0 | 5.90 | 6.79 | 6.02 | 6.69 | 6.14 | 6.58 | 6.25 | 6.47 | 6.36 | 6.36 | 9.0 |
| 10.0 | 6.56 | 7.55 | 6.69 | 7.43 | 6.82 | 7.31 | 6.95 | 7.19 | 7.07 | 7.07 | 10.0 |
| 11.0 | 7.22 | 8.30 | 7.36 | 8.17 | 7.50 | 8.04 | 7.64 | 7.91 | 7.78 | 7.78 | 11.0 |
| 12.0 | 7.87 | 9.06 | 8.03 | 8.92 | 8.18 | 8.78 | 8.34 | 8.63 | 8.49 | 8.49 | 12.0 |
| 13.0 | 8.53 | 9.81 | 8.70 | 9.66 | 8.87 | 9.51 | 9.03 | 9.35 | 9.19 | 9.19 | 13.0 |
| 14.0 | 9.18 | 10.57 | 9.37 | 10.40 | 9.55 | 10.24 | 9.73 | 10.07 | 9.90 | 9.90 | 14.0 |
| 15.0 | 9.84 | 11.32 | 10.04 | 11.15 | 10.23 | 10.97 | 10.42 | 10.79 | 10.61 | 10.61 | 15.0 |
| 16.0 | 10.50 | 12.08 | 10.71 | 11.89 | 10.91 | 11.70 | 11.11 | 11.51 | 11.31 | 11.31 | 16.0 |
| 17.0 | 11.15 | 12.83 | 11.38 | 12.63 | 11.59 | 12.43 | 11.81 | 12.23 | 12.02 | 12.02 | 17.0 |
| 18.0 | 11.81 | 13.58 | 12.04 | 13.38 | 12.28 | 13.16 | 12.50 | 12.95 | 12.73 | 12.73 | 18.0 |
| 19.0 | 12.47 | 14.34 | 12.71 | 14.12 | 12.96 | 13.90 | 13.20 | 13.67 | 13.44 | 13.44 | 19.0 |
| 20.0 | 13.12 | 15.09 | 13.38 | 14.86 | 13.64 | 14.63 | 13.89 | 14.39 | 14.14 | 14.14 | 20.0 |
| 21.0 | 13.78 | 15.85 | 14.05 | 15.61 | 14.32 | 15.36 | 14.59 | 15.11 | 14.85 | 14.85 | 21.0 |
| 22.0 | 14.43 | 16.60 | 14.72 | 16.35 | 15.00 | 16.09 | 15.28 | 15.83 | 15.56 | 15.56 | 22.0 |
| 23.0 | 15.09 | 17.36 | 15.39 | 17.09 | 15.69 | 16.82 | 15.98 | 16.54 | 16.26 | 16.26 | 23.0 |
| 24.0 | 15.75 | 18.11 | 16.06 | 17.84 | 16.37 | 17.55 | 16.67 | 17.26 | 16.97 | 16.97 | 24.0 |
| 25.0 | 16.40 | 18.87 | 16.73 | 18.58 | 17.05 | 18.28 | 17.37 | 17.98 | 17.68 | 17.68 | 25.0 |
| 26.0 | 17.06 | 19.62 | 17.40 | 19.32 | 17.73 | 19.02 | 18.06 | 18.70 | 18.38 | 18.38 | 26.0 |
| 27.0 | 17.71 | 20.38 | 18.07 | 20.06 | 18.41 | 19.75 | 18.76 | 19.42 | 19.09 | 19.09 | 27.0 |
| 28.0 | 18.37 | 21.13 | 18.74 | 20.81 | 19.10 | 20.48 | 19.45 | 20.14 | 19.80 | 19.80 | 28.0 |
| 29.0 | 19.03 | 21.89 | 19.40 | 21.55 | 19.78 | 21.21 | 20.15 | 20.86 | 20.51 | 20.51 | 29.0 |
| 30.0 | 19.68 | 22.64 | 20.07 | 22.29 | 20.46 | 21.94 | 20.84 | 21.58 | 21.21 | 21.21 | 30.0 |
| | cos | sin | cos | sin | cos | sin | cos | sin | cos | sin | |
| | + 311 -
- 229 -
- 131 +
+ 49 + | | + 312 -
- 228 -
- 132 +
+ 48 + | | + 313 -
- 227 -
- 133 +
+ 47 + | | + 314 -
- 226 -
- 134 +
+ 46 + | | + 315 -
- 225 -
- 135 +
+ 45 + | | |





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